

## Fuzzy Linguistic Optimization on Multi-Attribute Machining

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### Abstract

Most existing multi-attribute optimization researches for the modern CNC (computer numerical control) turning industry were either accomplished within certain manufacturing circumstances, or achieved through numerous equipment operations. Therefore, a general deduction optimization scheme proposed is deemed to be necessary for the industry.

In this paper, four parameters (cutting depth, feed rate, speed, tool nose runoff) with three levels (low, medium, high) are considered to optimize the multi-attribute (surface roughness, tool wear, and material removal rate) finish turning. Through FAHP (Fuzzy Analytic Hierarchy Process) with eighty intervals for each attribute, the weight of each attribute is evaluated from the paired comparison matrix constructed by the expert judgment. Additionally, twenty-seven fuzzy control rules using trapezoid membership function with respective to seventeen linguistic grades for each attribute are constructed. Considering thirty input and eighty output intervals, the defuzzification using center of gravity is thus completed.

The TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) is moreover utilized to integrate and evaluate the multiple machining attributes for the Taguchi experiment, and thus the optimum general deduction parameters can then be received. The confirmation experiment for optimum general deduction parameters is furthermore performed on an ECOCA-3807 CNC lathe. It is shown that the attributes from the fuzzy linguistic optimization parameters are all significantly advanced comparing to those from benchmark. This paper not only proposes a general deduction optimization scheme using orthogonal array, but also contributes the satisfactory fuzzy linguistic approach for multiple CNC turning attributes with profound insight.

**Keywords:** computer numerical control, orthogonal array, fuzzy deduction, Technique for Order Preference by Similarity to Ideal Solution

### 1. Introduction

Machining operations have been the core of the manufacturing industry since the industrial revolution [28]. The existing multi-attribute optimization researches for CNC (computer numerical controlled) turning were either simulated within particular manufacturing

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circumstances [3, 8, 20, 22], or achieved through numerous frequent equipment operations [16, 23]. Nevertheless, these are regarded as computing simulations, and the applicability to real world industry is still uncertain. Therefore, a general deduction optimization scheme without equipment operations is deemed to be necessarily developed.

Surface roughness, tool life, and cutting force are commonly considered as manufacturing goals [7] for turning operations in many of the existing researches. It is also recognized that lighter cutting force often results to better surface roughness and tool life. This is why smaller cutting conditions conclude toward to be optimum [19] in many of the researches. As the flexibility and adaptability needs increased, the stability of modern CNC machines is now designed robust. Since the productivity concern becomes more critical than the cutting force in the industry, this paper proposes material removal rate (MRR) instead of the cutting force. The machining process on a CNC lathe is programmed by speed, feed rate, and cutting depth, which are frequently determined based on the job shop experiences. However, the machine performance and the product characteristics are not guaranteed to be acceptable. Therefore, the optimum turning conditions have to be accomplished. It is mentioned that the tool nose run-off will affect the performance of the machining process [29]. Therefore, the tool nose run-off is also selected as one of the control factors in this study.

Taguchi method, an experimental design method, has been widely applied to many industries. It can not only optimize quality characteristics through the setting of design parameters, but also reduce the sensitivity of the system performance to sources of variation [1, 8, 10, 13].

The AHP [20] method can be used to express experts' opinions, but cannot model human thinking. Therefore, FAHP [15, 26], a fuzzy extension of AHP, was developed to solve hierarchical imprecise problems. Therefore, the group decision by FAHP (Fuzzy Analytic Hierarchy Process) utilized in this study will assist to receive impersonal weights in matching the need from the modern industry. Besides, the Taguchi method adopts a set of orthogonal arrays to investigate the effect of parameters on specific quality characteristics to decide the optimum parameter combination. These kinds of arrays use a small number of experimental runs to analyze the quality effects of parameters as well as the optimum combination of parameters.

To achieve the general optimization, it is necessary to first describe the dynamic behavior of the system to be controlled. Because of the number, complexity and unclear, vague nature of the variables of the dynamic systems that may influence the decision maker's decision, fuzzy set theory is the most suitable solution [30, 31]. Fuzzy linguistic models permit the translation of verbal expressions into numerical ones [2]. Therefore, the input output relationship of the process can be described by the collection of fuzzy control rules involving linguistic variables rather than a complicated dynamic mathematical model.

