The Inter-Temporal Stability of Real Estate Returns: An Empirical Investigation.

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Stephen L Lee

Department of Land Management
University of Reading
Whiteknights
Reading
RG6 6AW
England

+44 (0) 118 931 6338 (PHONE)
+44 (0) 118 931 8172 (FAX)

Abstract

This paper examines one of the central issues in the formulation of a sector/regional real estate portfolio strategy, i.e. whether the means, standard deviations and correlations between the returns are sufficiently stable over time to justify using *ex-post* measures as proxies of the *ex-ante* portfolio inputs required for MPT. To investigate these issues this study conducts a number of tests of the inter-temporal stability of the total returns of the 19 sector/regions in the UK of the IPDMI. The results of the analysis reveal that the theoretical gains in sector and or regional diversification, found in previous work, could not have been readily achieved in practice without almost perfect foresight on the part of an investor as means, standard deviations and correlations, varied markedly from period to period.

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Introduction

A number of studies have examined the potential gains from diversifying across real estate sectors and/or regions using the techniques of modern portfolio theory (MPT), see for example Lee and Byrne (1998) for a extensive review. It is well known, however, that *ex-ante* optimal mean-variance efficient portfolio selection and management requires accurate or, at a minimum, unbiased forecasts of assets' expected returns, variances and covariances. Perhaps the simplest method to generate expectational values of the required inputs to the portfolio choice problem is to rely on their historic (*ex-post*) values. The use of *ex-post* variance/covariance structure as a proxy for the *ex-ante* structure can be justified only if the structure is stable over time. Stationarity implies that a time series' first and second moments are well defined and there is no problem in computing unconditional means, variances and covariances based on observations over some sample period. In essence, an assumption of inter-temporal stationarity of real estate returns has been typically made for the sake of convenience and exposition of the benefits of sector and regional diversification. If mean returns, variances, and covariances, however, are not inter-temporally stationary, portfolio selection based on historical parameters would produce sub-optimal results.

However, while the issue of inter-temporal stability of equity market comovements has been examined extensively in the literature (see Cheung and Ho, 1991) little research as been undertaken into real estate returns data. With only a two studies alluding to the problem in sector and or regional returns using the NCREIF quarterly returns for the US (Meyer and Webb, 1991 Mueller and Laposa, 1995). While only one study has compared the similarity of *ex-post* and *ex-ante* efficient frontiers (Pagliari, Webb and Del Casino, 1995), again based on NCREIF data, without any formal tests of the equality of mean returns and variance covariance matrices. One study employed formal tests of stability (Eichholtz, 1996) this on securitised property in various countries of the world, but no study has used real estate data from the UK.

The purpose of this study is to fill this gap. It conducts a number of tests of stationarity on the mean, covariance and the correlation matrices of returns for 19 sector/regions in the UK for the period January 1987 to December 1996. The advantage of using a battery of tests for investigating the inter-temporal stability of the inputs to the portfolio decision has the virtue that the disadvantages associated with the application of a single technique are often compensated for in another test.

The paper is organised as follows: the following section reviews the previous research into the inter-temporal stability of real estate returns. Section 2 then describes the data. Section 3 outlines a number of tests for stability and presents the empirical results; and Section 4 concludes the study.

Previous Studies

The use of the *ex-post* values for the means and variance-covariance/correlation matrices between the returns as a proxy for an *ex-ante* estimate of the inputs into MPT models has attracted extensive academic interest in the equity market and raised questions about the inter-temporal stability and predictability of these matrices. Some studies have concentrated on the correlation matrix, others on the covariance matrix, while a few have included an examination of mean return vector as well. While the methodologies encountered in the literature are wide-ranging and include: (i) the normal distribution test for equality between individual correlation coefficients, (ii) principal components analysis, (iii) the Box M test; (iv) cluster analysis; (v) the Jennich Chi-squared test and (vi) the correlation between similar matrices (e.g. variance-covariance and correlation matrices) for two different periods. In comparison only one study has formally tested for inter-temporal stability in real estate data and this in international property securities, Eichholtz (1996).

A few studies have tested the problem indirectly. Meyer and Webb (1991) for example, analysed a ten-year period from 1978 to 1988 using NCREIF returns for Office, Retail, R&D office, and Warehouses. Their portfolio optimisation found different mixes for different time periods. Warehouse and R&D/office was the strongest in the 1978 to 1983 sub-period, while during the 1983 to 1988 period Warehouse and Retail had the best risk/return. They concluded that returns move differently during different time periods and thus a single portfolio allocation strategy may not be optimal during different sub-periods. This conclusion was endorsed by Mueller and Laposa (1995) and Pagliari, Webb and Del Casino (1995).

Mueller and Laposa (1995) while not directly attempting to investigate the inter-temporal nature of real estate returns allude to the problem in a study of the allocation institutional investors should make to the different property-types; Retail, Office, Apartments and Warehousing, using quarterly data in the US. In particular Mueller and Laposa argue that property-type returns have gone through cycles in the past and estimating future returns depends on the cyclic movement within each property-type. In order to identify which property-type to invest in the future investors need to be aware of how each property-type as performed in different phases of the cycle. The authors therefore divided the NCREIF returns data from 1978:1 to 1994:3 into different periods based on three cyclic indicators: total returns, capital appreciation and GDP growth. They then constructed efficient frontiers in each of the sub-periods and found different allocations both in terms of assets chosen and portfolio weights during the different periods. Mueller and Laposa concluded that property-type returns do indeed go through different cycles (based on supply and demand). In developing a future allocation investors therefore need to be aware of the current and future phases of the cycle in order to determine future portfolio compositions. In addition the authors noted that while it is easy to determine the best returns *ex-post* determining the correct mix in the future is more difficult.

In contrast Pagliari, Webb and Del Casino (1995) set out to investigate whether the strict application of MPT offered superior returns to naive or average-mix (market weight) strategies for institutional investors. Using NCREIF sector and regional real estate data over the period 1978:1 to 1992:4, the authors showed that the use of MPT can lead to mixed results. That is, MPT portfolios constructed from *ex-post* data and extrapolated into future periods, may or may not outperform naive and average-mix strategies. Further, in none of the sub-periods examined in this study did the *ex-ante* MPT-based portfolio strategies generate portfolios that were on the *ex-post* mean-variance efficient frontier! The authors concluded that the effective application of an MPT-based portfolio strategy depends heavily on the accuracy of the inputs (mean returns, standard deviations and correlation coefficients for each of the property-types) to the portfolio optimisation process and that the use of historical inputs as a proxy for future values may lead to sub-optimal results.

