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STOCHASTIC MODELLING OF PUMP-STORAGE HYDROELECTRIC POWER PLANTS, Part II

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Original scientific paper

Two parts of this paper represent a contribution to implementation of a pump-storage hydroelectric power plant stochastic model into a power plant system reliability model. Part II provides the method for determining the probability distribution of required pumping capacity of a pump-storage plant with natural inflow, and then the insertion of pump-storage hydro plants into the reliability model of the entire electrical energy production system. The inflow is thereby treated as a random variable, and stochastic modelling relies on the well-known method of constant and variable energy production. The model developed is suitable to create an additional decision-making criterion for studies related to expansion planning strategy of generating capacities in power systems. The stochastic models presented in this paper are illustrated by simple numerical examples.

Key words: *convolution, density function, distribution function, method of constant and variable energy production, probability function, reliability, stochastic inflow*

Stohastički model crpno-akumulacijskih hidrelektrana, II. dio

Izvorni znanstveni članak

Ovaj članak u dva dijela predstavlja doprinos implementiranju stohastičkog modela crpno-akumulacijskih hidroelektrana u model pouzdanosti sustava elektrana. U drugom dijelu dana je metoda određivanja razdiobe vjerojatnosti potrebne snage crpljenja crpno-akumulacijskog postrojenja s prirodnim dotokom, a zatim je prikazan način uvrštenja crpno-akumulacijskih hidroelektrana s prirodnim dotokom u model pouzdanosti cjelokupnog sustava elektrana. Pri tome se dotok tretira kao slučajna varijabla, a stohastičko modeliranje oslanja se na poznatu metodu konstantne i varijabilne energije. Razvijeni model je pogodan za stvaranje pomoćnog kriterija odlučivanja pri izradi studija za planiranje izgradnje proizvodnih kapaciteta u elektroenergetskom sustavu. Prikazani stohastički modeli ilustrirani su jednostavnim brojčanim primjerima.

Ključne riječi: *funkcija gustoće, funkcija razdiobe, funkcija vjerojatnosti, pouzdanost, konvolucija, metoda konstantne i varijabilne energije, stohastički dotok*

1 Introduction
Uvod

In order to evaluate the reliability characteristics of the electricity generating systems which in addition to thermal plants with unlimited available energy includes also hydroelectric plants, beside the stochastic outages of generating units, the limitation of the – from stochastic inflows dependent – available amounts of primary energy (water) must be also taken into account. Therefore the proposed model counts on the – from the literature known – so-called convolution method for determining the reliability characteristics of thermal power plant systems (see the Appendix), that has been modified by the consideration of the probability function of the available energy of hydro power plants, as described in [4].

2 Determining the probability distribution of the required pumping capacity of the pumped-storage plant with natural hydraulic inflow
Utvrđivanje vjerojatnosti distribucije potrebnog crpnog kapaciteta reverzibilnog postrojenja s prirodnim hidrauličkim dotokom

The required pumping capacity of a pumped-storage plant with natural hydraulic inflow into upper storage depends on, according to equation (12, Part I), the amount of incoming water, as well as on the available outage-determined pumping capacity. Therefore, by determining the probability distribution of the inflow-determined and pump-outage-determined pumping capacity, the inflow probability density function and the outage-dependent available pumping capacity probability density function of the entire power plant is used. The latter is determined using the well-known two-stage model of stochastic behaviour of the pumps (the same as in case of outage-determined available capacity of the generating units in turbine operating mode). In this way, the discrete probability density function of the available pumping capacity, given by the *Dirac's* delta function, takes the following shape:

$$f\left(P_{Mp,i}\right)=\Pr\left(P_{Mp,i}\right)\cdot\delta\left(P_{Mp}-P_{Mp,i}\right);\ i=0,1,2,\dots,n. \qquad (1)$$

Here is i the index of certain pumping capacity stages, i.e. the number of pumps ready for operation, and $\Pr_{(PMp,i)}$ the probability of certain available capacity stages according to the expression:

$$\Pr\left(P_{Mp,i}\right)=\Pr\left(iP_0\right)=\binom{n}{i}\Pr\left(B\right)^i\Pr\left(A\right)^{n-i};$$
$$i=0,1,2,\dots,n \qquad (2)$$
$$\binom{n}{i}=\frac{n!}{i!\left(n-1\right)!};\quad \binom{n}{n}=1.$$

which applies to n identical pumping units with equal capacities P_0, as well as equal probabilities of outage states Pr(A) and operating states Pr(B).

To each capacity volume of n pumps of the pump-storage plant m capacity stages are added, so that each stage can be identified by means of indices i=0,1,2,…n, and j=1,2,…,m.

A specific pumping capacity will be achieved based on the following two operating cases (of course, on the assumption that gross head and efficiency rates are constant):

Case I:

The number of pumps i ready for operation, i.e. their capacity is sufficient for pumping as much water as required, which, together with the natural inflow water, is required for variable generation $W_{vt,max}$. The falling section for growing inflows in the function $P_p=f(Q)$, shown in Fig. 2, Part I applies for this operation mode.

The probability that the outage- and inflow-determined pumping capacity P_p lies in the interval with marginal values $P_{p,i,j}$ and $P_{p,i,j+1}$ within certain capacity volumes i, equals the probability that: a) the outage-determined pumping capacity P_{Mp} is at least as high as necessary for pumping the inflow-determined amount of water Q_p **AND** b) the inflow Q falls within the interval with margins $Q_{i,j}$ and $Q_{i,j+1}$.

The limit values $Q_{i,j}$ and $Q_{i,j+1}$ correspond to the pumping capacity stages $P_{p,i,j}$ and $P_{p,i,j+1}$, where this dependence is given by the linear section of the curve $P_p=f(Q)$. These events are mutually independent, and therefore:

$$\begin{aligned}&\Pr\left(P_{p,i,j} < P_p \le P_{p,i,j+1}\right) = \\ &\Pr\left(P_{Mp} \ge P_{Mp,i}\right)\cdot \Pr\left(Q_{i,j} < Q \le Q_{i,j+1}\right); \\ &i = 0,1,2,\dots,n;\ j = 1,2,\dots,m.\end{aligned} \tag{3}$$

$$\begin{aligned}&F^I\left(P_{p,i,j+1}\right) - F^I\left(P_{p,i,j}\right) = \\ &F^*\left(P_{Mp,i}\right)\cdot\left[F^*\left(Q_{i,j}\right) - F^*\left(Q_{i,j+1}\right)\right]; \\ &i = 0,1,2,\dots,n;\ j = 1,2,\dots,m.\end{aligned} \tag{4}$$