With all the viewpoints above, this paper considers four parameters (cutting depth, feed rate, speed, tool nose runoff) with three levels (low, medium, high) to optimize the multi-attribute CNC finish turning. The fuzzy control rules using triangle membership function with respective to seventeen linguistic grades for each attribute are additionally constructed. The defuzzification is then quantified using center of gravity. For multiple machining attributes, the preference value from TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [12] is moreover introduced to Taguchi experiment, and thus the optimum general deduction parameters can then be received. This paper definitely proposes a fuzzy deduction general optimization approach and satisfactory fuzzy linguistic technique for improving multiple machining attributes in CNC turning with profound insight.

## 2. Methodology

In this paper, the linguistic variable quantification, multi-attribute integration, and parameter optimization for general deduction CNC turning operations are proposed using fuzzy set theory, TOPSIS (Technique for Order Preference by Similarity to Ideal Solution), and Taguchi method respectively. They are described as below.

**2.1. Fuzzy set theory**

Let  $X$  be an universe of discourse,  $\tilde{A}$  is a fuzzy subset of  $X$  if for all  $x \in X$ , there is a number  $\mu_{\tilde{A}}(x) \in [0,1]$  assigned to represent the membership of  $x$  to  $\tilde{A}$ , and  $\mu_{\tilde{A}}(x)$  is called the membership function of  $\tilde{A}$ . A trapezoid fuzzy number  $\tilde{A}$  can be defined by a triplet  $(a, b, c, d)$  (Fig. 1) [9]. The membership function is defined as

$$\mu_{\tilde{A}}(x:a,b,c,d) = \begin{cases} \frac{x-a}{b-a} & a < x \leq b \\ 1 & b < x \leq c \\ \frac{x-d}{c-d} & c < x \leq d \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

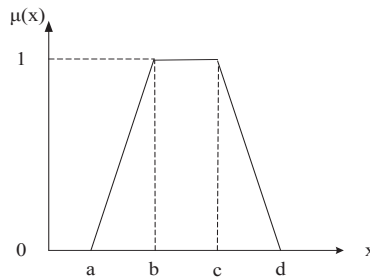


Figure 1. Trapezoid fuzzy numbers

In this paper, the two most important parameters for each attribute are primarily concluded through literature review. Additionally, twenty-seven fuzzy control rules for each attribute using trapezoid membership function with respective to seventeen linguistic grades will be constructed following IF-THEN rules.

To eliminate the computation, thirty input (parameter) and eighty output (attribute) intervals are considered to prepare the defuzzification. Through Cartesian product, the degree of membership for both input and output can thus be attained as

$$R = \text{Input} * \text{Output} \quad (2)$$

Here, “Input” describes the parameter, “Output” represents the attribute, and  $R$  denotes the fuzzy relation between the parameter and attribute.

The “OR” rules are then utilized for combining rules for maximum degree of membership as

$$\mu_{R1} + \mu_{R2} = \max\{\mu_{R1}, \mu_{R2}\} \quad (3)$$

where,  $R1$  and  $R2$  symbolize for the two rules.

In this study, the average value using center of gravity is determined to represent the fuzzy set as

$$F(x_i) = \frac{\sum_i x_i * \mu_{\tilde{A}}(x_i)}{\sum_i \mu_{\tilde{A}}(x_i)} \quad (4)$$

where  $F(x_i)$  is the final rating of activity,

$\mu_{\tilde{A}}(x_i)$  describes the membership function of fuzzy set  $\tilde{A}$ .

**2.2. Multi-attribute integration**

Hwang and Yoon [12] developed TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) to assess the alternatives before multiple-attribute decision making. TOPSIS considers simultaneously the distance to the ideal solution and negative ideal solution

regarding each alternative, and also selects the most relative closeness to the ideal solution as the best alternative [5].

When the alternative set for multi-attribute decision and evaluation attribute set are described as  $A = \{a_i \mid i = 1, 2, \dots, m\}$  and  $\{g = g_j \mid j = 1, 2, \dots, n\}$  respectively; the computational steps of TOPSIS can be expressed as

Step 1: This step involves a matrix based on all the information available that describes a material's attributes, and is called a "decision matrix". Each row of this matrix is allocated to one alternative, and each column to one attribute. The decision matrix can be stated as

$$D = \begin{matrix} & X_1 & X_2 & \cdot & X_j & X_n \\ A_1 & \begin{bmatrix} x_{11} & x_{12} & \cdot & x_{1j} & x_{1n} \\ x_{21} & x_{22} & \cdot & x_{2j} & x_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ A_i & \begin{bmatrix} x_{i1} & x_{i2} & \cdot & x_{ij} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ A_m & \begin{bmatrix} x_{m1} & x_{m2} & \cdot & x_{mj} & x_{mn} \end{bmatrix} \end{bmatrix} \end{matrix} \end{matrix} \quad (5)$$

Step 2: Obtain the normalized decision matrix  $r_{ij}$ . This can be represented as:

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^m x_{ij}^2}} \quad (6)$$

where  $r_{ij}$  represents the normalized performance of  $A_i$  with respect to attribute  $X_j$ .

