

Calculating Center-Glass Performance Indices of Windows with a Diathermanous Layer

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ABSTRACT

The fenestration chapter of the 2005 ASHRAE Handbook—Fundamentals (ASHRAE 2005) has long included methods for determining the U-factor and solar heat gain coefficient (SHGC), or window performance indices, using the radiative and convective heat transfer coefficients around a glazing layer. The present work examines the errors inherent in applying these standard calculation methods to window systems that include a single diathermanous layer (such as a shading layer), and new equations for calculating the performance indices are derived. Furthermore, the radiative heat transfer coefficients used in these calculations can be difficult to determine in the presence of a diathermanous layer. Therefore, a new and stable method of calculating radiative heat transfer coefficients is also presented. The effects of using the existing procedures are demonstrated using industry-standard software.

INTRODUCTION

Mounting a shading device adjacent to the indoor surface of a window, such as a venetian blind, is common practice for providing privacy and controlling glare and daylighting. The presence of these shading devices will also affect the solar heat gain (SHG) and thermal transmittance (U-factor) of the window system. Due to the complexity of the systems, however, reliable and approximate methods of predicting the potential solar and thermal benefits of some shades have only recently been developed (Collins 2004; Huang et al. 2006; Wright and Kotey 2006; Tasnim and Collins 2004). Research aimed at broadening the scope of such methods is ongoing (Wright and Collins 2004).

Existing analysis procedures are based on the assumption that all of the layers in a window are opaque to long-wave radiation layers. Therefore, separation can occur between the solar and thermal aspects of the problem. Solar radiation can be traced through a series of glass layers, and transmitted, reflected, and absorbed amounts of radiation can be determined. The absorbed radiation in each layer can then be used as input to an energy balance around each glazing layer. In this scenario, each glazing layer can only communicate thermally with the layers adjacent to it. Unfortunately, a shading layer such as a venetian blind is diathermanous by virtue of its openness. *Diathermanous* refers to a material that transmits both solar and long-wave radiant energy. The addition of such a layer means that glazing layers can also communicate with layers that are not adjacent, and significant complexity results in the heat transfer analysis.

One needs only to examine industry-standard software to experience the magnitude of this problem. Figures 1 and 2 show the calculated U-factor and SHGCs as a function of incident solar intensity for two windows using FRAMEplus 5.1 (Wright 1994), WINDOW 5.2 (Finlayson et al. 1993), and WIS 3.01 (Rosenfeld et al. 2000). The first window is a generic triple glazing, while the second has a diathermanous center layer. The details of the windows are provided in Table 1. The three programs are consistent in the case of the generic triple glazing, and in general, each does an excellent job of predicting the performance of systems without diathermanous layers. The only significant difference is that the U-factor predicted by FRAMEplus is specific to the environmental conditions, while WIS and WINDOW present a nighttime U-factor; that is, FRAMEplus factors in the influence of solar-heated window layers on the heat transfer coefficients that are used to

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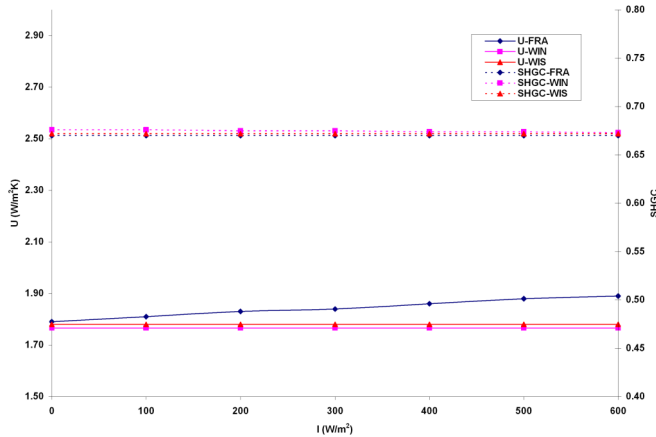


Figure 1 *U-factor and SHGC comparison of FRAMEplus, WINDOW, and WIS for a triple glazing. System properties are given in Table 1.*