The value of the pump capacity distribution function at the position $(i, j+1)$ is:

$$\begin{aligned}&F^I\left(P_p\right) = F^I\left(P_{p,i\ j+1}\right) = \\ &F^*\left(P_{Mp,i}\right)\cdot\left[F^*\left(Q_{i,j}\right) - F^*\left(Q_{i,j+1}\right)\right] - F^I\left(P_{p,i,j}\right); \\ &i = 0,1,2,\dots,n; \quad j = 1,2,\dots,m\end{aligned} \tag{5}$$

Case II:

$i<n$ pumps are ready for operation and the amounts of water Q from inflow are below or equal to the amounts corresponding to pump capacities in operation $P_{Mp,i}$. At the same time, it means that the flows of pumps in operation are insufficient to transport amounts of water required for energy generation $W_{vt,max}$. In this inflow area, for each i a specific characteristic $P_p=P_{Mp,i}=const.$ (Fig. 2, Part I) applies, and the available pumps operate at full load. Subsequently, only the discrete outage- and inflow-determined pumping capacity stages occur:

$$P_{p,i} = P_{Mp,i} = iP_0; \quad i = 0,1,2,\dots,n-1. \tag{6}$$

where P_0 denotes the maximum capacity of equal pumps.

The probability of the occurrence of these discrete capacity stages equals the product of probability of the mentioned independent single events.

$$\begin{aligned}&Pr^{II}\left(P_{p,i}\right) = \\ &= Pr\left(P_{Mp} = P_{Mp,i}\right)\cdot Pr\left\{Q \le \left[Q_t^{(i)} = f^{-1}\left(P_{Mp,i}\right)\right]\right\}; \\ &i = 0,1,2,\dots,n-1.\end{aligned} \tag{7}$$

Therefore, the density function and the probability distribution function for Case II at position i is:

$$\begin{aligned}&f^{II}\left(P_{p,i}\right) = f\left(P_{Mp,i}\right)\cdot F\left[Q_t^{(i)} = f^{-1}\left(P_{Mp,i}\right)\right]; \\ &i = 0,1,2,\dots,n-1.\end{aligned} \tag{8}$$

$$\begin{aligned}&F^{II}\left(P_p\right) = \sum_{i=0}^{l} f^{II}\left(P_{p,i}\right) \quad \text{za} \quad P_{p,i} \le P_p < P_{p,i+1}; \\ &l = 0,1,2,\dots,n-1.\end{aligned} \tag{9}$$

In equations (7) to (9) the function $Q=f^{1}(P_p)$ is the inverse of function (12, Part I).

Since the above mentioned operating cases are mutually exclusive, the total probability of the occurrence of an inflow- and outage-determined pumping capacity from the interval with limit values $P_{p,i,j}$ and $P_{p,i,j+1}$ equals the sum of probabilities according to (3) and (7). Consequently, the distribution function of stochastic variable P_p is:

$$F(P_p) = F^I(P_p) + F^{II}(P_p). \tag{10}$$

In the probability density function:

$$f(P_p) = \frac{\mathrm{d}F(P_p)}{\mathrm{d}P_p}, \tag{11}$$

at positions $P_{p,i}$ due to expression (8) discrete jumps appear, the example of which for four pumps is given in Fig. 1:

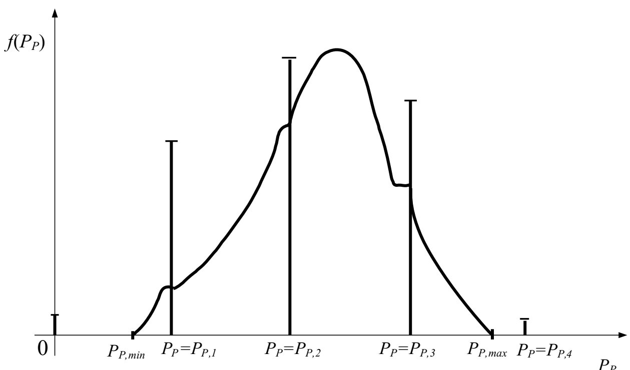

***Figure 1** Probability density function of outage- and inflow-determined pumping capacity*

***Slika 1.** Funkcija vjerojatnosti gustoće pumpnog kapaciteta određenog prekidom i dotokom*

The probability density function (11) meets the following requirement:

$$\int_{-\infty}^{\infty} f(P_p)\mathrm{d}P_p = \int_{0}^{P_{p,\max}} f(P_p)\mathrm{d}P_p = 1. \tag{12}$$

The numerical calculation of probability density function, which is given by (11), is shown in Example 1.

3
Insertion of the pump-storage hydroelectric plants with natural inflow into the reliability model
Umetanje reverzibilnog postrojenja hidroelektrane s prirodnim dotokom u model pouzdanosti

Fig. 3 of Part I shows the way the pump-storage hydroelectric plant with its generation and with pumping energy required for that generation – though assuming 100 % reliable machines in pump drive and in turbine drive – modifies the load duration curve. At this place, the same should be achieved within a framework of the reliability model, taking into account the stochastic inflows and the outages of machines as well in turbine drive as in pump drive.

Case I: $Q_{\max} \le Q_t$, i.e. the maximum probable inflow in the observed week is less or equals the flow-through corresponding to the weekly energy which can be produced by the plant without natural inflow during r working days with maximum capacity in time t_{vt} (as mentioned in Part I, t_{vt} is the maximum possible duration for using the maximum capacity in turbine drive, thus by energy production in pump-storage drive, disregarding the possible additional generation from natural inflow).

The required pumping energy is to be produced by power plants covering the constant load, owing to the pumps with their outage- and inflow-determined available capacities increase the load duration curve in its low-load area. Accordingly, the random variable "capacity deficiency" P_D (see the Appendix) can be in this case interpreted as the sum of two independent random variables "load" P_L and "outage- and inflow-determined pumping capacity" P_p:

$$P_D = P_L + P_p . \tag{13}$$

The outage- and inflow-determined pumping capacity is described by density function (11) and presented in Fig. 1. It must be for the plant r (from total R) in the first step discretized and therewith brought to well-known, regarding the computation, convenient shape:

$$f\left(P_{p,i}^{r}\right) = \Pr\left(P_{p,i}^{r}\right) \cdot \delta\left(P_p - P_{p,i}^{r}\right); \quad i = 0,1,2,\dots,n; \quad r = 1,2,\dots,R. \tag{14-a}$$

Index i relates here also to the discrete pumping capacity stages.