Step 3: Assume that the weight of each attribute is  $\{w_j \mid j = 1, 2, \dots, n\}$ , the weighted normalized decision matrix  $V = [v_{ij}]$  can be found as

$$V = w_j \bullet r_{ij} \quad (7)$$

here,  $\sum_{j=1}^n w_j = 1$

Step 4: Develop the "ideal" (best) and "negative ideal" (worst) solutions in this step. The ideal and negative ideal solution can be expressed as:

$$A^+ = \left\{ \left( \max_i v_{ij} \mid j \in J \right), \left( \min_i v_{ij} \mid j \in J' \mid i = 1, 2, \dots, m \right) \right\} \quad (8)$$

$$= \{v_1^+, v_2^+, \dots, v_j^+, \dots, v_n^+\}$$

$$A^- = \left\{ \left( \min_i v_{ij} \mid j \in J \right), \left( \max_i v_{ij} \mid j \in J' \mid i = 1, 2, \dots, m \right) \right\} \quad (9)$$

$$= \{v_1^-, v_2^-, \dots, v_j^-, \dots, v_n^-\}$$

Step 5: Determine the distance measures. The separation of each alternative from the ideal one is given by n-dimensional Euclidean distance from the following equations:

$$S_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2}, \quad i = 1, 2, \dots, m \quad (10)$$

$$S_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}, \quad i = 1, 2, \dots, m \quad (11)$$

Step 6: The proximity of a particular alternative to the ideal solution is expressed in this step

as follows:

$$C_i^+ = \frac{S_i^-}{S_i^+ + S_i^-}, i = 1, 2, \dots, m; 0 \leq C_i^+ \leq 1 \quad (12)$$

Step 7: A set of alternatives is made in descending order according to the preference value indicating the most preferred and least preferred feasible solutions.

### 2.3. Taguchi method

The Taguchi method is a robust design method technique [24, 25], which provides a simple way to design an efficient and cost effective experiment. In order to efficiently reduce the numbers of conventional experimental tasks, the orthogonal array [4, 11] by using design parameters (control factors) in column and standard quantities (levels) in row is proposed and further adopted. The performance measure, signal-to-noise ratio (S/N) [14] proposed by Taguchi is used to obtain the optimal parameter combinations. The larger S/N means the relation to the quality will become better. The lower quality characteristic will be regarded as a better result when considering the smaller-the-best quality. The related S/N ratio is defined as

$$S/N = -10 \left( \log \sum_{i=1}^n \frac{y_i^2}{n} \right) \quad (\text{dB}) \quad (13)$$

where  $n$  is the number of experiments for each experimental set, and  $y_i$  expresses the quality characteristic at the  $i$ -th experiment. On the contrary, the larger quality characteristic will have better result t when considering the larger-the-best quality, therefore, by taking the inverse of quality characteristic into Eq. (13), the related S/N ratio can also be deduced and shown in Eq. (14).

$$S/N = -10 \left( \log \sum_{i=1}^n \frac{1/y_i^2}{n} \right) \quad (\text{dB}) \quad (14)$$

In this study, the preference value using TOPSIS for multiple CNC machining attributes is introduced to the Taguchi experiment as the S/N ratio. Therefore, it is judged as the quality of larger-the-best. In addition to the S/N ratio, a statistical analysis of variance (ANOVA) [6] can be employed to indicate the impact of process parameters. In this way, the optimal levels of process parameters can be estimated.

## 3. Research Design

Surface roughness, tool wear, and material removal rate (MRR) are considered major attributes in this paper. Four parameters with three levels are selected to optimize the multi-attribute finish turning based on the orthogonal array. Additionally, twenty-seven fuzzy control rules with respective to seventeen linguistic grades for each attribute are constructed. Considering thirty input and eighty output intervals, the defuzzifierion using center of gravity is thus completed. The TOPSIS is moreover utilized to integrate multiple machining attributes for the Taguchi experiment, and thus the optimum general deduction parameters can then be received.

