*Saša Žiković**
*Mario Pečarić***

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MODELLING EXTREME EVENTS: APPLICATION TO ZAGREB STOCK EXCHANGE

In the paper we analyse the performance of Value at Risk (VaR) models at extreme quantiles: 0.99, 0.995 and 0.999 for both long and short positions in Croatian, Zagreb stock exchange index - CROBEX. Backtesting shows that none of the usually employed VaR models correctly forecasts the risk during the ongoing global and domestic financial crisis. The only exceptions are the extreme value based models which correctly forecast the true level of upside and downside risk. We also investigate the closeness of fit of theoretical distributions to the extreme tails of CROBEX returns. Results show that generalised Pareto distribution, which has a sound theoretical foundation, provides the best fit to both tails of CROBEX returns. We find that distribution tails differ significantly, with the right tail having a higher tail index, indicative of more extreme events.

Key words: Extreme value theory, Value at Risk, Emerging markets, CROBEX, Zagreb stock exchange

1. Introduction

Statistical distributions have been used for a long time to describe the behaviour of financial returns; it is often assumed that these financial returns are normal-

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* S. Žiković, doc. dr. sc., Faculty of Economics University of Rijeka (e-mail: szikovic@efri.hr)

** M. Pečarić, prof. dr. sc., Faculty of Economics University of Split (e-mail: mario.pecaric@efst.hr)

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ly distributed. However, empirical research provides evidence that empirical distributions of financial returns often have fatter tails than implied by the normality assumption; thus, large financial returns are more likely than the normal distribution implies. Such extremes can have a devastating impact upon economic wealth and the stability of financial system. Empirical evidence suggests that current VaR estimates are inadequate since they do not model the tails of a portfolio's return distribution realistically and incorrectly assess the probability of extreme events. The danger is that such risk models are prone to fail just when they are needed the most – in large market moves, when largest losses occur. Estimation of risk associated with rare events with limited data is inevitably problematic, and these difficulties increase as the events concerned become rarer. The key to estimating the distribution of extreme events is the extreme value theorem (EVT), which governs the distribution of extreme values, and shows how this distribution looks like in the limit, as the sample size increases. It is important to be aware of the limitations implied by the adoption of the extreme value paradigm. EVT models are developed using asymptotic arguments, which should be kept in mind when applying them to finite samples. EVT models are derived under idealized circumstances, which need not be true for a process being modelled. McNeil (1998) studies the estimation of the tails of loss severity distributions and the estimation of the quantile risk measures for financial time series using extreme value theory. McNeil and Frey (2000) study the estimation of tail-related risk measures for heteroskedastic financial time series. Gencay et al. (2003) compare the performance of unconditional EVT to those of other methods like GARCH, VCV and Historical simulation. They find that GARCH and GPD models are preferable for most quantiles. Gencay and Selcuk (2004) use VCV, Historical simulation and unconditional EVT model to calculate and compare VaR estimates in emerging markets. While having mixed results at the usual 99% level EVT model is found to be more accurate at higher quantiles. Maghyereh, Al-Zoubi (2006) investigate performance of a range of models, including EVT, to estimate VaR in seven Middle East and North Africa countries. Unconditional EVT model was again among top ranking models but skewed-t APARCH model was found to perform better in some cases. Measuring of market risk on Croatian Zagreb Stock Exchange (ZSE) has not been as extensively studied. Žiković (2006) analyses the benefits of using time weighted (BRW) simulation and obtains much better results than by using plain historical simulation. Jurun et al. (2007) conclude that using assumption of heavy tailed distribution, such as Student's t-distribution in GARCH model, it is possible to forecast market risk much more precisely than under normality assumption. Žiković (2007a, b) tests a wide range of VaR models on transitional markets of 2004 and 2007 EU new member states as well as Croatia and concludes that traditional VaR models are incapable of providing adequate risk coverage specified under Basel 2.

The aim of this paper is, firstly, to identify the asymptotic distribution of extreme positive and negative daily returns for the Zagreb Stock Exchange (ZSE) index (CROBEX) over the period 2007 to 2010 by employing extreme value theory. Secondly, the aim is to investigate the relative performance of a wide array of VaR models on CROBEX index during the period of increased market stress, for both long and short trading positions. All of the above mentioned studies as well as the great majority of risk related studies in general measure VaR only for long trading positions. To the best of our knowledge this is the first paper that measures the risk for an emerging country taking into account both long and short trading positions. This is also the first study of asymptotic distribution of tails for the emerging country stock index. We also introduce extreme value theory to VaR calculation on Croatian financial market. Besides testing the performance of unconditional extreme value model on an emerging market which is not a novelty we introduce a conditional extreme value model and test its performance. The rest of the paper is organised as follows: Section 2 presents a theoretical background on extreme value theory and extreme value VaR estimation. Section 3 presents a description of the data and tail fitting results. In section 4 VaR backtesting results are presented and their implications discussed. Finally, section 5 summarizes our main findings.