The next step is the calculation of load probability function after insertion of the pump drive of r-th plant using the recursive convolution integral (see the Appendix):

$$F_{Lp,r}^{*}\left(P_L\right) = \int_{-\infty}^{\infty} F_{L,r-1}^{*}\left(P_L - P_p\right) \cdot f_p\left(P_p\right) \mathrm{d}P_p . \tag{15-a}$$

The discrete form of this expression, i.e. the load probability function $R_{Lp,r}(P_L)$ modified by the outage- and inflow-determined available capacity in pump drive (moved to the right toward higher loads), arises through insertion of multi-stage discrete probability density function of available, outage- and inflow-determined pumping capacity (14-a):

$$R_{Lp,r}\left(P_L\right) = \sum_i R_{L,r-1}\left(P_L - P_{p,i}^{r}\right) \cdot \Pr\left(P_{p,i}^{r}\right); \quad i = 0,1,2,\dots,n; \quad r = 1,2,\dots,R. \tag{15-b}$$

The required pumping energy is given by the probability function $F^*(W_{pm,r})$ determined in section 3, Part I. This energy should be placed onto surface available for that between the curves $R_{L,r-1}(P_L)$ and $R_{Lp,r}(P_L)$ in the area of low loads. The function $W' = f_1(p)$, describing the energy content of the available surface, will be determined according to equation:

$$W' = T \int_{P_{L,\min}}^{p} \left[R_{L,r-1}(P_L) - R_{L,r}(P_L) \right] \mathrm{d}P_L ; \quad P_{L,\min} \le P_L < P_{L,\max} + P_{p,n}^{r} . \tag{16}$$

The upper limit p in (16) is the capacity value below which – with the purpose of achieving the generation $W_{vt,\max}$ in peak-load area – the pump drive is to be inserted into the model.

Upon the placement of this energy of stochastic character the weighting of both functions $R_{Lp,r}(P_L)$ and $R_{L,r-1}(P_L)$ by adequate probabilities is necessary (Fig. 2). The weighting is to be carried out by means of the following adapted version of equation (D-18):

$$R_{Lr}^{p}\left(P_L\right) = R_{L,r-1}\left(P_L\right) \cdot \left[1 - F^{*}\left(W_{pm,r}\right)\right] + R_{Lp,r}\left(P_L\right) \cdot F^{*}\left(W_{pm,r}\right) \tag{17}$$

and starts at $P_L = P_{L,\min}$, *following the logic described by storage hydroelectric plants* [4]. Therewith the transition between the curves $R_{Lp,r}(P_L)$ and $R_{L,r-1}(P_L)$ becomes continuous.

From Fig. 2 is also evident the way of turbine drive modelling of the pump-storage plant. The load probability function $R_{Lt,r}(P_L)$ reduced by outage-dependent available capacity in turbine drive (moved to the left toward lower loads) is obtained according to the adapted version of equation (15-a), recursive from function $R_{Lt,r-1}(P_L)$, which represents the load before insertion of the pump-storage plant r:

$$R_{Lt,r}\left(P_L\right) = \sum_i R_{Lt,r-1}\left(P_L + P_{M,i}^{r}\right) \cdot \Pr\left(P_{M,i}^{r}\right); \quad i = 0,1,2,\dots,n; \quad r = 1,2,\dots,R. \tag{15-c}$$

About obtaining the function (15-c), into (15-a) instead of $f_p(P_p)$ it should be substituted the multi-stage discrete probability density function of outage-dependent available capacity in turbine drive:

$$f\left(P_{M,i}^{r}\right) = \Pr\left(P_{M,i}^{r}\right) \cdot \delta\left(P_M - P_{M,i}^{r}\right); \quad i = 0,1,2,\dots,n; \quad r = 1,2,\dots,R. \tag{14-b}$$

For determining the energy content of the surface between $R_{Lt,r-1}(P_L)$ and $R_{Lt,r}(P_L)$ the equation (D-17) is to be used.

Although the inflow-dependent variable generation of the pump-storage plant underlies probability distribution, it will be always supplemented to the value $W_{vt,\max}$ with the generation from the pump-storage operation. Therefore, the

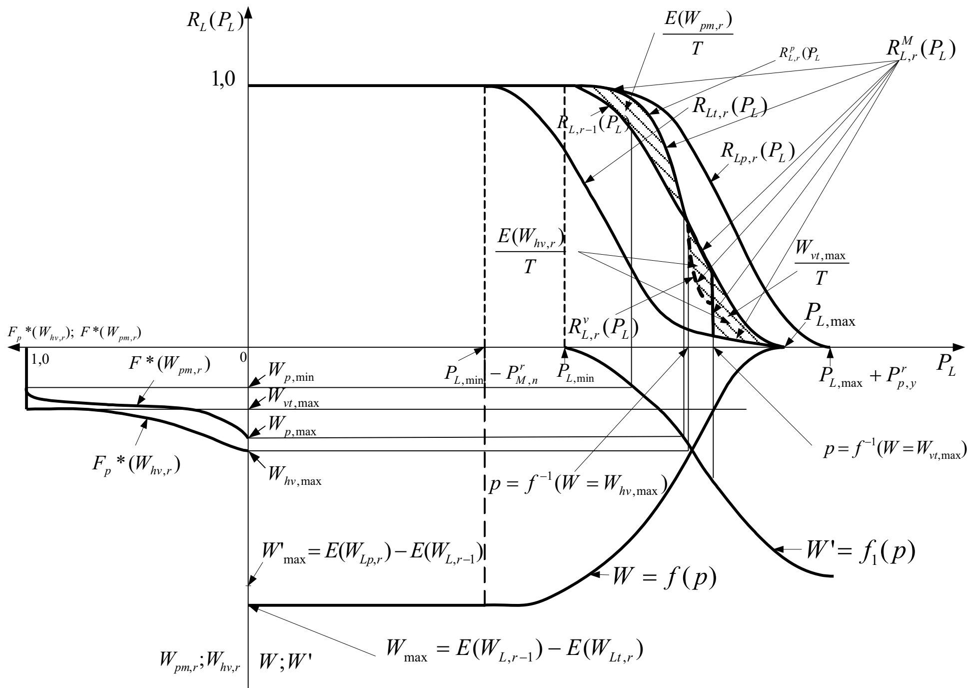

Figure 2 *Insertion of the pump-storage plant with natural inflow into the reliability model*
Slika 2. *Umetanje reverzibilnog postrojenja s prirodnim dotokom u model pouzdanosti*

total generation is given by expression (3, Part I) as a determined value and will be inserted in the peak-load area.