The Mueller and Laposa and Pagliari, Webb and Del Casino studies therefore both indicate that real estate data is unstable and that portfolio allocations will change markedly from period to period. This is easily seen by the changes in allocations found by Mueller and Laposa and Pagliari, Webb and Del Casino for the minimum risk and maximum returns portfolios as shown in Table 1.

Table 1: Portfolio Allocation in the Max Return and Minimum Risk Portfolios for Different Periods: US Data.

	Min Risk Allocation %					Max Return Allocation %				
Panel A										
Period	A	O	R&D	R	W	A	O	R&D	R	W
1978:1 - 1984:4			8	47	43		100			
1985:1 - 1990:2	46	24		29					100	
1990:3 - 1992:3				75	25	100				
Panel B										
Period										
1978:1 - 1982:4			28	72			100			
1983:1 - 1987:4		5			95				100	
1988:1 - 1992:4	86				15				100	

Note: A = Apartment: O = Office; R&D = R&D Office; R = Retail; W = Warehouse.
Source: Panel A Mueller and Laposa Exhibit 6 and Panel B Pagliari, Webb and Del Casino Exhibits 4, 7, and 10.

As can be seen in Table 1 the allocation of assets in both the minimum risk and the maximum return portfolio show dramatic shifts between each sub-period. For example, in Panel A of Table 1 in each sub-period the maximum return portfolio is different in each period. The allocation moved from Offices through Retail and to Apartments as the economic and real estate cycles change. Furthermore, even though the data in each study covers much of the same period there are noticeable differences between the results from both studies especially in the minimum risk portfolios. In Panel A of Table 1, for example, the allocation in the Mueller and Laposa study is almost equally split between Retail and Warehouses, while Panel B based on the results from Pagliari, Webb and Del Casino shows an allocation of almost three quarters to Retail and the rest in R&D Offices. Yet the difference in time period covered in both studies in the first sub-period is only two years. This indicates that even for real estate rapid shifts in portfolio compositions can take place in a very short period of time.

Even without any formal tests the US real estate data show all the characteristics of inter-temporal instability. Whether this is due to instability in the means, variances or covariances, was not formally tested in either study. However, Pagliari, Webb and Del Casino observed that the *ex-ante* minimum variance portfolios were fairly consistent in their risk suggesting some stability in the covariance matrix of returns. Unfortunately, the same consistency is not observed with regard to the returns.

The only study to formally test for the equality of variance-covariance matrices and correlation matrices using real estate data is that of Eichholtz (1996). Using monthly property company indices from Datastream for nine countries: Belgium, France, Italy, the UK, Australia, Japan, Singapore, Canada and the US, over the period February 1973 through May 1993, except for Canada. Eichholtz tested for inter-temporal stability in the covariance, correlation and variance matrices using Jennrich's (1970) Chi-squared test. Dividing the data into four equal sub-periods of sixty-one months, the author found that the null hypothesis of stability could not be rejected for the correlation matrix at the conventional statistical levels, except when comparing the first and second periods with the third. In comparison, the covariance matrices were unstable regardless of time period considered. Furthermore, adjacent periods were no less significant than non-adjacent periods suggesting that correlation and covariance matrices do not change gradually over time but are subject to abrupt shifts in return patterns. Eichholtz arguing that this implies that the variance of returns will therefore be less stable than correlations between countries, as correlation coefficients measure the degree of integration between markets and that integration will not change suddenly. Eichholtz therefore next tested the variances of returns for instability. He found that all countries indeed show instability in the variances of returns between most sub-periods. This suggests structural changes in covariance structures are occurring rather than a uniform change in the level of covariance. Eichholtz concluded that the instability in covariance structures limits the use of standard portfolio models in determining the allocation in international security investments.

In summary only one study as formally tested for the equality in real estate returns and that using data on securitised markets across the world. In comparison the few studies using direct property data have all alluded to the problem indirectly, without any formal tests of equality. All studies noticing that property returns fluctuate over the economic or real estate cycles and all concluding that that the use of historic (*ex-post*) data gives sub-optimal results when used *ex-ante*. No study, however, as tested the equality of means, covariance or correlation matrices on direct real estate data, and none in the UK. The following sections therefore set out to correct this omission.

Data

Monthly returns for 19 sectors/regions were collected from the Investment Property Databank Monthly Index (IPDMI). The data is based on the performance of 2481 properties from the records of over 50 property funds valued at £4962.5m at the end of 1996 (IPD, 1997). The 19 sector/regional indices are: Greater London Retail (GLRET), Inner South East Retail (ISERET), Outer South East Retail (OSERET), South East Retail (SERET), Midlands and West Retail (MWRET), North England and Scottish Retail (NESRET), Central London Offices (CLOFF), Outer London Offices (OLOFF), Inner South East Offices (ISEOFF), Outer South East Offices (OSEOFF), South East Offices (SEOFF), Midlands and West Offices (MWOFF), North England and Scottish Offices (NESOFF), London Industrial (LIND), Inner South East Industrial (ISEIND), Outer South East Industrial (OSEIND), South East Industrial (SEIND), Midlands and West Industrial (MWIND), and North England and Scottish Industrial (NESIND). The data period covers January 1987 to December 1996, a total number of 120 monthly returns.

The data was also broken down into four equal sub-periods, of 30 months each. The first period from January 1978 through to June 1989. The second period running from July 1989 through to December 1991. The third period from January 1992 to June 1994. While the last period is based on data from July 1994 until December 1996. The first period covering the Bull market in the UK real estate market. The second the market collapse. The third period characterised by the market recovery. While the fourth and final sub-period covers the recent past which has been one of steady progress. The four sub-periods therefore provide data on the most recent Bull and Bear markets and covers the whole gamut of investor experience in the UK real estate market. The descriptive statistics for which are shown in Table 1.