2. Extreme value theory

Presuming n observations of P&L time series, if X is IID drawn from some unknown distribution $F(x) = P(X \leq x)$, estimating extreme value (EV) VaR posses a significant problem because the distribution $F(x)$ is unknown. Help comes from Fisher-Tippett theorem (1928), which can be considered to have the same status in EVT as the central limit theorem has in the study of sums. The theorem describes the limiting behaviour of appropriately normalised sample maxima. We denote the maximum of the first n observations by $M_n = max(X_1,\ldots, X_n)$. Assuming that we can find sequences of real numbers $a_n > 0$ and b_n such that $(M_n - b_n)/a_n$ the sequence of normalized maxima, converges in distribution:

$$P\left\{\left(M_n - b_n\right)/a_n \leq x\right\} = F^n(anx + b_n)\xrightarrow[n\to\infty]{} H(x) \tag{1}$$

for some non-degenerate distribution function $H(x)$. If this condition holds we say that F is in the maximum domain of attraction of H: $F \in MDA\ (H)$. It was shown by Fisher & Tippett (1928) that:

$F \in MDA(H) \Rightarrow H$ is of the type H_ξ for some ξ.

Thus, if we know that suitably normalized maxima converge in distribution, then the limit distribution must be an extreme value distribution. It shows that as *n* gets large the distribution of tail of *X* converges to the generalized extreme value distribution (GEV):

$$H_{\mu,\sigma,\xi}(x) = \begin{cases} \exp\left(-\left[1+\xi(x-\mu)/\sigma\right]^{-1/\xi}\right) & if \quad \xi \neq 0 \\ \exp\left(-e^{-(x-\mu)/\sigma}\right) & \quad\;\; \xi = 0 \end{cases} \tag{2}$$

where *x* satisfies the condition $1 + \xi(x-\mu)/\sigma > 0$. GEV distribution has three parameters: location parameter (μ), which is a measure of central tendency, scale parameter (σ), which is a measure of dispersion and tail index (ξ), which is a measure of the shape of the tail. The Fisher-Tippett theorem tells us that fitting of the GEV distribution should be done on data on sample maxima. Although this is not a problem when dealing with hydrology or meteorology it might present a serious problem when dealing with financial data. Using only sample maxima would lead to serious waste of information. Since there is only one maxima in any sample period we are disregarding all other extreme events and thus limiting our data set. For this reason the most widely accepted method of using EVT in finance is based on modelling the behaviour of extreme values above a high threshold. This method is usually named peaks over threshold approach (POT). POT approach extracts extremes from a sample by taking the exceedances over a predetermined threshold *u*. An exceedance of the threshold *u* occurs when a realization is higher than the threshold, $X_t > u$ for any *t* in $t = 1, 2, ..., n$. An excess over *u* is defined by $y = X_i - u$. Provided a high threshold *u*, the probability distribution of excess values of *X* over threshold *u* can be defined as:

$$F_u(y) = P\left(X - u \leq y \mid X > u\right) \tag{3}$$

which represents the probability that the value of *X* exceeds the threshold *u* by at most an amount *y* given that *X* exceeds the threshold *u*. Balkema, de Haan (1974) show that under MDA conditions given in equation (1) the generalised Pareto distribution (GPD) is the limiting distribution for the distribution of the excesses, as the threshold tends to the right endpoint. A positive measurable function $\sigma(u)$ can be found such that:

$$\lim_{u\uparrow\infty} \sup_{0\le x\le\infty} \left|F_u(x)-G_{\xi,\sigma(u)}(x)\right| = 0 \quad \text{iff} \quad F\in MDA(H_\xi)$$

This theorem suggests that for sufficiently high threshold u, the distribution function of the excess observations may be approximated by the GPD. As the threshold u gets larger, the excess distribution $F_u(y)$ converges in limit to the GPD, which is defined as:

$$G_{\xi,\sigma,\mu}(x) = \begin{cases} 1-\left(1+\xi\dfrac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}} & if \quad \xi\neq 0 \\ 1-e^{-(x-\mu)/\sigma} & if \quad \xi = 0 \end{cases} \qquad x\in\begin{cases} [\mu,\infty] & if \quad \xi\ge 0 \\ [\mu,\mu-\sigma/\xi] & if \quad \xi<0 \end{cases} \tag{4}$$

