FORCED CONVECTION FROM A PAIR OF SPHERES: A THREE-TEMPERATURE PROBLEM

Seyed Sepehr Mohaddes Foroushani [1 *], John L. Wright [1], David Naylor [2]

[1] Dept of Mechanical & Mechatronics Eng, University of Waterloo
(200 University Ave West, Waterloo, ON, N2L 3G1, Canada)

[2] Dept of Mechanical & Industrial Eng, Ryerson University (Toronto, Canada)

[*] Correspondence author: Fax: +1 519 888 4567 Email: ssmohadd@uwaterloo.ca

Keywords: Convection, Multi-temperature heat transfer, CFD

ABSTRACT

Interaction of spherical particles in a fluid flow is important in combustion, chemical processes and air pollution. In this study, a recently developed technique for characterizing multi-temperature convective heat transfer is used to investigate convection from a pair of spheres with different surface temperatures. The technique entails numerical solutions of the full set of governing equations and subsequent solutions of the energy equation with perturbed boundary conditions. Steady-state heat transfer in intermediate Reynolds number flows over spheres in side-by-side and tandem arrangements in a water tunnel is studied. The results are expressed in terms of local and total Nusselt numbers. The variation of sphere-sphere and sphere-fluid Nusselt numbers with flow conditions is examined.

NOMENCLATURE

A	surface area
C_d	drag coefficient
D	diameter
h	heat transfer coefficient
k	thermal conductivity
L	center-to-center spacing
Nu	Nusselt number
Pr	Prandtl number
Q	heat transfer rate
Re	Reynolds number
T	temperature
δ	*denotes small change*

Subscripts

i	*at source* i
ij	*from source* i *to source* j

INTRODUCTION

Interaction of spherical particles in a fluid flow is important in the analysis of particulate and multiphase flow systems in such areas as combustion, chemical processes and air pollution. In many cases, the single sphere data does not provide an accurate prediction of the system behaviour [1] and interaction of spheres moving in close proximity must be examined. This interaction has been studied by various authors. Kim *et al.* [2], for instance, numerically studied three-dimensional flow over two identical spheres arranged side by side in a uniform stream at moderate Reynolds numbers. Tal *et al.* [3,4] studied fluid flow and heat transfer in assemblies of spherical particles. Patnaik [5] and Raju and Sirignano [6] studied the interactions between two moving vaporizing droplets in tandem arrangement for Re<200. Yoon and Yang [7] studied flow-induced forces on two identical nearby spheres in various arrangements at Re=300. Prahl *et al.* [8] further examined the interaction between two spheres in tandem at Re=300 using both steady and pulsating inflow conditions. Liang *et al.* [9] studied the drag force in various multi-particle arrangements for 30<Re<106. The interaction between groups of droplets in low to moderate Reynolds numbers is of special importance in combustion. A comprehensive review of the theory of droplet array combustion has been recently published by Sirignano [10]. Kleinstreuer *et al.* [11] computed the transient velocity and temperature fields around interacting vaporizing fuel droplets for an initial Reynolds number of 100. Zhu and Dunn-Rankin [12] used a spectroscopy technique to perform temperature measurements in a stream of combusting droplets.

Juncu [1] has studied unsteady convective heat and mass transfer from two spheres in tandem.

In most of the published studies, *e.g.* references [3,4], the array of spheres has been assumed to be isothermal, *i.e.* no temperature difference between individual particles. Based on this assumption, the problem of convective heat transfer becomes a *two-temperature* problem. Nevertheless, if the particles are at different surface temperatures, a *multi-temperature* convection problem occurs. The set of heat transfer coefficients needed for fully characterizing a multi-temperature convection problem cannot be obtained using conventional methods, *e.g.* CFD.

In the present work, the three-temperature problem of forced convection from a pair of spheres, with different temperatures, is examined using a recently developed technique, dQdT. The flow of water, Pr=7, in a 1.2m×1.2m×2.4m water tunnel is considered in a range of intermediate Reynolds numbers, 0<Re<250. The spheres are placed in side-by-side and tandem arrangements, with the center-to-center spacing between the spheres equal to twice their diameter, L/D=2. The present study is conducted as part of a feasibility study for the experimental implementation of the dQdT technique in a water tunnel.

