Achievable Rate Regions of Two-Way Relay Channels

by

Liang Dong

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

With the fast development of communication networks, cooperative communication has been more widely used in many different fields, such as satellite networks, broadcast networks, internet and so on. Therefore relay channels have been playing a pivotal role since their definitions were proposed by Van-der Meulen. However, the general achievable rate region of a relay channel is still unknown which inspires more people to persistently work on. There are several different kinds of coding schemes proposed by people after relay channels came into our lives. Until now, the two most commonly used coding strategies of relay channels are Decode-and-Forward and Compress-and-Forward. In this thesis we will provide a way to obtain the achievable rate region for two-way relay channels by using decode-and-forward coding.

With the knowledge of basic information theory and network information theory, we will focus our study on the achievable rates of relay channels. Most of the previous study of relay channels are aiming to find a more general achievable rate region. In this thesis, an intuitional way will be used to study four-terminal relay channels. This method makes a good use of the information from three-terminal relay channels by separating a four-terminal relay channel into two parts: (1). a three-terminal relay channel; (2). a common end node. The final achievable rate region is obtained by combing together the separate achievable rates of the two parts. We split the complex model to two easier ones, this idea may give help for doing researches on more complicated channels.

Eliminating interferences is also a difficulty in the study of relay channels. Comparing with the achievable rate regions of two-way two-relay channels which have already been proved, we found that it is feasible to separate a two-way two-relay channel into a three-terminal relay channel and an common end node. Therefore, we apply this method to all two-way four-terminal relay channels. After fixing two different source nodes, all of the possible transmission schemes are presented in this thesis. However not all of the four-terminal channels can be separated into two parts. By studying the schemes failed to be decomposed to a three-terminal relay channel and a common end node, we found that these schemes are infeasible for message transmission. Thus our method can still be used to study on feasible two-way relay channels.

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Table of Contents

List of Figures

Chapter 1

Introduction

1.1 Background

As the size of global communication networks are expanding faster and wider, mail is gradually replaced by fax, telegrams and then telephones, mobile phones. The distance of communication is therewith increased from within a foot to worldwide, and from point to point wire communication links to wireless communication networks. The communication techniques have been developed rapidly.

In the 1980's, the first generation of wireless communication systems (1G) using frequency division multiple access (FDMA) came into use. Since the transmission rate was only 9.6kbps, the business at that time was limited only in voice service. In the 1990's, narrowband was proposed, and time division multiple access (TDMA) and code division multiple access (CDMA) were the two main access techniques in the second generation of wireless communication system (2G). 2G provided a transmission rate from 9.6-28.8kbps, therefore, low speed data service came into use, such as dial-up internet access. In the 2000's, the third generation of wireless communication systems (3G) technology was approached nearly to everyone, more people held mobile phones and a higher percentage of people owned computers. The transmission rate was highly increased up to 384kbps. As the rate is high enough to transmit images, music, and videos from user to user, therefore, mobile internet access, mobile TV technologies, and FaceTime became widely spread. Nowadays, we are in an age of the fourth generation of wireless communication systems (4G), the new generation of technology has higher peak rates. Wider channel frequency bandwidth is Hertz and larger capacity of simultaneous data transfers. It not only means faster services, but also a worldwide integration of different industries, such as finance, medicaleducation, transportation, and different networks such as local area network (LAN), internet, broadcast networks, satellite networks.

With the rapidly developed communication technologies, cooperative transmission gradually shows its importance. We want a wider, higher rate and more reliable communication when carrying through a long distance point to point communication. However it is hard not to consider the attenuation and interference problems. As the networks around us are growing larger and more complicated, the transmitted message will be heard not only by its objective node, its neighbors will also perceive that message. Then people began to consider how to let the neighborhood help this transmission. Thus the helpful neighborhood nodes became relay nodes. The concept of relay channels was first brought forward by Van-der Meulen [17], [13]. He introduced a channel model not only the source node and the destination node, but also the intermediate helper, relay node, and gave out a fundamental achievable rates boundary. Later Cover and El Gamal in [4] proposed several fundamental channel schemes, the most discussed different coding strategies are Decode-and-forward (DF) schemes, [4, Theorem 1], and Compress-and-Forward (CF) schemes, [4, Theorem 6]. In the DF relay channel, the relay node first decodes the received information and then forwards to the destination. In the CF relay channel, the relay node quantizes the received message and then transfers the compression message.

When the communication becomes even more complex, a one-way single-relay channel extend to a one-way multiple-relay channel. When there are two users in a channel need to communicate with each other, a two-way relay channel is engendered. Two-way relay channels are widely used in wireless communication and the achievable rate region is always the most concerned problem. But an achievable rate region for a general two-way relay channel is still unknown. In a network model, for a better use of each node, a node may play different roles, such as, a source node of one way may also be the destination node or relay node for the other way.

