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OPTIMISATION OF DECAY FACTOR IN TIME WEIGHTED (BRW) SIMULATION: IMPLICATIONS FOR VAR PERFORMANCE IN MEDITERRANEAN COUNTRIES

ABSTRACT

In this paper we propose an optimisation approach to determining the optimal decay factor in time weighted (BRW) simulation. Testing of BRW simulation with different decay factors and competing VaR models is performed on a sample of nine Mediterranean countries, over a four year period that includes the ongoing financial crisis. After optimisation the BRW simulation is among the best performing tested VaR models, second only to EVT approaches. Optimising the decay factor in regards to Lopez function results in decay factor estimates that are higher than usually employed 0.97 and 0.99. The optimal decay factors are stable over time and provide significantly better backtesting results than the standard assumptions.

Key words: *Risk management, Value at Risk, time weighted (BRW) simulation, optimisation, decay factor, Mediterranean*

1. INTRODUCTION

Although we now have at our disposal advanced VaR estimation techniques such as conditional extreme value models, there exists a need for some less sophisticated, computationally less time consuming and costly VaR models. Such models are in demand by less conservative investors or when serving as a quick approximation to the true level of risk an investor is facing. The existing approaches to estimating market risk for a portfolio of securities can be divided into three groups: fully parametric methods based on an econometric model for volatility dynamics and the assumption of conditional normality e.g. RiskMetrics and GARCH family of models; non-parametric models; and models based on extreme value theory (EVT). The nonparametric approach represents the most widely used and simplest method of calculating VaR. The main representative in this group of models is the historical simulation. The whole concept is built on the premise that potential changes in the risk factors are identical to the observed changes in the risk factors over a historical period i.e. that history regularly repeats itself. Modelling the risk factors underlying the changes in portfolio value significantly lowers the computational time since the number of relevant risk factors is considerably smaller than the number of financial instruments in the portfolio. Historical simulation assumes that the

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historically observed factor changes used in the simulation are taken from independent and identical distributions (IID) which are the same as the distribution applicable to the forecasts. The main strength of the historical simulation is that it, ex ante, does not presume any specific distribution of the data. Hendricks (1996) used simulated spot foreign exchange portfolios to show that with departures from normality historical simulation provided good estimates of the 99th percentile. When using historical simulation a trade-off is made between long observation periods which potentially violate the assumption of IID and short observation periods which reduce the precision of the estimate. A more realistic setting which violates the IID assumption would be that returns from the recent past better represent today portfolio's risk than returns from the more distant past. Based on this setting in the paper from 1998 Boudoukh, Richardson, and Whitelaw, BRW hereafter, introduced a generalization of the historical simulation which assigns a relatively higher amount of probability to returns from the more recent past. In practical application this modification makes a significant difference in forecasting performance (see e.g. Boudoukh, Richardson and Whitelaw, 1998, Pritsker, 2001, Žiković 2006). Boudoukh, Richardson and Whitelaw (1998) test the performance of BRW simulation on USD/DEM exchange rate, spot oil prices and S&P500 index. They find that it performs better than the parametric models and the historical simulation and at the same time produces independent VaR errors. Žiković (2006) found that BRW simulation with decay factor set to 0.99 is superior to historical simulation for a range of confidence levels in small and illiquid markets of EU candidate states. The most comprehensive study analysing the behaviour and characteristics of BRW and historical simulation can be found in Pritsker (2001). Pritsker found that BRW and historical simulation adjust slowly to changes in the true level of risk. He concludes that correlation of the VaR estimates with the true VaR is fairly high for the BRW simulation in contrast to the historical simulation. BRW model moves with the true VaR in the long run but is slow to respond promptly to changes in the level of risk. As a result, VaR estimates based on historical simulation and BRW are not very accurate. Apart from the mentioned papers, BRW simulation is not extensively used in the mainstream VaR literature. What is even more interesting is that after the original paper from 1998 we could not find any papers calculating VaR with different decay factors from the values originally suggested.

The goal of this paper is to present an optimisation procedure to determining the optimal decay factor for BRW simulation and explore the benefits of such an optimisation on VaR forecasts. Contributions of this paper are several: development of an optimisation approach to estimation of optimal decay factor in BRW simulation, analysis of the stability of the optimal decay factors and identification of the benefits to VaR estimation from the optimisation of the decay factor. VaR models that are analyzed are: time weighted (BRW) simulation with different decay factors, parametric GARCH model with GARCH specification and distribution that has the highest Akaike information criterion (AIC) value, unconditional EVT approach using Generalized Pareto distribution (GPD) (see Longin, 2000) and conditional quantile EVT-GARCH approach (McNeil, Frey, 2000). We test the optimisation procedure and its benefits on a sample of nine stock indexes from EuroMed region. We analyse the following stock market indexes (France – CAC, Italy – MIB 30, Spain – IBEX, Greece – FTASE, Turkey – XU 100, Egypt – CASE, Croatia – CROBEX, Malta – MALTEX, Morocco – MOSEMDX). The analysed group of stock indexes is very heterogeneous comprising stock indexes from developed countries such as France, Italy and Spain as well as emerging markets such as Turkey, Egypt and Morocco.