where ξ is the shape parameter, σ is the scale parameter, and μ is the location parameter. In order to estimate the tails of the loss distribution, the result that, for a sufficiently high threshold u, $Fu(y) \approx G\xi,\beta(u)(y)$ is used. An approximation of $F(x)$, for $X>u$ is:

$$F(x) = \left[1-F(u)\right]G_{\xi,\sigma,u}(x-u)+F(u) \tag{5}$$

Estimate for $F(x)$ is obtained by:

$$\hat{F}(x) = 1-\frac{k}{n}\left(1+\hat{\xi}\frac{x-u}{\hat{\sigma}}\right)^{-\frac{1}{\xi}} \quad \text{provided that} \quad G_{\xi,\sigma,u}(x) = 1-\left(1+\xi\frac{x-u}{\sigma}\right)^{-\frac{1}{\xi}} \tag{6}$$

where k represents the number of exceedences over the threshold u and n number of observations, $\hat{\xi}$ and $\hat{\sigma}$ are the maximum likelihood estimators of ξ and σ. This equation can be inverted to obtain a quantile of the underlying distribution, which is actually VaR. For $cl \ge F(u)$ VaR is calculated as:

$$VaR_{cl} = q_{cl}(F) = u+\frac{\sigma}{\xi}\left(\left(\frac{1-cl}{\overline{F}(u)}\right)^{-\xi}-1\right) = u+\frac{\sigma}{\xi}\left(\left(\frac{1-cl}{k/n}\right)^{-\xi}-1\right) \tag{7}$$

To remedy the problems of unconditional estimation that is traditional in EVT McNeil and Frey (2000) developed a conditional quantile EVT approach under the assumption that the tail of the conditional distribution of the GARCH is approximated by a heavy-tailed distribution. They underline the conditional quantile problem and apply EVT to the conditional return distribution by using a two-stage method, which combines GARCH model with EVT in applying the residuals from the GARCH process.

3. Data analysis and behaviour of the tails

CROBEX index data set is composed of 750 daily returns, which are collected for the period of three years, 02.01.2007 - 04.01.2010, including the latest financial crisis and its' effects on global stock markets. The calculated VaR figures are for a 1-day ahead horizon at 99, 99.5 and 99.9 percent confidence levels. VaR models that are tested in this paper are: Normal simple moving average (VCV) VaR, RiskMetrics, Historical simulation with rolling windows of 100, 250 and 500 days, BRW (time weighted) simulation with decay factors of 0.97 and 0.99, GARCH parametric model, unconditional EVT approach using GPD and McNeil, Frey (2000) conditional EVT approach. Table 1 gives a summary of descriptive statistics and normality test for the entire analysed sample daily log returns.

Table 1

SUMMARY DESCRIPTIVE STATISTICS FOR CROBEX RETURNS
IN THE PERIOD 02.01.2007 – 04.01.2010

CROBEX	02.01.2007 - 04.01.2010
Descriptive statistics	
Mean	-0,00061
Median	0,00009
Minimum	-0,10764
Maximum	0,14779
St.Dev.	0,02003
Skewness	-0,02518
Kurtosis	9,92
Normality tests	
Lilliefors	0,096
(p value)	0,00
Shapiro Wilk/Francia	0,912
(p value)	0,00
Jarque-Bera	1.499,28
(p value)	0,00
Unit Root tests	
ADF (AR + drift)	-19,48
P-P (AR + drift)	-24,38

Source: Authors' calculation

Skewness and excess kurtosis of the series are significantly different from zero. The distribution of CROBEX returns is slightly negatively skewed and has far fatter tails than assumed under normality. The negative skew and fat tails can be attributed to the global financial crisis and severe market crashes during 2008. In line with the characteristics of the moments of the series normality tests confirm that the CROBEX returns are not normally distributed. Ljung-Box and Engle's ARCH test show that there is significant autocorrelation and ARCH effects present in CROBEX returns i.e. volatility tends to cluster together, meaning that CROBEX returns are not IID. These findings are troubling for VaR models based on normality assumption, as well as for the nonparametric and semi-parametric approaches that are based on IID assumption, such as historical simulation and BRW approach. Since elementary assumptions of such VaR models are not satisfied, VaR figures obtained for such models cannot be trusted. By modelling the series as an ARMA(2,1)-GARCH(1,1)-t process we managed to remove autocorrelation and heteroskedasticity from the data making the innovations of the process IID.

