Are Property Prices Non-Linear? An Investigation of the Behaviour of US REITs and UK Property Company Shares

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Abstract

Linear models of market performance may be misspecified if the market is subdivided into distinct regimes exhibiting different behaviour. Price movements in the US Real Estate Investment Trusts and UK Property Companies Markets are explored using a Threshold Autoregressive (TAR) model with regimes defined by the real rate of interest. In both US and UK markets, distinctive behaviour emerges, with the TAR model offering better predictive power than a more conventional linear autoregressive model. The research points to the possibility of developing trading rules to exploit the systematically different behaviour across regimes.

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1. Introduction

The purpose of this paper is to consider the behaviour of publicly traded US and UK real estate performance using non-linear regime-based modelling techniques. Such techniques have been applied in areas of economics and finance with some success (see Engel, 1991; Dacco' & Satchell, 1995). They are particularly useful in trying to conceptualise situations where behaviour differs in distinct market states. Under such circumstances, the conventional linear model, which allows for just one market state, will not be able to fully explain the movement of prices and returns and hence fails to generate useable trading rules or informed portfolio allocation decisions.

Real estate markets are generally considered to be strongly cyclical in nature. It is plausible that the structure of market behaviour differs across boom and bust phases. For example, it is possible that in a property downturn, capital values fall more sharply, and with less volatility, than they rise in a property upturn. In equity markets, it is known that falling prices are more volatile than rising prices, the so called leverage effect, see Black (1976). A regime-switching model may be able to capture these behavioural differences. Knowledge of the state variable - the variable or set of variables that determine the regimes - might point to distinctly different investment strategies.

One approach to regime modelling is based on the use of Markov switching models, as popularised by Hamilton (1989)[2]. This technique treats the state variable as unobservable which, from a forecasting viewpoint, leads to serious difficulties (see Dacco' and Satchell *op cit.*). Accordingly, we have preferred to use threshold autoregressive (TAR) models which have the advantage that the state variable is observable in many of the possible specifications. In this paper, we consider the real rate of interest as a state variable. While this is only one of many possible variables defining regimes, there are strong *a priori* reasons for anticipating interest rate effects. This intuition is confirmed by the results presented below.

This research tests for the existence of distinct interest-rate regimes in the US REIT and UK property company markets and compares the behavior of these investments over time. An earlier study examined the UK property company sector using a TAR approach (Lizieri, & Satchell, 1997b). These researchers found distinct market regimes separated by values of the real interest rate. However, their results are hard to interpret due to the trending nature of the price series. This paper addresses the trending issue directly in the model specification. Although there are some dissimilarities between REITs and property companies, both are considered as highly liquid real estate investments in their respective countries and might be seen as complementary assets in an international diversification strategy. It will thus be interesting to determine whether or not these two publicly-traded real estate investment vehicles exhibit similar behavioural traits.

The remainder of the paper is structured as follows. In section two we discuss the influence of the real rate of interest on the commercial real estate sector. Next we review the underlying fundamentals of TAR (Threshold Autoregressive) modelling and forecasting and the literature on threshold models. Section four describes the data employed in our study. In section five, we present results of our comparison of US and UK markets. To anticipate, we find that there are distinct regimes defined by the level of real interest rates, with both markets exhibiting broadly similar behaviours. Finally, we conclude by pointing to future research directions.

2. Commercial Real Estate and the Real Rate of Interest

[2] For a real estate example see Maitland-Smith (1996).

It is our contention that one of the major drivers of commercial real estate markets is the real rate of interest. We would not contend that the real interest rate is the *only* exogenous variable influencing real estate behavior. Other macro variables may be of significance and could be used to segment the data series. Nonetheless, we consider interest rates to be of major significance and hence anchor our investigation of non-linear structures upon them. In conventional appraisal (valuation) models, rents are capitalised using a rate which is dependent, *inter alia*, on nominal interest rates. In high interest rate environments, increased yields will lead to a decrease in the capital value of commercial property. This impact is magnified due to the extensive use of financing for real estate investments. On the other hand, inflation is often seen as having a beneficial effect on real estate investments, as owners benefit from increasing income and capital growth, while the real value of their historic debt is eroded.

For simplicity, defining the real rate of interest as $R_t = (N_t - F_t)$ where N_t is the nominal rate and F_t is the expected inflation rate, it can be seen, on the basis of our previous argument, that the real rate has an unequivocally negative impact on property values.[3]

The links between private and public (in the UK, the direct and indirect) real estate markets are complex and are subject to lags. The performance of the REIT and property company sectors will, ultimately, be based on the performance in the underlying direct market. For example, in the UK, property investment companies are valued by equity analysts on a discounted Net Asset Value (NAV) basis. Hence their share prices will reflect changing property values. As rising real interest rates depress capital values, so NAV will be marked down. Clearly the actual price will vary on a day by day basis depending upon the amount of the discount and on general stock market movements. Returns on individual properties and developments will be affected by interest rate impacts on project-specific borrowing. The impact of corporate borrowing (the gearing ratio) will magnify any changes. Developer-Trader companies will, in similar fashion, be affected by capital growth and the cost of borrowing and their share price will vary accordingly. Most credit-driven markets will be subject to real interest rate effects, and the property sector should be no exception. As a result of valuation inertia, it is possible that the response to interest rate changes will be more rapid in the property equity[4] market than in the underlying direct investment market.

Recent North American studies have indicated that interest rates have a significant impact on real estate markets. For example, Ling & Narnjo (1995) sought to identify key macro-economic risk factors or 'state variables' that affect US real estate returns. Using both fixed-coefficient and time-varying models, they report that interest rate variables (the real Treasury Bill rate and the term structure of interest rates) were significant, albeit dominated by growth in consumption. The real interest rate was particularly significant for real estate stocks (CRSP Real Estate Investment Trust data). McCue & Kling (1994), using a VAR approach, find significant links between US real estate stock prices, inflation and nominal rates (US Treasury 3 month T-bills), with nominal rates explaining over 36% of the variation in the real estate series. Notwithstanding the crudity of the VAR approach, it is an excellent technique for isolating those "drivers" that influence the variable of interest, in this case US REIT prices. Thus the work of McCue & Kling is useful in identifying inflation and nominal rates as relevant to our study.