### 3.1. Evaluation of multi-attribute weights

The hierarchy structure constructed using KJ method in this paper is shown as Figure 2. The “Zero” level denotes the evaluation of multiple weights in CNC turning. The “First” level represents the major attribute aspects. The “Second” level signifies the major parameter indexes.

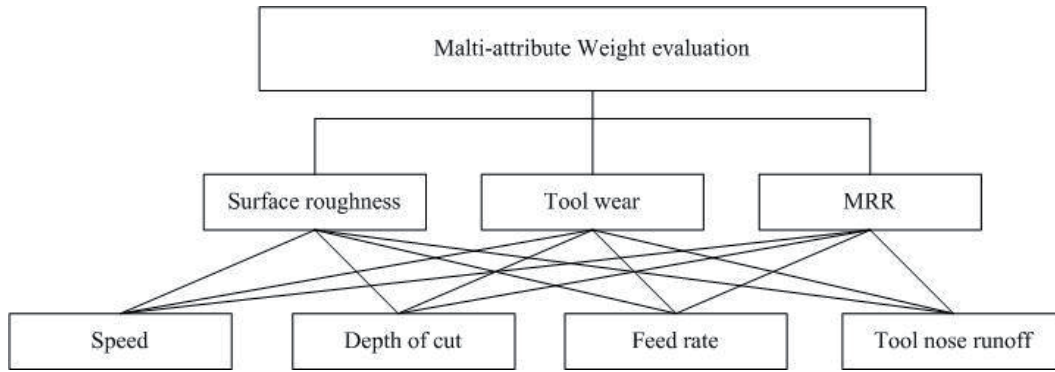


Figure 2. Hierarchy Structure

With the viewpoints on the three attributes, this paper inquired for 15 experts in the real-world industry. Through the examination on the CI (Consistency Index) and CR (Consistency Ratio), 12 from the 15 questionnaires were selected. The Excel program is proposed to evaluate the effects and weights of the parameters to the individual attribute. The trapezoid fuzzy numbers were then derived, and the defuzzification equations [27] were used to receive the fuzzy weights. The normalized overall attribute weights as well as the parameter weights under each attribute are thus shown in Table 1-Table 4.

	Fuzzy weight	Defuzzification weight	Normalized weight
Surface roughness	(0.330416,0.398186,0.459241)	0.3959	0.4583
Tool wear	(0.155931,0.157651,0.162013)	0.1585	0.1835
MRR	(0.235212,0.304062,0.389108)	0.3095	0.3582

Table 1. Overall attribute weights

	Fuzzy weight	Defuzzification weight	Normalized weight	Hierarchy connection
Speed	(0.294888,0.327293,0.360801)	0.3277	0.3703	0.169706
Cutting depth	(0.160830,0.184987,0.215550)	0.1871	0.2115	0.096917
Feed rate	(0.274640,0.293955,0.305562)	0.2914	0.3293	0.150918
Tool nose runoff	(0.063416,0.076082,0.096611)	0.0787	0.0889	0.040763

Table 2. Parameter weights under surface roughness

	Fuzzy weight	Defuzzification weight	Normalized weight	Hierarchy connection
Speed	(0.279897,0.300781,0.312073)	0.2976	0.3320	0.060914
Cutting depth	(0.173985,0.214716,0.262842)	0.2172	0.2423	0.044456
Feed rate	(0.252842,0.294177,0.343541)	0.2969	0.3311	0.060765
Tool nose runoff	(0.067514,0.081633,0.105336)	0.0848	0.0946	0.017364

Table 3. Parameter weights under tool wear

	Fuzzy weight	Defuzzifierion weight	Normalized weight	Hierarchy connection
Speed	(0.155205,0.186764,0.226945)	0.1896	0.2242	0.080308
Cutting depth	(0.259537,0.270312,0.273584)	0.2678	0.3166	0.113413
Feed rate	(0.247211,0.272425,0.301716)	0.2738	0.3237	0.115943
Tool nose runoff	(0.094499,0.112517,0.136798)	0.1146	0.1355	0.048533

Table 4. Parameter weights under material removal rate

**3.2. Construction of orthogonal array**

In this study, the four turning parameters (A-speed, B-cutting depth, C-feed rate and D-tool nose runoff ) [18] with three different levels (low, medium, and high) are constructed for the deduction optimization of machining operation. The three levels of speed, cutting depth, and feed rate are considered according to the machining handbook suggested by the tool manufacturer.