To find which distribution provides the best fit to tails of CROBEX returns we fit fat tailed, positively skewed distributions: lognormal, gamma, inverse Gaussian (IG) and generalized Pareto (GPD), along with exponential distribution as a benchmark, to the empirical tails. As stated earlier EVT methods are applicable over a high threshold with the most problematic element being the choice of the suitable threshold. By setting the threshold too high we are left with only a few data points and increase parameter uncertainty. By setting the threshold too low we are losing the theoretical justification for the application of extreme value theory. We fit the selected fat tailed distributions to 2.5% left and right tail of the return distribution. Distributions are fitted using maximum likelihood estimation. The results of parameter estimation with standard errors given in parenthesis are presented in Table 2.

Table 2.

MAXIMUM LIKELIHOOD PARAMETER ESTIMATES AND STANDARD ERRORS FOR THE TESTED DISTRIBUTIONS

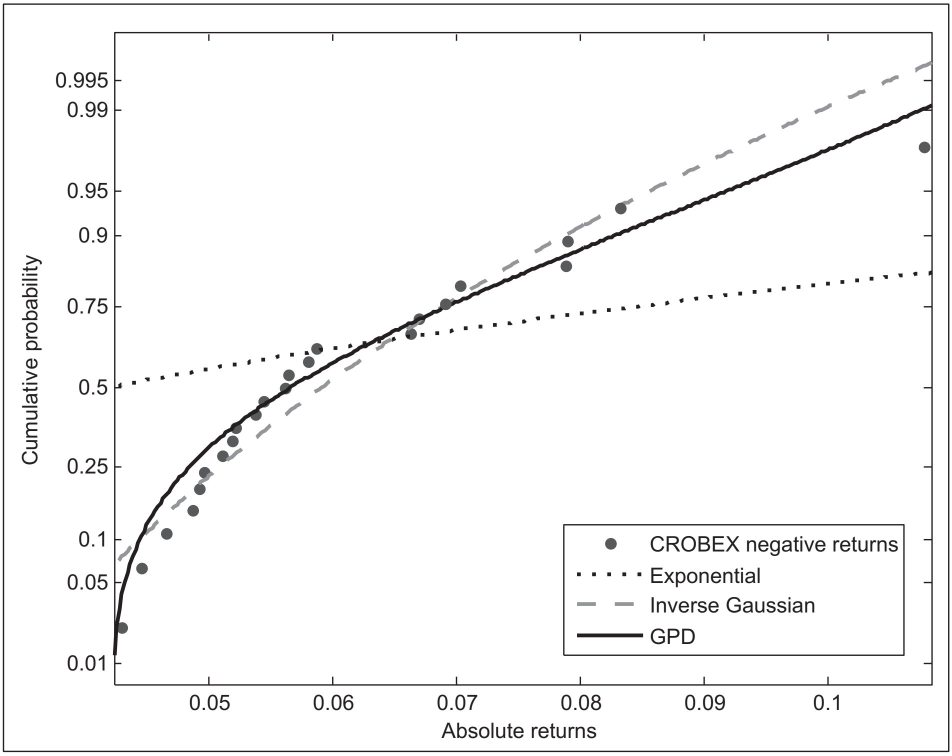
Negative returns					
Distribution	Lognormal	Exponential	Gamma	IG	GPD
Parameters	μ = -2.83	μ = 0.0606	a = 18,99	μ = 0.0606	ξ = -0.024
	(0.048)	(0.0126)	(5.552)	(0.0029)	(0.101)
	σ = 0.228		b = 0.0032	λ = 1.184	σ = 0.023
	(0.035)		(0.0009)	(0.349)	(0.006)
					k = -0.042
Log likelihood	66.93	41.48	66.11	66.95	69.2
Positive returns					
Distribution	Lognormal	Exponential	Gamma	IG	GPD
Parameters	μ = -3.042	μ = 0.0515	a = 6.748	μ = 0.0515	ξ = 0.356
	(0.076)	(0.0107)	(1.943)	(0.004)	(0.319)
	σ = 0.366		b = 0.0076	λ = 0.370	σ = 0.012
	(0.056)		(0.0023)	(0.109)	(0.005)
					k = 0.033
Log likelihood	60.98	45.22	58.72	60.88	69.89

Source: Authors' calculation

For both left and right tail (long and short trading positions), GPD provides the best fit in the tail followed by the lognormal and inverse Gaussian distribution. The exponential distribution that was used as a benchmark does not provide a close fit to the tail regions of the distribution. Right tail of the distribution belongs to Fréchet domain of attraction and it does not even have a finite fourth moment since the estimated tail index is greater than 0.25. The left tail is not significantly different from zero implying that it has a medium tail belonging to Gumbel domain of attraction. The two distributions providing the best fit to the empirical left and right tails, along with the worst fit - exponential distribution are plotted in figures 1 and 2. We use the three parameter form of the GPD with the location parameter set to the threshold value.