Mueller & Pauley (1995) examined the effect of interest rates movements on REIT prices in the period 1972-1993 using a regression-based approach. They found weak negative correlations with

[3] We are aware of the cross-product term when working with unlogged data.

[4] In this paper, when we refer to the property or real estate equity market we refer to the UK Property Company and US REIT markets that are publicly traded on stock markets, also known as the indirect or public markets. These markets differ from private ownership of land and buildings - sometimes known as equity ownership in the US. It is the indirect market, traded on the stock exchange, that is the focus of our study.

short, medium and long-term interest rate variables. However, in examining sub-periods, they reported inconsistent betas and suggested that this implies that '*REIT price movements cannot be adequately explained by interest-rate movements*' (p324). The study does not decompose interest rates into real and anticipated inflation elements. Their conclusion, however, hints that there may be different effects on REIT prices from periods of rising and falling interest rates. Liang *et al* (1995) present results of a two index model analysis of REIT portfolios that suggest both market and interest rate betas are time-varying. Finally, Liang & Webb (1995), using a similar methodology, argue that a large part of the market risk associated with mortgage REITs is derived from interest rate uncertainties.

From a different perspective, Hendershott's (1995) analysis of rental determination in the Sydney office market is based on a model where changes in the real (risk free) interest rate drive equilibrium real rents. His dynamic adjustment model is thus strongly dependent upon real interest rate changes. This result suggests that there seem to be strong *a priori* grounds for examining the linkage between interest rates and property performance.

Given the likely impact of interest rates, an initial autoregressive model might relate price changes to current and lagged values of the price level of property Y_t and the real interest rate X_t. We formulate this below as:

$$\Delta Y_t = \alpha_0 + \Sigma \alpha_j Y_{t-j} + \Sigma \beta_j X_{t-j} + \upsilon_t \qquad [2.1]$$

where υ_t is assumed, for convenience, to be independently identically distributed normal with mean 0, variance σ^2 or ~ iid $N(0,\sigma^2)$.[5]

Since the price level is measured in nominal terms, Y_t is likely to be trending upwards. Equation 2.1 could thus be augmented to include a time-trend variable. Whilst this is a perfectly plausible model of ΔY_t it may not capture the periods where Y_t is highly volatile, where Y_t is either trending up or down or where Y_t is mean-reverting. From a financial perspective, it is important to isolate such behaviour as each regime can be associated with different investment strategies. For example, in a world where values are trending up or down, portfolio insurance would seem appropriate, whilst in a mean-reverting state it might be profitable to follow some form of contrarian strategy, such as rebalancing. The disadvantage of the formulation in [2.1] is that it admits only one such state owing to its linearity. For this reason we examine a TAR formulation.

3. TAR Modelling

Threshold models were introduced by Tong & Lim (1980) and are extensively discussed by Tong (1983, 1990). The basic idea may be explained as follows. We start with a linear model for a series Y_t and then allow the parameters of the model to vary according to the values of a finite number of past values of an associated series Z_t. In most applications, the associated series coincides with the series under investigation, in which case we have a Self-Exciting Threshold Autoregressive (SETAR) model.

Tong (1983) proposed a grid searching algorithm based on the use of the Akaike (1973) information criterion. We fit n models for n possible values of a threshold value, z, choosing the model (that is, the value of z) with the minimum Akaike criterion. Tong argues that in determining the final choice of model, one need not adhere strictly to the values of the structural parameters selected by the

[5] Equation 2.1 could be rewritten in terms of an error-correction formulation. This would leave all variables in differences except for the levels components of I(1) variables that correspond to a "long-run" equilibrium. We donot pursue this parameterization.

Akaike criterion. Rather the Akaike criterion is used as a guide to select a small sub-class of plausible models which may then be examined by diagnostic checks to assess whether the fitted model shares the main characteristics of the data.[6]

A more recent discussion of related techniques to locate and determine thresholds can be found in Chen (1995). Applications of TAR models to GNP can be found in Tiao & Tsao (1994), Peseran & Potter (1994) and Potter (1995). Yadav, Pope & Paudyal (1994) and Cao & Tsay (1994) provide applications for equity markets and Lizieri & Satchell (1997b) utilise a TAR model for UK commercial property company prices.

A two-regime TAR model may be estimated by fitting each of the two components separately to the appropriate subset of observations and using standard least squares exactly as in the case of a linear autoregressive model.[7] The estimation of coefficients, therefore, presents little difficulty. The evaluation of results is a more difficult problem.

It is possible to test the model against a standard linear model as the null. This test, however, does not have a standard null distribution. Two reasons are relevant here. Firstly, under the null linear hypothesis, the threshold parameter z is not identified. This means that the large sample likelihood surface is flat under the null hypothesis with respect to the threshold parameter. The asymptotic likelihood function has no unique maximum and is not locally quadratic. Secondly, the score with respect to z is identically zero when evaluated at the null hypothesis. Either of these conditions is sufficient to render standard asymptotic theory inapplicable.

Various attempts have been made to avoid this difficulty - see, for example, Petrucelli & Davies (1986), Moeanaddin & Tong (1988), Chan & Tong (1990) and Chan (1991). These last two approaches constrain the estimated residual variance to be constant across regimes. This seems unsatisfactory for financial data: one of our specific aims in this paper is to model stock price volatility. To address these hypothesis testing issues relating to the threshold variable for our particular TAR model would involve some major developments in statistical theory.