In this thesis, we will look into all possible message transmission paths of three-terminal and four-terminal relay channels and we will consider their relations and achievable rate regions to help us better understand relay channels and find a more general achievable rate region for relay channels.

1.2 Thesis outline and Contributions

The thesis is organized as follows.

Chapter 1 mainly introduces the background, motivations, contributions and the outline of this thesis.

Chapter 2 introduces the definitions and theorems that we will use through this thesis. We will use their formal definitions. The preliminary knowledge is separated into two parts, consisting of the basic information theory and the network information theory. In the first part, we begin with the most fundamental definitions such as Entropy, and several significant terminologies are introduced for a better understanding of the Shannon network coding theorem. In the second part, we use information theory in communication networks. Three most widely used channel models are presented, including multiple-access channels, broadcast channels, and relay channels. In addition, several coding strategies including the Slepian-Wolf coding, and the network coding is also clarified. Relay channels are discussed in particular, especially two relay channel schemes: decode-and-forward (DF) relay channels and compress-and-forward (CF) relay channels.

Chapter 3, the achievable rate regions of one-way single-relay channels and two-way single-relay channels are introduced at the beginning of this chapter. With the foundation of these two channel models, we present two schemes of two-way two-relay channels with a little difference on message transmission paths. If we omit the symmetric schemes, these two channels are all the possible schemes of a two-way two-relay channel models. We obtain the achievable rates by using a way that separate each of the scheme into two parts, one is a complete three-terminal relay channel and the other one is a common end node. We use three steps to get the achievable rates. Firstly, find the achievable rates of the three-terminal channel. In the first relay scheme, depicted in Figure 3.3-I, it is a two-way single-relay channel, in the second relay scheme, it is a three-terminal relay channel with different relay node for each way. If we want to get an achievable rate region without interference, we will face a 'deadlock' problem identified in [21], and later Ponniah and Xie fixed this problem by adding one constrain to the previous proved achievable rate regions [14]. The theorem mentioned in the paper will also be introduced. We will show that this (three-terminal channel) + (a common end node) method is feasible to generate two-way two-relay channels from three-terminal channels by the same result with [14]. And this arouse our interest that whether all of the four-terminal channel model can be separated into these two parts? Or to say can all of the four-terminal channels be generated from a known three-terminal channels? This will be talked about in the next chapter.

Chapter 4 introduces 36 two-way relay channel schemes with fixed two source nodes, exclude the schemes we discussed in chapter 3. Since we will focus on the four-terminal relay channels generated from three-terminal relay channels, we introduce all of the possible schemes of three-terminal channels before showing the 36 schemes. This is for a later use of generating four-terminal relay channel models. The achievable rate region for each scheme is introduced, including the way to get rid of the interference. Then the 36 schemes are classified into 6 families and the feasible ones are in 9 groups, each group represents a kind of channel model. The detailed reasons for some schemes to be infeasible is also introduced.

Chapter 5, we do a conclusion of the whole thesis, including the model we proposed, a three-terminal relay + a node channel model; and if this model can be applied to all of the two-way relay channels. Then we discuss the potential future works.

Chapter 2

Preliminaries

In this chapter, several basic but important definitions, theorems and results will be introduced for later utility throughout this thesis. We will talk about both the basic information theory and the network information theory, and give definitions to the terminologies which are crucial in our study. [6]

2.1 Basic Information theory

2.1.1 Entropy

Entropy

In information theory, entropy is an expected value (average amount) of the information in each message throughout a channel. It is a measure of the uncertainty of a random variable. Thus if the entropy of message A is larger than B's, then message A is more uncertain than message B, it means message A contains more information.

Definition 2.1.1 *The entropy H(X) of a discrete random variable X is defined by*

$$H(X) = -\sum_{x\in\mathcal{X}} p(x) \log\, p(x).$$

Joint Entropy

In 2.1.1, we defined the entropy of a single random variable, here we define the entropy of a pair of random variables (X,Y). The pair of random variable can be extended to a set of random variables $(X_1, X_2, ..., X_n)$.

Definition 2.1.2 *The joint entropy H(X, Y) of a pair of discrete random variable (X, Y) with a joint distribution p(x,y) is defined as*

$$H(X,Y) = -\sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p(x,y) \log\, p(x,y).$$

The joint entropy of a set of variables is greater than or equal to any of the individual entropy of random variable in that set.

$$H(X_1, X_2, ..., X_n) \geq max[H(X_1), H(X_2), ..., H(X_n)]$$

Conditional Entropy

The conditional entropy defines the entropy of one random variable at the condition of having previous knowledge about the other random variable.