Figure 1.

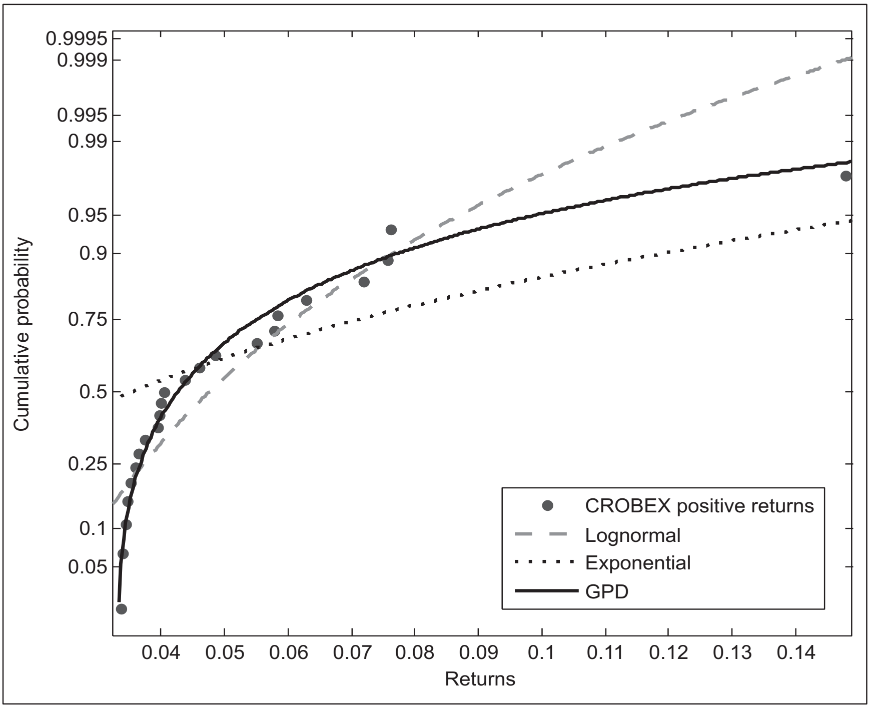
PERFORMANCE OF GPD, INVERSE GAUSSIAN AND EXPONENTIAL DISTRIBUTION COMPARED TO EMPIRICAL 2.5% LEFT TAIL OF CROBEX RETURNS



Source: Authors' calculation

Figure 2.