There are also problems associated with TAR forecasting. As discussed in De Goojer & Kumar (1992), the distribution of multi-step forecasts need not be unimodal. Therefore computing conditional expectations may not be appropriate. In general, multi-step-ahead forecasting is influenced by whether at time t one is "sitting on a peak or a trough."[8]

In the case of a non-linear autoregressive process of order one, for more than two step-ahead forecasts, Pemberton (1987) showed that, in contrast to the linear case, the prediction depends on the noise distribution and the prediction error itself appears to depend on the process value at time t.

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[6] Tsay (1986, 1989) proposed a test based on the portmanteau test of non-linearity of Petrucelli and Davis (1986). Both are based on arranged autoregression and predictive recursive residuals. The main idea here is to transform a TAR model into a regular change point problem in linear regression. This is achieved by using the concepts of arranged autoregression. That is, instead of using the time index to control the flow of data, arranged autoregression uses the magnitude of the threshold variable. The advantage of the Tsay procedure over the Tong algorithm is that it does provide diagnostic statistics to assess the need for a threshold model. However, there is a strong assumption that the only conceivable non-linearity is a TAR model. We refer to Tsay (1989) for a discussion and to Brock *et al* (1991) for some simulation results. In this paper we follow the Tong procedure.

[7] Tong (1990) showed that for an ergodic SETAR model, the set of estimated coefficients has an asymptotic multivariate normal distribution. This result does not take into account the sampling properties of the threshold parameter, but assumes that it is known *a priori*. In the case of Gaussian errors, the log likelihood function is equivalent to the conditional sum of squares.

[8] Moeanaddin & Tong (1988) demonstrate with Wolff's sunspot series that multi-step-ahead SETAR models outforecast linear models over "troughs" although the reverse was true over "peaks".

Even for a one-step-ahead forecast, it seems that no uniformity exists in the evidence presented on the forecasting ability of non-linear models. For example, Dacco' & Satchell (1995) showed that it requires only a small misclassification when forecasting which regime the world will be in to lose any advantage from knowing the correct model specification relative to (incorrectly) assuming that the process is a random walk.

For all these notes of caution, the TAR approach offers potential gains in the understanding of the operation of markets. This will be particularly true where the price behaviour in different regimes is distinctive. Under such a scenario, the conventional linear formulation will be misspecified and hence fail to capture the underlying characteristics of the market. The TAR model may thus improve both our understanding of market processes and our forecasting ability.

In what follows we shall restrict our analysis to the situation where there is a single period lag and two regimes. It would be possible to include more lags in the model and consider more regimes. However, this is likely to introduce more complexity than clarity. We use a one period lag on the assumption that REIT and property equity prices will quickly adjust to changing interest rate environments. In the direct investment market, inertia may mean that the adjustment process is slower and a longer lag would be appropriate.

We shall now consider a threshold autoregression (TAR) model which accounts for an underlying time trend. In this structure we shall assume that:

$$\Delta Y_t = \alpha_i + \beta_i Y_{t-1} + \gamma_i T + \sigma_i \upsilon_t \quad \text{if} \quad X_{t-1} \in R_i \qquad [3.1]$$

where Y_t is the log of the price index, T is a time trend (T = 1,2,3 ... n), i = 1, 2 and (α_i, β_i, γ_i, σ_i) are the parameters associated with the regime R_i. and υ_t is the error term at time t. There may be more than two regions, the R_i needs to be mutually exclusive and exhaustive and the errors, υ_t are iid ~ N (0,1). The underlying linear time-trend is captured by the γ coefficient, while the β coefficient relates to the autoregressive response.

The above model is simple and admits a wide range of different behavioural characteristics. We list some below, but many more are possible:

(i)	$\alpha_i = 0$, $\beta_i = 0$,	a random walk;
(ii)	$\beta_i < 0$,	a mean - reverting process around a trend;
(iii)	$\beta_i > 0$,	a mean-averting process around a trend;
(iv)	$\gamma_1 > 0$, $\gamma_2 < 0$,	regimes which alternately trend up and down;
(v)	σ_1 large, σ_2 small,	a world with high and low volatility states around the trend.

To identify the regions R_1 and R_2, we assume that these will be determined by real interest rates. Thus if X_{t-1} < some threshold value z, we might expect a highly volatile mean reverting market, if X_{t-1} > z, we might expect the opposite, a low volatility, downward trending market.

The individual regression equations for the two regimes are estimated by conditional least squares, which is consistent and, as noted above, gives estimates which are asymptotically normal with the usual covariance matrix. As mentioned earlier, the only complexity comes from the estimation of z, the threshold coefficient. This value is estimated by carrying out a gridsearch on the total residual sum of squares. For the reasons outlined above, we do not report standard errors for the threshold estimate $\hat{z}$.

We contrast this TAR formulation with a more standard autoregressive linear formulation:

$$\Delta Y_t = \alpha + \beta Y_{t-1} + \gamma T + \upsilon_t \qquad [3.2]$$

To some extent, the simple autoregressive formulation is a "straw man" which does not represent a state of the art model[9]. It might be possible to construct a model based on fundamental analysis principles that would give better data fits and predictions. Such a causal approach would certainly be preferable in the private, direct equity real estate market. Here, however, we are dealing with publicly traded real estate securities with relatively high frequency data. With such data, the simple linear form seems more appropriate. Simple autoregressive models in this form are frequently used in equity analysis. The linear autoregressive formulation provides us with a benchmark to evaluate the existence of regimes. Although further research could examine regimes within the context of a multifactor model, this lies outside the scope of the present analysis and may introduce more noise than clarity.

In discussing the significance of the coefficients in our models, several points need to be considered. Firstly, it may be thought that our tests are of the Dickey-Fuller (DF) kind with non-standard asymptotic distributions and critical values nearer three than two. Such a thought would be quite logical as we are testing for a random walk. That this is *not* the case is due to the result of West (1988) who shows that, in an AR(1) model with drift, a t-test of the hypothesis $\beta = 1$ will be asymptotically standard normal as long as the intercept (or trend, if the trend is being modelled) is non-zero under the null. Thus the appropriate t-statistic for a one-sided test would be -1.645. If we wished to parameterize the trend γ as $\gamma = \gamma_0 (1-\beta)$ such that the intercept under H_0 would also be zero then it follows that the t-tests would have DF type distributions of great complexity. Simulation would be required to determine the critical values in this case. However, we do not assume such a parameterization here.