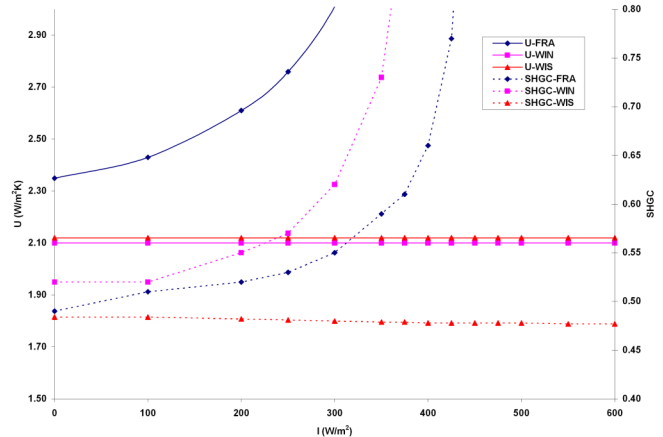


Figure 2 *U-factor and SHGC comparison of FRAMEplus, WINDOW, and WIS for a triple glazing with diathermanous center layer. System properties are given in Table 1.*

Table 1. Model Conditions for Software Comparison

	τ_s/ρ_s	τ_v/ρ_v	$\tau_{bw}/\rho_{bw}/\varepsilon$	k (W/m·K) or h (W/m ² ·K)	Thickness (mm)
Indoor glass	0.83/0.07	0.90/0.08	0.00/0.16/0.84	0.9	3.05
Air				Wright,1996	12.7
Middle layer clear glass	0.83/0.07	0.90/0.08	0.00/0.16/0.84	0.9	3.05
Middle layer diathermanous	0.40/0.05	0.40/0.05	0.41/0.05/0.54	0.9	3.05
Air				Wright,1996	12.7
Outdoor glass	0.83/0.07	0.90/0.08	0.00/0.16/0.84	0.9	3.05
	T (°C)	h (W/m ² ·K)			
Indoors	21	8			
Outdoors	-18	30			

calculate the U-factor.¹ In this regard, it is noted that all three pieces of software produce similar results when the solar intensity is zero. The effect of the calculation procedure is evident in the case of the second window. Both WINDOW and FRAMEplus produce results that are obviously incorrect. The results from WIS seem stable, but it is uncertain as to whether they are accurate. A more detailed explanation of why the software behaves in this manner will be presented in the following sections.

PERFORMANCE INDICES

The first step in analyzing the center-of-glass (CoG) region of a fenestration system is to determine the solar/optical

¹. A comprehensive discussion of how U-factor and inward-flowing fraction are affected by daytime and nighttime conditions can be found on pages 44–47 of Hollands et al. (2001).

performance of the system. This analysis follows the reflected, absorbed, and transmitted components of incident solar radiation and accounts for multiple reflections within the glazing system. This results in the fenestration's solar transmission (τ_s) and the absorbed solar flux in each layer (S_i).

The heat flow and glazing layer temperatures are found using a one-dimensional analysis. Each surface (i.e., two per glass layer) is assigned a node, and nodal temperatures are estimated. Radiative heat flux rates (J_i) and convective heat transfer coefficients (h_i) are then calculated based on fundamental relations and empirical correlations, respectively. An energy balance is subsequently formed at each node that includes convective, radiative, and conductive heat transfer as well as the absorbed incident flux determined in the solar/optical balance. Finally, the energy balance is used to determine new nodal temperatures, and the process is repeated until convergence occurs. Once converged, the user is usually left

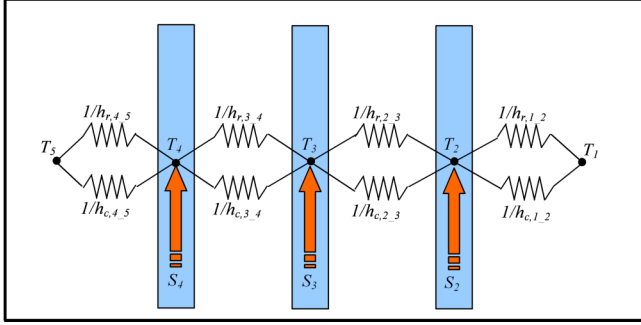


Figure 3 Thermal resistance network for a typical window without diathermanous layers.

with the nodal temperatures, convective heat transfer coefficients and convective heat flux, radiative heat transfer coefficients and surface radiosities, and nondimensional heat transfer indices such as the Nusselt (Nu) and Rayleigh (Ra) numbers.

The calculation of these parameters has been covered in a number of references and will not be reexamined here. The reader is referred to Edwards (1977) for a description of how to perform the optical analysis of fenestration and Hollands and Wright (1980) or Rubin (1982) for details of the energy balance procedure. Details of the standard performance indices calculation are given in Wright (1998). It is sufficient to say that the reader, when calculating window performance indices, is in possession of the data from a converged energy analysis.