PERFORMANCE OF GPD, LOGNORMAL AND EXPONENTIAL DISTRIBUTION COMPARED TO EMPIRICAL 2.5% RIGHT TAIL OF CROBEX RETURNS



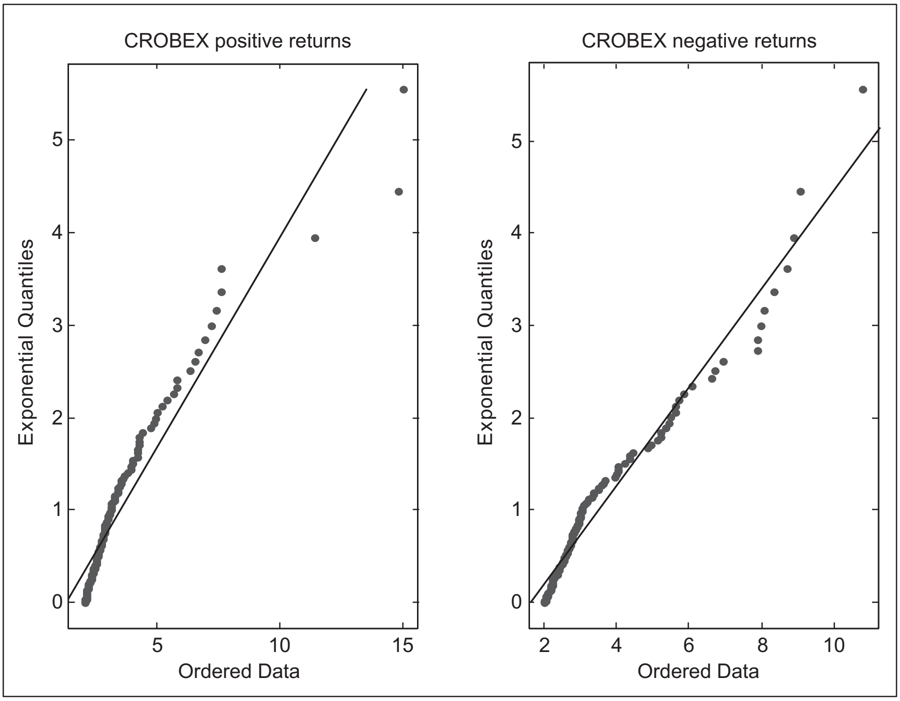
Source: Authors' calculation

Besides providing the best fit to the empirical 2.5% tails GPD has solid foundations in the mathematical theory of the behaviour of extremes and as such does not simply represent ad hoc curve fitting. It is possible that by trial and error, some other distribution can be found which fits the analysed tail data even better. Such a case can be found in Burneckia, Kukla, Weron (2000), where they find that for property claim services (PCS) indices lognormal distribution is superior to GPD in the tail region. One should keep in mind that such a distribution is an arbitrary, without any mathematical justification, and extrapolating beyond the available data set would be highly questionable.

To estimate EVT risk measures it is necessary to estimate EVT parameters – μ, σ, and in the case of Fréchet distribution the tail index (ξ). Estimation of the tail index is the most problematic element of EVT estimation. As a first step before model fitting is undertaken, a number of exploratory graphical methods can be used to obtain preliminary information about the tails of the data. In statistics, a quantile-quantile (QQ) plot is a convenient visual tool to examine whether a sample comes from a specific distribution. Specifically, the quantiles of an empirical distribution are plotted against the quantiles of a hypothesized distribution. If the sample comes from the hypothesized distribution or a linear transformation of the hypothesized distribution, the QQ plot is linear. The QQ-plot against the exponential distribution (distribution with a medium-sized tail) is a very useful instrument in identifying heavy tails. If the analysed data is from an exponential distribution, the points on the graph would lie along a straight line. If there is a concave presence, this would indicate a fat tailed distribution, whereas a convex departure is an indication of short tailed distribution. There are also other purely graphical techniques, such as mean excess function plot that can be used in identifying the shape of the tail (see McNeil (1998) for more details).

Figure 3.

QQ PLOT OF CROBEX POSITIVE AND NEGATIVE RETURNS AGAINST EXPONENTIAL DISTRIBUTION



Source: Authors' calculation

In case of positive returns there is clear evidence of concave shape for observations above the 3% threshold. For negative returns the results are more ambiguous and it would be hard to come to any confident conclusion. In this paper we opted for a purely statistical approach to threshold estimation where the value of threshold has been chosen as the value which minimizes Anderson-Darling statistic as proposed by Coronel-Brizio and Hernandez-Montoya (2005). The use of Anderson-Darling statistic is due to the fact that the corresponding weighting function puts more weight in the tails of the distribution. Under the assumption that a tail of the distribution follows a Pareto law, the asymptotic distribution of Anderson-Darling statistic is known and we can use this distribution as a reference to determine an estimate of the cut off using a statistical approach. Although we use maximum likelihood method for fitting the generalized Pareto distribution to excesses data over a high threshold other methods such as the method of probability weighted moments can be used. In plotting the Hill estimator as well as Anderson-Darling maximum likelihood estimation of GPD parameters we choose to end the plot at the third order statistic, thus omitting two most extreme observations since they significantly differ from the rest of the sample and may be treated as erratic.

Table 3.

MAXIMUM LIKELIHOOD ESTIMATES OF GPD TAIL INDEX AND SCALE PARAMETER, THRESHOLD BASED ON ANDERSON-DARLING STATISTIC

	Positive returns			Negative returns		
	estimate	s.e.	threshold value	estimate	s.e.	threshold value
Tail index	0,1877	0,1771	3,4784	-0,0641	0,1119	2,9557
Sigma	1,6932	0,3890		1,9921	0,3258	

Source: Authors' calculation

The exclusion of the two most extreme positive and negative observations along with the use of Anderson-Darling statistic made a huge difference to parameter estimation compared to full sample ad hoc 2.5 and 97.5 quantile estimation. Truncated series of extreme losses with Anderson-Darling threshold has a higher tail index i.e. is more extreme than a series of positive extremes. Both positive and negative truncated extremes are significantly different from zero i.e. fat tailed, putting them both in Fréchet domain of attraction. Modelling of high quantiles for such a distribution with light or middle tail distributions such as: normal, exponential, gamma or lognormal, would result in serious underprediction of risk. High value of the estimated tail index for both tails makes CROBEX index a good candidate for EVT VaR models as it indicates that Croatian stock market experienced extreme gains and crashes over the recent period.