In different regimes, different trading rules may apply. If prices follow a random walk with positive trend, then a rule of buy and hold will dominate any rule which involves changes in portfolio weighting - the actual proportion of investible wealth placed in commercial real estate would depend upon that investor's utility function, the time horizon, the competing asset classes and the statistical properties of the errors in the random walk.

If we have a regime that is mean-reverting autoregressive, we would find that prices above the trend would revert to the trend. The same process would hold for prices below the trend. In this market, one would buy when prices were below trend and sell when they were above trend - a contrarian strategy. By contrast, in a mean-averting, explosive, environment one might buy (sell) on a trend.

Given the above remarks, it is possible to define a (not necessarily unique) trading strategy depending upon the states of the markets. If regime 1 were a random walk and regime 2 were mean-reverting, one would buy and hold in regime one and buy or sell in regime two depending on the behaviour of prices relative to the trend. Furthermore, one could use forecasts of what regime is likely to apply next period to make adjustments to portfolio weightings. It should be emphasised that the efficacy of such trading rules are conditional on there being a liquid market with comparatively low information and transaction costs and a model with sufficiently high R^2 with limited noise. The ability to implement such a strategy in the private direct-ownership real estate market is questionable. In public-traded securities markets as those of US REITs and, in particular, UK property company shares, market conditions permit the development of such techniques.

The presence of successful trading rules or autoregressive patterns in returns might seem to contravene market efficiency. The literature on market efficiency in the 1970s broadly treated efficiency as synonymous with (log) prices following a random walk. However, research in the last twenty years has evolved considerably. The question of efficiency has been broadened to address the question: are returns consistent with an intertemporal representative agent equilibrium in the asset

[9] We are grateful for the comments of one of our referees on this point.

markets? Discussions may be found in Fama & French (1988, 1989), McQueen & Thorley (1991), or Poterba & Summers (1986, 1988).

A number of papers have theoretically shown that such markets may have risk premia that are time-varying, autoregressive or predictable (see, for example, Abel, 1988, Balvers *et al*, 1990, and, for empirical evidence, Whitelaw, 1994). Since the optimal trading strategies of investors in such economies are, typically, extremely complex - far more complex that investors use in practice - contrarian and other trading strategies may be interpreted as robust approximations to the true dynamic trading strategy. These rules do not lead to "excess" returns since, in equilibrium, there will be a risk adjustment just as there occurs with the dynamic CAPM.

Before presenting the results of our tests of the TAR and autoregressive models, we turn to data issues.

4. Data Issues: Analysing the Public Real Estate Market

A major difficulty facing real estate research is the absence of timely and reliable indicators of direct market activity. The benchmark portfolio indices available in the UK are based on appraisals (valuations) rather than on transactions - as are published US indices such as the NCREIF index.. This causes a number of problems - notably questions of the reliability of those valuations (Lizieri & Venmore-Rowland, 1991, 1993; Brown, 1992; Matysiak & Wang, 1995) and smoothing and autocorrelation due to asynchronicity of valuations, temporal aggregation and the valuation process (see Blundell & Ward, 1987; Brown, 1991; Geltner, 1991, 1993; Newell & MacFarlane, 1994; Barkham & Geltner, 1995; Brown & Matysiak, 1996, for example). Although it is possible to "unsmooth" the direct indices, there remains a coverage problem.

In the UK, the only generally available monthly indices (the IPD Monthly Index and the Richard Ellis Monthly Index) both start in 1986. Furthermore, these indices are dominated by valuations of unit trust properties (properties held in a unitised trust for smaller pension funds and charities pooling their resources) which may not be fully representative of the typical commercial real estate investment market. Accordingly, we have preferred to use price data for property company shares in the UK and REITs in the US. These have the advantage that they are publicly traded, transaction-based, transparent and reported on a frequent basis. The use of price data allows to mark commercial property to market and thus use risk management systems based on value at risk concepts.

Nonetheless, the use of stock market property indices to measure property performance is also problematic. US research has indicated that Real Estate Investment Trust indices (such as NAREIT) are strongly correlated with common stock indices which has led some authors to question whether they measure real estate performance at all. Similar results have been found for UK property company share prices (see Mengden & Hartzell, 1986; Gyourko & Keim, 1992; McCue & Kling, 1994; Campeau, 1995).

Evidence strongly suggests that there are common stock market factors which partially drive the performance of real estate securities. Gyourko & Keim (*op cit*) found a contemporaneous correlation between their REIT index and the S&P 500 of 0.65 for the period 1978-1990. Barkham & Geltner report contemporaneous correlations between unsmoothed direct market indices and equity real estate of 0.48 in the US (1975-1992) and of 0.62 in the UK (1970-1992). Lizieri & Satchell (1997a) record a correlation of 0.829 between the FT-Property Index and the FT-All Share index in the period 1972-1992. Nonetheless, as argued above, there must be a logical link between the performance of real estate in the direct and indirect markets. Therefore, the focus of this paper is on REIT and property *company* performance. Furthermore, anecdotal evidence suggests that many institutional investors have, in recent years, sought to switch their real estate exposure away from holding direct investments with its attendant illiquidity, high levels of specific risk and high management and transaction costs, in favour of indirect investment vehicles.

Since our focus is on equity performance itself, and not on the links between equity real estate and the underlying commercial real estate market, we have not attempted to remove the effects of gearing (leverage) from the equity real estate indices as was carried out by Barkham & Geltner (*op cit*). They studied price discovery in property markets and employed an approach using the weighted average cost of capital and debt/equity ratios. It is interesting to note that Barkham & Geltner's unleveraged UK series had a *higher* contemporaneous correlation with the stock market than the geared series (0.74 compared to 0.62). Campeau (*op cit*) obtained similar results. These results confirm our contention that the real estate sector of the stock market is more interest-rate sensitive than the stock market in general.