METHODOLOGY

The dQdT technique: A technique for characterizing three-temperature convection problems called dQdT is used in this study. This technique entails a numerical solution of the full set of governing equations and consequent solutions of the energy equation with perturbed boundary conditions. A *paired* heat transfer coefficient, h_{ij}, characterizing the influence of the temperature of one heat source, T_i, on heat transfer from another source, Q_j, is obtained as shown in Equation 1. The term paired is used here to designate heat transfer between one source and another designated heat source, as opposed to *overall*, which is used to designate heat transfer between one source and all the other sources.

$$h_{ij} = -\frac{1}{A_j}\frac{\partial Q_j}{\partial T_i}\Big|_{h_{ij}=\text{const}} = -\frac{1}{A_j}\frac{\delta Q_j}{\delta T_i} \qquad (1)$$

In Equation 1, A_j is the surface area of source j, δQ_j denotes the observed changed in the rate of heat transfer at source j as a result of a change, δT_i, in the temperature of source i. The condition of h_{ij}=const is satisfied by solving the energy equation while the flow field and the fluid properties are fixed, *i.e.* prevented from changing after a perturbation is introduced in the temperature boundary conditions.

A heat transfer coefficient, h_{ij}, calculated using Equation 1, is converted to dimensionless form, *i.e.* the paired Nusselt number Nu_{ij}, using Equation 2.

$$Nu_{ij} = \frac{h_{ij}D}{k} \qquad (2)$$

The paired heat transfer coefficients calculated by dQdT can also be used to characterize the thermal resistor model of three-temperature heat transfer problems. Characterizing thermal resistor networks in terms of a set of resistances (heat transfer coefficients) is significantly beneficial in the analysis of thermal systems and in decreasing the computational expense of modeling such systems, *e.g.* in building energy simulation.

Numerical solutions: The commercial CFD code ANSYS FLUENT is used to solve the equations of continuity, momentum and energy in steady state. Constant thermophysical properties are assumed. The SIMPLE scheme is used for pressure-velocity coupling, along with 2nd order discretization schemes for all the governing equations. A schematic of the side view of the computational domain is shown in Figure 1. Uniform velocity and temperature profiles are set at the inlet of the domain, while zero gauge pressure is set as the boundary condition at the outlet. The domain sides, representing the four walls of the water tunnel, are modeled as adiabatic impermeable no-slip walls. The boundary conditions at the spheres are zero velocity and constant temperature.

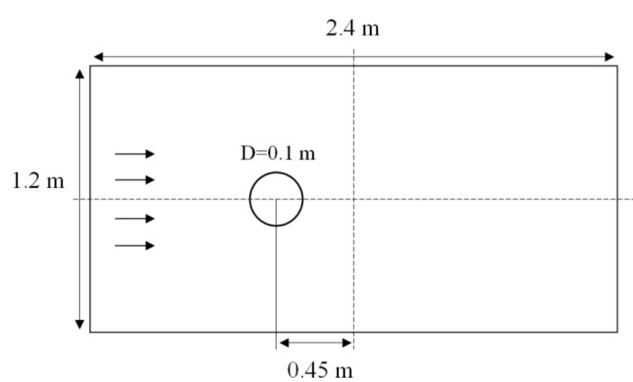

Figure 1
Schematic of the computational domain (side view)

For every Reynolds number, the full set of governing equations is first solved to obtain the temperature and velocity fields. Heat transfer rates at the spheres are then computed by integrating the temperature field. To obtain paired heat transfer coefficients defined by Equation 1, the temperature boundary condition at one of the isothermal heat sources, say one of the spheres, is perturbed. Next, the energy equation is solved and the new heat transfer rate at the unperturbed source, the other sphere, is calculated.

Verification: The present numerical solutions are verified by comparison to previously published results. Table 1 provides a comparison of the drag coefficient of side-by-side spheres as predicted by the current solutions and the numerical results of Kim *et al.* [2]. Note that these results are for a free stream, *i.e.* with the boundary conditions at the sides of the domain set as symmetric (zero normal gradient).

Table 1
Drag Coefficient of Side-by-Side Spheres (L/D=2)