For fenestration without diathermanous layers, the calculation of standard window performance indices is relatively simple. Because window glass is opaque to long-wave radiation, a layer of glass can only communicate thermally with the layers adjacent to it. The thermal network for the system, shown in Figure 3, is a string of series resistors where each resistor is given by $1/(h_{c,i,j} + h_{r,i,j})$. Examination of this system reveals that there are three unknown temperatures (T_2 , T_3 , and T_4) and four unknown heat fluxes ($q_{1,2}$, $q_{2,3}$, $q_{3,4}$, and $q_{4,5}$) and the following equations:

$$\begin{aligned} q_{1,2} &= \frac{T_1 - T_2}{R_{1,2}} & q_{2,3} &= \frac{T_2 - T_3}{R_{2,3}} \\ q_{3,4} &= \frac{T_3 - T_4}{R_{3,4}} & q_{4,5} &= \frac{T_4 - T_5}{R_{4,5}} \end{aligned} \quad (1)$$

$$q_{1,2} + S_2 = q_{2,3} \quad q_{2,3} + S_3 = q_{3,4} \quad q_{3,4} + S_4 = q_{4,5} \quad (2)$$

Solving for $q_{1,2}$ in terms of T_1 , T_5 , S_2 , S_3 , S_4 , and the R s is performed by first combining the heat fluxes given in Equation 1 to give

$$q_{1,2}R_{1,2} + q_{2,3}R_{2,3} + q_{3,4}R_{3,4} + q_{4,5}R_{4,5} = (T_1 - T_5). \quad (3)$$

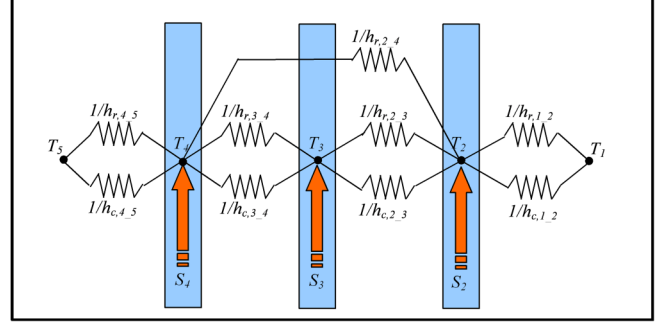


Figure 4 Thermal resistance network for a typical window where the center glazing is a diathermanous layer.

Then $q_{2,3}$, $q_{3,4}$, and $q_{4,5}$ are removed by substituting Equation 2 into Equation 3. After reorganization, we are left with

$$\begin{aligned} q_{1,2} &= U_{tot}(T_1 - T_5) - S_2(R_{2,3} + R_{3,4} + R_{4,5}) - S_3(R_{3,4} + R_{4,5}) - S_4(R_{4,5}) \\ &= U_{tot}(T_1 - T_5) - S_2N_2 - S_3N_3 - S_4N_4 \end{aligned} \quad (4)$$

where

$$U_{tot} = \left(\sum_{i=1}^{N-1} R_{i,i+1} \right)^{-1} \quad (5)$$

$$N_i = U_{tot} \left(\sum_{j=i}^{N-1} R_{j,j+1} \right). \quad (6)$$

When the system includes a diathermanous layer, the thermal network becomes more complex, as shown in Figure 4. A thermal resistor must be inserted to connect the layers on either side of it. The new resistor is shown as being radiative heat transfer only.² Examination of this system reveals that there are three unknown temperatures (T_2 , T_3 , and T_4) and five unknown heat fluxes ($q_{1,2}$, $q_{2,3}$, $q_{3,4}$, $q_{4,5}$, and $q_{2,4}$) and the following equations:

$$\begin{aligned} q_{1,2} &= \frac{T_1 - T_2}{R_{1,2}} & q_{2,3} &= \frac{T_2 - T_3}{R_{2,3}} & q_{3,4} &= \frac{T_3 - T_4}{R_{3,4}} \\ q_{4,5} &= \frac{T_4 - T_5}{R_{4,5}} & q_{2,4} &= \frac{T_2 - T_4}{R_{2,4}} \end{aligned} \quad (7)$$

$$\begin{aligned} q_{1,2} + S_2 &= q_{2,3} + q_{2,4} & q_{2,3} + S_3 &= q_{3,4} \\ q_{3,4} + q_{2,4} + S_4 &= q_{4,5} \end{aligned} \quad (8)$$

Solving for $q_{1,2}$ in terms of T_1 , T_5 , S_2 , S_3 , S_4 , and the R s is performed by first combining (most of) the heat fluxes given in Equation 7 to give

$$q_{1,2}R_{1,2} + q_{2,3}R_{2,3} + q_{3,4}R_{3,4} + q_{4,5}R_{4,5} = (T_1 - T_5). \quad (9)$$

² The analysis can be expanded to include a convectively based resistor that would represent the presence of a porous layer.