UK Data

To construct a real interest rate series, it would be possible to compare and analyze coupon rates and projected redemption yields of UK index-linked gilts with those of conventional government fixed interest securities. Depending on the magnitude of the average duration of loans taken out to purchase commercial property, if such data were available, we could find the appropriate real interest rate to use in the direct market. In the real estate equity market (the focus of our analysis) shorter term interest rates would be appropriate. There are many difficulties with such an approach, however, not the least being that index-linked gilts were not available before 1983; there are further difficulties in synchronising the two series, see Robertson & Symons (1994) not to mention the absence of index-linked gilts in the US. Finally, finding reliable information on the average time period of loans on commercial property would be problematic enough if one could decide what was the appropriate aggregation methodology. Therefore, we have created a real interest rate series using current inflation and short-term interest rates.

Current monthly inflation was measured by the change in the headline Retail Price Index (RPI). We noted the arguments concerning the inclusion of mortgage costs and certain tax items in the headline rate and considered using the underlying rate. However, the more consistent time-series available for the headline rate proved persuasive in our choice. As an indicator of nominal interest rates, we used the three month Treasury Bill rate, taken from Datastream International. The Datastream series records the London discount rate, reporting the middle rate between bid and offer on a daily basis. We used the daily closing rate. In certain market states, it may be that the bid-offer spread may have some significance in influencing sector performance. We believe that any such effect will be dominated by changes in underlying nominal and real rates and leave investigation of this issue for later research. We use the three-month T-bill rate as this constitutes the market's benchmark interest rate indicator. While a longer duration bond may be more appropriate for analyzing direct equity investment in real estate, the short-term indicator seems more appropriate for real estate stock performance.

As our index of UK real estate performance, we used the Datastream International UK Property Price Index. This index includes the share prices of UK property companies (excluding property agents) listed on the London Stock Exchange, weighted according to market capitalization. It is similar in construction to the more commonly used FT Property Index. However, we preferred the Datastream index since the FT index was reorganised in 1994, necessitating the merging of two series. For the data series, we collected monthly data for the period January 1975 to August 1995. Earlier data would be distorted by legislative and planning controls on real estate in the early 1970s. Market capitalization as a share of the overall equity market has varied over time, although not as markedly as US REIT data. Analysis is complicated by changing compositions but the average share of the total stock market for a consistent sub-sample of major companies over the period 1980-1995 was 2.85% In mid-1995, the estimated market capitalisation of the sample was approximately £16.1bn (US$ 24.5bn).

In examining the data, it was evident that there was a spectacular fall and subsequent rise in the property index around 12/90 whilst there have been high levels of real interest rates since about 12/90. In the direct (private) property market, the boom years of the late 1980s resulted in overbuilding and consequent falls in rental and capital values. The property sector has emerged only slowly from this "crisis" phase. To some extent, the fall in values had been anticipated in the property equity market. In the early 1990s, the high real interest rate environment would be expected to adversely affect the property sector. However, property companies that survived the crash would have, in many instances, successfully completed corporate restructuring and thus be recovering. Perhaps more importantly, the performance of the property company sector in the 1991-1995 period appears to be driven largely by the general rise in equity prices. In this particular period, it seems that sector specific factors are overwhelmed by general equity market movements disguising the effects of interest rate structures. Nonetheless, tests on the data excluding the 1991-1995 period generated similar results to tests for the full period of analysis.

The US Data

As our index of real estate performance for the US, we had use of several of the NAREIT share price and return indices: equity, equity (without healthcare), mortgage, hybrid and the total NAREIT index which are reported in the *1995 REIT Handbook: The Complete Guide to the Real Estate Investment Trust Industry*.[10] Equity REITs are those REITs whose primary investment (over 75%) are in direct ownership of the real estate asset, mortgage REITs are those publicly traded REITs which are invested in real estate primarily through mortgages, and the hybrids are REITs invested in real estate as both an equity position and on a loan basis. In common with other authors, our analysis has used the Equity REITs without healthcare NAREIT index. The type of properties and forms of ownership held by these constituent REITs are most similar to the portfolios of UK property companies.

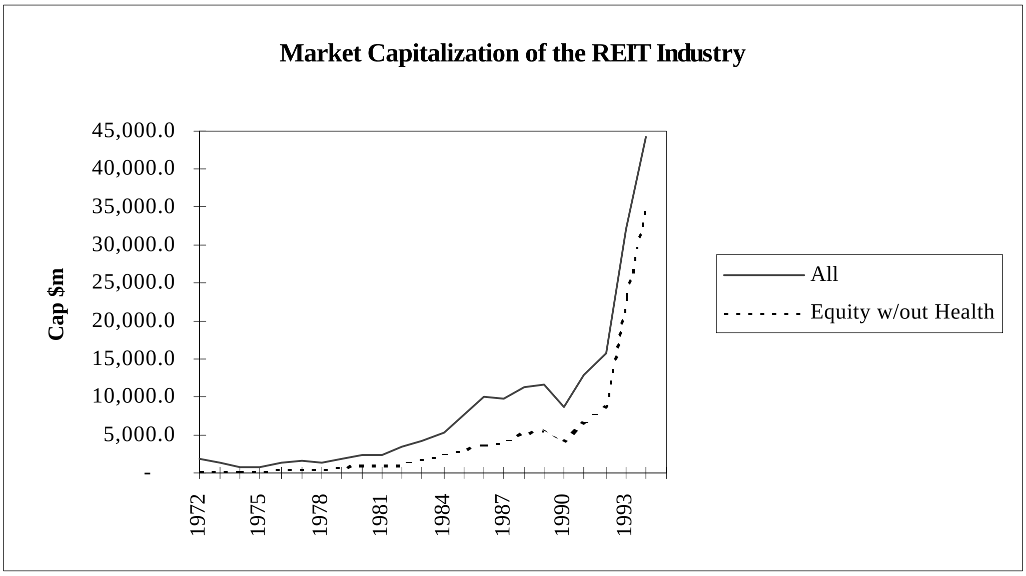
As many researchers have pointed out, there has been an explosive growth of the REIT market, and in particular the equity REIT market, in the last 5 years (for example see Pagliari and Webb 1995, Han and Liang 1995 and Corgel *et al* 1995). As detailed in Figure 1, below, the market capitalization of the industry has gone from $1.88 billion in 1972 to $44.31 billion in 1994 for the total index with a substantial amount of that growth in the equity index (without healthcare). This index has increased in nominal terms from $377 million in 1972 to $35.59 billion in 1994.