The rest of the paper is organised as follows: in section 2 of the paper, the characteristics of time weighted (BRW) historical simulation approach to measuring VaR are discussed. In section

3 we present the optimisation procedure for obtaining optimal decay factor values and section 4 discusses the optimisation results. Section 5, analyses and compares the performance of optimal decay factor BRW model with other VaR models on a sample of nine stock indexes from Mediterranean countries at 99% confidence level. The final section summarizes the conclusions.

2. TIME WEIGHTED (BRW) HISTORICAL SIMULATION

Historical simulation (HS VaR) drastically simplifies the procedure for computing VaR, since it does not make any direct distributional assumption about portfolio returns. Due to its simplicity and speed investors often rely on VaR figures obtained by historical simulations. Under the historical simulation approach the value of VaR is calculated as the 1*00cl'th* percentile or the *(T+1)cl'th* order statistic of the set of portfolio returns. The time series of historical portfolio returns is constructed just by using the current portfolio holdings and historical asset returns.

Historical simulation VaR can than be expressed as:

$$HS - VaR_{T+1|T}^{cl} \equiv r_w((T+1)cl) \qquad (1)$$

where $r_w((T+1)cl)$ is taken from the set of ordered portfolio returns $\{r_w(1), r_w(2), ..., r_w(T)\}$. If *(T+1)cl* is not an integer value then the two adjacent observations can be interpolated to calculate the VaR. Historical simulation has a number of shortcomings, which have been well recorded (see Pritsker, 2001). Perhaps most importantly, historical simulation does not properly incorporate conditionality into the VaR forecasting framework. The only source of dynamics in the historical simulation comes from the movement of the observation window with the passing of time. Unfortunately, in practice this source of conditionality is minor. Another shortcoming of the historical simulation is that it assigns equal probability weight of *1/N* to each observation. This means that the historical simulation estimate of a specific confidence level (*cl*) corresponds to the *N(1-cl)* lowest return in the *N* period rolling sample. Because a crash is the lowest return in the *N* period sample, the *N(1-cl)* lowest return after the crash, turns out to be the *(N(1-cl)-1)* lowest return before the crash. If the *N(1-cl)* and *(N(1-cl)-1)* lowest returns happen to be very close in magnitude, the crash actually has almost no impact on the historical simulation estimate of VaR. From the equation for historical simulation it can be seen that HS VaR changes significantly only if the observations around the order statistic $r_w((T+1)cl)$ change significantly. Although historical simulation makes no explicit assumptions about the distribution of portfolio returns, an implicit assumption is hidden behind the procedure: the distribution of portfolio returns doesn't change within the window. From this implicit assumption several problems may arise in using this method in practice. From the assumption that all the returns within the observation window used in historical simulation have the same distribution, it follows that all the returns of the time series also have the same distribution: if $y_{t\text{-window}},...,y_t$ and $y_{t+1\text{-window}},...,y_{t+1}$ are IID, then also y_{t+1} and $y_{t\text{-window}}$ has to be IID, by the transitive property. Forecasts of historical simulation VaR are meaningful only if the historical data used in the calculations have the same distribution. Another problem connected with the historical simulation is the fact that for the empirical quantile estimator to be consistent, the size of observation window must go to infinity. The length of the window must satisfy two contradictory properties: it must be large enough, in order to make statistical inference significant, and it must short enough, to avoid the risk of taking observations outside of the current volatility cluster. Clearly, there is no easy solution to this problem. If the market is moving from a period of low volatility to a period of high volatility, VaR forecasts based on the historical simulation will under predict the true risk of a position since it will take some time before the observations from the low volatility period leave the observation window. Finally, VaR forecasts based on historical simulation may present predictable jumps, due to the discreteness of extreme returns. If VaR of a portfolio is computed using a rolling window of *N* days and today's return is a large negative number, it is easy to predict that the VaR estimate will jump upward, because of today's observation. The same effect (reversed) will reappear exactly after *N* days, when the large observation drops out of the observation window.