Re	C_d	
	Present study	Kim *et al.* [1]
50	1.67	1.70
100	1.16	1.18
200	0.95	0.96

RESULTS & DISCUSSION

Side by side arrangement: When two spheres of equal diameter are placed side by side, they "see the flow" identically. In other words, they are placed symmetrically with respect to the flow. Therefore, assuming the fluid properties to vary negligibly with temperature, heat transfer coefficients of the two spheres will be equal, regardless of their surface temperatures, T_L and T_R. Nevertheless, each sphere is in thermal communication not only with the fluid, but also with the other sphere. Thermal interaction between the spheres is function of the driving temperature difference, distance between the spheres and the thickness of the thermal boundary layer developed around each sphere. The latter is itself function of flow conditions and fluid properties. If the spacing between spheres is small enough, roughly smaller than the combined thickness of the two thermal boundary layers, the two boundary layers interfere. In this situation, a change in the surface temperature of one sphere will affect the temperature field in the vicinity of the other sphere, and thus the rate of heat transfer at the second sphere. On the other hand, if the spheres are spaced far apart, *i.e.* if the spacing is larger than the combined boundary layer thickness, two independent thermal boundary layers develop. In this case, changing the surface temperature of one sphere will not alter the heat transfer at the other sphere. The sphere-sphere Nusselt number, Nu_{LR}, characterizes this interaction. In Figure 2, Nu_{LR} is plotted versus Re. As Re increases, the boundary layer thickness decreases and thus, at a fixed spacing, the thermal interaction between the two spheres becomes weaker. Heat transfer between spheres and the fluid, on the other hand, is enhanced as Re increases. This is well known from the theory of external forced convection. This trend is captured by the paired sphere-fluid Nusselt number, $Nu_{L\infty}$, also shown in Figure 2.

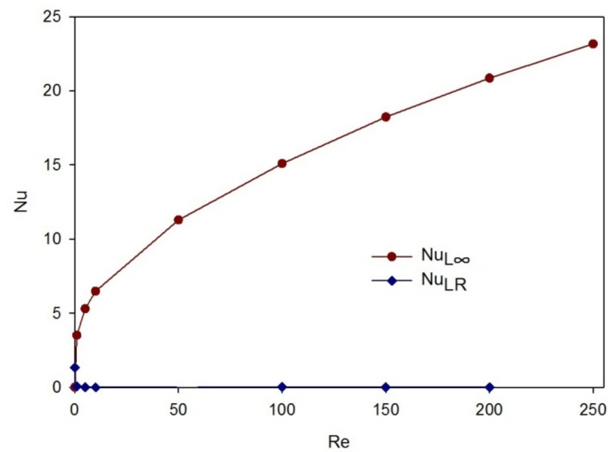

Figure 2
Variation of sphere-sphere and sphere-fluid Nu with Re (side-by-side)

Figure 3 shows the distribution of the overall Nusselt number of a sphere in side-by-side arrangement as well the local distribution of the paired sphere-fluid Nusselt number at Re=100. At Re=100, sphere-sphere heat transfer is zero. Therefore, heat transfer at a sphere is chiefly between the sphere and the fluid. It is therefore expected that paired sphere-fluid and overall sphere Nusselt numbers, obtained from dQdT and the baseline CFD solution respectively, be equal ($Nu_{L\infty}=Nu_L$). Maximum and minimum heat transfer coefficients are observed at $\theta=0$ and $\theta=140°$, stagnation and separation points respectively.

This three-temperature problem can be represented by a thermal resistor network such as the one shown in Figure 4. A set of three paired Nusselt numbers, Nu_{ij}, characterizes this thermal network for any given combination of geometry, fluid, flow, and temperature boundary conditions. As mentioned previously, with constant fluid properties, the two sphere-fluid Nusselt numbers are equal and thus the thermal network of Figure 4 is characterized by two Nusselt numbers. It is noteworthy that prior to the development of the dQdT technique, there were no tools, numerical or experimental, available for determining the three Nusselt numbers (resistors) of this network.

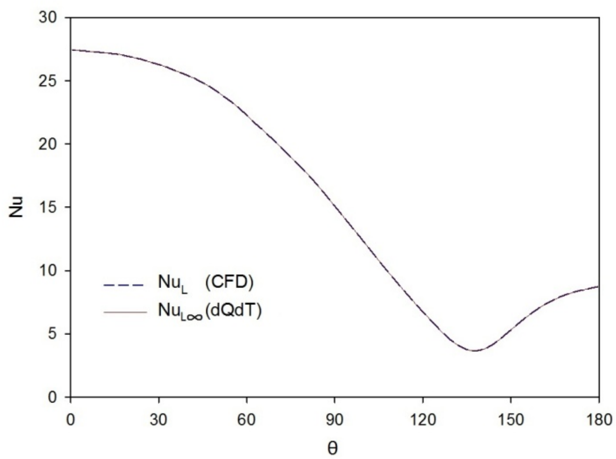

Figure 3
Local sphere-fluid and overall Nu
(side-by-side, Re=100)