As of 1994, total asset values reported in the NAREIT index were $88.17 billion, an increase of $27.09 billion or 44% from the previous year. The breakdown between types of REITs in

[10] This index is published by the National Association of Real Estate Investment Trusts (NAREIT), which is the national, not-for-profit trade organization representing the real estate industry.

the index was as follows: 205 equity REITs with a reported value of $62.06 billion (70.4% of total asset value), 32 Mortgage REITs with a reported value of $21.78 billion (24.7%) and 23 hybrid REITs with a reported value of $4.34 billion (4.9%).

Figure 1: REIT Capitalization



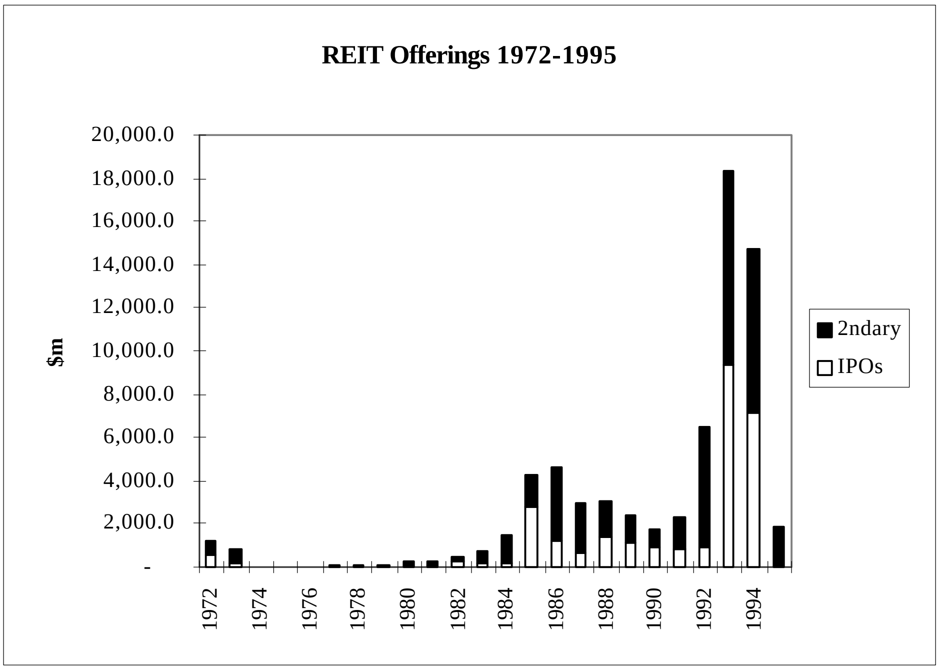
Source: *1995 REIT Handbook*

As indicated earlier, there has been a significant change in the REIT marketplace in the last few years. In some ways this makes the index difficult to use as a proxy for real estate investments. First, a survivorship bias exists. Glascock and Hughes (1995) found that only 16 of the 245 REITs in the NAREIT index existed for their entire study period (1972-1991). Examining the number of offerings (Figure 2) it is obvious that some years were much more active than others and that activity varied by type of REIT (equity, mortgage or hybrid). The last two years have seen a much higher number of equity REIT security offerings than in previous years.

The purpose of this paper, however, is to try to forecast the return for securitized real estate by isolating interest rate regimes. Therefore, it is important to use what the market is using to proxy the prices of securitized real estate. This objective is best achieved by employing the NAREIT data series since that is the most widely quoted series available for investors considering investing in the US public real estate market.[11]

[11] There are two other REIT indices available in the US, the Wilshire index and the Lehman Brothers index. (See Giliberto and Sidoroff,1995, for a comparison of the three indices). Our primary motivation for using NAREIT are twofold. First, the NAREIT index has the longest series (NAREIT starts in 1971, Wilshire in 1977 and Lehman Bothers in 1991) so we can attempt to analyze the shifts in the interest rate regimes over a longer time period. Second, this index covers the most REITs and has the largest market capitalization.

Figure 2: REIT Public Offerings 1972-1995



Source: *1995 REIT Handbook*

Current US inflation was measured by the change in the Consumer Price Index reported at the end of each month.[12] The 3-month treasury bill return series (annualized and for new issues) reported by the Department of the Treasury[13] was used to proxy nominal interest rates. Monthly data for the period January 1972 to March 1995 was used in the analysis, although our US models begin in 1973 (to allow for calculation of an annual inflation figure) with the UK models beginning in 1975.

5. Results

As noted above, our initial linear model for the two data sets was:

$$\Delta Y_t = \alpha + \beta Y_{t-1} + \gamma T + \upsilon_t \qquad [3.2]$$

where Y_t is the log of the price index at time t, T is a time trend (T = 1,2,3 ... n) and υ_t is an error term, $E(\upsilon_t) = 0$. We contrast this with a two regime TAR model where the regimes are defined in terms of the real interest rate X_t:

$$\Delta Y_t = \alpha_i + \beta_i Y_{t-1} + \gamma_i T + \sigma_i \upsilon_t \qquad \text{if } X_{t-1} \in R_i \qquad [3.1]$$

[12] The index was taken from various issues of the Federal Reserve Bank of St. Louis publication. This index is seasonally adjusted and was set to 100 in 1983.
[13] The data were taken from various issues of *Economic Indicators* published by the US Government Printing Press.

