Laminar Free Convection from a Pair of Horizontal Cylinders: A Three-Temperature Problem

Seyed Sepehr Mohaddes Foroushani [1], David Naylor [2] and John L. Wright [1]

[1] *Department of Mechanical & Mechatronics Engineering, University of Waterloo, Waterloo, Canada*
[2] *Department of Mechanical & Industrial Engineering, Ryerson University, Toronto, Canada*

Email: *ssmohadd@uwaterloo.ca*

ABSTRACT

The formulation of multi-temperature convection problems in terms of a single driving temperature difference and a single heat transfer coefficient does not properly reflect the physics of the problem. Paired heat transfer coefficients, which designate both the source and the sink of heat transfer, are a suitable alternative. A numerical technique, namely dQdT, is proposed to compute such paired heat transfer coefficients. The dQdT technique entails a numerical solution of the governing equations and consequent solutions of the energy equation with perturbed boundary conditions. In the present study, dQdT is applied to the three-temperature problem of steady-state laminar free convection from two horizontal cylinders, with equal diameters but different surface temperatures, aligned vertically at a center-to-center spacing of two diameters. A number of moderate Rayleigh numbers, $2\times10^4<Ra<2\times10^5$, are considered. The baseline solutions are validated against experimental data from the literature. A grid convergence study is performed to assess the discretization error. The utility of dQdT in characterizing this three-temperature free convection problem, specifically in quantifying the interaction of the cylinders, is demonstrated. It has also been shown that the paired heat transfer coefficients obtained through dQdT provide detailed information about the local interaction of the cylinders, which are not available otherwise.

1. INTRODUCTION

Free-convective heat transfer from vertical arrays of horizontal cylinders has been the subject of numerous experimental and numerical studies. The vast attention this configuration has received is due to its widespread application in the industry in such areas as heating and refrigeration, nuclear reactors, and electronics cooling. Although studies of free convection from arrays of horizontal cylinders usually assume the same temperature for all the cylinders, this assumption is an idealization of the realistic performance conditions in the industry. The case where cylinders are at different temperatures has also been examined in a few studies, possibly starting with the work of Sparrow and Niethammer in 1981 [1]. Sparrow and Niethammer [1] performed experiments on the interaction of two asymmetrically heated horizontal cylinders aligned vertically, for a range of moderate Rayleigh numbers, $2\times10^4<Ra<2\times10^5$, to examine effects of spacing and temperature imbalance. Paykoc *et al* [2] studied the problem by supplying the same electrical power to both cylinders, leading to a difference in the surface temperatures. The effect of spacing on heat transfer at the cylinders was examined for $1.5\times10^4<Ra<3\times10^5$. Razelos [3] conducted an experimental study of the problem for $1.3\times10^2<Ra<1.3\times10^3$, leading to an empirical correlation for the temperature imbalance leading to maximum heat transfer enhancement at the upper cylinder. More recently, Yoon *et al* [4] used numerical simulation to investigate the effects of sizing on free convection from a vertical array of two asymmetrically heated cylinders in a square enclosure for $10^3<Ra<10^5$.

As reported by Sparrow and Niethammer [1], and confirmed by subsequent work of other researchers, for a center-to-center spacing of at least two diameters, the presence of the upper cylinder has almost no effect on heat transfer at the lower cylinder. The lower cylinder, on the other hand, can significantly influence heat transfer at the upper cylinder. Sparrow and Niethammer [1] suggested two mechanisms working in opposite directions for this influence. First, the acceleration of the fluid arriving at the upper cylinder due to the buoyancy created by the lower cylinder tends to enhance heat transfer at the upper cylinder. Second, the lower cylinder, acting as a preheater, heats the fluid approaching the upper cylinder, which degrades heat transfer at the upper cylinder.

In the present study, a new technique is used to examine free convection from a pair of horizontal cylinders with a temperature imbalance. The utility of this technique in investigating multi-temperature free convection problems, particularly in quantifying the mutual influence of the heat sources, is demonstrated.