**3.3. Fuzzy control rules**

The twenty-seven fuzzy control rules with respective to seventeen linguistic grades for each attribute in this paper are constructed under the following considerations.

(1) Surface roughness

The seventeen linguistic grades for surface roughness are determined. From the existing literature [23], it is found that the surface roughness can be expressed as  $R_i(i = a, zD, t, p, q, 3z) = C_i V^{mi} f^{ni}$ , where the machining speed ( $V$ ) and feed rate ( $f$ ) are concluded as priority parameters to surface roughness. Therefore, the fuzzy rules can be described as shown in Table 5.

Rules	Speed	Feed rate	Deduction
1	Low	Low	medium
2	Low	Medium	large
3	Low	High	largest
4	Low	Low	medium
5	Low	Medium	large
6	Low	High	largest
7	Low	Low	medium
8	Low	Medium	large
9	Low	High	largest
10	Medium	Low	small
11	Medium	Medium	medium
12	Medium	High	large
13	Medium	Low	small
14	Medium	Medium	medium
15	Medium	High	large
16	Medium	Low	small
17	Medium	Medium	medium
18	Medium	High	large
19	High	Low	smallest
20	High	Medium	small



21	High	High	medium
22	High	Low	smallest
23	High	Medium	small
24	High	High	medium
25	High	Low	smallest
26	High	Medium	small
27	High	High	medium

Table 5. Fuzzy rules for surface roughness

## (2) Tool wear

Since less tool wear results better tool life, the tool life is used to describe the tool wear in this study. The modified Taylor equation  $TV^{1/n} f^{1/m} d^{1/l} = C'$  [22] is often utilized to express the tool life, where the machining speed ( $V$ ) and feed rate ( $f$ ) are found as major parameters to the tool wear. In this study, the machining speed ( $V$ ), feed rate ( $f$ ), and cutting depth ( $d$ ) are concluded as priority parameters to the tool wear. Therefore, the fuzzy rules can be described as shown in Table 6.

Rules	Speed	Cutting depth	Feed rate	Deduction
1	Low	Low	Low	smallest
2	Low	Low	Medium	extreme small
3	Low	Low	High	small
4	Low	Medium	Low	smaller
5	Low	Medium	Medium	smaller
6	Low	Medium	High	a little smaller
7	Low	High	Low	much smaller
8	Low	High	Medium	medium
9	Low	High	High	much larger
10	Medium	Low	Low	small
11	Medium	Low	Medium	smaller
12	Medium	Low	High	smaller
13	Medium	Medium	Low	much smaller
14	Medium	Medium	Medium	medium
15	Medium	Medium	High	much larger
16	Medium	High	Low	a little larger
17	Medium	High	Medium	larger
18	Medium	High	High	super small
19	High	Low	Low	much smaller
20	High	Low	Medium	medium
21	High	Low	High	much larger
22	High	Medium	Low	a little larger
23	High	Medium	Medium	larger
24	High	Medium	High	super large
25	High	High	Low	small
26	High	High	Medium	extreme large
27	High	High	High	largest

Table 6. Fuzzy rules for tool wear

## (3) Material removal rate

The material removal rate can be expressed as  $MRR = 1000fdV$ . As the experimental results [8], the surface speed ( $V$ ) has the least effect to the MRR. Therefore, the depth of



cut ( $d$ ) and feed rate ( $f$ ) are considered major parameters for MRR. The seventeen linguistic grades for tool wear are determined. Therefore, the fuzzy rules can be described as shown in Table 7.

Rules	Feed rate	Cutting depth	Speed	Deduction
1	Low	Low	Low	smallest
2	Low	Low	Medium	super small
3	Low	Low	High	small small
4	Low	Medium	Low	super small
5	Low	Medium	Medium	large
6	Low	Medium	High	much smaller
7	Low	High	Low	smaller
8	Low	High	Medium	a little smaller
9	Low	High	High	a little larger
10	Medium	Low	Low	small small
11	Medium	Low	Medium	smaller
12	Medium	Low	High	a little smaller
13	Medium	Medium	Low	much smaller
14	Medium	Medium	Medium	medium
15	Medium	Medium	High	much larger
16	Medium	High	Low	a little larger
17	Medium	High	Medium	larger
18	Medium	High	High	large large
19	High	Low	Low	a little smaller
20	High	Low	Medium	a little larger
21	High	Low	High	larger
22	High	Medium	Low	much larger
23	High	Medium	Medium	large
24	High	Medium	High	super large
25	High	High	Low	large large
26	High	High	Medium	extreme large
27	High	High	High	largest