Definition 2.1.3 *If (X, Y)∼p(x,y), the conditional entropy H(Y | X) is defined as follows*

$$\begin{aligned} H(Y \mid X) &= \sum_{x\in\mathcal{X}} p(x) H(Y|X=x) \\ &= -\sum_{x\in\mathcal{X}} p(x) \sum_{y\in\mathcal{Y}} p(y|x) \log p(y|x) \\ &= -\sum_{x\in\mathcal{X}}\sum_{y\in\mathcal{Y}} p(x,y) \log p(y|x) \\ &= -E \log p(Y \mid X) \end{aligned}$$

The chain rule shows the relationship of entropy, conditional entropy and joint entropy.

Theorem 2.1.1 *Chain rule*

$$H(X, Y) = H(X) + H(Y|X).$$

Mutual Information

Definition 2.1.4 *Consider two random variables X and Y with a joint probability mass function p(x,y) and marginal probability mass functions p(x) and p(y). The mutual information I(X;Y) is the relative entropy between the joint distribution and the product distribution p(x)p(y),*

$$I(X; Y) = \sum_{x \in \mathcal{X}} \sum_{y \in \mathcal{Y}} p(x, y) \log \frac{p(x, y)}{p(x)p(y)}$$

For a better understanding of the entropy, joint entropy, conditional entropy and mutual information, we use the Figure 2.1 to describe their mutual relationships.

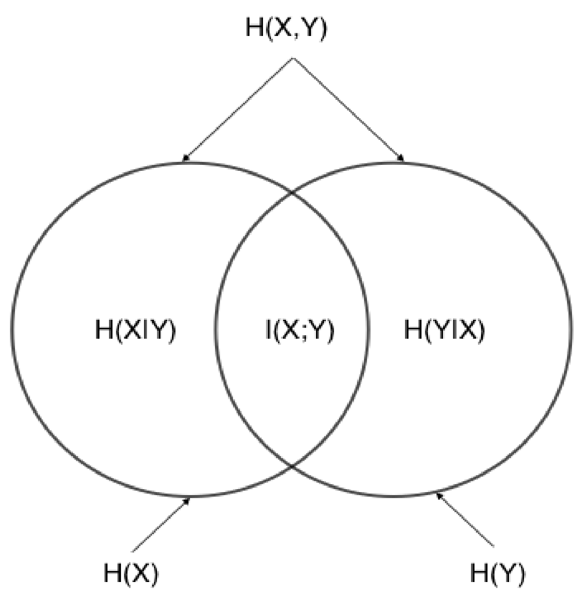

Figure 2.1: Relationship between entropy and mutual information

2.1.2 Asymptotic Equipartition Property (AEP)

According to the law of large numbers, for independent, identically distributed (i,i,d) random variables, we have

$$\frac{1}{n}\sum_{i=1}^{n} X_i \to E(X) \qquad as\ n \to \infty$$

In Asymptotic Equipartition Property (AEP), if $X_1, X_2, ..., X_n$ are i,i,d random variables and p($X_1, X_2, ..., X_n$) is the probability of this random variable sequence. Then we can get the following theorem 2.1.3,

Theorem 2.1.3 *If X_1, X_2,...are i.i.d $\sim$ p(x), then*

$$-\frac{1}{n}\log_2 p(x_1, x_2, ..., x_n) \to H(X) \ \ \textit{with probability approaching}\ 1.$$

2.1.3 Typical Sequences

Definition 2.1.5 *A typical sequence ${A_\epsilon}^{(n)}(x_1, x_2, ..., x_n) \in \mathcal{X}^n$ is within the following property:*

$$2^{-n(H(x)+\epsilon)} \leq p(x_1, x_2, ..., x_n) \leq 2^{-n(H(x)-\epsilon)}$$

note that p($x_1, x_2, ..., x_n$) $\approx 2^{-n(H(x)-\epsilon)}$.

For example, when we talk about the bias of a coin, the probability of getting each side is based on typical sequences.

Theorem 2.1.4 (Property of typical sequence)

1. *If $(x_1, x_2, ..., x_n) \in {A_\epsilon}^{(n)}$, then*

$$H(X) - \epsilon \leq -\frac{1}{n}\log_2 p(x_1, x_2, ..., x_n) \leq H(X) + \epsilon$$

Since H(X) is the mathematical entropy, while $-\frac{1}{n}\log_2 p(x_1, x_2, ..., x_n) = -\frac{1}{n}\sum_{i+1}^{n}\log_2 p(x_i)$, which is the empirical entropy. Equation (2.10) shows that the empirical entropy of typical sequences is very close to the mathematical entropy, and all typical sequences are nearly the same in probability.