2. PROBLEM STATEMENT

Steady-state laminar free convection from two horizontal isothermal cylinders of the same diameter, but at different surface temperatures, aligned vertically at a center-to-center spacing of two diameters is examined. The upper-cylinder Rayleigh number, defined by Equation 1, is used for a dimensionless presentation of the examined cases.

$$\mathrm{Ra} = \left[\frac{g\,\beta(T_U - T_\infty)D^3}{\upsilon^2}\right]\mathrm{Pr} \qquad (1)$$

In Equation 1, g is the gravitational acceleration, β is the thermal expansion coefficient, T_U is the temperature of the upper cylinder, T_∞ is the temperature of the far-field ambient, D is the cylinders diameter, υ is the kinematic viscosity, and Pr is the Prandtl number.

The imbalance between the temperature of the upper cylinder, T_U, and the temperature of the lower cylinder, T_L, is characterized by a temperature ratio, r, as shown in Equation 2.

$$r = \frac{T_L - T_\infty}{T_U - T_\infty} \qquad (2)$$

A range of moderate Rayleigh numbers, $2\times10^4<\mathrm{Ra}<2\times10^5$, and three temperature ratios, $r\epsilon\{0.5,1,2\}$, were examined at Pr=0.7.

3. METHODOLOGY

3.1 The dQdT Technique

When dealing with multi-temperature convection, *i.e.* convection problems involving more than two isothermal heat sources, it is useful to use *paired* heat transfer coefficients to characterize the problem. The term paired is used to refer to heat transfer between one source and another designated heat source. *Overall* heat transfer coefficients, on the other hand, correspond to heat transfer between one source and all the other sources. A paired heat transfer coefficient, h_{ij}, is defined by Equation 3.

$$h_{ij} = -\frac{1}{A_j}\frac{\partial Q_j}{\partial T_i}\bigg|_{h_{ij}=\mathrm{const}} \qquad (3)$$

In Equation 3, A_j is the surface area of source j.

A technique is developed for calculating the paired heat transfer coefficient, h_{ij}. This technique, which gives heat transfer coefficients as the ratio between finite differences between heat transfer rates and temperatures, δQ and δT, is dubbed dQdT. The dQdT technique entails a numerical solution of the full set of governing equations and consequent solutions of the energy equation with perturbed boundary conditions. Using dQdT, the condition of h_{ij}=const in Equation 3 is satisfied by solving the energy equation while the flow field and the fluid properties are fixed, *i.e.* prevented from changing after a perturbation is introduced in the temperature boundary conditions. Thus, to calculate h_{ij}, a solution of the full set of governing equations (the *baseline* solution) is first obtained. Q_j is then computed by integrating the temperature gradient at source j. Next, T_i is perturbed by some finite amount, δT_i, and the energy equation only is solved again, using the velocity field and temperature-dependent fluid properties from the baseline solution. The new rate of heat transfer at source j will differ from the baseline case by some amount, δQ_j. The paired heat transfer coefficient h_{ij} can then be calculated as shown in Equation 4.

$$h_{ij} = -\frac{1}{A_j}\frac{\delta Q_j}{\delta T_i} \qquad (4)$$

Note that h_{ij} quantifies the influence of the temperature of one heat source, T_i, on heat transfer from another source, Q_j. h_{ij} is converted to dimensionless form, the paired Nusselt number Nu_{ij}, using Equation 5.

$$Nu_{ij} = \frac{h_{ij}D}{k} \qquad (5)$$

3.2 Numerical Solution

The commercial CFD code ANSYS FLUENT 14.0 [5] was used to solve the steady-state equations of the conservation of mass, momentum and energy, using constant fluid properties and the Boussinesq approximation. The SIMPLE scheme was used for pressure-velocity coupling, along with second-order discretization schemes for all of the equations.

Assuming large length-to-diameter ratios, a 2D model of the configuration of is considered. Due to symmetry, and in order to reduce the computational expense, solutions are obtained around half cylinders, with the line connecting the cylinder centers set as a symmetry boundary condition. The other three sides of the domain are set as pressure boundary conditions at zero gauge pressure and the far-field ambient temperature, T_∞. The cylinders are modeled as impermeable no-slip isothermal walls. A schematic of the computational domain is shown in Figure 1a.