We start with the results of the linear model. For the full US data set from December 1972 to March 1995 we found:

$$\Delta Y_t = \underset{(1.6567)}{0.0866} - \underset{(-1.4706)}{0.0174\, Y_{t-1}} + \underset{(1.4035)}{0.0002\, T} \qquad [5.1]$$

Figures in parentheses are t-statistics Adj $R^2 = 0.004$ RMSE = 0.0408

The UK results for the full data set from January 1975 to August 1995 we found:

$$\Delta Y_t = \underset{(2.1391)}{0.2125} - \underset{(-1.9741)}{0.0333\, Y_{t-1}} + \underset{(1.3343)}{0.0002\, T} \qquad [5.2]$$

Adj $R^2 = 0.011$ RMSE = 0.0668

These two results suggest at best a weak autoregressive process with limited forecasting ability. None of the US coefficients are significant. The UK results indicate mean reversion about the trend line and upward drift. The generated equations do not seem to point to effective trading rules. Can a TAR formulation provide stronger results and usable trading rules?

The full sample US REIT two-regime TAR results are shown in Panel A of Table 1. We observe two regimes separated by a threshold real interest rate of 2.87%. The market is in Regime 1, the lower interest rate regime, 64% of the time. The regimes are persistent, the probability of remaining in Regime 1 is 0.965 and of remaining in Regime 2 is 0.947. As detailed in the table, the behavior of the market is quite distinct across the two regimes.

In Regime 1, with lower real rates, we observe mean reverting behavior about a positive trend - that is positive and significant values of α and γ and a negative and significant value of β. The market moves upward around its trend line with slower or faster growth corrected the next period. By contrast, in the second, higher interest rate, regime, we observe random walk behavior around a negative trend: only the γ coefficient is significant and values fall with little volatility down the trend line. The magnitude of the intercept and the steepness of the slope of the trend are sufficient that the overall price volatility in the second regime is greater than in the first regime, capturing the leverage effect well known in equity markets since the work of Black (1976). Volatility around the trend line (measured by the sigma coefficient) is, however, lower in the second regime.

The two regimes differ with respect to the fit between forecast and actual values. For Regime 1, the correlation between actual and fit is 0.229; for Regime 2 the correlation rises to 0.355. Both offer higher correlations than the linear model's 0.0903. These results strongly suggest that the linear model is misspecified and that the TAR model offers superior forecasting ability. Although the relatively low R^2 values indicate that the model will not provide accurate point estimates of price changes, the correlations are such that it should theoretically be possible to generate useable trading rules. Given the high persistence of regimes, one might adopt a contrarian strategy in regime one and sell in regime two. The effectiveness would depend on the holding period, which possibly requires further analysis. Operationalising rules would require a forecast of interest rates (perhaps using the yield curve or, in the UK, the spread between index-linked and conventional gilts). These, and related, questions will be the subject of a further paper.

The full sample UK property company results (shown in Panel B of Table 1) are remarkably similar to the US REIT behavior. The threshold value separating regimes is higher at 5.452% with 75% of observations falling into the first, lower real rate regime. The first regime is persistent (with a probability of remaining in the regime being 0.931) while the probability of remaining in regime 2 is lower (P_{22} = 0.797).

Once again we observe distinct behavior in the two regimes. The behavior follows the same pattern as the US REITs index. Regime 1 shows mean reversion about a positive trend with significant α and γ coefficients and a significant and negative β coefficient. Regime 2 follows a random walk around a negative trend: the γ coefficient is strongly negatively significant. In the UK case, the absolute value of the γ trend coefficient is larger in Regime 2 than in Regime 1. In aggregate, prices fall more sharply in Regime 2 than they rise in Regime 1 - again consistent with the Black (1976) leverage effect.

In the UK, the correlation between forecast and actual values is higher for Regime 2 (0.342) than for Regime 1 (0.199) although both are higher than for the linear model (0.148). Furthermore, the root mean square error in Regime 2 is lower than that for Regime 1. This implies that knowledge of regime tells us when price changes may be forecastable. This, once again, points to the possibility of developing operational trading rules.

The early part of the analysis period coincides with considerable instability in financial markets with the impact of oil price shocks and the breakdown of the Bretton Woods currency framework being reflected in global interest rates and inflation. To test the stability of the results, both US REIT and UK Property Company performance were reanalyzed for a sub-sample of our data beginning in January 1980. The results, shown in Panels A & B of Table 2, are remarkably consistent with the findings over the whole time-series. In both UK and US cases, the same distinct patterns of behavior are observed across high and low interest regimes.

For the US REIT data, the threshold real interest rate was estimated to be 2.71%. In the lower interest rate regime, we observe mean reverting behavior around a rising trend with significant α, β and γ coefficients. In Regime 2, as before, we find random walk around a statistically significant downward trend. The correlations of forecast to actual values are higher for the two TAR regimes (at 0.263 and 0.331) than for the linear autoregressive model (0.141).

We obtain similar results for the UK subsample. The regimes are defined by a real interest rate of 5.41%. As with the US results, in the lower interest regime prices mean revert about a positive trend while, at higher real interest rates, prices follow a random walk around a descending trend line. In the full time series, the first TAR regime had a lower correlation between forecast and actual values than the second, higher interest rate regime. For the shorter time series, both regimes have comparable correlation coefficients (0.319 and 0.336, respectively) and both outperform the linear model's correlation of 0.185.

6. Conclusions

This paper has presented two models of commercial real estate performance for both the US and the UK. The interest rate regime-based TAR model seems to offer advantages over the more conventional linear autoregressive process. Firstly, in both markets, the correlation between forecast and actual values is higher for the TAR model than for the linear autoregressive model. Secondly, and perhaps more significantly, the TAR model identifies distinct patterns of behavior in each regime. This implies that the linear model may be prone to misspecification errors. The results largely confirm the contention that the real rate of interest plays a significant role as an indicator of real estate performance in both the UK and the US.