The modified probability function after the insertion of the r-th pump-storage plant with natural inflow is:

$$R_{L,r}^{M}(P_L) = R_{L,r}^{p}(P_L); P_L < \left[p = f^{-1}\left(W = W_{vt,\max}\right)\right],$$
$$R_{L,r}^{M}(P_L) = R_{Lt,r}(P_L); P_L \geq \left[p = f^{-1}\left(W = W_{vt,\max}\right)\right]. \tag{18}$$

The shaded surface in Fig. 2, which symbolizes the required pumping energy, is proportional to the expected value of the pumping energy:

$$E\left(W_{pm}\right) = T \cdot \int_{0}^{P_{L,\max}} \left[R_{L,r}(P_L) - R_{L,r-1}(P_L)\right] \mathrm{d}P_L. \tag{19}$$

Case II: $Q_{\max} > Q_t$

In this case by certain probability the amounts of the inflow $Q > Q_t$ allow also generation of variable energy greater than $W_{vt,\max}$, whereas the generation equal to $W_{vt,\max}$ still appears too with certainty (because of the supplementary pump-storage operation). The probability function of variable energy, obtained from density function of variable generation by exclusive usage of the natural inflow shown in Fig. 4, Part I, should now be utilized by weighting the surface available between both of the curves $R_{L,r-1}(P_L)$ and $R_{Lt,r}(P_L)$ for placement of energy production of the pump-storage plant. The load probability function after insertion of the variable generation of plant r is:

$$R_{L,r}^{v}(P_L) = R_{L,r-1}(P_L) \cdot \left[1 - F_p^{*}\left(W_{hv,r}\right)\right] +$$
$$+ R_{Lt,r}(P_L) \cdot F_p^{*}\left(W_{hv,r}\right). \tag{20}$$

This function is depicted in Fig. 2 with broken line, together with $F^{*}_{p}(W_{hv,r})$.

The final load probability function, modified by the plant r with natural inflow, is described by equations:

$$R_{L,r}^{M}(P_L) = R_{L,r}^{p}(P_L); \quad P_L < \left[p = f^{-1}\left(W = W_{hv,\max}\right)\right],$$
$$R_{L,r}^{M}(P_L) = R_{L,r}^{v}(P_L); \quad \left[p = f^{-1}\left(W = W_{hv,\max}\right)\right] \leq P_L,$$
$$P_L < \left[p = f^{-1}\left(W = W_{vt,\max}\right)\right], \tag{21}$$
$$R_{L,r}^{M}(P_L) = R_{Lt,r}(P_L); \quad P_L \geq \left[p = f^{-1}\left(W = W_{vt,\max}\right)\right].$$

The shaded surface in the head of the load diagram in Fig. 2 is proportional to the expected variable generation of r-th pump-storage plant with natural inflow for $Q_{\max} > Q_t$:

$$E\left(W_{hv,r}\right) = T \cdot \int_{0}^{P_{L;\max}} \left[R_{L,r}^{p}(P_L) - R_{L,r}^{M}(P_L)\right] \mathrm{d}P_L \tag{22}$$

In the case that according to criterion in section 3, Part I, the plant with natural inflow also has to generate constant energy with certain probability, it is necessary to perform an additional modification of the load probability function. However, discussion of such case exceeds the frame of this paper. (Remark: The apparent contradiction – constant generation together with pumping energy – is of course easy to explain. Namely, it is not a matter of deterministic case, therefore although in reality they are mutually exclusive, in the reliability model these two amounts of energy appear with correspondent probability.)

In Example 2 a numerical proof of above derivations is given.

4
Examples
Primjeri

4.1
Example 1: Determination of probability distribution for inflow- and outage-determined pumping capacity of pump-storage plant with natural inflow

Primjer 1: Određivanje distribucije vjerojatnosti za dotokom i prekidom određen pumpni kapacitet reverzibilnog pumpnog postrojenja s prirodnim dotokom

Input data required for calculation are:

Maximum turbine flow-through (designed size): Q_A=100 m^3/s; gross (net) head: H_b=H_n=56,631555 m; efficiency rate of the plant in turbine drive: η_t=0,9; efficiency rate of the plant in pump drive: η_p=0,8; useful storage volume: V_k=4 hm^3; number of equal generating units 4; number of working days in the observed week: r=6; rated capacity of generating units in turbine drive: $P_{t,N}$=25 MW; rated capacity of the pumps in pump drive: $P_{p,N}$=30 MW; maximum capacity of the plant in turbine drive: $P_{h,\max}$=50 MW; outage probability of generating units in turbine drive: $\Pr_t(A)$=0,1; outage probability of the pumps in pump drive: $\Pr_p(A)$=0,1; peak-load duration during working days in the observed week: rt_v=130 h; maximal duration of using variable energy of hydroelectric plant during working days: rt_{vh}=30 h. The density function and the distribution function of natural inflow given in Fig. 3 are considered as known.

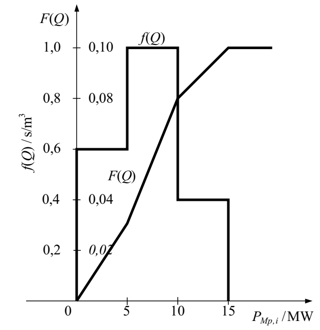

Figure 3 *Density function and distribution function of natural inflow*
Slika 3. *Funkcija gustoće i funkcija raspodjele prirodnog dotoka*

Shortened calculation procedure:

The density function of the outage-determined pumping capacity has been obtained on the basis of equation (2) which applies to n pumps of the same capacity P_0 and same probability of outage and operating state Pr(A) and Pr(B), until the probability function of the outage-determined pumping capacity by summing the discrete values of the density function according to expression $F^*\left(P_{Mp}\right)=\sum_{P_{Mp}>P} f\left(P_{Mp,i}\right)$ (Fig. 4).

Characteristics P_p=f_i (Q), when both, or respectively only one pump is in operation, are given in Fig. 5.

The amount of inflow Q_t (by which the pump drive becomes dispensable) and $Q_t^{(1)}$ (by which the maximum pumping capacity during the operation of one pump is achieved) are calculated according to equation (12, Part I) for P_p=0, i.e. P_p=$P_{Mp,1}$. The maximum flow of the pumps $Q_{p,\max}$ for Q=0 and maximum pumping capacity required for

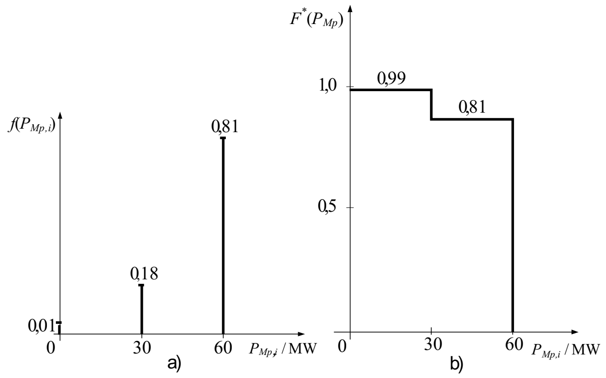

Figure 4 *Density function of outage-determined pumping capacity (a) and probability function of outage-determined pumping capacity (b)*
Slika 4. *Funkcija gustoće pumpnog kapaciteta određenog prekidom (a) i funkcija vjerojatnosti pumpnog kapaciteta određenog prekidom (b)*