A grid convergence study was conducted using the Richardson-extrapolation scheme recommended by the *Journal of Fluids Engineering* [6]. Using three non-uniform unstructured triangular grids with average sizes of 0.04D, 0.02D and 0.01D, and based on the sum of total heat transfer rates at the two cylinders, a grid convergence index of 2% was obtained. The medium grid (0.02D) was used to obtain the subsequent results. The structure of the mesh is shown in Figure 1b. The local Nusselt number at the upper cylinder at $Ra=10^5$ and r=2, as obtained from the three grids, is shown in Figure 2. Note that these Nusselt numbers are the "traditional" overall Nusselt numbers, defined by Equation 6.

$$Nu_j = \left[\frac{Q_j/A_j}{(T_j - T_\infty)}\right]\frac{D}{k} \qquad (6)$$

3.3 Validation

The numerical solutions were validated against the experimental data reported by Sparrow and Niethammer [1]. In Table 1, the ratio of the upper-cylinder overall Nusselt number, Nu_U, to the total Nusselt number of an isolated horizontal cylinder, Nu_0, as measured by Sparrow and Niethammer [1], is shown for $Ra=10^5$ and different temperature ratios. Note that Nu_U and Nu_0 are both computed based on Equation 6. Close agreement is observed between the present CFD solutions and the experimental data.

r	**Nu_U/Nu_0**	
	CFD (present)	Experiment [1]
0.5	0.99	0.98
1.0	0.83	0.86
2.0	0.36	0.39

Table 1 – Upper-cylinder overall Nusselt number ($Ra=10^5$)

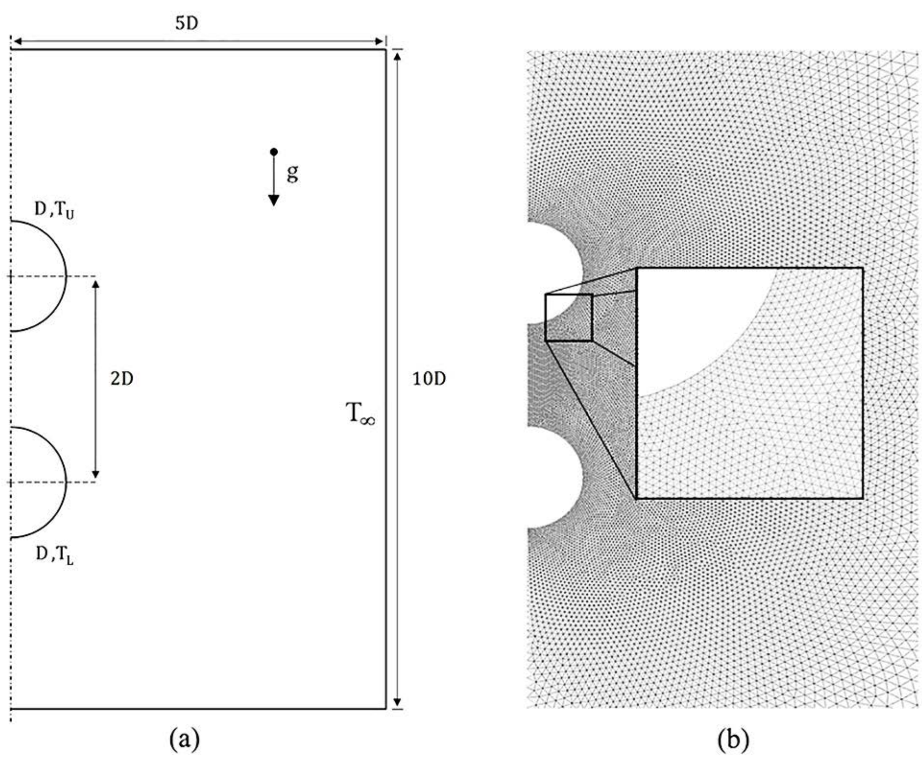

Figure 1 – Schematic of the computational domain (a) and mesh structure (b)