Descriptive Statistics

A number of points emerge from an inspection of the descriptive statistics in Table 1. First, the average returns vary noticeably across the sectors and regions. In individual terms the best returns were achieved by Industrial properties in the North England and Scotland (NESIND), the Midlands (MWIND) and the South East (SEIND) with a mean monthly return of 1.40, 1.29 and 1.17 per cent per month respectively. That is Industrial properties out of London offered the best performance. The performance of Central London Offices (CLOFF) performed the worst, earning has it did an average return of only 0.53 per cent per month, closely followed by Inner South East Offices (ISEOFF) and Inner South East Retail (ISERET), which both achieved returns of only 0.62 and 0.63 per cent per month, respectively. The best returns achieved outside London and the South East especially in the peripheral regions of the UK. Such returns, however, were not necessarily at the expense of higher risk levels.

Table: 2 Descriptive Statistics

	Overall			Average Return				Standard Deviation				RPUR			
	Mean	SD	RPUR	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4	Period 1	Period 2	Period 3	Period 4
GLRET	0.76	1.05	0.72	1.69	-0.21	0.91	0.64	1.04	0.64	0.97	0.37	1.62	-0.33	0.93	1.71
ISERET	0.63	1.03	0.61	1.35	-0.21	0.99	0.39	0.84	0.79	1.09	0.53	1.60	-0.27	0.91	0.74
OSERET	0.71	0.90	0.79	1.41	0.09	0.98	0.36	0.72	0.64	1.03	0.32	1.96	0.14	0.95	1.12
SERET	0.71	0.94	0.75	1.33	-0.01	1.09	0.42	0.82	0.71	1.05	0.38	1.62	-0.02	1.04	1.13
MWRET	0.79	0.86	0.92	1.47	0.04	1.12	0.55	0.49	0.66	1.00	0.29	2.99	0.06	1.12	1.88
NESRET	0.81	0.90	0.90	1.22	0.00	1.35	0.65	0.58	0.69	1.05	0.36	2.10	0.00	1.29	1.81
CLOFF	0.53	1.61	0.33	2.27	-1.04	0.29	0.61	1.17	1.18	1.40	0.24	1.94	-0.88	0.20	2.59
OLOFF	0.70	1.29	0.54	1.71	-0.05	0.59	0.53	1.42	0.91	1.29	0.70	1.21	-0.05	0.46	0.75
ISEOFF	0.62	1.28	0.49	1.71	-0.34	0.66	0.45	1.14	1.03	1.36	0.45	1.50	-0.33	0.49	1.01
OSEOFF	0.73	1.27	0.57	1.81	0.29	0.71	0.09	1.28	1.14	1.24	0.47	1.41	0.26	0.57	0.19
SEOFF	0.85	1.50	0.57	2.09	0.23	0.71	0.37	1.66	1.54	1.27	0.43	1.26	0.15	0.55	0.87
MWOFF	1.00	1.23	0.81	1.86	0.67	0.91	0.55	1.62	1.10	1.06	0.34	1.15	0.61	0.86	1.64
NESOFF	1.03	1.38	0.74	2.35	0.59	0.79	0.36	1.35	1.40	1.15	0.39	1.75	0.43	0.69	0.93
LIND	0.99	1.28	0.77	2.22	0.36	0.87	0.53	1.39	1.11	1.09	0.37	1.59	0.32	0.80	1.43
ISEIND	0.86	1.25	0.69	2.19	0.20	0.75	0.29	0.90	1.05	1.27	0.43	2.45	0.19	0.59	0.68
OSEIND	1.06	1.25	0.85	2.53	0.50	0.59	0.62	1.12	0.99	0.94	0.45	2.27	0.50	0.62	1.37
SEIND	1.17	1.37	0.86	2.60	0.54	1.12	0.44	1.29	1.06	1.25	0.43	2.02	0.51	0.89	1.03
MWIND	1.29	1.40	0.92	2.55	0.72	1.39	0.51	1.49	1.14	1.29	0.44	1.71	0.63	1.07	1.16
NESIND	1.40	1.29	1.08	2.62	0.80	1.59	0.59	1.37	1.05	1.03	0.35	1.91	0.76	1.55	1.65

Note: Greater London Retail (GLRET), Inner South East Retail (ISERET), Outer South East Retail (OSERET), South East Retail (SERET), Midlands and West Retail (MWRET), North England and Scottish Retail (NESRET), Central London Offices (CLOFF), Outer London Offices (OLOFF), Inner South East Offices (ISEOFF), Outer South East Offices (OSEOFF), South East Offices (SEOFF), Midlands and West Offices (MWOFF), North England and Scottish Offices (NESOFF), London Industrial (LIND), Inner South East Industrial (ISEIND), Outer South East Industrial (OSEIND), South East Industrial (SEIND), Midlands and West Industrial (MWIND), North England and Scottish Industrial (NESIND).

Although portfolio theory associates higher risk with higher return, the worst risk (standard deviation) was in the Central Office market in London (CLOFF) which had the worst average monthly returns. All the Retail property and almost all the Industrial property showed lower risk levels than Offices. As a result when the mean return is divided by the standard deviation, the return per unit risk (RPUR) shows the dramatic under-performance of Offices and highlight the superior performance of Industrial even more. The worst RPUR sector/region being Central London Office market (CLOFF) with a figure of 0.33, substantially less than the next worst Inner South East Offices (ISEOFF) at 0.49, which was closely followed by Outer London Offices (OLOFF) at 0.54. The best performing sector/region was, not unsurprisingly, North England and Scottish Industrial (NESIND) with a RPUR of 1.08, closely followed the Midlands Industrials and Retail (MWIND, MWRET) with RPUR figures of 0.92 each. The general impression from Table 2 is that property other than Offices achieved good risk adjusted performance, especially outside London and the South East.