Then $q_{4,5}$, $q_{3,4}$, and $q_{2,3}$ are removed by substituting Equation 8 into Equation 9. Unfortunately, the resulting equation still has the $q_{2,4}$ term included.

$$q_{1,2}(R_{1,2} + R_{2,3} + R_{3,4} + R_{4,5}) - q_{2,4}(R_{2,3} + R_{3,4}) + S_2(R_{2,3} + R_{3,4} + R_{4,5}) + S_3(R_{3,4} + R_{4,5}) + S_4(R_{4,5}) = (T_1 - T_5) \quad (10)$$

To remove $q_{2,4}$, we recognize that

$$q_{2,4}R_{2,4} = (T_2 - T_4) = q_{2,3}R_{2,3} + q_{3,4}R_{3,4}. \quad (11)$$

Substituting in the energy balances of Equation 8, we get:

$$q_{2,4} = q_{1,2} \frac{(R_{2,3} + R_{3,4})}{(R_{2,3} + R_{3,4} + R_{2,4})} + S_2 \frac{(R_{2,3} + R_{3,4})}{(R_{2,3} + R_{3,4} + R_{2,4})} + S_3 \frac{(R_{3,4})}{(R_{2,3} + R_{3,4} + R_{2,4})} \quad (12)$$

Finally, substituting Equation 12 into Equation 10 and reorganizing into the general form of Equation 4, we get:

$$q_{1,2} = U_{tot}(T_1 - T_5) - S_2 U_{tot} \left(R_{2,3} + R_{3,4} + R_{4,5} - \frac{(R_{2,3} + R_{3,4})^2}{R_{2,4} + R_{2,3} + R_{3,4}} \right) - S_3 U_{tot} \left(R_{3,4} + R_{4,5} - \frac{R_{3,4}(R_{2,3} + R_{3,4})}{R_{2,4} + R_{2,3} + R_{3,4}} \right) - S_4 U_{tot}(R_{4,5}) = U_{tot}(T_1 - T_5) - S_2 N_2 - S_3 N_3 - S_4 N_4 \quad (13)$$

where

$$U_{tot} = \left(R_{1,2} + R_{2,3} + R_{3,4} + R_{4,5} - \frac{(R_{2,3} + R_{3,4})^2}{R_{2,4} + R_{2,3} + R_{3,4}} \right)^{-1} \quad (14)$$

The performance indices for any system of $N-2$ layers (1 being the indoors, N being the outdoors) that contains a single diathermanous layer located at $i = k$ can be expressed in a generic form. The inward flowing fraction of each layer is given by

$$N_i = U_{tot} \left(\sum_{j=i}^{N-1} R_{j,j+1} \right) \text{ for } N > i > k, \quad (15)$$

$$N_i = U_{tot} \left[\left(\sum_{j=i}^{N-1} R_{j,j+1} \right) - \left(\frac{R_{k,k+1}(R_{k-1,k} + R_{k,k+1})}{R_{k-1,k+1} + R_{k-1,k} + R_{k,k+1}} \right) \right] \text{ for } i = k, \text{ and} \quad (16)$$

$$N_i = U_{tot} \left[\left(\sum_{j=i}^{N-1} R_{j,j+1} \right) - \left(\frac{(R_{k-1,k} + R_{k,k+1})^2}{R_{k-1,k+1} + R_{k-1,k} + R_{k,k+1}} \right) \right] \text{ for } 1 < i < k, \quad (17)$$

and U_{tot} is given by

$$U_{tot} = \left[\left(\sum_{j=1}^{N-1} R_{j,j+1} \right) - \frac{(R_{k-1,k} + R_{k,k+1})^2}{R_{k-1,k+1} + R_{k-1,k} + R_{k,k+1}} \right]^{-1}. \quad (18)$$

RADIATIVE HEAT TRANSFER COEFFICIENTS

In order to apply Equations 15 to 18, the long-wave radiant energy exchange (radiosities) from each surface must be recast in the form of thermal resistors or radiation heat transfer coefficients. To do this, however, extreme caution must be exercised. Consider the radiosity (J) balance of the window cavity shown in Figure 5. For clarity, convective heat transfer has been omitted.