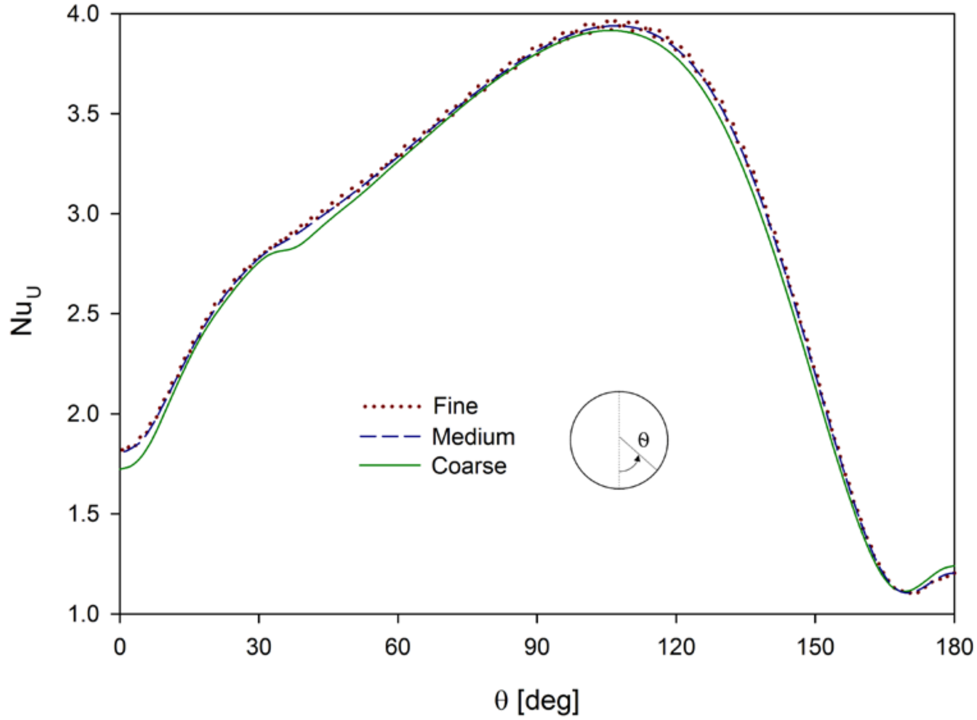

Figure 2 – Local Nusselt number at the upper cylinder ($Ra=10^5$, $r=2$)

4. Results and Discussion

This problem comprises three isothermal heat sources: the upper cylinder at T_U, the lower cylinder at T_L, and the far-field ambient fluid at T_∞. Correspondingly, six paired heat transfer coefficients may be defined to quantity the mutual influences of the heat sources. In Figure 3, the two paired Nusselt numbers related to the interaction between the two cylinders are plotted against the upper-cylinder Rayleigh number, at different temperature ratios. As known from previous studies, *e.g.* reference [1], the upper cylinder has no effect on heat transfer at the lower cylinder, which is reflected by $Nu_{UL}=0$. The influence of the lower cylinder on heat transfer at the upper cylinder follows a different trend. It can be seen from Figure 3 that Nu_{LU}, characterizing the effect of T_L on Q_U, increases with both Ra and r, which is, again, in agreement with the previously published results, *e.g.* reference [1].

In Figure 4, $Nu_{L\infty}$ and $Nu_{\infty L}$, quantifying the interaction between the lower cylinder and the fluid, are plotted against Ra for various values of r. The same general trend, *i.e.* enhancement as Ra and r increase, is observed for both Nusselt numbers. Note that $Nu_{\infty L}>Nu_{L\infty}$, meaning that Q_L is more sensitive to T_∞ than Q_∞ is to T_L. The situation is reverse at the upper cylinder where $Nu_{\infty U}<Nu_{U\infty}$, as seen in the plots of Figure 5. It is noteworthy that, in previous work on forced convection from pairs of spheres [7], similar results were observed for “upstream” and “downstream” bodies.

Comparing $\{Nu_{L\infty},Nu_{\infty L}\}$ and $\{Nu_{U\infty},Nu_{\infty U}\}$ a stronger interaction between the upper cylinder and the fluid is observed compared to the interaction of the lower cylinder and the fluid. This is contrary to the aforementioned results for forced convection (reference [7]), possibly because here the upper cylinder, the “downstream” body, itself generates buoyancy and enhances the flow.

It is noteworthy that $Nu_{U\infty}>Nu_{L\infty}$ implies higher sensitivity of the total heat transfer from the pair, Q_∞, to the temperature of the upper cylinder, which can be of practical significance.