There is also a noticeable difference in the risk and return characteristics in the various sub-periods. The first period shows very good positive returns of almost 2% per month, in all markets, with an average risk of only just over 1%. The risk adjusted returns (RPUR) levels were therefore very favourable especially in Central London Offices and Industrial properties across the whole of the UK. The second period in contrast shows a low average return of less than 0.2% per month, with the South East and London showing negative returns for both Retail and Office properties, with only Industrial offering positive returns in all regions. That is the so called property market crash in the UK was an Office sector phenomenon concentrated principally in Central London with a knock on effect into the Retail sector in the South of England. These low or negative returns when coupled with an average risk (standard deviation) of 1% per month leading to very poor RPUR results. The period of recovery, shows once again positive returns in all sector/regions, but with an average return of just under 1% per month, half that of the boom period, along with a higher risk of over 1.1% and hence a poor risk adjusted performance level. The most recent period characterised by low but steady returns of just under 0.5% per month and an average risk level of 0.41% per month. Leading to good RPUR figures during this sub-period in all sectors/regions, except for Retail and Industrials in the Inner South East Retail (ISERET, ISEIND), and North England and Scottish Offices (NESOFF). All three show RPUR values less than one due to poor average returns, rather than high risk levels, in comparison with the other sectors/regions.

However, although individual risk and return characteristics are important. The attractiveness, or otherwise, of an asset class as a diversifier in a portfolio is influenced more by its correlation with the other asset classes, than its individual features. It is through the less than perfect positive correlation between assets that diversification is achieved. An analysis of the correlation coefficients between assets can give some clues as to the assets that will have a positive allocation in a mixed-asset portfolio. The simple inspection of a 19 by 19 correlation matrix, however, is unlikely to provide a clear indication of the assets which will offer the efficient investment combination in such a multivariate problem. A number of simplifications needed to be made to facilitate the analysis.

Eichholtz et al (1995) for example suggest counting the number of correlation coefficients that are significantly different from zero and one. Meric and Meric (1989) suggest that the attractiveness of an asset can be indicated by calculating what they call a dependency index. Where such an index is an average of the correlation coefficients between an asset returns and the returns of the other asset's. The number of significant positive and negative correlations and the dependency indices of the 19 sector/regions are presented in Table 3

Table 3 Dependency Indices and the Number of Correlation Coefficients Significantly Different from Zero and Insignificantly Different from One

	Period 1 Dependency		Period 2 Dependency		Period 3 Dependency		Period 4 Dependency	
Sector/region	Index	Rank	Index	Rank	Index	Rank	Index	Rank
GLRET	0.15	1	0.53	8	0.85	12	0.45	10
ISERET	0.37	7	0.47	2	0.85	13	0.52	16
OSERET	0.43	13	0.55	9	0.84	10	0.58	19
SERET	0.41	10	0.60	15	0.87	19	0.57	18
MWRET	0.27	4	0.49	5	0.86	16	0.51	15
NESRET	0.36	6	0.51	6	0.86	18	0.47	11
CLOFF	0.17	2	0.44	1	0.81	4	0.22	1
OLOFF	0.44	14	0.47	3	0.86	17	0.22	2
ISEOFF	0.48	19	0.56	11	0.84	9	0.44	8
OSEOFF	0.47	18	0.56	10	0.83	7	0.48	13
SEOFF	0.37	8	0.59	13	0.82	6	0.32	4
MWOFF	0.26	3	0.48	4	0.80	2	0.27	3
NESOFF	0.39	9	0.62	16	0.86	15	0.36	5
LIND	0.30	5	0.63	17	0.84	8	0.39	7
ISEIND	0.42	12	0.66	19	0.85	11	0.44	9
OSEIND	0.44	16	0.59	14	0.82	5	0.53	17
SEIND	0.41	11	0.65	18	0.76	1	0.48	12
MWIND	0.44	15	0.58	12	0.86	14	0.48	14
NESIND	0.46	17	0.51	7	0.81	3	0.37	6
Average	0.37		0.55		0.84		0.43	
Correlations	No.	%	No.	%	No.	%	No.	%
Positive	162	94.7	169	98.8	171	100.0	170	99.4
Negative	9	5.3	2	1.2	0	0.0	1	0.6
Sig. Positive	103	60.2	152	88.9	171	100.0	108	63.2
Sig. Negative	0	0.0	0	0.0	0	0.0	0	0.0
Insig. from One	2	1.2	13	7.6	170	99.4	8	4.7

As seen in Table 3 the number of positive correlations is very large and not less than or 95% of the total. With no negative correlations in periods 2 and 3. Also the vast majority of the positive correlations are significantly different from zero at the

5% level. While none of the negative correlations are significantly different from zero. Although only a few of the correlation coefficients are insignificantly different from one, except in period 3 when all but one the correlations are insignificantly different from one. Indicating that any diversification benefits that may have accrued from sector and regional diversification over this period will have been relatively small, and almost non-existent in period 3.

These observations are confirmed in the dependency indices. The average correlation increasing from 0.37 in period 1 through 0.55 in period 2 and peaking at 0.84 in period 3, before settling down again to 0.43 in period 4. That is during periods of divergent growth the correlation between sectors and regions is relatively low, while during periods of decline markets have a tendency to move together. Confirming the observations of Pagliari, Webb and Del Casino and Mueller and Laposa, both finding increasing correlation coefficients during the periods of worst performance.

Looking at the individual sector/regions there is a marked change from period to period in the assets that show the highest and lowest level of correlation. For example, in the first sub-period the sector/region most correlated with the other sector/regions is Inner South East Offices (ISEOFF) with an average correlation of 0.48 with the other assets. The least dependent was Greater London Retail (CLRET), 0.15 . However, by the fourth period the sector/region most related the other areas is Outer South East Retail (OSERET), 0.58 and the least related is Central London Offices (CLOFF), 0.22.

As will be appreciated from the Table 3, the rankings switch round from period to period with little continuity from one period to the next. This instability is confirmed by calculating the Spearman rank correlation between the various sub-periods as shown in Table 4. In all cases the rank correlations are insignificantly different from zero, at even the 20% significance level. The portfolio attractiveness of a sector/region in one period, as indicated by its portfolio risk characteristics (correlation) with the other sector/regions, can quickly disappear in a subsequent period. This suggests that in the case of direct real estate data, unlike the findings of Eichholtz (1966) for the securitised property market, the correlation matrices are likely to exhibit inter-temporal instability, even in adjacent periods.