A more realistic setting which violates the IID assumption assumes that the returns from the recent past better represent today portfolio's risk than returns from the more distant past. Based on this setting in the paper from 1998 Boudoukh, Richardson, and Whitelaw introduced a generalization of the historical simulation which assigns a relatively higher amount of probability to returns from the more recent past. The BRW approach combines exponential smoothing and historical simulation, by applying exponentially declining probability weights to past returns of the portfolio. After the probability weights are assigned, VaR is calculated from the empirical cumulative distribution function weighted by the modified probability weights. Historical simulation method can be considered as a special case of the more general BRW model in which the decay factor (λ) is set equal to 1. Under the BRW approach, the most recent return receives probability weight of just over 1% for $\lambda = 0.99$ and a weight of over 3% for $\lambda = 0.97$. In both cases, this means that if the most recent observation is the worst loss of the *N* days, it automatically becomes the VaR estimate at 1% confidence level. The BRW method appears to remedy one of the main problems of historical simulation since very large losses are immediately reflected in VaR forecasts. The simplest way to implement BRW approach is to construct a history of *N* hypothetical returns that the portfolio would have earned if held for each of the previous *N* days, $r_{t-1},\ldots, r_{t-N}$ and then assign exponentially declining probability weights $w_{t-1},\ldots, w_{t-N}$ to the return series[1]. Given the probability weights, VaR at the specific confidence level can be approximated from $G(.;\, t;N)$, the empirical cumulative distribution function of r based on the return observations $r_{t-1},\ldots, r_{t-N}$.

$$G(x;t,N) = \sum_{i=1}^{N} 1_{\{r_{t-i} \le x\}} w_{t-i} \tag{2}$$

Because the empirical cumulative distribution function, unless smoothed, for example via kernel smoothing, is discrete, a VaR figure at the *cl* confidence level will typically not correspond to a particular return from the return history. Instead, the BRW solution for VaR at the specific confidence level can be between a return that has a cumulative distribution that is less than *cl*, and one that has a cumulative distribution that is higher than *cl*. These returns can be used as estimates of the BRW VaR model at specific confidence level. The estimate that understates VaR at the *cl* percent confidence level (upper limit) is given by Pritsker (2001):

$$BRW^{u}(t \mid \lambda, N, cl) = \inf(r \in \{r_{t-1},\ldots r_{t-1-N}\} \mid G(r;t,N) \ge cl) \tag{3}$$

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[1] The weights sum to 1 and are exponentially declining at rate λ $(0 < \lambda \le 1)$

$$\sum_{i=1}^{N} w_{t-i} = 1$$

$$w_{t-i-1} = \lambda w_{t-i}$$

and the estimator of lower limit is given by:

$$BRW^{o}(t \mid \lambda, N, cl) = \sup(r \in \{r_{t-1}, \ldots r_{t-1-N}\} \mid G(r; t, N) \leq cl) \tag{4}$$

where λ is the exponential weight factor, N is the length of the history of returns used to compute VaR, and cl is the VaR confidence level. $BRW^{u}(t \mid \lambda, N, cl)$ is the lowest return of the N observations whose empirical cumulative probability is greater than cl, and $BRW^{o}(t \mid \lambda, N, cl)$ is the highest return whose empirical cumulative probability is less than cl. The main issue in evaluation of BRW VaR is the extent to which VaR forecasts based on the BRW method respond to changes in the underlying risk factors. It is important to know under what circumstances risk estimates increase when using the $BRW^{u}(t \mid \lambda, N, cl)$ estimator. The result is provided in the following proposition:

Proposition: If $r_t > BRW^{u}(t, \lambda, N)$ then $BRW^{u}(t+1, \lambda, N) \geq BRW^{u}(t, \lambda, N)$. (5)

Proof:

When BRW VaR is estimated for returns during time period *t+1*, the return at time *t−N* is dropped from the sample, the return at time *t* receives weight $\frac{1-\lambda}{1-\lambda^{N}}$ and the weight on other returns is λ times their earlier values. Consequently, *r(cl)* is defined as:

$$r(cl) = \{r_{t-i}, i = 1, \ldots N \mid G(r_{t-1}; t, N) \leq cl\} \tag{6}$$

To verify this proposition, it suffices to examine how much probability weight the VaR estimate at time *t+1* places below $BRW^{u}(t, \lambda, N)$, (see Žiković, 2006):

Case 1: $r_{t-N} \notin r(cl)$ - in this case, since by assumption, $r_t \notin r(cl)$ then:

$G(BRW^{u}(t, \lambda, N); t+1, \lambda, N) < \lambda G(BRW^{u}(t, \lambda, N))$. Therefore, (7)

$$BRW^{u}(t+1, \lambda, N) = \inf(r \in \{r_t, \ldots r_{t-1-N}\} \mid G(r; t+1, \lambda, N) \geq cl) \geq BRW^{u}(t, \lambda, N) \tag{8}$$