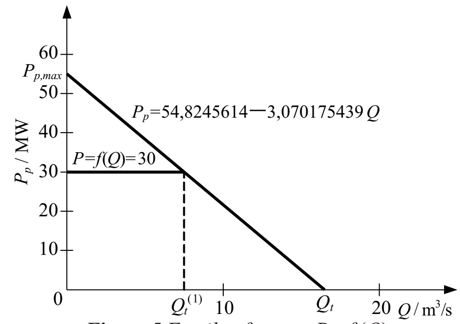

Figure 5 *Family of curves P_p=f_i(Q)*
Slika 5. *Obitelj krivulja P_p=f_i(Q)*

that flow are obtained from expressions (5) and (2):

Q_t=13,85714286 m^3/s
$Q_t^{(1)}$=8,085714284 m^3/s
$Q_{p,\max}$=78,94736842 m^3/s
$P_{p,\max}$=54,8245614 MW

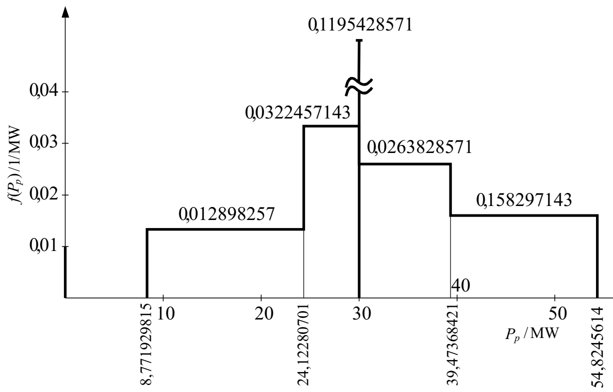

Figure 6 *Probability density function of inflow- and outage-determined pumping capacity*
Slika 6. *Funkcija gustoće vjerojatnosti dotokom i prekidom određenog pumpnog kapaciteta*

Considering the two possible operation cases, the probability density function determined by inflow and pumping unit availability is defined by means of equations (5), (9), (10) and (11). That curve is shown in Fig. 6.

4.2 Example 2: Insertion of pump-storage plant with natural inflow into the reliability model

Primjer 2: Umetanje reverzibilnog pumpnog postrojenja s prirodnim dotokom u model pouzdanosti

Input data required for calculation are:

The data of the pump-storage plant and the inflow density function in the observed week (Fig. 3) are the same as in Example 1, while the load is represented by probability function obtained based on load duration curve in Fig. 8, Part I:

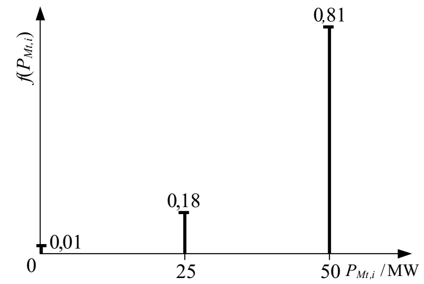

Figure 7 Density function of the outage-dependent plant capacity in turbine drive

Slika 7. Funkcija gustoće kapaciteta postrojenja ovisnog o prekidu u turbinskom pogonu

$$R_{L,0}(P_L) = 1; \qquad -\infty \le P_L \le 250 \text{ MW}$$

$$R_{L,0}(P_L) = 2,666666667 - 0,066666667 \cdot P_L; \qquad 250 \le P_L \le 400 \text{ MW}$$

$$R_{L,0}(P_L) = 0; \qquad 400 \text{ MW} \le P_L \le \infty$$

By means of equation (2) is installed the discrete probability density function of the outage-dependent plant capacity in turbine drive (Fig. 7).

The probability density function of the inflow- and outage-determined pumping capacity, obtained as well in Example 1 is also necessary (Fig 6).

Shortened calculation procedure:

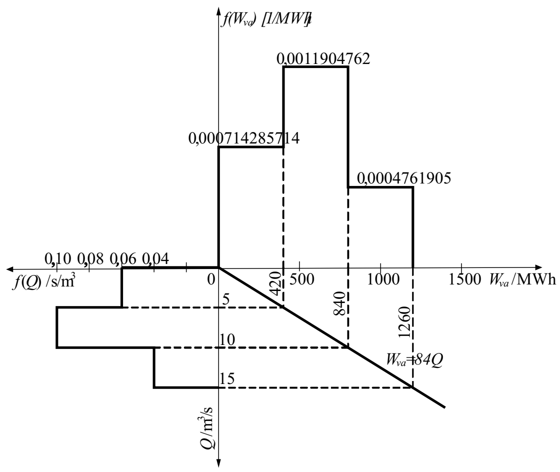

Figure 8 Determination of the density function of variable generation by exclusive usage of natural inflow

Slika 8. Određivanje funkcije gustoće varijable generacije pomoću ekskluzivnog korištenja prirodnog dotoka

The insertion of the observed plant into the reliability model of power plant system (Fig. 10) is carried out in the following steps:

First of all, according to the rules from Example 1, Part I, from the inflow density function the variable generation density function $f(W_{va})$ of the pump-storage plant by exclusive usage of natural inflow is calculated (Fig. 8), and then from this function the density function $f(W_{pm})$ and the probability function $F^*(W_{pm})$ of required pumping energy (Fig. 9).