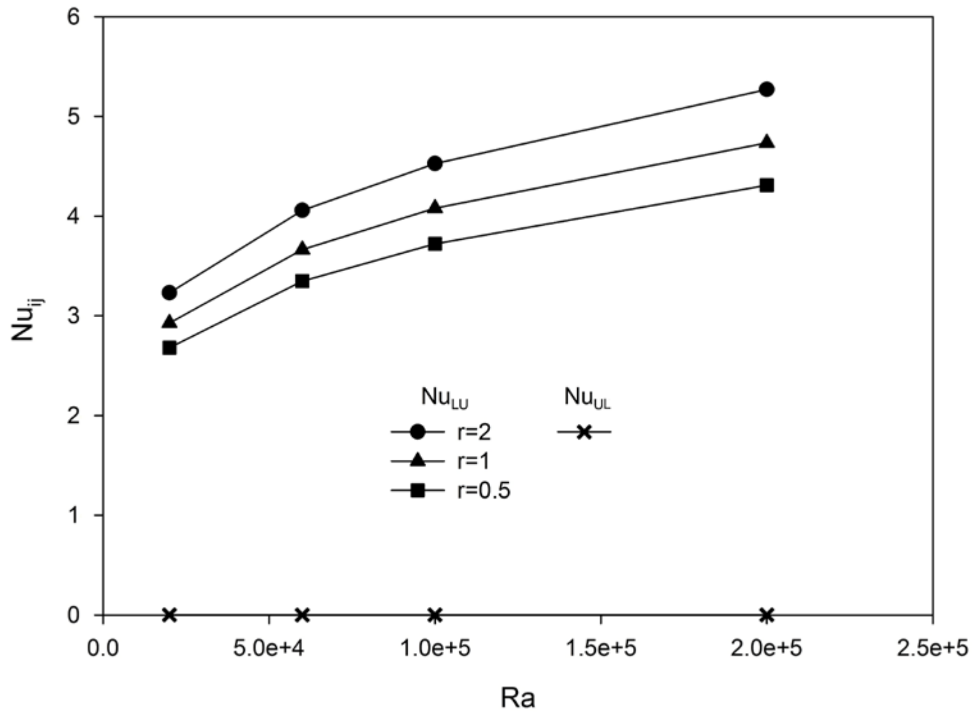

Figure 3 – Cylinder-cylinder Nusselt numbers

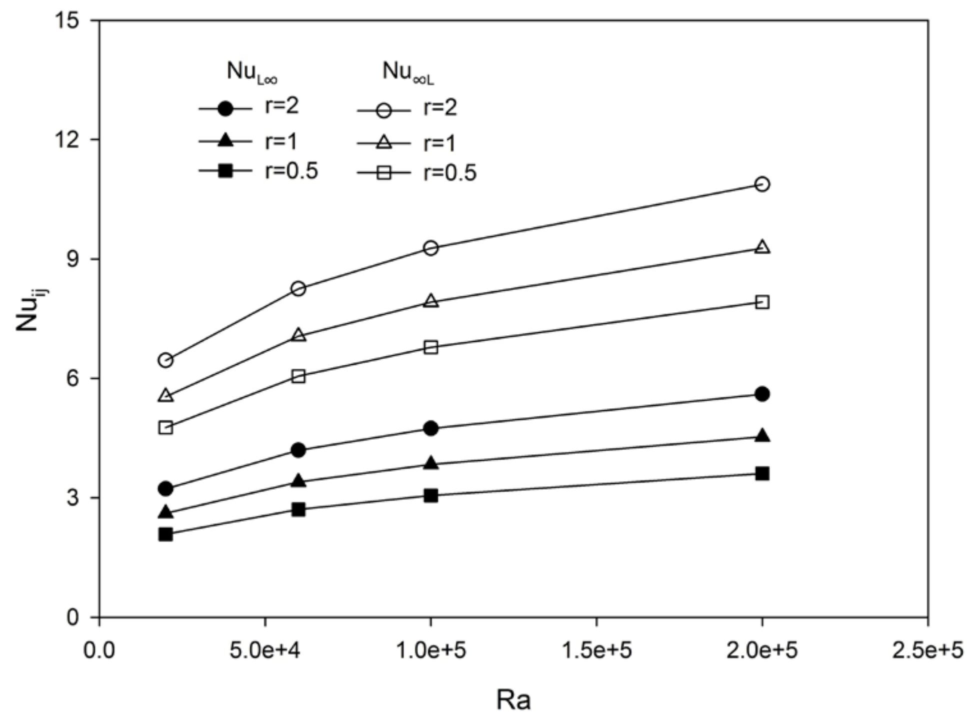

Figure 4 – Cylinder-fluid Nusselt numbers at the lower cylinder

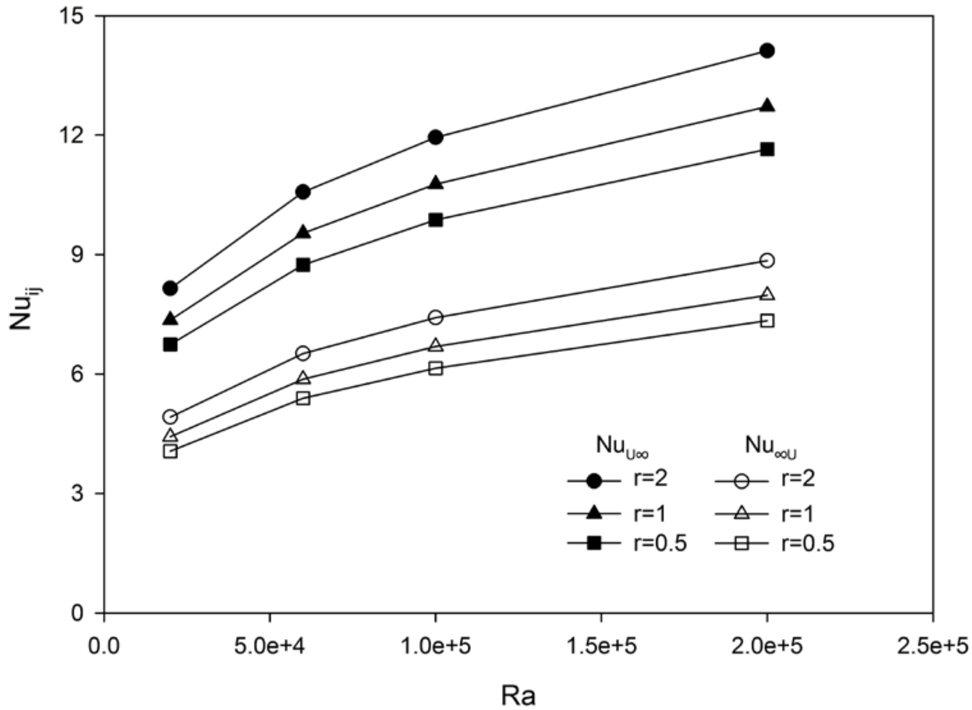

Figure 5 – Cylinder-fluid Nusselt numbers at the upper cylinder

Aside from providing a quantified measure of the interaction between the cylinders, the dQdT results presented in Figures 3 to 5 are not different in essence from the previously published results obtained via conventional techniques. Nevertheless, dQdT can be used to obtain further insight into the physics of the problem – information that is generally not available through conventional methods. For example, dQdT can be used to obtain local paired heat transfer coefficients if the division by A_j is removed from Equations 3 and 4 and the total heat transfer rate, Q_j, is replaced by the local heat flux, q_j. These local coefficients will reveal the local influence of heat sources on each other. In Figure 6 the distribution of paired Nusselt numbers, Nu_{ij}, and overall Nusselt numbers, Nu_j, on the cylinders are shown for $Ra=10^5$ and r=2. Note that for the two curves pertaining to the lower cylinder, the horizontal axis corresponds to angular location on the lower cylinder, while for the other three curves, the horizontal axis shows location on the upper cylinder.