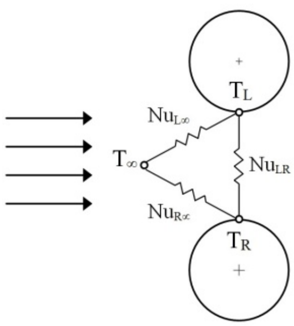

Figure 4
Thermal network of the side-by-side arrangement

Tandem arrangement: When arranged in tandem, the spheres will see the flow differently, *i.e.* they are placed asymmetrically with respect to the flow. Due to the blocking effect of the upstream sphere, Q_d is smaller than Q_u. This difference increases with the Reynolds number. Moreover, due to their asymmetric arrangement, the spheres are expected to have asymmetric effects on each other. This asymmetry in thermal communication between two heat sources is referred to as *preferential advection*. To demonstrate this effect further, sphere-sphere Nusselt numbers are plotted against Re in Figure 5. In this figure, Nu_{du} corresponds to the heat transfer coefficient obtained by perturbing the surface temperature of the downstream sphere, T_d, and computing the resulting change in the heat transfer rate at the upstream sphere, Q_u, according to Equation 1. Hence, Nu_{du} is a measure of the influence of T_d on Q_u. Similarly, Nu_{ud} characterizes the influence of T_u on Q_d. At Re=0, the pure conduction limit, $Nu_{ud}=Nu_{du}$. However, for Re>0, the influence of T_u on Q_d is much larger than that of T_d on Q_u which is zero for Re<50. For 50<Re<200, via recirculation between the spheres, T_d can affect Q_u and hence the non-zero values of Nu_{du}. Maximum Nu_{ud} also occurs in this interval, indicating the enhancement of the influence of the T_u on Q_d by the recirculating zone.

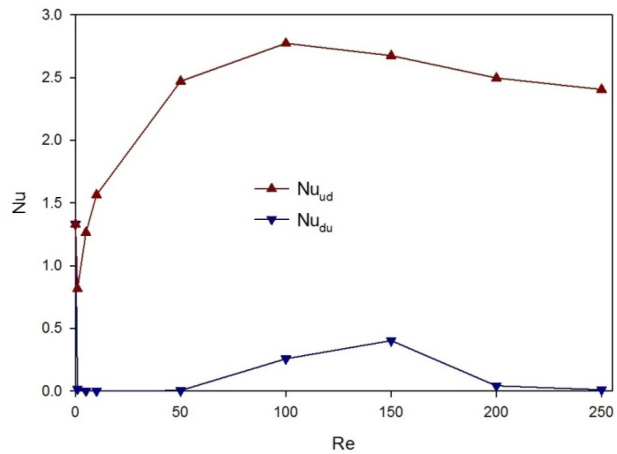

Figure 5
Variation of sphere-sphere Nu with Re (tandem)

In Figure 6, sphere-fluid Nusselt numbers are plotted versus the Reynolds number. Corresponding to each sphere, two distinct Nusselt numbers are obtained: one by perturbing the sphere surface temperature and observing the resulting change in the total heat transfer (from both spheres) to the fluid, and the other by perturbing the temperature of the approaching free stream and computing the resulting change in the heat transfer rate at the sphere of interest. See Equation 1. Preferential advection is observed between spheres and the fluid too. Note that in Figure 6, the sphere-fluid curve of Figure 2 is reproduced for comparison. It is seen that with Nu_{du} substantially zero, $Nu_{\infty u}$ is very close to $Nu_{\infty L}$, which is another indication of the small thermal communication *from* the downstream sphere *to* the upstream sphere. Then difference between $Nu_{\infty d}$ and $Nu_{\infty L}$, on the other hand, is considerable due to significant thermal communication from the upstream sphere to the downstream sphere, as reflected by Nu_{ud} values of Figure 5.

The observed difference between Nu_{ij} and Nu_{ji} indicates that the tandem arrangement cannot be represented by the thermal network of Figure 4. In fact, the paired heat transfer coefficients calculated through dQdT can be used to determine whether a multi-temperature system can be properly modeled as a thermal resistor network.