Table 4: The Spearman Rank Correlation Between the Four Sub-Periods

	Period 1	Period 2	Period 3	Period 4
Period 1	1.000			
Period 2	0.302	1.000		
Period 3	-0.086	-0.075	1.000	
Period 4	0.239	0.205	0.298	1.000

Note: All correlations are insignificantly different from zero at even the 20% level.

The variability in risk and return characteristics between the various sub-periods indicates that the portfolio parameters needed as inputs into the portfolio programming problem are likely to be unstable. The following section tests this by applying a number of tests of temporal stability.

Tests of Stability

Portfolio Compositions

Following the work of Meyer and Webb (1991), Mueller and Laposa (1995) and especially that of Pagliari, Webb and Del Casino (1995) the stability, or otherwise, of the portfolio holdings on the efficient frontier in various time periods are examined next. However, unlike the previous authors a number of optimal *ex-post* portfolios are examined, rather than constructing the whole efficient frontier. This is done to make comparisons across different periods easier to evaluate.

The first portfolio identified is the maximum return portfolio. Such a portfolio will identify whether mean returns are changing markedly across the sub-periods. As the risk characteristics of the individual assets are ignored in the identification of such a portfolio. The second group of optimal portfolios are all *ex-post* (tangency) portfolios, identified by the following maximisation problem:

$$\text{Max } \theta \equiv \frac{\overline{R}_p - R_f}{\sigma_p} \qquad (1)$$

Where:

$\overline{R}_p$ = the expected return of portfolio p,

R_f = the risk - free rate of return,

σ_p = the standard deviation of the portfolio.

The weights in these portfolios then are the ones offering the highest *ex-post* mean return per unit risk (RPUR). Furthermore the composition of such a tangency portfolio, as shown by Tobin (1958), is independent of the investors' preference structure. Note also that θ in the above formulation is, in fact, the *ex-post* Sharpe (1966,1994) performance measure. A number of different risk-free were used. First the risk free rate is set to zero. Comparisons across different time periods therefore are unaffected by changes in 90 day Treasury-Bills in each sub-period. The second approach using the average risk free rate over each the sub-periods under study. This portfolio would represent the 'best' allocation investors could have achieved in anyone period.

Finally, the weights of the minimum variance portfolio (MVP) in each period were identified. This portfolio may shed some light on the stability of the risk characteristics of the data across sub-periods. As this strategy assumes that there is no useful asset-specific information in the vector of mean returns because it is not required as an input to solve the portfolio problem. Pagliari, Webb and Del Casino (1995) suggest that such a portfolio is qualitatively more stable, in its risk characteristics, than other portfolios on the efficient frontier. The risks and returns for each optimal portfolio in the four sub-periods shown in Tables 5.

Table 5: The Risk and Return Characteristics for the Optimal Portfolios

	Period 1		Period 2		Period 3		Period 4	
	Return	Risk	Return	Risk	Return	Risk	Return	Risk
Max Return	2.62	1.40	0.80	1.05	1.59	1.04	0.65	0.37
Sub-period Risk Free	2.03	0.58	0.80	1.05	1.59	1.04	0.63	0.21
Zero Risk Free	1.73	0.47	0.75	0.88	1.58	1.04	0.60	0.19
MVP	1.44	0.42	-0.07	0.54	0.86	0.92	0.57	0.18

An inspection of Table 5 reveals that risk and returns vary across the various sub-periods in line with the figures in Table 2 for the individual assets. Returns fall from period 1 to 2 rising again in period 3 and levelling off in the fourth and final period. In addition there appears to be little similarity in the portfolio risk characteristics of the portfolio solutions, even for the MVPs, in adjacent periods. That is risks and returns are unstable.

In particular it is worth noting the during period 2 and especially period 3 most of the optimal portfolios are made up of investment in one sector/region the North England and Scottish Industrial (NESIND). This dominance by NESIND suggested by the results in Table 2 above and stems from two factors. First the individual return characteristics of NESIND in periods 2 and 3 were such that it also had the highest RPUR in both periods by a considerable margin. Second as discussed previously the average correlation increased from period 1, such that by period 3 all correlations were significantly positive and more importantly insignificantly different from one. In such a situation there is little to be gained from diversification! Therefore in both periods 2 and 3, the superior risk adjusted performance of NESIND dominated all other individual investments opportunities and a 100% holding in NESIND would be an efficient and optimal portfolio, except at the lowest risk levels.

In order to see whether similar solutions occur in each sub-period two issues need to be addressed. First to what extent do the same assets appear in each sub-period? Second for those assets that are contained in each sub-period solution, to what extent are they included in like proportions? In order to do these calculations the number of asset and their weights in each optimal portfolio for each period were identified. From which it is possible to find those assets that are in common in each period, that is overlap, and to what extent the asset weights are similar between periods The results presented in Table 6.

Table 6: Number of Asset Holdings and Asset Overlap for the Optimal Portfolios

	Number of Asset Holdings				Number of Overlapping Asset Holdings					
Periods	**Adjacent**				**Non-Adjacent**					
Optimal Portfolios	**P1**	**P2**	**P3**	**P4**	**P1/2**	**P2/3**	**P3/4**	**P1/3**	**P2/4**	**P1/4**
Max Return	1	1	1	1	1	1	0	1	0	0
Sub-period Risk Free	8	1	1	4	1	1	0	1	0	2
Zero Risk Free	8	3	2	8	1	1	2	1	2	6
MVP	5	6	5	8	4	2	3	2	2	3

As shown in Table 6 the MVP in period 1 contains five assets, in comparison the MVP in period 2 is made up of six investments. While the number of assets in the MVPs that overlap, or are common in both periods is four. In other words four of the five assets which make up the MVP in period 1 are also part of the six assets that form the MVP in period 2. Similar calculations apply for the other optimal portfolios. Using these raw numbers *portfolio overlap, weight* and *similarity* indices were calculated.