Table 7. Fuzzy rules for material removal rate

### 3.4 Defuzzification

In this paper, the three parameter levels are selected based on the Taguchi experimental method, therefore, each triangle membership function is related to the peak point of its fuzzy area. Considering thirty input and eighty output intervals, the defuzzification of seventeen linguistic grades using center of gravity can then be completed.

Since two major parameters are considered for each machining attribute, the input (parameter) membership functions are regard as the intersection of two fuzzy sets, and the height of fuzzy set is considered as. The degree of membership for input (parameter) and output (attribute) can be described as shown in Table 8 and Table 9 respectively. Utilizing the average value of the fuzzy set to represent the entire set, we then have the quantified result for the fuzzy item of seventeen linguistic grades as shown in Table 10.

Linguistic level	Low	Medium	High
Range	[0,12.5,15]	[0,2.5,27.5,30]	[15,17.5,30]

Table 8. Trapezoid ranges of three linguistic levels for parameters

Linguistic level	Smallest	Extreme small	Super small	Small small	Small
Range	[0,2.5,5]	[0,2.5,7.5,10]	[5,7.5,12.5,15]	[10,12.5,17.5,20]	[15,17.5,22.5,25]
Smaller	Much smaller	A little smaller	Medium	A little larger	Much larger
[20,22.5,27.5,30]	[25,27.5,32.5,35]	[30,32.5,37.5,40]	[35,37.5,42.5,45]	[40,42.5,47.5,50]	[45,47.5,52.5,55]
Larger	Large	Large large	Super large	Extreme large	Largest
[50,52.5,57.5,60]	[55,57.5,62.5,65]	[60,62.5,67.5,70]	[65,67.5,72.5,75]	[70,72.5,77.5,80]	[75,77.5,80]

Table 9. Trapezoid ranges of seventeen linguistic levels for attributes

Linguistic level	Smallest	Extreme small	Super small	Small small	Small	Smaller	Much smaller	A little smaller
Defuzzification	1.666667	5	10	15	20	25	30	35
Medium	A little larger	Much larger	Larger	Large	Large large	Super large	Extreme large	Largest
40	45	50	55	60	65	70	75	78.33333

Table 10. Quantified results for linguistic results

#### 4. Results and Discussion

By considering the parameter combinations of the nine sets of experiment based on the  $L_9(3^4)$  orthogonal array, the quantified results from fuzzy deduction for the machining attributes are determined and shown as Table 11.

Attributes Experiment	Surface Roughness	Tool Wear	MRR
1	40	1.6667	1.6667
2	60	20	30
3	70.8854	45	65
4	60	30	20
5	40	55	55
6	20	35	45
7	40	65	45
8	9.1146	45	35
9	60	70	70

Table 11. Fuzzy Deduction Results

With the fuzzy deduction results, the original decision matrix can then be formulated as D, the transformation by Eq. (6), the normalized decision matrix is found as R

$$D = \begin{bmatrix} 40 & 1.6667 & 1.6667 \\ 60 & 20 & 30 \\ 70.8854 & 45 & 65 \\ 60 & 30 & 20 \\ 40 & 55 & 55 \\ 20 & 35 & 45 \\ 40 & 65 & 45 \\ 9.1146 & 45 & 35 \\ 60 & 70 & 70 \end{bmatrix} \quad R = \begin{bmatrix} 0.275320 & 0.012200 & 0.012179 \\ 0.412981 & 0.146146 & 0.219219 \\ 0.487905 & 0.329000 & 0.474974 \\ 0.413000 & 0.219000 & 0.146000 \\ 0.275320 & 0.401901 & 0.401901 \\ 0.137660 & 0.255755 & 0.328828 \\ 0.275320 & 0.474974 & 0.328828 \\ 0.062736 & 0.328828 & 0.255755 \\ 0.412981 & 0.511511 & 0.511511 \end{bmatrix}$$