- If it is assumed that $h_{r,2,4}$ can be neglected, the radiative heat transfer coefficients are the differences in the front and back radiosities divided by the temperature difference between the layers: $h_{r,2,3} = (J_{2f} - J_{3b}) / (T_2 - T_3)$ and $h_{r,3,4} = (J_{3f} - J_{4b}) / (T_3 - T_4)$. Using Equations 5 and 6, the system U-factor and inward-flowing fractions can then be determined. Because layer 3 is diathermanous, however, T_2 can equal T_3 while $J_{2f} \neq J_{3b}$, or T_3 can equal T_4 while $J_{3f} \neq J_{4b}$. Essentially, while the emitted radiation from the two layers will be equal when there is no temperature difference, there is still a reflected and/or transmitted component from both layers due to the radiosity of the third surface transmitting energy through the diathermanous layer. By extension, there can also be radiative heat transfer in one direction while the temperature difference suggests it should be in the other. Division by zero and negative heat transfer coefficients can result, both of which can have a significant impact on the calculation of U and N_i .
- If $h_{r,2,4}$ is not neglected, the problem still exists. Here, the radiative heat transfer coefficient between layers 2 to 4 would be the difference in radiosity between these surfaces that is transmitted through surface 3 divided by their temperature difference: $h_{r,2,4} = (J_{2f} - J_{4b}) \tau_3 / (T_2 - T_4)$. The remaining heat transfer coefficients are modified to

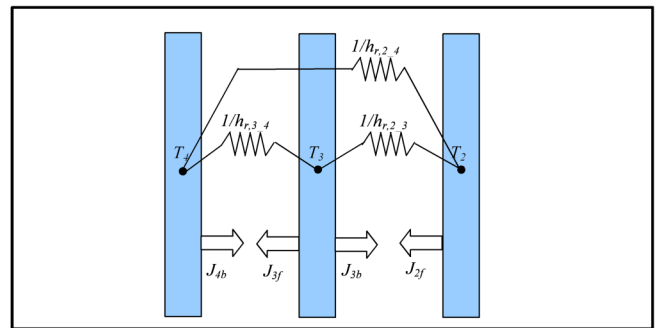


Figure 5 Thermal resistance network for a typical window where the center glazing is a diathermanous layer.

remove this transmitted radiation: $h_{r,2_3} = (J_{2f}(1 - \tau_3) - (J_{3b} - J_{4b}\tau_3))/(T_2 - T_3)$ and $h_{r,3_4} = ((J_{3f} - J_{2f}\tau_3) - J_{4b}(1 - \tau_3))/(T_3 - T_4)$. In this scenario the situation, for example, where T_2 can equal T_3 while $J_{2f} \neq J_{3b}$ is still present. Inter-reflection of emitted radiation from a third surface can still result in heat transfer in the presence of no temperature difference or heat transfer in a direction opposite to that suggested by the temperature difference.

Adding more detail to the resistance network cannot account for the inter-reflection that occurs between surfaces. For example, a resistance network that includes the terms $h_{r,2_4}$, $h_{r,2_3b}$, $h_{r,2_3f}$, $h_{r,3f_4}$, and $h_{r,3b_4}$ (a five-resistor network) was derived and the results compared to the two- and three-resistor networks discussed above. Figure 6 demonstrates this comparison. The U-factor calculated is from outdoor glass to indoor glass and includes radiative heat transfer coefficients only. The long-wave optical properties of the layers are the same as those presented in Table 1. The indoor and outdoor glass temperatures are 293 K and 273 K, respectively. Adding complexity to the network increases the stability of the analysis near the discontinuity, but errors continue to occur when small temperature differences are present between layers.

To reduce the infinite number of inter-reflections that occur in the system to absolute terms of long-wave radiation exchange between two surfaces, a more elegant approach is proposed. A more stable analysis involves revisiting the radiosity balance with only one surface emitting at a time. If it were assumed that all of the surfaces except (for example) surface 2 had a temperature of 0 K, then only surface 2 would have an emitted component of radiosity based on its known temperature. The radiosity of all other surfaces would only include reflected components. In reference to Figure 5, the following relations apply.