4. Backtesting results

In this section the backtesting results for eleven VaR models are presented and their performance is analysed according to different criteria. Performance of each VaR model is evaluated separately for long and short position in the CROBEX index, based on several performance tests. Overall summary results are very useful to see how tested VaR model fare with standard backtesting framework based on the complete testing sample. Kupiec test and Christoffersen independence test are used to identifying VaR models that are acceptable to regulators, and actually provide the desired level of safety both to individual investors and regulators. It often happens that more than one VaR model is deemed adequate and the problem of ranking the models arises. To overcome this shortcoming of the backtesting measures forecast evaluation can be used, such as Lopez size-adjusted loss function. A loss function can allow for the sizes of tail losses to influence the final rating of VaR model. VaR model that generates higher tail losses would generate higher values under this size adjusted loss function than a VaR model that generates lower tail losses, ceteris paribus.

Kupiec and Christoffersen independence (IND) test backtesting results, at 5% significance level, for tested VaR models at 99, 99.5 and 99.9% confidence levels are presented in tables 4 and 5.

Table 4.

KUPIEC TEST BACKTESTING RESULTS AT 99, 99.5 AND 99.9% CONFIDENCE LEVELS, PERIOD 02.01.2007 - 04.01.2010

VaR models	Positive returns			Negative returns		
	99%	99,5%	99,9%	99%	99,5%	99,9%
HS 100						
HS 250						
HS 500						
BRW λ=0,97						
BRW λ=0,99						
Normal VCV						
RiskMetrics						
GARCH	+					
EVT GARCH	+	+		+	+	+
GPD	+	+	+	+	+	+

Grey areas mark the VaR models that satisfy Kupiec test for positive/negative CROBEX returns and selected confidence level, at 5% significance level.

Source: Authors' calculation

Table 5.

CHRISTOFFERSEN INDEPENDENCE (IND) TEST BACKTESTING RESULTS AT 99, 99.5 AND 99.9% CONFIDENCE LEVELS, PERIOD 02.01.2007 - 04.01.2010

VaR models	Positive returns			Negative returns		
	99%	99,5%	99,9%	99%	99,5%	99,9%
HS 100						
HS 250		+	+			
HS 500	+	+	+			+
BRW λ=0,97				+		
BRW λ=0,99			+			
Normal VCV			+		+	
RiskMetrics						+
GARCH	+	+	+	+	+	+
EVT GARCH	+	+	+	+	+	+
GPD	+	+	+	+	+	+

Grey areas mark the VaR models that satisfy Christoffersen IND test for positive/negative CROBEX returns and selected confidence level, at 5% significance level.

Source: Authors' calculation

Backtesting results for both long and short trading position in CROBEX index are equally disappointing. With the exception of EVT models none of the tested VaR models provides adequate risk coverage for any of the tested high quantiles. Besides satisfying the basic Basel criteria – the Kupiec test, EVT models also satisfy the independence criteria i.e. EVT VaR errors do not bunch together making them IID. Although independence of VaR errors is not required under Basel rules, in practice it is of vital importance. Dependence of VaR errors i.e. their bunching is crucial for the stability of an institutional investor since bunched VaR errors can erase capital reserves much faster than slight underestimation of risk. Out of the widely used VaR models, GARCH is the only model that, although providing falsely optimistic risk estimates, yields independent VaR forecasts for both long and short positions at all of the tested quantiles. Historical simulation model with observation period of 500 days (HS 500) satisfied Christoffersen independence test only for short position while failing the test for long trading position. All other models besides failing the basic Kupiec test also failed the independence test. These characteristics make non EVT VaR models extremely dangerous if relied upon by investors and risk managers since they pose a dual treat – they are providing overly optimistic risk forecasts and at the same time their VaR forecast errors are dependent. Taking into account the length of the backtesting period and consistency of results we can confidently conclude that when including into the backtesting period the global financial crisis, only conditional and unconditional EVT models perform satisfactory, while other, widespread VaR models tend to seriously underpredict the true level of risk in Croatian market.

Besides providing adequate risk coverage a good VaR model has to yield forecasts as close as possible to the true level of risk. An ideal model would neither under or overstate the true level of risk. A risk measure that would satisfy the Kupiec test but yield excessively high risk forecasts is unacceptable by any investor since it would require unnecessary high reserves. By employing Lopez test and calculating average VaR values we identify which VaR model gives the closest fit to the true level of risk and as such is the most acceptable by investors. The results are presented in table 6.

Table 6.