The weighted decision matrix is then found as

$$V = W \times R = \begin{bmatrix} 0.4583 & 0.1835 & 0.3582 \end{bmatrix} \times \begin{bmatrix} 0.275320 & 0.012200 & 0.012179 \\ 0.412981 & 0.146146 & 0.219219 \\ 0.487905 & 0.329000 & 0.474974 \\ 0.413000 & 0.219000 & 0.146000 \\ 0.275320 & 0.401901 & 0.401901 \\ 0.137660 & 0.255755 & 0.328828 \\ 0.275320 & 0.474974 & 0.328828 \\ 0.062736 & 0.328828 & 0.255755 \\ 0.412981 & 0.511511 & 0.511511 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1262 & 0.0022 & 0.0044 \\ 0.1893 & 0.0268 & 0.0785 \\ 0.2236 & 0.0603 & 0.1701 \\ 0.1893 & 0.0402 & 0.0523 \\ 0.1262 & 0.0737 & 0.1440 \\ 0.0631 & 0.0469 & 0.1178 \\ 0.1262 & 0.0872 & 0.1178 \\ 0.0288 & 0.0603 & 0.0916 \\ 0.1893 & 0.0939 & 0.1832 \end{bmatrix}$$

After determining the ideal and negative ideal solution, the separation of each alternative from the ideal and negative ideal solution can then be achieved. Therefore, the closeness to the ideal solution and the preference value (Table 12) are derived for each experiment in the orthogonal array.

Experiment	Preference value
1	0.700666
2	0.665754
3	0.574367
4	0.508084
5	0.496200
6	0.453552
7	0.396507
8	0.353598
9	0.274613

Table 12. Preference value

Introducing the preference value as the signal to noise ratio (S/N) for multiple machining attributes for larger-the-best expectation, the results of factor responses are calculated and listed in Table 13. The mean effects for S/N ratios are then drawn by MINITAB 14 and shown as Figure 3. Therefore, the optimum fuzzy deduction multi-attribute turning parameters are found to be A ( Medium ) , B ( High ) , C ( Medium ) , and D ( Medium ) .

Parameter \ Level	A	B	C	D
Low	0.4012	0.3931	0.5876	0.4890
Medium	0.5165	0.5312	0.3748	0.5208
High	0.5567	0.5501	0.5120	0.4646
Delta	0.1555	0.1571	0.2128	0.0561
Rank	3	2	1	4

Table 13. Result of factor responses

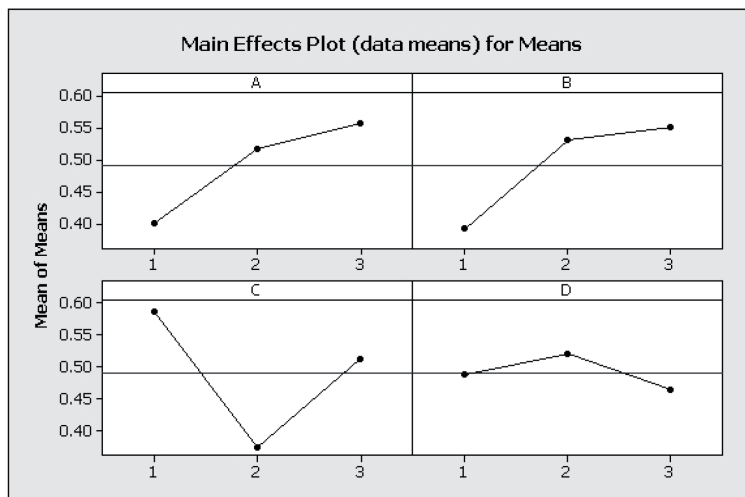


Figure 3. Plot of main effects

## 5. Confirmation Experiment

The finishing diameter turning operation of S45C ( $\phi 45\text{mm} \times 250\text{mm}$ ) work piece on an ECOCA-3807 CNC lathe is arranged for the experiment. The TOSHIBA WTJNR2020K16 tool holder with MITSUBISHI NX2525 insert is utilized as the cutting tool. The four turning parameters (speed, cutting depth, feed rate, and tool nose runoff) with three different levels (low, medium, and high) (Table 14) are experimentally distinguished for the machining operation on the basis of  $L_9(3^4)$  orthogonal array. In Table 14, the three levels of speed, cutting depth, and feed rate are identified from the machining handbook suggested by the tool manufacturer. The tool nose runoff is positioned by using different shims located under the tool holder and determined by measuring the tip after face turned the work piece. When the tool nose is set approximately 0.1mm higher (lower) than the center of the work piece, it is regard as “High (Low)”. When the tool nose is set within  $\pm 0.03\text{mm}$ , it is considered as “Medium”.