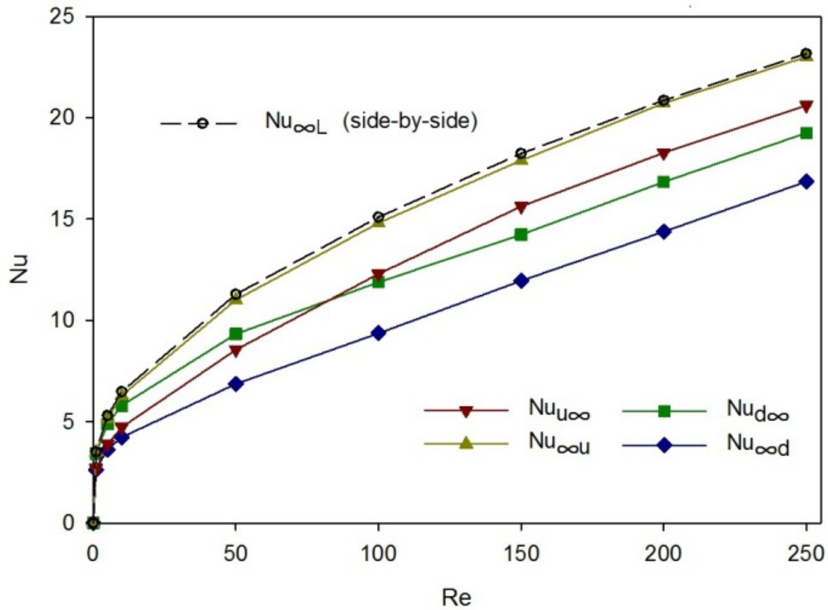

Figure 6
Variation of sphere-fluid Nu with Re

Local paired heat transfer coefficients on a plane parallel to the flow are shown in Figure 7. Note that the angular positions on the periphery of the two spheres are measured in opposite directions. As expected, Nu_{du}, characterizing the influence of T_d on Q_u, is maximum at θ_u=0, smoothly decaying to zero at $\theta_u \approx 60^\circ$. The local effect of T_u on heat transfer at the downstream sphere, on the other hand, is more complicated. Starting from $Nu_{ud} \approx 1$ at θ_d=0, Nu_{ud} reaches its maximum at $\theta_d \approx 60^\circ$, the counterpart of [$\theta_u \approx 60^\circ$; Nu_{du}=0]. Note that $\theta_d \approx 60^\circ$ is also an inflection point, beyond which Nu_{du} decreases to its minimum at $\theta_d \approx 150^\circ$, the second inflection point, where flow separation occurs.

It is noteworthy that following the convention in heat transfer, a temperature ratio must be introduced to characterize the temperature arrangement of this three-temperature problem. The overall heat transfer coefficients, obtained from measurement or a numerical solution, will then be a function of this temperature ratio. Using dQdT, however, the obtained paired heat transfer coefficients are independent of the temperature ratio. In other words, the paired Nusselt numbers presented in this work can be used for any temperature arrangement. This is an important advantage of dQdT in characterizing multi-temperature forced convection.

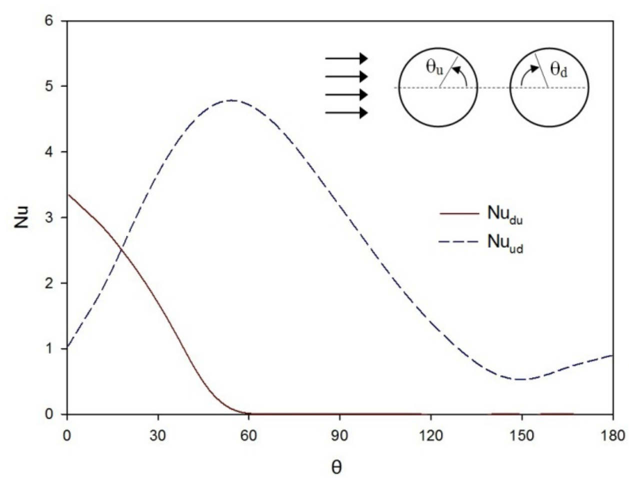

Figure 7
Local sphere-sphere Nu (Re=100, tandem)

Remarks on experimental dQdT: This study was conducted to assess the feasibility of implementing dQdT in a water tunnel study of flow over nearby spheres. To avoid complexities of unsteady flow and transition to turbulence, it is preferable to perform an experimental dQdT study in the steady laminar regime. Moreover, when performing measurements, the heat transfer response to a temperature perturbation must be at least an order of magnitude larger than the associated errors to provide meaningful data for calculating paired heat transfer coefficients. Note that dQdT is in essence a sensitivity analysis. The presented results indicate little thermal communication between the spheres in the steady laminar regime. For 0<Re<250, Pr=7 and L/D=2, sphere-sphere Nusselt numbers are at least one order of magnitude smaller than sphere-fluid Nusselt numbers. In side-by-side arrangement, Nu_{LR} is essentially zero for Re>1. In tandem arrangement, spacing and Reynolds number have variable effects on sphere-sphere interaction depending on the flow structure, especially the size and strength of the recirculating zone between the spheres. Nevertheless, for 0<Re<250, the obtained sphere-sphere Nusselt numbers are very small. Based on the presented results, successful implementation of experimental dQdT in intermediate Reynolds numbers does not seem feasible.