LOPEZ TEST RANKING AND AVERAGE VAR VALUES OF COMPETING VAR MODELS, PERIOD 02.01.2007 - 04.01.2010

Lopez test	Positive returns			Negative returns		
	99%	99,5%	99,9%	99%	99,5%	99,9%
HS 100	8,26	8,20	10,16	11,25	9,19	11,15
HS 250	8,25	7,19	7,13	9,25	9,17	8,10
HS 500	10,34	8,23	4,14	17,37	12,23	9,10
BRW λ=0,97	8,26	8,22	10,21	10,26	12,24	14,21
BRW λ=0,99	4,21	6,21	8,21	8,22	10,21	14,21
Normal VCV	12,29	11,23	7,16	16,36	13,28	12,18
RiskMetrics	4,18	7,15	6,09	10,25	11,19	9,09
GARCH	3,11	4,07	3,02	9,14	6,08	5,03
EVT GARCH	1,09**	2,06**	1,01	-0,94**	0,024**	0,00**
GPD	-6,97	-2,99	-1,00**	-3,97	-2,98	-1,00
Average VaR (%)	99%	99,5%	99,9%	99%	99,5%	99,9%
EVT GARCH	4,40	4,83	6,26	4,82	5,53	7,15
GPD	11,36	13,92	21,05	7,87	8,88	11,05

** marks VaR model with the lowest Lopez value i.e. smallest deviation from expected number of failures

Source: Authors' calculation

For both long and short trading position EVT models yield the lowest Lopez size adjusted score, making them, by this criterion, the best VaR models since they minimise the deviation between recorded and expected VaR failure rate. At all of the tested confidence levels, with the exception of short trading position at 99,9% confidence level for which it failed the Kupiec test, conditional EVT GARCH model yields lower Lopez test value than the unconditional GPD model, providing a better approximation to the true level of risk. Besides being favoured by the Lopez test results the conditional EVT GARCH model yields approximately 50% lower VaR forecasts than the unconditional GPD model. This is an important and welcomed characteristic since it drastically decreases the capital requirement for market risk and thus lowers the operating costs.

5. Conclusion

We find that both theoretically and empirically generalised Pareto distribution (GPD) fits the extreme tails of CROBEX index return distribution better than any other tested fat or medium tailed tested distribution. It could happen that by trial and error, some other distribution can be found which fits the analysed tail data even better. One should keep in mind that such a distribution is an arbitrary choice, without any mathematical justification, and extrapolating beyond the available data set would be highly questionable.

Our VaR backtesting results for long positions are similar to Gencay et al. (2003), Gencay and Selcuk (2004) and Žiković (2007a, b) for emerging markets but are even more disappointing for the widely used VaR models. Such results can naturally be attributed to the ongoing global financial crisis which is included in the backtesting period. Both simpler and more sophisticated VaR models consistently fail their task, for both long and short trading positions, with none of the models giving adequate risk coverage at any of the tested quantile. We can safely conclude that widespread VaR model should not be used for risk measurement purposes at high quantiles in case of the CROBEX index. Usually employed models provide investors and risk managers in the Croatian market with falsely optimistic information about the true levels of risk they are exposed to. Taking into account the length of the backtesting period and consistency of results we can confidently conclude that when including into the backtesting period the global financial crisis, only conditional and unconditional EVT models perform satisfactory. The performance of the two EVT models with regards to minimising the deviation from the expected number of VaR exceedances is similar but the real difference can be seen in the average VaR values they yield. Conditional EVT GARCH model yields approximately 50% lower VaR forecasts than the unconditional GPD EVT model, making it a preferable model for investors operating on the Croatian stock market.

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MODELIRANJE EKSTREMNIH DOGAĐAJA: PRIMJENA NA ZAGREBAČKU BURZU

SAŽETAK

U ovom se radu analizira uspješnost modela rizične vrijednosti (Value at Risk - VaR) mjereno pri ekstremnim kvantilima distribucije: 0.99, 0.995 i 0.999 za duge i kratke pozicije u dioničkom indeksu Zagrebačke burze – CROBEX-u. Testiranje VaR modela ukazuje na činjenicu da niti jedan od modela koji se uobičajeno koriste ne predviđa ispravno stvarnu razinu rizika tijekom aktualne globalne financijske krize. Jedinu iznimku predstavljaju modeli zasnovani na teoriji ekstremnih vrijednosti koji uspješno predviđaju rizik kratkih i dugih pozicija. U radu je istražena i prilagodljivost teorijskih distribucija krajnjim (ekstremnim) repovima distribucije prinosa na CROBEX indeks. Rezultati pokazuju da Generalizirana Pareto distribucija, koja ima snažne teorijske osnovice u teoriji ekstremnih vrijednosti, najbolje opisuje ekstremne repove distribucije CROBEX-a. Primjetno je da se distribucija lijevog i desnog repa distribucije CROBEX-a značajno razlikuju, s time da desni rep distribucije ukazuje na znatno ekstremnije skokove.

Ključne riječi: Teorija ekstremnih vrijednosti, Rizična vrijednost, Tržišta u razvoju, CROBEX, Zagrebačka burza