Level \ Parameter	High	Medium	Low
A: speed (m/min)	250	200	150
B: cutting depth (mm)	3	2	1
C: feed rate (mm/rev)	0.4	0.3	0.2
D: tool nose runoff (mm)	0.1	± 0.03	-0.1

Table 14. Parameters and levels

The surface roughness ( $R_a$ ) of machined work pieces are measured on the MITSUTOYO SURFTEST at three different sections of 40mm, 80mm, and 120mm from the face, therefore, the average data are received as the attribute of surface roughness. The tool wearing length  $V_{B2}$  (mm) in Fig. 4 is selected and scaled on the 3D SONY COLOR VIDEO electronic camera. To reduce the costly and time-consuming experiments, this study employs the tool wear ratio (tool wear length per unit material removal volume) instead of the tool life to demonstrate the tool wear status of turning under specific parameter combination. The tool wearing length is then divided by the volume of material removed as the tool wear ration ( $\text{mm}^{-2}$ ), which is utilized as the indicator of tool wear in this study. And, the MRR ( $\text{mm}^3/\text{min}$ ) is calculated using  $MRR = 1000fdV$ . Here;  $f$  (mm per revolution) denotes the feed rate,  $d$  (mm) describes the cutting depth, and  $V$  (m/min) presents the surface speed of the turning operation.

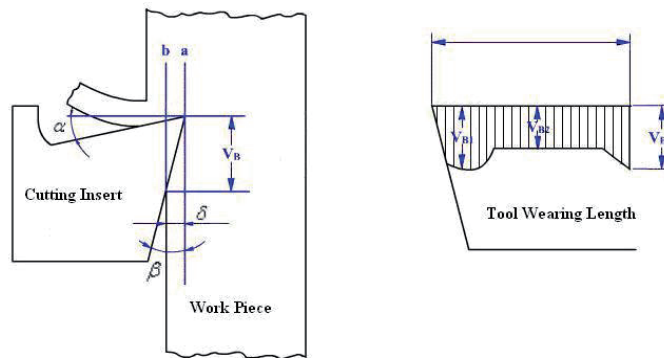


Figure 4. Tool wear length

To verify the applicability of the optimum result achieved by our proposed general multi-attribute optimization technique, the machining operations under both fuzzy Linguistic optimization parameters and benchmark parameters; A (medium), B (medium), C (medium), D (medium), which are often introduced into the confirmation experiment in many of the studies [23, 17] for comparison to the optimum parameters, are performed on the CNC lathe. The machined results are concluded and listed in Table 6. From Table 15, it is observed that the surface roughness, tool wear ratio, and MRR under fuzzy deduction parameters are significantly improved by 55.23%, 22.83% and 37.5% respectively, and the overall multi-attribute preference value is also improved by 31.6% from the benchmark parameters. It is shown that our proposed general deduction optimization technique can really advance the multiple machining attributes without compromise.

	Surface roughness	Tool wear ratio	MRR	Preference order
Fuzzy Deduction	0.4133 $\mu\text{m}$	3.38 E-07 $\text{mm}^{-2}$	0.0075 $\text{mm}^3/\text{min}$	0.843219
Benchmark	0.9233 $\mu\text{m}$	4.38 E-07 $\text{mm}^{-2}$	0.012 $\text{mm}^3/\text{min}$	0.640723

Table 15. Confirmation results

## 6. Concluding Remarks

In this paper, the fuzzy linguistic scheme was proposed and applied to achieve the optimum CNC finish turning parameters under the considerations of multiple attributes. A confirmation experiment of the optimum general deduction parameters was conducted to indicate the effectiveness of the proposed fuzzy Linguistic optimization method. Through the confirmation test for the proposed method, the experimental results validate the potency that all the attributes can be greatly advanced from our fuzzy Linguistic optimization technique. The considered attributes in the general deduction optimization are found valuable to be possibly extended for the real-world machining industry.

Parameter optimization is a hard-solving issue because of the interactions between parameters. This paper not only proposes a fuzzy deduction general optimization approach using orthogonal array, but also contributes the satisfactory fuzzy linguistic technique for improving multiple machining performances in CNC turning with profound insight. The competition of manufacturing industry will then be economically excited through the proposed development in this study.

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