Based on the results shown in Figure 6, the cylinder-fluid Nusselt number and the overall Nusselt number at the lower cylinder are equal: $Nu_{\infty L}=Nu_L$. Note these two Nusselt numbers are defined according to Equation 5 and Equation 6 respectively. Also note that at the lower cylinder, the cylinder-cylinder Nusselt number is zero: $Nu_{UL}=0$. Furthermore, at the lower cylinder, both maximum heat transfer and maximum sensitivity to the ambient temperature are observed at $\theta=0°$. At the upper cylinder, however, the cylinder-cylinder Nusselt number, Nu_{LU}, is nonzero. Note that Nu_{LU} is comparable in magnitude to $Nu_{\infty U}$. Moreover, while heat flux at the upper cylinder, q_U, is maximum at $\theta\approx110°$, q_U is most sensitive to both T_∞ and T_L at $\theta=0°$. Moving from $\theta=0°$ to $\theta=180°$ on the upper cylinder, the influence of T_∞ and T_L on q_U diminishes. Such detailed information about the interaction of heat sources in a multi-temperature convection problem, in this case the two cylinders, cannot be inferred from overall heat transfer coefficients and Nusselt numbers.

5. Conclusion

This study was conducted to demonstrate the utility of a new technique, namely the dQdT technique, in the study of multi-temperature free convection. dQdT was used to examine the three-temperature problem of steady-state free-convective heat transfer from a pair of asymmetrically heated, vertically aligned horizontal cylinders. Moderate Rayleigh numbers and different temperature arrangements were considered. Numerical solutions were validated against experimental data from the literature. Through paired Nusselt numbers obtained from dQdT, it was shown that while the upper cylinder has no effect on heat transfer at the lower cylinder, the lower cylinder can influence heat transfer at the upper cylinder depending on the flow rate and the temperature ratio. The observed trends are in agreement with the reported results in the literature. It was further shown that total heat transfer from the pair is more sensitive to the temperature of the upper cylinder than to that of the lower cylinder. Likewise, compared to its effect on heat transfer at the lower cylinder, the ambient temperature has a larger effect on heat transfer at upper cylinder. The utility of dQdT in providing detailed information about the local interaction of the cylinders was also demonstrated.

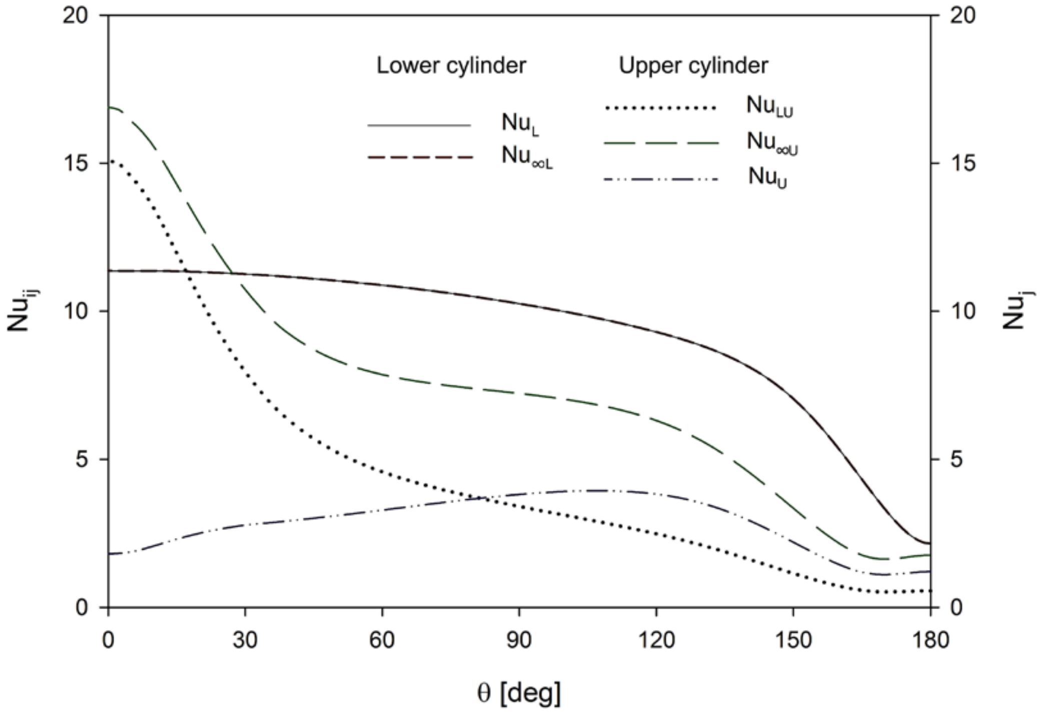

Figure 6 – Local paired and overall Nusselt numbers at the two cylinders

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