For example, the *portfolio overlap* index for the MVPs in periods 1 and 2 is defined as the ratio of the number of assets that overlap in period 1 and 2 {Np1 ⋂ Np2} to the number of assets at the union {Np1 ⋃ Np2} between period 1 and 2. Where the union between period 1 and 2 {Np1 ⋃ Np2} is the equal to the number of assets in period 1 {Np1=5} plus the number of assets in period 2 {Np2=6} minus the number of assets in common between period 1 and 2 {Np1 ⋂ Np2=4}which in this case is equal to 5+6-4=7. That is for the MVP in periods 1and 2 while 11 assets appear in both periods, four assets are common to both results, leaving seven different assets which appear in only one solution. The ratio of the overlap between period 1 and 2 {Np1 ⋂ Np2} to the union between period 1 and 2 {Np1 ⋃ Np2} is therefore 4/7 = 57.1%. In other words 57.1% of assets contained in the MVP in period 1 are also contained in the MVP for period 2. Other *portfolio overlap* indices calculated in a same way. The results shown in panel A Table 7.

The calculation of portfolio *overlap indices,* however, only address one facet of the question as to the similarity or dissimilarity between portfolio compositions. When two portfolios contain the same assets the portfolio overlap index will be 100%. However, the weights within such portfolios could vary markedly leading to important investment implications.

Table 7: Portfolio Overlap, Weight and Similarity Indices

	Adjacent			Non-Adjacent		
Periods	**P1/2**	**P2/3**	**P3/4**	**P1/3**	**P2/4**	**P1/4**
Panel A	**Portfolio Overlap Index**					
Max Return	100.0	100.0	0.0	100.0	0.0	0.0
Sub-period Risk Free	12.5	100.0	0.0	12.5	0.0	20.0
Zero Risk Free	10.0	25.0	25.0	11.1	22.2	60.0
MVP	57.1	22.2	30.0	25.0	16.7	30.0
Panel B	**Portfolio Weight Index**					
Max Return	100.0	100.0	0.0	100.0	0.0	0.0
Sub-period Risk Free	1.3	100.0	0.0	1.3	0.0	1.1
Zero Risk Free	2.9	55.7	6.9	2.5	20.6	33.6
MVP	46.4	28.4	16.6	12.3	18.9	25.1
Panel C	**Portfolio Similarity Index**					
Max Return	100.0	100.0	0.0	100.0	0.0	0.0
Sub-period Risk Free	0.2	100.0	0.0	0.2	0.0	6.2
Zero Risk Free	0.3	13.9	1.7	0.3	4.6	20.2
MVP	26.5	6.3	5.0	3.1	3.2	7.5

To test the similarity between the weights attached to assets held in common by two portfolios a *portfolio weight* index can be constructed. Where such an index is measured by summing the minimum weight attached to each asset that overlap between two portfolio solutions. For example Panel A of Table 7 shows a *portfolio overlap* index of 57.1% between for the MVPs in periods 1 and 2. In panel B of Table 7, on the other hand, the sum of the minimum weights found in the MVPs in periods 1 and 2 for the assets that are common in both periods is only 46.4%. In other words only 46.4% of the holdings of the MVP in period 1 also appear in the MVP in period 2 of the assets in common in periods 1 and 2. The other values for the *portfolio weight* indices calculated in the same way.

Finally Panel C of Table 7 shows that multiplying the *portfolio overlap* indices by the *portfolio weight* indices gives the proportion of assets common to both periods with similar weights, a *portfolio similarity* index. That is in the case of the MVPs in periods 1 and 2, only 26.5% of the assets in period 1 are also in period 2 with similar portfolio weights.

The results from Tables 7 indicate a lack of overlap and hence persistence in the composition of the optimal portfolios from adjacent period to period. The overlap becoming even less for non adjacent periods. Except for periods 1 and 4 which shows a high degree of overlap in asset compositions. In addition the portfolio weights held by common assets in any two periods is generally low, except for the maximum return portfolios in periods 1, 2 and 3. Resulting from the dominance of North England and Scottish Industrial (NESIND) in these periods. While not surprisingly the amount of asset weights prescribed in any one period to the same assets in the same proportions in a subsequent period (Panel C) are very low.

When looking at the four optimal portfolios across adjacent and non adjacent periods as we move down the efficient frontier, from the highest return portfolio to the lowest risk portfolio, two features are noticeable. First the composition of the holdings in the portfolios at the highest end of the efficient frontier are subject to sudden and abrupt changes. The maximum return portfolios showing the greatest shifts in overlap from 100% to zero. Indicating that assets mean returns are particularly subject to rapid change, suggesting that the mean return vector is likely to display a great deal of instability. Secondly at the lower end of the frontier the composition appears to be somewhat more stable, both in terms of assets and portfolio weights, especially for adjacent periods. Although even here the similarity index is generally very small, which suggests that the variances are also changing over time. Persistence in portfolio compositions and holdings varying from period to period indicating a lack of temporal stability in the portfolio inputs. The inter-temporal instability in the means, variances or covariance investigated further in the next section.

Equality of Risks and Returns

Covariance

Most of the previous studies which have tested the equality of two or more covariance matrices across different time periods among equity markets (Kryzanowski and To, 1987; Kaplanis, 1988; Meric and Meric, 1989; Tang, 1995, and Wahab and Lashgari, 1993, for example) have used the Box M test (Box, 1949). The details of the Box M test is given in Morrison (1976) and Mardia, Kent and Bibly (1979) and only a brief discussion is given below. The test statistic is given by:

$$Q_1 = MC^{-1} \qquad (2)$$

where

$$M = \sum_{i=1}^{k} n_i \ln|S| - \sum_{i=1}^{k} n_1 \ln S_i ,$$

and

$$C^{-1} = 1 - \frac{2p^2 + 3p - 1}{6(p+1)(k-1)} \left(\sum_{i=1}^{k} \frac{1}{n_i} - \frac{1}{\sum n_i} \right)$$

where k denotes the number of matrices tested for equality (k=2); $n_i = N_1$ and N_2 denote the number of observations in the first and second sub-periods to be evaluated; p is the number of sector/regions in the covariance matrix (p=19) and S_i is the covariance matrix computed from returns over the i^{th} sub-period. Box (1949) shows that Q_1 is distributed asymptotically as a Chi-square with ½(k-1)p(p-1) degrees of freedom. The null hypothesis is the two covariances are equal (inter-temporally stationary).