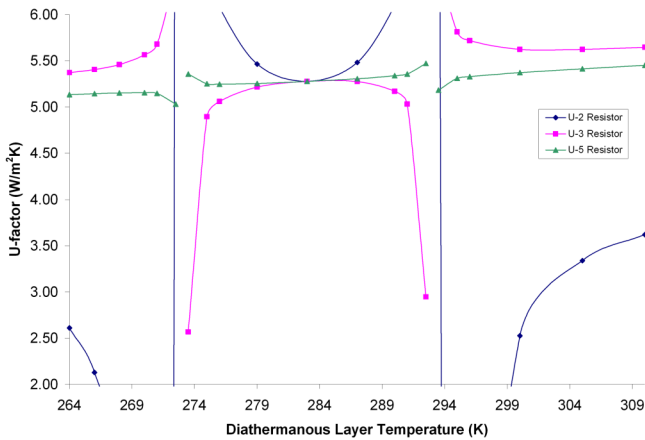


Figure 6 U-factor comparison by radiosity reduction technique. Convection is omitted.

$$\begin{aligned} J_{2f}^{(2)} &= \varepsilon_{2f}\sigma T_2^A + \rho_{2f}J_{3b}^{(2)} \\ J_{3b}^{(2)} &= \rho_{3b}J_{2f}^{(2)} + \tau_3J_{4b}^{(2)} \\ J_{3f}^{(2)} &= \rho_{3f}J_{4b}^{(2)} + \tau_3J_{2f}^{(2)} \\ J_{4b}^{(2)} &= \rho_{4b}J_{3f}^{(2)} \end{aligned} \quad (19)$$

The superscript (2) refers to the surface from which the radiant energy entered the system, while the subscripts f and b refer to front and back, respectively. $J_{4b}^{(2)}$ can be interpreted to be the radiosity of surface 4 that results from emission from surface 2. Continuing this procedure for the remaining three surfaces, we get

$$\begin{aligned} J_{2f}^{(3b)} &= \rho_{2f}J_{3b}^{(3b)} \\ J_{3b}^{(3b)} &= \varepsilon_{3b}\sigma T_3^A + \rho_{3b}J_{2f}^{(3b)} + \tau_3J_{4b}^{(3b)} \\ J_{3f}^{(3b)} &= \rho_{3f}J_{4b}^{(3b)} + \tau_3J_{2f}^{(3b)} \\ J_{4b}^{(3b)} &= \rho_{4b}J_{3f}^{(3b)} \end{aligned} \quad (20)$$

$$\begin{aligned} J_{2f}^{(3f)} &= \rho_{2f}J_{3b}^{(3f)} \\ J_{3b}^{(3f)} &= \rho_{3b}J_{2f}^{(3f)} + \tau_3J_{4b}^{(3f)} \\ J_{3f}^{(3f)} &= \varepsilon_{3f}\sigma T_3^A + \rho_{3f}J_{4b}^{(3f)} + \tau_3J_{2f}^{(3f)} \\ J_{4b}^{(3f)} &= \rho_{4b}J_{3f}^{(3f)} \end{aligned} \quad (21)$$

$$\begin{aligned} J_{2f}^{(4)} &= \rho_{2f}J_{3b}^{(4)} \\ J_{3b}^{(4)} &= \rho_{3b}J_{2f}^{(4)} + \tau_3J_{4b}^{(4)} \\ \text{and} \quad J_{3f}^{(4)} &= \rho_{3f}J_{4b}^{(4)} + \tau_3J_{2f}^{(4)} \\ J_{4b}^{(4)} &= \varepsilon_{4b}\sigma T_4^A + \rho_{4b}J_{3f}^{(4)} \end{aligned} \quad (22)$$

With this information, the direct radiative transfer between surfaces can be easily determined as a three-resistor system. Consider $h_{r,2_4}$. It is the difference in the net radiosity originating from surface 2 that reaches surface 4, and the net radiosity originating from surface 4 that reaches surface 2, divided by the temperature difference between surfaces 2 and 4.

$$h_{r,2_4} = \frac{(J_{3f}^{(2)} - J_{4b}^{(2)}) - (J_{3b}^{(4)} - J_{2f}^{(4)})}{(T_2 - T_4)} \quad (23)$$

Similarly,

$$h_{r,2_3} = \frac{(J_{2f}^{(2)} + J_{4b}^{(2)} - J_{3b}^{(2)} - J_{3f}^{(2)}) - (J_{3b}^{(3b)} + J_{3f}^{(3f)} - J_{2f}^{(3b)} - J_{2f}^{(3f)})}{(T_2 - T_3)} \quad (24)$$

$$h_{r,4_3} = \frac{(J_{2f}^{(4)} + J_{4b}^{(4)} - J_{3b}^{(4)} - J_{3f}^{(4)}) - (J_{3f}^{(3b)} + J_{3f}^{(3f)} - J_{4b}^{(3b)} - J_{4b}^{(3f)})}{(T_2 - T_3)} \quad (25)$$

This method has been dubbed the R^∞ method because it inherently catches all of the radiant transfer between surfaces, regardless of the number of inter-reflections that have occurred in the process.