Case 2: $r_{t-N} \in r(cl)$ - in this case, since $r_t \in r(cl)$ by assumption, then:

$$G(BRW^{o}(t, \lambda, N); t+1, \lambda, N) < \lambda G(BRW^{o}(t, \lambda, N)). \tag{9}$$

Therefore:

$$BRW^{o}(t+1, \lambda, N) = \sup(r \in \{r_t, \ldots r_{t-1-N}\} \mid G(r; t+1, \lambda, N) \leq cl) \leq BRW^{o}(t, \lambda, N) \tag{10}$$

The proposition shows that when losses at time *t* are bounded below the BRW VaR estimate at time *t*, the BRW VaR estimate for time *t+1* will indicate that risk at time *t+1* is no greater than it was at time *t*. To understand the importance of this, it suffices to examine the case when today's VaR estimate for tomorrow's return is conditionally correct, but since risk changes with returns, tomorrow's return will influence risk for the day after tomorrow. Under these circumstances, one might wonder: what is the probability that a VaR estimate that is correct today will increase tomorrow? The answer provided by the proposition is that tomorrow's VaR estimate will not increase with probability *1−cl*. For example, if *cl* is equal to 1%, then a VaR estimate that is correct today will not increase tomorrow with probability 99%. Although the BRW approach suffers from the explained logical inconsistency, this approach still represents a significant improvement over the historical simulation, since it drastically simplifies the assumptions needed in the parametric models and it incorporates a more flexible specification than the historical simulation. BRW quantile estimator can be expressed as:

$$\hat{q}_{t+1,cl} = \sum_{j=t-N+1}^{t} y_j I\left(\sum_{i=1}^{N} f_i(\lambda;N) I(y_{t+1-i} \le y_j) = cl\right) \tag{11}$$

where $f_i(\lambda;N)$ are the weights associated with return y_i and $I(\cdot)$ is the indicator function. If $f_i(\lambda;N) = 1/N$, BRW quantile estimator equals the historical simulation estimator. The main difference between BRW approach and historical simulation is in the specification of the quantile process. With historical simulation each return is given the same weight, while with the BRW approach returns have different weights, depending on how old the observations are. Strictly speaking, none of these models is completely nonparametric, since a parametric specification is proposed for the quantile. Boudoukh, Richardson and Whitelaw in their original paper set λ equal to 0.97 and 0.99, as in their framework no statistical method is available to estimate this unknown parameter. In the next section we present an optimisation approach to determining the optimal decay factor for the purpose of VaR estimation.

3. OPTIMISATION OF THE BRW DECAY FACTOR

The forecast evaluation approach to backtesting VaR models was suggested by Lopez (1998) and is motivated by the evaluation methods often used to rank the forecasts of macroeconomic models. This approach allows for ranking of different competing models, but does not give any formal statistical indication of model adequacy. In ranking them, it also allows to take account of any particular concerns one might have. For example, higher losses can be given greater weight because of greater concern about them. Furthermore, because they are not statistical tests, forecast evaluation does not suffer from the low power of standard tests such as the Kupiec test. This makes forecast evaluation approach very attractive for backtesting with the small data sets typically available in practice. The first input in a forecast evaluation is a set of paired observations of returns for each period and their associated VaR forecasts. The second input is a loss function that gives each observation a score depending on how the observed return compares to the VaR forecast. To implement forecast evaluation, it is necessary to specify the loss function. Lopez (1998) suggested a size-adjusted loss function:

$$C_t = \begin{cases} 1 + (L_t - VaR_t)^2 & \text{if} \quad L_t > VaR_t \\ 0 & \text{if} \quad L_t \le VaR_t \end{cases} \tag{12}$$

where L_t represents a loss and VaR_t calculated VaR values at time *t*. This loss function allows for the sizes of tail losses to influence the final rating of VaR model. VaR model that generates higher tail losses would generate higher values under this size adjusted loss function than a VaR model that generates lower tail losses, ceteris paribus. However, with this loss function, there is no longer a straightforward condition for the benchmark, and the benchmark has to be estimated by some other means. Under assumption that the observed returns are IID an empirical loss function and the value of the final score can be derived by repeating the operation a large number of times, and using the average final score as the estimate of the benchmark. However, if the VaR model is parametric, simpler and more direct approaches can be used to estimate the benchmark. For example, return data can be simulated under the assumption of a specific distributional form using Monte Carlo methods, and the average of final scores can be taken as the benchmark.