A major advantage of the TAR model advanced above is that it does not require a forecast of the real interest rate, performing well while only using the lagged interest rate X_{t-1}. While the R^2 values from the model are not high, it should be recalled that we are working with price changes, not levels. The observed correlations within regimes of 0.3 or above may be sufficient to develop trading rules. Since the behavior across regimes is so different, and since the regimes are highly persistent, knowledge of the likely regime could point to the adoption of distinct investment strategies.

In the high interest rate regimes, it seems that indirect property indices fall sharply down their trend line. In lower interest rate environments, prices exhibit mean-reverting behavior around an upward trend. In general, the results suggest that the price/return falls in high real interest environments are sharper than the rises associated with lower real rates. If this pattern can be translated to the underlying private real estate ownership market, it may imply that there are asymmetries in real estate performance. This would impact upon the way in which researchers should approach the analysis of real estate cycles. The results fit an intuitive view of the property boom and bust cycle with a rapid and severe downward "correction" being followed by a longer and more erratic climb back to a local peak.

It is quite feasible that there may be more than two interest rate regimes. The TAR approach can attempt to identify such regimes and incorporate longer lag structures. Similarly, as noted above, there might be other state variables that define relevant regimes for real estate. These might include other, non-interest rate macro-variables influencing demand and supply. TAR models do not require a single variable in defining regimes. The threshold variable could, for example, be derived from a vector of variables identified as relevant to price formation. Other extensions of the work might include investigation of regime-based structures in the direct, private ownership market. This poses problems in terms of data frequency and data reliability. Another avenue to explore might be to work with an unleveraged real estate equity market series or with an index with overall common stock market effects removed (but see Lizieri & Satchell, 1997a, for a critique of such an approach). We leave these for later research.

Our main intention in this paper has been to investigate whether regime-switching models can provide useful insights into the performance of publicly traded real estate. The fact that our TAR model identified clear regimes for both the US and UK data sets - and that the behavior patterns observed in high and low interest rate regimes were remarkably similar in both countries - is highly persuasive. Although the two-regime real interest rate model examined above may not fully describe changes in property prices, the very different behavior observed across regimes casts doubt on the applicability of linear, one-state models and their ability to generate usable, reliable and robust trading rules.

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Table 1: TAR Results For Full Sample

Panel A US Results 1973 - 1995

Real interest rate threshold value separating regimes: 2.870%

	Regime 1	T Statistic	Regime 2	T Statistic
α	0.2245	3.4056	-0.0557	-0.9759
β	-0.0050	-3.2963	0.0200	1.6488
γ	0.0006	3.3025	-0.0004	-2.7917
σ	0.0452		0.0292	
P(S1) = .643	P_{11} = .9649	P_{12} = .0351	P_{21} = .0532	P_{22} = .9468
Correlations:	S1 = 0.2293	S2 = 0.3552	AR = 0.0903	

Panel B: UK Results 1975 - 1995

Real interest rate threshold value separating regimes: 5.452%

	Regime 1	T Statistic	Regime 2	T Statistic
α	0.2943	2.9180	0.1186	1.1624
β	-0.0481	-2.7937	-0.0052	-0.3230
γ	0.0004	2.4816	-0.0006	-3.9068
σ	0.0686		0.0570	
P(S1) = .747	P_{11} = .9306	P_{12} = .0694	P_{21} = .2034	P_{22} = .7966
Correlations:	S1 = 0.1992	S2 = 0.3415	AR = 0.1476	

The table shows the coefficients and t-statistics for the TAR model for Regime 1 (below threshold real interest rates) and Regime 2 (higher interest rates). P(S1) is the proportion of time spent in Regime 1: P(S2) = 1 - P(S1). P_{11} shows the probability of remaining in Regime 1 in the next period, given that one is in Regime 1 now, P_{12} shows the probability of moving from Regime 1 to Regime 2 and so on. The correlations shown are between the forecast values and the actual values for TAR Regime 1 (S1), Regime 2 (S2) and for the linear autoregressive model (AR).

Table 2: TAR Results 1980 - 1995

Panel A: US Results

Real interest rate threshold value separating regimes: 2.712%

	Regime 1	T Statistic	Regime 2	T Statistic
α	0.3540	2.7523	-0.0877	-1.003
β	-0.0602	-2.8141	0.0189	1.2477
γ	0.0006	3.0291	-0.0004	-2.2687
σ	0.0412		0.0292	
P(S1) = .429	P_{11} = .9444	P_{12} = .0556	P_{21} = .0421	P_{22} = .9579
Correlations:	S1 = 0.2631	S2 = 0.3310	AR = 0.1412	

Panel B: UK Results

Real interest rate threshold value separating regimes: 5.408%

	Regime 1	T Statistic	Regime 2	T Statistic
α	0.7240	3.7601	0.0342	0.2190
β	-0.1015	-3.7230	0.0006	0.0259
γ	0.0004	2.5029	-0.0007	-3.5727
σ	0.0638		0.0597	
P(S1) = .585	P_{11} = .8718	P_{12} = .1282	P_{21} = .1786	P_{22} = .8214
Correlations:	S1 = 0.3193	S2 = 0.3355	AR = 0.1849	

The table shows the coefficients and t-statistics for the sub-period TAR model for Regime 1 (below threshold real interest rates) and Regime 2 (higher interest rates). P(S1) is the proportion of time spent in Regime 1: P(S2) = 1 - P(S1). P_{11} shows the probability of remaining in Regime 1 in the next period, given that one is in Regime 1 now, P_{12} shows the probability of moving from Regime 1 to Regime 2 and so on. The correlations shown are between the forecast values and the actual values for TAR Regime 1 (S1), Regime 2 (S2) and for the linear autoregressive model (AR).