Figure 7 is an update of Figure 6 including the results produced using the R^∞ method. As can be seen, the process is far more stable than any of the previously described techniques. Near the point where a division by 0 discontinuity would occur, the method behaves exceptionally well. Care must still be taken, however, to avoid division by zero when inserting this into a numerical routine.

At this point, it is useful to revisit the comparison of data produced using FRAMEplus, WINDOW, and WIS presented in Figure 2. This time, results predicted using the R^∞ method have been included. The conditions of the control are shown in Table 1. The results are shown in Figure 8.

While daytime U-factors can be easily calculated, standards call for the use of a nighttime U-factor (ISO 2000). Unfortunately, by doing this, information about the time-of-use performance characteristics, which may be useful to non-rating exercises such as building energy modeling, is lost. In this regard, both WIS and WINDOW quote a nighttime U-factor that is not a function of the incident solar irradiation. Under nighttime conditions, the diathermanous layer must be at some intermediate temperature between the indoor and outdoor glazings, and layers cannot be equal in temperature. Therefore, the calculations are not carried out for situations that would result in a discontinuity. Using Equation 18 to determine the U-factor, the R^∞ method predicts a nighttime U-factor that is within 1% of that predicted by WINDOW or WIS. FRAMEplus calculates a daytime U-factor using a two-resistor reduction of the radiosity balance. As such, it is extremely vulnerable to a division by 0 discontinuity, as demonstrated in Figure 8. Despite the presence of a discontinuity,

the R^∞ method can predict both nighttime and daytime U-factors with little problem.

SHGC calculations also rely on a thermal network of the system and are therefore also subject to the possibility of a discontinuity error. Not surprisingly, the SHGC predicted by FRAMEplus changes significantly for the given situation. The fact that WINDOW, like FRAMEplus, also uses a two-resistor reduction is demonstrated by the SHGC results. WIS again seems to account for this effect very well and even shows a slight reduction in the SHGC with increased solar irradiation. Unfortunately, the mechanisms behind the WIS solution routine are not known, so it is impossible to know if the results are by design or by luck. The R^∞ method also predicts the decrease in SHGC with increasing irradiation without problem. Regarding the effect of the corrected inward-flowing fraction calculations presented in Equations 15 to 17, the software predicts SHGCs within $\pm 2\%$ of R^∞ methods prediction for nighttime conditions.

CONCLUSIONS

The difficulties inherent in the calculation of window performance indices when the system includes a single diathermanous layer such as a window shade have been demonstrated, and a new method has been put forward for calculating U and SHGC in such systems. It is clear that the standard series resistance network is not sufficient for analyzing these types of systems.

There is a deeper problem, unfortunately, stemming from the calculation of radiative heat transfer coefficients for input into these equations. Traditional methods for calculating radiative heat transfer coefficients appear to work for some limited situations but have a tendency to give erroneous results under most circumstances. The problem can be traced to radiant heat transfer between two surfaces of similar temperature due to

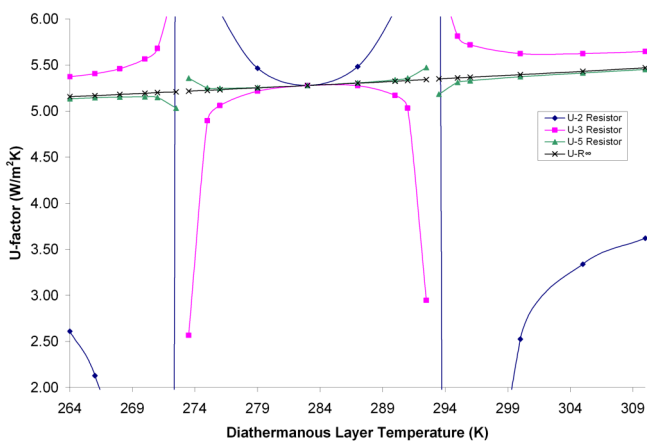


Figure 7 U-factor comparison by radiosity reduction technique, including new methodology. Convection effects ignored.