Here are needed the following functions:

$W_{va} = 84 \cdot Q$, MWh

$W_{pm} = 2083,333333 - 1,388888889 \cdot W_{va}$, MWh

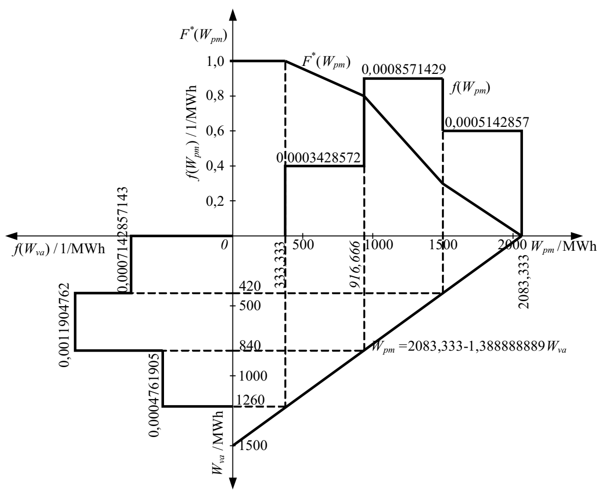

Figure 9 Determination of density function and probability function of required pumping energy

Slika 9. Određivanje funkcije gustoće i funkcije vjerojatnosti potrebne pumpne energije

Table 1-a/Tablica 1-a

P_L/MW	0	250	260	270	280	290
$R_{Lp,1}(P_L)$	1	1	0,99771228	0,98384561	0,96255353	0,91123424
P_L/MW	300	310	320	330	340	350
$R_{Lp,1}(P_L)$	0,84456757	0,7779009	0,71123424	0,64456757	0,57790090	0,51123424

Table 1-b/Tablica 1-b

P_L/MW	360	370	380	390	400	410
$R_{Lp,1}(P_L)$	0,44456757	0,3779009	0,31123424	0,24456757	0,17790090	0,11352195
P_L/MW	420	430	454,824561			
$R_{Lp,1}(P_L)$	0,06072195	0,0153474	0			

1. Determining the function $R_{Lp,1}(P_L)$ by convolution in accordance with equation (15) of the function $R_{L,0}(P_L)$ with the density function $f(P_p)$ of inflow- and outage-determined pumping capacity from Fig. 6, discretized through (14-a).

The function $R_{Lp,1}(P_L)$ is given in Table 1.

2. Calculation of the function $W'=f_1(p)$ according to (16) – Table 2.

Table 2-a/Tablica 2-a

P/MW	0	250	260	270	280	290
W'/MWh	0	0	55,253254	207,76192	445,29729	731,92361
P/MW	300	310	320	330	335.5	340
W'/MWh	1030,7971	1329,6706	1628,5441	1927,4177	2083,3333	2226,2912

Table 2-b/Tablica 2-b

P/MW	350	360	370	380	390	400
W'/MWh	2525,1647	2824,0382	3122,9117	3421,7853	3720,6588	4019,5323
P/MW	410	420	430	440	454,824561	
W'/MWh	4263,1525	4409,5147	4470,8556	4483,0383	4580,2017	

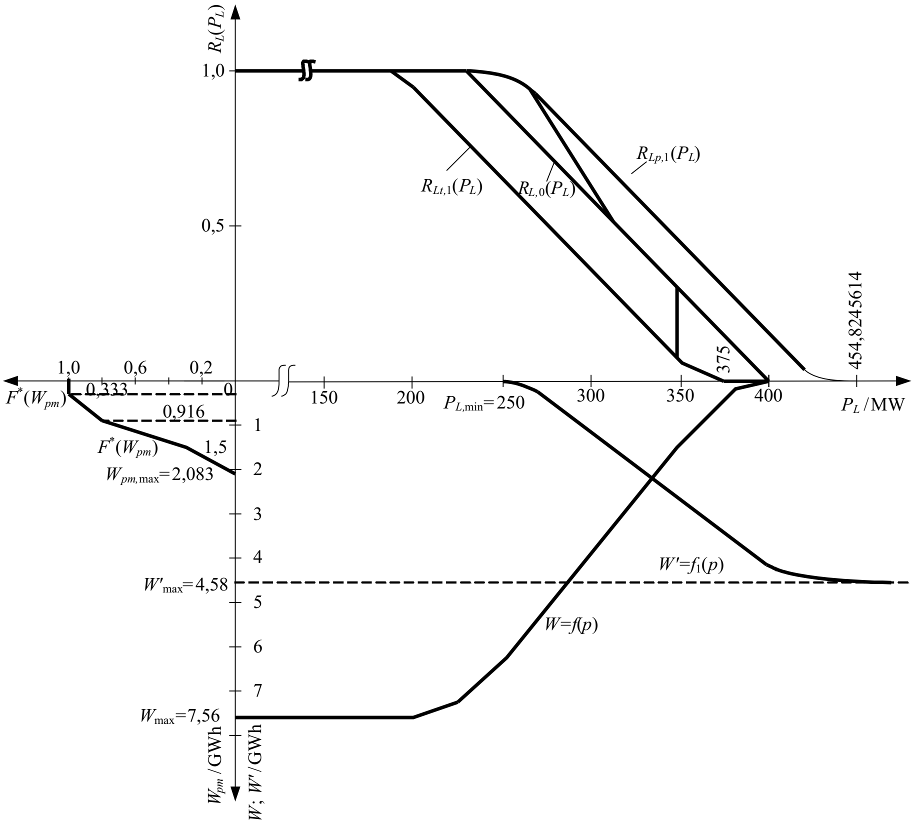

Figure 10 *Insertion of the pump-storage plant with natural inflow into the reliability model of power plant system*
Slika 10. *Umetanje reverzibilnog pumpnog postrojenja s prirodnim dotokom u model pouzdanosti sustava elektrane*

3. Insertion of the required pumping energy, described with the probability function $F^*(W_{pm})$ (Fig. 9), by means of weighting the surface between the curves $R_{Lp,1}(P_L)$ and $R_{L,0}(P_L)$ according to equation (17). This results in the function $R^p_{L,1}(P_L)$ shown in Tab. 3.

Table 3-a/Tablica 3-a

P_L/MW	0	250	260	270	280
$R^p_{L,1}(P_L)$	1	1	0,99771228	0,98384561	0,95631349
P_L/MW	290	300	310	320	330

Table 3-b/Tablica 3-b

$R^p_{L,1}(P_L)$	0,88692238	0,79158404	0,67934320	0,57494286	0,48093169
P_L/MW	335	340	400		
$R^p_{L,1}(P_L)$	0,43392610	0.4	0		

Table 4/Tablica 4

P_L /MW	0	200	225	250	350	375	400
$R_{Lt,1}(P_L)$	1	1	0,865	0,7	0,0333333333	0,0016666667	0

Table 5/Tablica 5

P/MW	400	375	350	250	225	200	0
W/GWh	0	0,3465	1,323	6,363	7,2765	7,56	7,56

4. Determining the probability function $R_{Lt,1}(P_L)$ by convolution - according to (15-c) - of the original load probability function $R_{L0}(P_L)$ with the density function of outage-determined plant capacity in turbine drive from Fig. 7. The values of the calculated function are listed in Tab. 4.

5. Calculation of the function $W=f(p)$ (Tab. 5) from (D-17).

6. Insertion of the determined variable generation $W_{vt,\max}$ given by expression (3, Part I) at value $p = f^{-1}(W = W_{vt,\max})$:

$W_{vt,\max} = 1,5$ GWh; $p = f^{-1}\left(W = 1,5\right) = 346$ MW.