CONCLUSION

The three-temperature problem of convection from two spheres with the same size but different surface temperatures was studied using the dQdT technique. Intermediate Reynolds number flow of water over spheres in side-by-side and tandem arrangements was considered. It has been shown that due to the symmetry of the side-by-side arrangement with respect to the flow, Nusselt numbers corresponding to sphere-fluid heat transfer are the same at the two spheres. In tandem arrangement, however, the blocking effect of the upstream sphere leads to a large difference between heat transfer rates at the two spheres. Moreover, considerable difference was observed between the Nusselt numbers associated with any two of the isothermal sources. This difference is due to preferential advection, caused by the asymmetric position of the spheres with respect to the flow. Sphere-sphere Nusselt numbers are significantly smaller than sphere-fluid Nusselt numbers. This difference can impose a practical barrier to the experimental implementation of dQdT in the steady laminar flow regime. It has been shown that the side-by-side arrangement can be represented by a three-node thermal network, with the associated resistors obtained by dQdT. In tandem arrangement, since preferential advection is significant, the problem cannot be formulated in terms of a thermal network. dQdT provides the tool for determining whether a multi-temperature convection problem can be represented as a thermal network.

ACKNOWLEDGMENTS

The authors acknowledge the financial support from the Smart Net-Zero Energy Buildings Strategic Network (SNEBRN) of the Natural Sciences and Engineering Research Council of Canada (NSERC) and the University of Waterloo.

REFERENCES

[1] Juncu, G, 2007, “Unsteady forced convection heat/mass transfer around two spheres in tandem at low Reynolds numbers,” *Int. J. Thermal Sciences*, 46, pp. 1011-1022.

[2] Kim, I., Elghobashi, S., and Sirignano, W., 1993, “Three-dimensional flow over two spheres placed side-by-side,” *J. Fluid Mech.*, 246, pp. 456-488.

[3] Tal, R., Lee, D.N., Sirignano, W.A., 1983, “Hydrodynamics and heat transfer in sphere assemblages – cylindrical cell models,” *Int J. Heat Mass Transfer*, 26(9), pp. 1265-1273.

[4] Tal, R., Lee, D.N., Sirignano, W.A., 1984, “Heat and momentum transfer around a pair of spheres in viscous flow,” *Int. J. Heat Mass Transfer*, 27(11), pp. 1953-1962.

[5] Patnaik G., “A numerical solution of droplet vaporization with convection,” Ph.D. Dissertation, Carnegie-Mellon University, Dept. of Mechanical Engineering; 1986.

[6] Raju, M.S., and Sirignano, W.A., 1990, “Interaction between two vaporizing droplets in an intermediate Reynolds number flow,” *Physics of Fluids*, 2(10), pp. 1780-1896.

[7] Yoon, D-H., and Yang, K-S., 2007, “Flow-induced forces on two nearby spheres,” *Physics of Fluids*, 19.

[8] Prahl, L., Jadoon, A., and Revstedt, J., 2009, “Interaction between two spheres placed in tandem arrangement in steady and pulsating flow,” *Int. J. Multiphase Flow*, 35, pp. 963-969.

[9] Liang, S.-C., Hong, T., and Fan, L.-S., 1996, “effects of particle arrangements on the drag force of a particle in the intermediate flow regime,” *Int. J. Multiphase Flow*, 22(2), pp. 285-306.

[10] Sirignano, W.A., 2014, “Advances in droplet array combustion theory and modeling,” *Progress in Energy and Combustion Science*, 42, pp. 54-86.

[11] Kleinstreuer, C., Comer, J.K., and Chiang, J., 1993, “Fluid dynamics and heat transfer with phase change of multiple spherical droplets in a laminar axisymmetric gas stream,” *Int. J. Heat Mass Transfer*, 14(3), pp. 292-300.

[12] Zhu, J.Y., and Dunn-Rankin, D., 1992, “temperature characteristics of a combusting droplet stream,” *24th Int. Symposium on Combustion*, pp. 1473-1481.