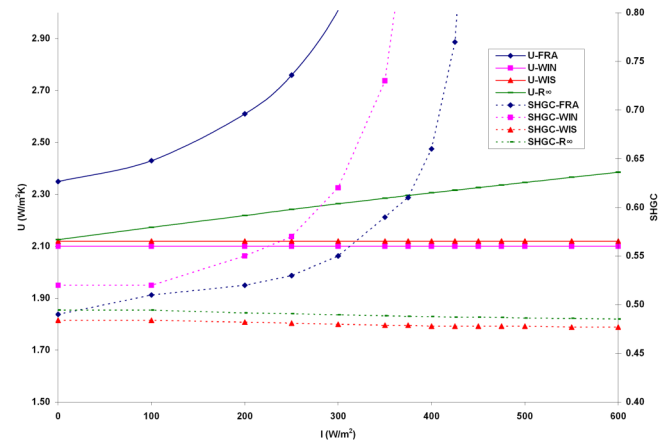


Figure 8 U-factor and SHGC comparison of R^∞ method versus FRAMEplus, WINDOW, and WIS for a triple glazing with diathermanous center layer. System properties are given in Table 1.

reflected heat transfer from a third surface at a third temperature. The possibility exists for heat transfer to occur from cold to hot (negative h 's), when no temperature difference exists between layers (h 's of infinity) or at levels that are not representative of the actual heat transfer rates (positive but incorrect h 's). A more stable method of determining the radiative heat transfer between layers, called the R^∞ method, has been presented.

Inclusion of inaccurate h coefficients into traditional U-factor and SHGC calculations has also been demonstrated in some of the most popular industry-standard window analysis packages. Only WIS appears to give stable results in this respect. Both WINDOW and FRAMEplus need some modification before they can be used to determine the performance indices of windows that include diathermanous layers.

ACKNOWLEDGMENT

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NOMENCLATURE

Variables

h	=	heat transfer coefficient (W/m ² ·K)
J	=	radiosity (W/m ²)
k	=	conductivity (W/m·K)
Nu	=	Nusselt number (dimensionless)
q	=	heat flux (W/m ²)
Ra	=	Rayleigh number (dimensionless)
S	=	absorbed solar energy (W/m ²)
SHG	=	solar heat gain (W)
SHGC	=	solar heat gain coefficient (dimensionless)
T	=	temperature (K)
U	=	U-factor or thermal transmissivity (W/K)
Δ	=	difference in
ε	=	emittance (dimensionless)
τ	=	transmittance (dimensionless)
ρ	=	reflectance (dimensionless)

Subscripts

b	=	back
f	=	front
i	=	layer i
lw	=	long wave
s	=	solar
v	=	visible

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APPENDIX: DETERMINING PERFORMANCE INDICES USING PERTURBATION

An alternative approach to determining the performance indices is via the perturbation of thermal inputs. The method works as follows.

- The system energy balance is solved using the conditions of interest.

- To determine the inward-flowing fraction of layer i , the previous system is re-solved with 1 W of additional solar absorption added to i . The inward-flowing fraction will be the fraction of added energy that is seen in the flux from the indoor glazing—i.e., $(q_{2-1,new} - q_{2-1,old})/\Delta S_i$. This must be repeated for each layer.
- To determine the U-factor, the original system is re-solved by adding 1 K to the outdoor-to-indoor air temperature difference. It can be shown that the U-factor is the difference in the flux from the indoor (or outdoor) glazing—i.e., $(q_{2-1,new} - q_{2-1,old})/(\Delta T_{new} - \Delta T_{old})$.

The method assumes that the convective and radiative heat transfer coefficients or thermal resistances change by an insignificant amount as a result of the perturbation.

While this method seems reasonable, it does not work in a satisfactory manner. For a triple glazing, the system must be solved four times to determine the appropriate performance indices. If computation time is important, this might be a limit-

ing factor. More importantly, however, the method is very sensitive to changing thermal resistances. Consider a triple glazing with a U-factor of $2.0 \text{ W/m}^2\cdot\text{K}$ exposed to a ΔT of 20 K (colder outside). Assuming that the U-factor is constant for a small perturbation, $(q_{2-1,new} - q_{2-1,old})/(\Delta T_{new} - \Delta T_{old}) = (2.00 \times 21 - 2.00 \times 20)/1 = 2.00$. Unfortunately, a 1 K increase in the outdoor-to-indoor air temperature difference will cause a 0.5% or $0.01 \text{ W/m}^2\text{K}$ reduction in the overall U-factor for this window. The previous calculation, therefore, will be $(q_{2-1,new} - q_{2-1,old})/(\Delta T_{new} - \Delta T_{old}) = (1.99 \times 21 - 2.00 \times 20)/1 = 1.79$; an error of 10.5%. This error does not change with the starting ΔT_{old} or with $\Delta T_{new} - \Delta T_{old}$. That is, increasing or decreasing the outdoor-to-indoor air temperature by different amounts consistently creates an error on the same order as that already shown. It is further noted that the same error is present, although smaller, in the inward-flowing fraction calculations. For a typical double and single glazing, the same analysis results in errors of approximately 7% and 0%, respectively.