We propose the optimisation of the decay factor for BRW simulation with regards to minimising the Lopez size-adjusted function. The decay factor that minimizes the Lopez size adjusted function for a given time series is chosen as the optimal since it minimizes the deviation (positive or negative) between observed and expected VaR exceedances while taking into account the size of those exceedances. In this manner we are treating over conservative and inadequate VaR forecasts equally. Optimisation procedure can be written as:

$$\lambda_{opt} = \min_{\lambda} |C_t| \qquad C_t = \begin{cases} 1 + (L_t - \hat{q}_{t,cl})^2 & if \quad L_t > \hat{q}_{t,cl} \\ 0 & if \quad L_t \leq \hat{q}_{t,cl} \end{cases} \tag{13}$$

$$\hat{q}_{t,cl} = \sum_{j=t-N}^{t} y_j I\left(\sum\nolimits_{i=1}^{N} f_i(\lambda; N) I(y_{t-i} \leq y_j) = cl\right)$$

The optimisation procedure is straightforward. The proposed algorithm runs through decay factor values and at each step calculates the VaR and records the VaR performance in the backtesting period at the selected confidence level. After finishing its runs through decay factor values, having recorded VaR performance for each decay factor, it searches for the VaR model with the lowest Lopez size-adjusted value in absolute terms. The decay factor that was used in the VaR model with the lowest Lopez size-adjusted value is chosen as the optimal since it produces the lowest possible deviation from the realised level of risk i.e. number of exceedances and their size.

4. BRW DECAY FACTOR OPTIMISATION RESULTS

Based on presented optimisation procedure the obtained optimal values of decay factor for time weighted BRW simulation are calculated for the analysed stock indexes during the consecutive time periods of the latest 500, 1,000 and 1,500 days.

Table 1

Optimal decay factor values for tested indexes at 99% confidence level and different time frames

Period		CAC	IBEX	MIB 30	FTASE	XU 100	CASE	CROBEX	MALTEX	MOSEMDX
1500 days	**Optimal λ**	0,994	0,993	0,996	0,997	0,996	0,991	0,995	0,998	N/A
	Lopez score	1,18	1,22	2,18	7,22	2,34	6,45	6,32	2,15	N/A
	Average VaR %	3,39	3,08	3,05	3,27	5,41	4,34	3,84	2,27	N/A
	λ = 0.99									
	Lopez score	5,21	3,25	3,21	12,25	4,36	8,45	11,37	6,15	N/A
	Average VaR %	3,21	2,96	2,84	3,12	5,31	4,31	3,63	2,22	N/A
	λ = 0.97									
	Lopez score	22,30	19,32	17,25	20,31	13,48	20,55	17,46	15,19	N/A
	Average VaR %	2,75	2,61	2,43	2,86	4,83	3,79	3,10	1,93	N/A
1000 days	**Optimal λ**	0,995	0,993	0,996	0,996	0,998	0,991	0,995	0,998	0,991
	Lopez score	5,18	4,20	6,18	10,20	1,22	6,33	6,22	4,12	2,12
	Average VaR %	2,95	2,87	2,79	3,26	4,78	4,61	2,97	2,46	3,08
	λ = 0.99									
	Lopez score	8,20	6,22	8,21	12,23	4,20	8,33	11,24	7,12	3,12
	Average VaR %	2,83	2,80	2,67	3,22	4,88	4,57	2,85	2,46	3,04
	λ = 0.97									
	Lopez score	21,24	17,26	18,24	18,27	10,29	15,38	14,28	12,14	11,16
	Average VaR %	2,52	2,53	2,39	2,98	4,49	3,95	2,66	2,14	2,67
500 days	**Optimal λ**	0,994	0,993	0,996	0,996	0,996	0,996	0,995	0,996	0,991
	Lopez score	6,15	5,17	5,15	7,15	1,13	3,22	10,20	0,04	1,05
	Average VaR %	3,48	3,63	3,19	3,81	5,16	4,87	3,32	2,64	3,30
	λ = 0.99									
	Lopez score	7,18	6,20	6,16	9,17	2,13	6,24	11,22	4,04	1,06
	Average VaR %	3,45	3,57	3,19	3,91	5,33	4,23	3,25	2,41	3,26
	λ = 0.97									
	Lopez score	14,20	10,22	12,18	12,20	4,19	11,29	12,24	6,06	6,08
	Average VaR %	3,13	3,32	2,86	3,66	4,94	3,51	3,07	2,06	2,86

Lopez optimal decay factor values show consistency over different time windows with minimal changes in their values, with the exception of Egyptian CASE index jump from 1,000 to 500 days time window. There is economic justification in optimising decay factor for each time series since once calculated these values do not change very often, and even when they change they do so by a very small amount. This is a very useful characteristic which allows the optimisation procedure not to be performed daily but far less frequently, resulting in lower computational time and costs. More developed Mediterranean countries (France, Italy, Spain and Greece) have a very stable optimal decay factor ranging from 0.993 for Spain's IBEX index to 0.997 for Greek FTASE index. Situation is similar with emerging and developing economies so we cannot point to any significant difference in the optimal decay factor based on wealth, development or size of the stock market. The highest decay factor value (0.998) was found for the Maltin MALTEX index (1,500 and 1,000 day window) and Turkish XU 100 index (1,000 day window). The lowest value of decay factor (0.991) is found for Moroccan MOSEMDX index (1,000 and 500 day window) and Egyptian CASE index (1,500 and 1,000 day window). The decay factor values are rounded to three decimal places since we found that further refinements of decay factor did not yield any significant improvements. In all of the cases optimal decay values are between 0.99 and 1, which signals that using lower decay factors, such as proposed 0.97 and 0.99 might results in unreliable VaR forecasts.