The load probability function modified by the observed pump-storage plant with natural inflow is defined through equation (18).

5 Conclusion
Zaključak

The presented stochastic modelling of pump-storage hydroelectric power plants is a very complex process based on operation analysis of such plants. Thereby the natural inflow into the upper storage is considered as a stochastic variable, as well as the availability of the machines in pump drive and turbine drive. The model development consists of the following phases: a) determining the probability distribution of the variable generation by exclusive usage of the inflow into the upper storage, b) determining the probability distribution of the variable generation from pump-storage operation, c) determining the probability distribution of the required pumping energy for pump-storage plants with natural inflow, d) determining the probability distribution of the required pumping capacity for pump-storage plants with natural inflow, and finally e) insertion of the pump-storage plants with natural inflow into reliability model of the entire power plant system. The ultimate goal of the described stochastic modelling is to create an additional decision-making criterion for studies related to expansion planning strategy of generating capacities in the power systems.

6 Appendix
Dodatak

Convolution method and the reliability characteristics of power plant system.

The generating units in the convolution method are represented with a two-step model of behaviour regarding operation outages and the stochastic available capacity of the i-th generating unit can be described by a discrete density function shown in Fig. D-1.

In Fig. D-1 are: $\Pr(B_i)$ operating state probability of i-th generating unit (availability), $\Pr(A_i)$ outage state probability of i-th generating unit (unavailability), $P_{N,i}$ rated capacity of the i-th generating unit. Availability and unavailability of generating units are defined in the way well known from the reliability theory (see e.g. [8]).

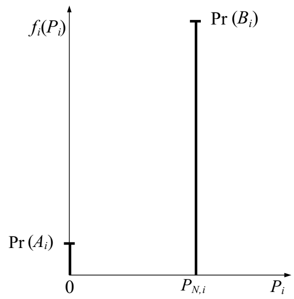

Figure D-1 *Density function of the available capacity of i-th generating unit*
Slika D-1. *Funkcija gustoće raspoloživog kapaciteta i-te proizvodne jedinice*

The discrete probability density function of available capacity $f_i(P_i)$ of the i-th generating unit can be mathematically expressed by means of Dirac's delta-function [2]:

$$f_i(P_i) = \Pr(A_i) \cdot \delta(P_i) + \Pr(B_i) \cdot \delta(P_i - P_{N,i}). \tag{D-1}$$

The load is given by the load probability function $F^*_L(P_L)=R_L(P_L)$ (Fig. D-2). This is obtained from the forecasted load duration curve (e.g. Fig. D-1, Part I) by exchanging the co-ordinate axes and scaling the time-axis to one.

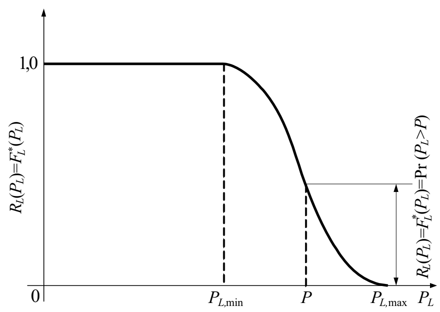

Figure D-2 *Load probability function*
Slika D-2. *Funkcija vjerojatnosti opterećenja*

The definition of capacity deficiency represents an initial base for the calculations. Capacity deficiency $P_D(t)$ of a power plant system at the arbitrary moment t is obtained as a difference between the load $P_L(t)$ and the capacity of the system consisting of m generating unit:

$$P_D(t) = P_L(t) - \sum_{i=1}^{i=m} P_i(t). \tag{D-2}$$

The density function of capacity deficiency $f_{D,i}(P_D)$ after one by one insertions of generating units can be obtained by means of convolution when the load P_L and the available capacities of the generating units P_i are realized as mutually independent variables.

The density function of the difference $P_D=P_L-P_i$ of two stochastic variables is to be determined by calculating it recursively as a convolution product of the densities $f_{L,i-1}(P_L)$ and $f_i(P_i)$:

$$f_{D,i}(P_D) = f_{L,i}(P_L) = f_{L,i-1}(P_L)\cdot f_i(P_i);\quad i=1,2,...,m. \tag{D-3}$$

$$f_{D,i}(P_D) = f_{L,i}(P_L) = f_{L,i-1}(P_D+P_i)\cdot f_i(P_i);\quad i=1,2,...,m. \tag{D-4}$$

$$f_{D,i}(P_D) = \int_{-\infty}^{\infty} f_{L,i-1}(P_D+P_i)\cdot f_i(P_i)\mathrm{d}P_i;\quad i=1,2,...,m. \tag{D-5}$$

Respectively, the density function of capacity deficiency after the insertion of the i-th generating unit is identical to the density function of the remaining load after the i-th insertion:

$$f_{L,i}(P_L) = \int_{-\infty}^{\infty} f_{L,i-1}(P_L+P_i)\cdot f_i(P_i)\mathrm{d}P_i;\quad i=1,2,...,m. \tag{D-6}$$

The load probability distribution function, which should be covered by the remaining generating units after the insertion of the i-th unit:

$$\begin{aligned} F_{L,i}(P_L) &= \int_{-\infty}^{\infty} f_{L,i}(P_L)\mathrm{d}P_L = \\ &= \int_{-\infty}^{P_L}\int_{-\infty}^{\infty} f_{L,i-1}(P_L+P_i)f_i(P_i)\mathrm{d}P_i\mathrm{d}P_L = \\ &= \int_{-\infty}^{\infty} f_{L,i-1}(P_L+P_i)f_i(P_i)\mathrm{d}P_i. \end{aligned} \tag{D-7}$$

The complementary function – the probability function $F^*_{L,i}(P_L)$ – results from the probability distribution function on the basis of the known fact that

$$\begin{aligned} &\int_{-\infty}^{\infty} f_i(P_i)\mathrm{d}P_i = 1, \\ &F^*_{L,i}(P_L) = 1 - F_{L,i}(P_L) = \\ &\int_{-\infty}^{\infty} F^*_{L,i-1}(P_L+P_i)f_i(P_i)\mathrm{d}P_i;\quad i=1,2,...,m. \end{aligned} \tag{D-8}$$

Taking (D-1) into account, a recursive formula can be derived for determining the probability function of load deficiency:

$$\begin{aligned} &R_{L,i}(P_L) = F^*_{L,i}(P_L) = R_{L,i-1}(P_L)\Pr(A_i) + \\ &+R_{L,i-1}(P_L+P_{N,i})\Pr(B_i);\quad i=1,2,...,m. \end{aligned} \tag{D-9}$$

After insertion of the last, m-th generating unit the equation (D-9) gives the probability function of load deficiency $R_D(P_D)$. That function determines the probability by which the deficiency exceeds the value P:

$$\begin{aligned} &R_{L,m}(P_L) = F^*_{L,m}(P_L) = R_D\left(P_D\right) = \\ &\Pr(P_D > P) = \int_{P}^{\infty} f_D(P_D)\mathrm{d}P_D. \end{aligned} \tag{D-10}$$

Fig D-3 shows the conceptual form of curves $R_{L,i}(P_L)$ after one by one convolutions, with a remark that in the systems of purely thermal plants (i.e. generating units without any restrictions on primary energy) the sequence of generating unit commitment is given on the basis of increasing marginal fuel costs, and the generating unit will be omitted from the sequence of commitment if in the observed week it is in repair.