The Chi-squared approximation of the test Box M statistic, however, is only appropriate for small dimensional problems $(p \leq 5)$, see Morrison (1976). For larger dimensional problems the test statistic follows an approximate F-distribution. The approach used here. Although Pearson (1969) showed that the two approximations produce similar results and concludes that for all practical purposes it is sufficient to employ the Chi-square test.

Correlation

In testing the hypothesis of equality of the correlation matrices of stock market Kryzanoswki and To (1987), Kaplanis (1988) used Jennrich's (1970) chi-square test procedure. An approach followed by Eichholtz (1996) when testing real estate security returns. Tang (1995) and Gibbons (1981) both show, however, that Box's M test can also be applied to test the equality of correlation matrices by first transforming the raw data into standard scores. As for two standardised random variables, their correlation equals their covariance. Hence, testing the hypothesis of equality of covariance matrices of these standardised returns, is equivalent to testing the hypothesis of equality of the correlation matrices of raw returns.

Mean

Morrison (1976) shows that the stationarity, or equality of two mean return vectors, can be tested by the following quantity Q_2, which follows an F-distribution with p and $N_1 + N_2 - p - 1$ degrees of freedom:

$$Q_2 = \frac{N_1 N_2 (N_1 + N_2 - p - 1)}{N_1 + N_2 (N_1 + N_2 - 2) p} * (\bar{X}_1 - \bar{X}_2)'(S_1 + S_2)^{-1}(\bar{X}_1 - \bar{X}_2) \qquad (3)$$

where $\bar{X}_1$ and $\bar{X}_2$ are the p-element vectors of the sample mean returns (p=19) computed over $N_1 = 1....t_1$ and $N_2 = 1.......t_2$ for the first (N_1) and second (N_2) sub-periods: S_1 and S_2 are p x p matrices of sums-of-squares and cross-products of the sample of p sector/regional returns estimated from sub-periods N_1 and N_2, respectively. The null hypothesis being tested is that the two means are equal (inter-temporally stationary). Rejection of the null achieved if $Q_2 \geq F_a ; p, N_1 + N_2 - p - 1$. This test having previously employed by Kryzanowski and To (1987) and Wahab and Lashgari (1993).

However, both the Q_1 and Q_2 statistics assume multivariate normality of the data. If this assumption is violated the power of both tests is less than would be expected theoretically. In particular the original development of the Q_2 statistic was dependent on the equality, or homogeneity of the unknown covariance matrix of both sets of data. This would imply that rejection of the null hypotheses may result from the lack of homogeneity of the joint covariance matrix rather than as a result of any significant difference in the two mean return vectors, a Type 1 error. Ito and Schull (1964), however, have shown that for the Q_2 statistic if the number of observations N_1 equals N_2 any lack of homogeneity of the covariance matrices S_1 and S_2, has no effect on the type 1 error probabilities of the tests. In other words if N_1 equals N_2 the Q_2 statistic is not a joint test of the equality of the means and covariance matrices. In addition if N_1 does not equal N_2 a formal test of the homogeneity of the mean return vectors given unequal covariance matrices is available, see Giri (1977). On the other hand the Q_1 statistic is an independent test of the equality of covariance matrices since it assumes unknown and potentially unequal mean return vectors (Giri, 1977).

Results

In testing the equality of the covariance and correlation matrices the data was divided into the four 30 month sub-periods identified previously and into two other periods. The first dividing the data into two 60 months periods, the data covering periods 1-2 and periods 3-4. The second based on the data from periods 1-3, and period 4. That is the covariance and correlation matrices based on the first 90 months were compared with the estimated covariance and correlation matrices based on the final 30 month sub-period. These later two divisions of the data designed to test whether data over longer periods is more stable than data over shorter periods. The results presented in Table 8, for the covariance and correlation matrices and Table 9 for the mean return vectors.

Table 8: Test of the Equality of Covariance and Correlation Matrices

Periods of	**Covariance**			**Correlation**		
Comparison	**Box's M**	**F**	**p-value**	**Box's M**	**F**	**p-value**
Adjacent						
P 1 v P 2	383.03	1.30	0.004	343.67	1.17	0.057
P 2 v P 3	380.43	1.29	0.004	408.45	1.39	0.000
P 3 v P 4	447.82	1.52	0.000	470.04	1.60	0.000
P 1-2 v P 3-4	711.70	3.11	0.000	362.73	1.59	0.000
P 1-3 v P 4	710.20	2.76	0.000	349.96	1.36	0.001
Non-Adjacent						
P 1 v P 3	547.97	1.86	0.000	560.63	1.91	0.000
P 2 v P 4	535.63	1.82	0.000	311.57	1.06	0.275
P 1 v P 4	703.12	2.39	0.000	311.52	1.06	0.275

As can be seen the evidence presented in Table 8 suggests that for the covariance matrix stationarity is rejected with almost complete certainty for all sub-period pairs both adjacent or non adjacent. In addition the rejection of stationarity is greater the longer the forecast period, which may indicate greater structural instability for longer as opposed to shorter periods with a gradual convergence to stability the shorter the horizon. For the correlation matrices the majority of the results reject the null hypothesis of equality, at conventional levels of statistical significance, except for a few of the periods especially for periods 1 and 2 with 4. Therefore unlike the results presented by Eichholtz (1996), for securitised property, direct real estate data exhibits instability in both covariance and correlation matrices. However, in comparison with the results for the covariance matrices the rejection of instability for the correlation matrices does not increases the longer the estimation period.

Table 9 presents the empirical results of tests of the inter-temporal stationarity of the vector of mean returns of the 19 sector/regional returns. As is apparent, the null hypothesis was rejected with almost complete certainty for any of the sub-period pairs. Two points are worthy of note. First the results for non adjacent periods are generally worse than for adjacent periods. Second the rejection is weaker for comparisons over longer periods, as in the case of the results for periods 1 and 2 against periods 3 and 4 and for periods 1-3 against period 4. Indeed it would be reasonable to expect the null hypothesis of stationarity between the means should not be rejected the longer the forecast period, since a general convergence in market returns is to be anticipated over time.