Optimised BRW model can provide practitioners with far better results than the ones we grew accustom to expect from this model.

5. VAR BACKTESTING COMPARISON

To test whether there is any practical advantage in optimising the decay factor we test the performance of optimised BRW simulation versus the usually assumed decay factors of 0.97 and 0.99 as well as an GARCH and conditional GDP and unconditional EVT-GARCH approach. Data used in the analyses of VaR models is the daily log returns from analysed indexes from Mediterranean countries. The returns are collected from Bloomberg for the period 01.01.2000 - 12.11.2008. The calculated VaR figures are for a one-day ahead horizon and 99% confidence level. To secure the same out-of-the-sample VaR backtesting period for all of the tested indexes, the out-of-the-sample data sets are formed by taking out 1,500 of the latest observations from each index. The rest of the observations are used as presample observations needed for VaR starting values and volatility model calibration. The only exception is the MOSEMDX index which started in 2002 and the analysis for a 1,500 days time frame is still not possible. That is why backtesting results for this index are based on a 1,000 days period. All of the analysed VaR models are tested in several ways to determine their statistical characteristics and ability to adequately measure market risk in the analysed markets. The first test in the evaluation of VaR performance is the Kupiec test, a simple expansion of the failure rate, which is prescribed by Basel Committee on Banking Supervision as the test for regulatory acceptance of a VaR model (see Kupiec, 1995). The second test is the Christoffersen (IND) independence test which tests whether VaR exceedances are IID (see Christoffersen, Hahn, Inoue, 2001). Although the independence of the VaR errors is not required under the Basel 2 rules, in practice it is of vital importance. The dependence of the VaR errors is crucial for the stability of any financial institution since bunched VaR errors can erase the capital reserves much faster than the slight underestimation of risk.

Kupiec and Christoffersen independence (IND) test backtesting results, at 5% significance level, for tested VaR models at 99% confidence level are presented in tables 2 and 3.

Table 2

Kupiec test backtesting results at 99% confidence level, 5% significance level, period: 1,500 days up to 12.11.2008.*

	CAC	IBEX	MIB 30	FTASE	XU 100	CASE	CROBEX	MALTEX	MOSEMDX
BRWλ=0,97									
BRWλ=0,99	+	+	+		+				+
BRWλ=opt	+	+	+		+	+	+	+	+
GARCH RM							+	+	
EVT GARCH	+	+	+	+	+	+	+	+	+
GPD	+	+	+	+	+	+	+	+	+

* Grey areas mark VaR models that satisfied Kupiec backtesting criterion

Table 3

Christoffersen independence (IND) test backtesting results at 99% confidence level, 5% significance level, period: 1,500 days up to 12.11.2008.*

	CAC	IBEX	MIB 30	FTASE	XU 100	CASE	CROBEX	MALTEX	MOSEMDX
BRWλ=0,97	+	+	+	+	+	+			+
BRWλ=0,99	+	+	+	+					+
BRWλ=opt	+	+	+	+	+	+	+	+	+
GARCH RM	+	+	+	+	+	+	+	+	+
EVT GARCH	+	+	+	+	+	+	+	+	+
GPD	+	+	+	+	+	+	+	+	

* Grey areas mark VaR models that satisfied Christoffersen independence backtesting criterion