In Fig. D-3 only the part of function $R_D(P_D)$ for which the capacity deficiency is greater than zero is certainly of interest.

Based on the function $R_D(P_D)$ it is possible to determine the reliability characteristics of the power plant system as well as the expected generations of the units.

Probability of capacity deficiency:

$$\Pr(D) = \Pr(P_D > 0) = R_D(0). \tag{D-11}$$

Expected value of total capacity deficiency duration in the week (T=168 h):

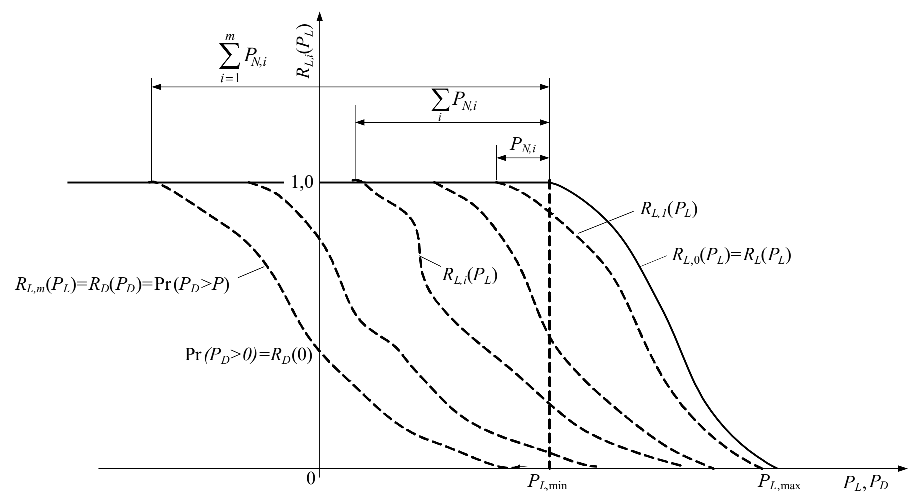

Figure D-3 *Conceptual form of curves $R_{L,i}(P_L)$*
Slika D-3. *Koncepcijski oblik krivulje $R_{L,i}(P_L)$*

$$E(T_{DK0}) = T \cdot \Pr(D). \qquad \text{(D-12)}$$

Expected value of energy deficiency in the week:

$$E(W_D) = T \int_0^{P_{L,\max}} R_D(P_D)\,\mathrm{d}P_D. \qquad \text{(D-13)}$$

Expected value of capacity deficiency:

$$E(P_D) = \frac{1}{\Pr(D)} \int_0^{P_{L,\max}} R_D(P_D)\mathrm{d}P_D = = \frac{\int_0^{P_{L,\max}} R_D(P_D)\mathrm{d}P_D}{\int_P^{\infty} f_D(P_D)\mathrm{d}P_D}. \qquad \text{(D-14)}$$

Expected generation of the i-th generating unit (the difference between required energies in the system before and after the insertion of the i-th generating unit):

$$E(W_i) = E(W_{L,i-1}) - E(W_{L,i}) = = T\int_0^{P_{L,\max}} \left[R_{L,i-1}(P_L) - R_{L,i}(P_L) \right] \mathrm{d}P_L. \qquad \text{(D-15)}$$

Placement of possible (variable) generation of the hydroelectric plant into the peak-load area.

Considering the hydroelectric plants with restricted available energy, two possible cases can appear concerning the placement of their possible (variable) energy into the peak-load area:

If the possible (variable) energy is given as a determined value, the transition between the curves $R_{L,i-1}(P_L)$ and $R_{L,i}(P_L)$ is excursive:

$$\begin{aligned} R_{L,i}^M(P_L) &= R_{L,i-1}(P_L); \quad P_L < p, \\ R_{L,i}^M(P_L) &= R_{L,i}(P_L); \quad P_L \geq p. \end{aligned} \qquad \text{(D-16)}$$

Here are: $R_{L,i}^M(P_L)$ the modified probability function (load duration curve) after the insertion of observed hydroelectric plant, i index of the plant which is in case of pump-storage plants changed to r; p the value of the load above of which the commitment of the i-th hydroelectric plant is necessary for the given variable generation $W_{hv,i}$.

For the determined generation $W_{hv,i}$ in the peak-load area it is valid:

$$W_{hm,i} = W_{hv,i} = T\int_p^{P_{L,\max}} \left[R_{L,i-1}(P_L) - R_{L,i}(P_L) \right] \mathrm{d}P_L. \qquad \text{(D-17)}$$

If the possible (variable) energy is given as a probability distribution, the transition between the curves $R_{L,i-1}(P_L)$ and $R_{L,i}(P_L)$ is continuous, in accordance with the modified probability function (load duration curve) after the insertion of the observed hydroelectric plant:

$$\begin{aligned} R_{L,i}^M(P_L) &= R_{L,i-1}(P_L) \cdot \left[1 - F^*(W_{hv,i}) \right] + \\ &+ R_{L,i}(P_L) \cdot F^*(W_{hv,i}). \end{aligned} \qquad \text{(D-18)}$$

Here one can see that the function $R_{L,i}^M(P_L)$ is obtained by weighting of the curves $R_{L,i-1}(P_L)$ and $R_{L,i}(P_L)$ with the corresponding probabilities in the peak-load area.

The expected value of possible (variable) generation of the i-th hydroelectric plant $E(W_{hv,i})$ is determined as a difference of expected energy contents of the curves $R_{L,i-1}(P_L)$ and $R_{L,i}^M(P_L)$:

$$\begin{aligned} E(W_{hm,i}) &= E(W_{hv,i}) = E(W_{L,i-1}) - E(W_{L,s}^M) = \\ &= T\int_0^{P_{L,\max}} \left[R_{L,i-1}(P_L) - R_{L,i}^M(P_L) \right] \mathrm{d}P_L. \end{aligned} \qquad \text{(D-19)}$$

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