Table 9 Tests of the Equality of Means

Periods of Comparison	Test of Means	
	Q_2	p-value
Adjacent		
P 1 v P 2	10.70	0.000000
P 2 v P 3	4.79	0.000015
P 3 v P 4	5.02	0.000001
P 1-2 v P 3-4	3.36	0.000618
P 1-3 v P 4	2.26	0.014980
Non-Adjacent		
P 1 v P 3	9.90	0.000000
P 2 v P 4	14.97	0.000000
P 1 v P 4	12.30	0.000000

In summary it as been found that stability could be rejected for the mean returns vector, the covariance matrices and the correlation matrices at the conventional statistical levels regardless of time period considered, except in a few cases. Furthermore, adjacent periods were no less significant than non-adjacent periods, suggesting that correlation and covariance matrices do not change gradually over time but are subject to abrupt shifts in return patterns as suggested by Eichholtz (1996). In addition the rejection of stationarity for the correlation matrices does not increase with longer forecasting periods as observed for the covariance matrices. This suggests that since the correlation coefficients measure the integration between markets there is some stability between the sectors and regions over time and that it is the shifts in the variance of returns that are leading to non stationarity in the data. Finally in comparison with the tests on the covariance and correlation matrices the rejection of the equality between the means are considerably higher. This echo's the well known dictum of Sharpe (1970) that historical returns may tell us something about covariances, less about risk (standard deviation) and almost nothing about expected return. That is the means are subject to the greatest change over time followed by the covariance matrix , with the least variability in the correlations, as in this case.

Implications

MPT has often been used to identify the sector and regional diversification benefits within a real estate portfolios through the generation of efficient frontiers. In particular MPT holds out the promise to fund managers of achieving the same level of returns over time with a portfolio of lower risk, as measured by the standard deviation of returns, in comparison with a fund which concentrates on returns alone. This promise can only be realised if the excepted risks and returns of the sector/regions can be successfully identified. The results above, however, show that the assumption that the mean returns vector, the variance-covariance matrix and the correlation structure between sectors and regions are constant over time can not be justified. Indeed the MPT inputs are subject to sudden and abrupt changes. Reliance on past values therefore is likely to produce sub-optimal results when used as the inputs in *ex ante* models.

In addition the work above identifies the variance-covariance matrix as the prime source of inter-temporal instability in portfolio risk. This suggests that models designed to forecast of the variance-covariance structure between assets need to be evaluated. However, previous attempts to forecast the correlation matrix and the covariance matrix in the equity market have met with little success. For example, Kaplanis (1988) in a study of international equity returns found the correlation matrix to be stable and examined four alternative ways of forecasting the *ex ante* correlations to take account of any structural shift in the matrix. The methods employed including: a simple historical model, a naïve overall-mean model, a Bayesian model which adjusts the correlations for prior beliefs about the true parameters, and one based on regressing the correlations in one period on the correlations in a previous period. The method that proved best on the basis of the Mean Square Error (MSE) and Mean Absolute Percentage Error (MAPE) metrics was the Bayesian model with the simple historic model being second best. In forecasting the covariances Kaplanis, who had found the covariance matrix to be unstable in different periods, attempted to reduce the sample noise within the matrices rather than estimate any structural changes. Testing the simple historical model against the overall-mean model, the regression based approach and the Single Index Model (SIM), with and without Bayesian adjustment, along with a Bayesian model based on two different priors. Again on the basis of MSE and MAPE Kaplanis found that the naive overall-mean model consistently outperformed the simple historical model implying that the differences in covariance matrices between periods is due to noise in the sample data. As a consequence the best method of forecasting future covariance matrices was the Bayesian models which were specifically designed to account for such noise, although none were particularly successful. A result supported by the findings of Wahab and Lashgari, (1993) again for international equities who compared the performance of the historical covariance matrix and the overall-mean covariance matrix against one based on perfect foresight in estimating the inputs for a mean-variance analysis. Of the two methods considered the overall-mean always came closest to replicating the actual values in the future than the one based on historical data. Both studies confirming the results of Eun and Resnick (1984) and Elton and Gruber (1987) in the US equity market. While Eun and Resnick (1992) found that multi-index models adjusted for firm size significantly outperformed the overall-mean model and the simple historical method. Again emphasising the need to account for sample noise in the series. However, the multi-index models failed to outperform the SIM, which suggests that while the multi-index models are useful in explaining the correlation structure they are not particularly useful for predicting future dependence between share prices.

The lack success in using different forecasting methods and the instability in the mean returns vector and the variance-covariance matrices indicates a different approach to modelling *ex-ante* portfolio inputs is required. In particular statistical models which explicitly incorporate the time varying nature of the variance-covariance matrix need to be evaluated. Such models exist including ARCH and GARCH models of various kinds (Bollersev, Engle and Wooldrige, 1988) and should prove an interesting area for future research.

Conclusions

This paper examines one of the central issues in the formulation of a sector/regional real estate portfolio strategy i.e. whether the means, standard deviations and correlations between the returns are sufficiently stable over time to justify using *ex-post* measures as proxies of the *ex-ante* portfolio inputs required for MPT. To investigate these issues this study conducts a number of tests of the inter-temporal stability of the total returns of the 19 sector/regions in the UK of the IPDMI. The results of the analysis reveal that the theoretical gains in sector and or regional diversification, found in previous work, could not have been readily achieved in practice without almost perfect foresight on the part of an investor as means, standard deviations and correlations, varied markedly from period to period. A finding supporting the results in the equity market and securitised real estate market. Thus at both a practical and theoretical level the emphasis now needs to focus on identifying the returns generating process within the various sectors and regions that are driving the markets in order to identify the appropriate portfolio input values.

Finally a number of other points are worth recording. First the results may be dependent on the specific time period the analysis of this paper. Second the analysis has been conducted at the aggregate sector/regional level, whereas fund managers usually invest in individual properties; a property-level analysis might have yielded very different conclusions. Next we have used administratively defined regions whereas a number of studies both in the USA and UK suggest that economical defined urban areas may be preferred. Classification by economic function may provide more meaningful diversification strategies to the portfolio manager than broadly defined regions and better portfolio diversification benefits, see Lee and Byrne (1998). Such areas, however, may be no more stable inter-temporally than administratively defined regions. Hence their use in MPT optimisers will be just as difficult.

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