Tested GARCH, EGARCH and GJR-GARCH models with Gaussian, T, skewed T and GED distribution performed unsatisfactory in Mediterranean stock markets, both developed and developing, providing satisfactory results only for CROBEX and MALTEX index. Such weak performance of this widely used VaR model can be attributed to the fact that the time period under consideration includes the ongoing global financial crisis. Since we are using a sufficiently long backtesting period of 1.500 days (almost six years of daily data) global financial crisis should not be used as an excuse and investors should seriously rethink the safety of their VaR models. Based on the obtained results we can safely say that it should not to be used in the tested stock markets for the purpose of risk measurement at high quantiles. The test reveal an absolutely supreme performance of conditional and unconditional EVT models that satisfied both tests for all of the tested indexes, with the only exception of the unconditional GPD model failing the Christoffersen independence test for MOSEMDX index. VaR model performance for the most developed Mediterranean countries; France, Italy and Spain is identical, with BRW simulation (optimal and 0.99 decay factor) and EVT models satisfying both employed tests. For developing Mediterranean countries VaR performance is similar to the developed ones since only the optimal decay factor BRW simulation and EVT models passed the two tests. The only exception is the Greek FTASE index for which only EVT models forecasted the true level of risk. Overall, the results are very consistent in pointing to the conclusion that for the time period under consideration only EVT models (especially the conditional EVT-GARCH model) perform satisfactory for all the tested stock indexes, while other VaR models tend to underpredict the true level of risk. The backtesting shows that performance of the BRW simulation depends, to a very large extent, upon the choice of decay factor. The BRW simulation with decay factor of 0.97 performs poorly and is not an adequate risk measure in any of the tested markets. Decay factor of 0.99 shows considerable improvements but still fails for four out of nine indexes (FTASE, CASE, CROBEX and MALTEX). BRW simulation with individually optimised decay factor brings a significant improvement over the usually used 0.99 decay factor and fails only once, in the case of Greek FTASE index. This makes the optimised BRW simulation second only to EVT approaches.

In the tested sample the optimised BRW simulation proved superior to the parametric GARCH estimation, both in developed and developing markets. The reasons for such a good performance of the optimised BRW model can be attributed to the high decay factors, in the range between 0.99 and 1.00. When using the BRW simulation with such high decay factors the observation window becomes very long since no cut-off level exist as in the case of the historical simulation. In this manner the model has a very long history from which to form the time weighted empirical cumulative distribution function and produce robust VaR forecasts. On the other hand information is updated but also lost much faster in the GARCH setup. These characteristics can work in favour of the BRW simulation and against GARCH in a situation where there are sudden bursts of volatility lasting only a couple of day. After these short bursts the excess volatility fades away only to appear again suddenly. In such instances the GARCH model cannot correctly conclude whether it is in a state of increased market stress or not. Upon visual inspection of the analysed indexes we find that exactly this is the case, especially in the period of global financial crisis. In the described circumstances the lower speed with which the BRW model with high decay factor reacts to the changes actually works in favour of the model since it does not automatically start to decrease VaR forecasts due to the calm periods between the short bursts of volatility. In situations where there is a clearly visible shift between periods of high and low volatility GARCH is obviously a preferred method. In situations where it is not easy to conclude about the characteristics of a certain period since calm and volatile days interchange very suddenly we conclude that the BRW simulation should be preferred to GARCH estimation. The results show that making even small adjustments to decay factor for example 0.001 in case of CASE index makes the difference between an acceptable and unacceptable VaR model. This is a clear proof that optimisation of the decay factor makes a huge difference in judging the performance of the BRW simulation. It can be concluded that studies evaluating the performance of the BRW model are flawed if they do not in some way optimise the decay factor. Taking ad hoc values is certainly not a reliable way of testing the performance of any VaR model.

With regards to independence of VaR exceedances results of the Christoffersen independence test are much better but still some VaR models such as the GPD EVT and BRW model fail in some cases, meaning that their VaR errors are not IID i.e. they tend to cluster together which makes them completely unusable in these circumstances. Since EVT and the optimal decay factor BRW simulation models are the best performing models according to Kupiec and independence test it is useful to know which model gives the closest fit to the true level of risk. The results are presented in table 4.

Table 4
Lopez test ranking of competing VaR models at 99% confidence level, period 1,500 days up to 12.11.2008.*

	CAC	IBEX	MIB 30	FTASE	XU 100	CASE	CROBEX	MALTEX	MOSEMDX
BRWλ=0,97	19,25	18,28	15,20	17,28	13,48	17,49	15,38	16,19	12,16
BRWλ=0,99	3,16	3,22	3,21	9,21	6,37	8,45	11,33	7,16	4,13
BRWλ=opt	1,18	1,22	2,18	7,22	2,34	6,45	6,32	2,15	2,12
GARCH RM	12,20	15,19	17,18	8,10	12,39	9,36	2,22	-0,91	9,12
EVT GARCH	3,13	6,14	-1,92	-2,93	5,31	-10,87	-9,88	-14,00	3,06
GPD	-6,89	-9,89	-11,96	-9,93	-14,00	-11,89	-13,99	-10,98	-1,95

* Grey areas mark VaR models yielding lowest Lopez score i.e. smallest deviation from expected values

In case of CAC, IBEX, XU 100 and CASE index optimal decay factor BRW simulation has the lowest Lopez size adjusted score, making it, by this criterion, the best VaR model since it minimises the deviation between recorded and expected VaR failure rate. For MIB 30, FTASE and MOSEMDX index the EVT models were the best models with regards to Lopez score function. Parametric GARCH model was the best performing VaR model for CROBEX and MALTEX index. When looking at the Kupiec, independence and Lopez test performance of non-EVT models is far worse than reported by similar studies which can be attributed to increased market stress and occurrence of extreme loses that cannot be accounted for by classical VaR models. The magnitude of losses that occurred in these markets under the parametric models using normality assumption are expected to occur once in a thousand years and in the historical simulation models periods of such high volatility and extreme losses simply fell out of the observation sample. The only models which overpredict the true level of risk in most of the indexes are the EVT models. The Lopez test results show that although EVT, especially the conditional EVT-GARCH version is superior to the optimised BRW model often the optimised BRW model provides a closer fit to the true level neither under or overpredicting it.

6. CONCLUSION

In this paper we present an optimisation approach to determine the optimal decay factor for the BRW simulation based on minimising the deviation (positive or negative) between observed and expected VaR exceedances while taking into account the size of those exceedances. The optimal decay factors obtained in this manner show consistency over different time windows with minimal changes in their values, which gives economic justification to their optimisation for each stock index since once calculated these values do not change very often, or they do so by a very small amount. This is a very useful characteristic which allows the optimisation procedure to be performed far less frequently. The optimal decay values are similar, for both developed and developing Mediterranean economies, ranging between 0.991 and 0.998, so we cannot point to any significant difference in the optimal decay factor based on wealth, development or size of the stock market. For the time period under consideration only the EVT models perform satisfactory for all of the tested Mediterranean stock indexes, while other VaR models tend to underpredict the true level of risk. Performance of the BRW simulation depends, to a very large extent, upon the choice of the decay factor. The BRW simulation with the decay factor of 0.97 performs poorly and is not an adequate risk measure in any of the tested markets. The decay factor of 0.99 shows considerable improvements but still fails for four out of nine indexes. The BRW simulation with the individually optimised decay factor brings a significant improvement over the usually used 0.99 decay factor. In the tested sample the optimised BRW simulation proved superior to the parametric GARCH estimation, both in developed and developing markets. The reasons for such a good performance of the optimised BRW model can be attributed to the high decay factors, in the range between 0.99 and 1.00. When using the BRW simulation with such high decay factors the observation window becomes very long since no cut-off level exist as in the case of the historical simulation. In this manner the model has a very long history from which to form the time weighted empirical cumulative distribution function and produce robust VaR forecasts. On the other hand information is updated but also lost much faster in the GARCH setup. These characteristics can work in favour of the BRW simulation and against GARCH in a situation where there are sudden bursts of volatility lasting only a couple of day. Among the non-EVT VaR models the optimal decay factor BRW simulation is a preferable method and as such presents a viable alternative when it comes to VaR estimation. The Lopez test results show that although EVT, especially the conditional EVT-GARCH version is superior to the optimised BRW model often the optimised BRW model provides a closer fit to the true level neither under or overpredicting it.

Optimisation of the decay factor makes a huge difference in judging the performance of the BRW simulation. It can be concluded that studies evaluating the performance of the BRW model are flawed if they do not in some way optimise the decay factor. Taking ad hoc values is certainly not a reliable way of testing the performance of any VaR model. In this paper we suggest optimising the decay factor with regards to the Lopez size adjusted function but there is no reason why optimisation of the decay factor with some other target function could not yield even better results. This possibility represents an interesting opportunity for future research.

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OPTIMIZACIJA FAKTORA OPADANJA U VREMENSKI PONDERIRANOJ (BRW) SIMULACIJI: POSLJEDICE ZA IZRAČUN VAR-A U MEDITERANSKIM ZEMLJAMA

SAŽETAK

U radu se predlaže optimizacijski pristup određivanju optimalnog faktora opadanja u vremenski ponderiranoj (BRW) simulaciji. Testiranje uspješnosti BRW simulacije sa različitim faktorima opadanja u odnosu na široki raspon VaR modela provedeno je na uzorku od devet mediteranskih zemalja tijekom razdoblja od četiri godine, uključujući i razdoblje aktualne svjetske financijske krize. Rezultati testiranja pokazuju da nakon provedene optimizacije BRW simulacija je među najuspješnijim testiranim VaR modelima zaostajući jedino za modelima temeljenim na teoriji ekstremnih vrijednosti. Optimiziranje faktora opadanja u odnosu na Lopezovu funkciju rezultira faktorima opadanja koji su viši od uobičajeno korištenih vrijednosti 0.97 i 0.99. Dobiveni optimalni faktori opadanja su izrazito stabilni tijekom testiranog razdoblja te rezultiraju značajno boljim VaR prognozama.

Ključne riječi: *Upravljanje rizicima, rizična vrijednost, vremenski ponderirana (BRW) simulacija, optimizacija, faktor opadanja, Mediteran*