2. *For n sufficiently large,*

$$Pr\{A_\epsilon^{(n)}\} > 1 - \epsilon$$

Thus, the probability of the typical set is very close to 1.

3. *If* $\mid A \mid$ *denotes the number of elements in the set A, for n sufficiently large,*

$$(1-\epsilon)2^{n(H(X)-\epsilon)} \leqslant \mid A_\epsilon^{(n)} \mid \leqslant 2^{n(H(X)+\epsilon)}$$

We can see from equation (2.12) that the number of typical sequences is almost 2^{nH}. Thus for n sufficient large, the empirical sequences will be as similar as typical sequence, the empirical entropy will converge to the mathematical entropy.

2.1.4 Markov Chain

We say a process satisfying Markov property, also characterised as memoryless, if the probability of future state depends only on the present state. A Markov chain, is a stochastic process if each of the random variable only depend on the preceding one, but conditionally independent from all the other random variables.

Definition 2.1.6 *A discrete stochastic process* $X_1, X_2, ...$ *is said to be a Markov Chain or a Markov Process if for n=1,2,...,*

$$Pr(X_{n+1} = x_{n+1} | X_n = x_n, X_{n-1} = x_{n-1}, ..., X_1 = x_1)$$
$$= Pr(X_{n+1} = xn + 1 | X_n = x_n)$$

for all $x_1, x_2, ..., x_n, x_{n+1} \in \mathcal{X}$.

2.1.5 Channel Capacity

When a sender A communicates with a receiver B, despite there are interferences around the physical signal process, we say it is a successful communication if B receives a message which is the same with A sent. [6]

Discrete Channel

Definition 2.1.7 *A discrete channel, denoted by $(\mathcal{X}, p(y|x), \mathcal{Y})$, consists of two finite sets $\mathcal{X}$ and $\mathcal{Y}$ and a collection of probability mass functions $p(y|x)$, one of each x belongs to $\mathcal{X}(x \in \mathcal{X})$, such that for every x and y, $p(y|x) \geq 0$, and for every x, $\sum_{y\in\mathcal{Y}} p(y|x) = 1$, with the interpretation that $\mathcal{X}$ is the input and $\mathcal{Y}$ is the output of the channel.*

Discrete Memoryless Channel (DMC)

The simplest point to point discrete memoryless channel is showed in Figure 2.2 .

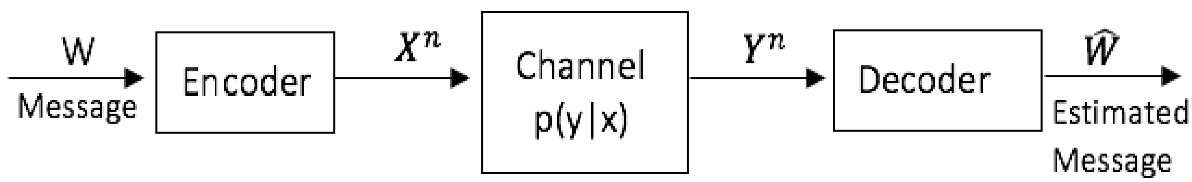

Figure 2.2: Discrete memoryless channel

Definition 2.1.8 *The n-th extension of the the discrete memoryless channel (DMC) is the channel $(\mathcal{X}^n, p(y^n|x^n), \mathcal{Y}^n)$ where*

$$p(y_k|x^k, y^{k-1}) = p(y_k|x^k)$$

Remark *If the channel is used without feedback, i.e. $P(x_k|x^{k-1}, y^{k-1}) = P(x_k|x^{k-1})$, the channel transition function for the n-th extension of the discrete memoryless channel reduces to*

$$\begin{aligned} P(y^n|x^n) &= \prod_{k=1}^{n} p(y_k|x_k) \\ &= p(y_1|x_1)p(y_2|x_2)...p(y_n|x_n) \end{aligned}$$

Channel Capacity of DMC

Definition 2.1.9 *We define the "information" channel capacity of a discrete memoryless channel as follows,*

$$C = \max_{p(x)} I(X;Y),$$

where the maximum is taken over all possible input distributions p(x).

Definition 2.1.10 *An (M, n) code for the channel* $(\mathcal{X}, p(y|x), \mathcal{Y})$ *consists of the followings;*

1. An index set $\{1, 2, ..., M\}$;

2. An encoding function X^n: 1, 2, ..., M$\rightarrow \mathcal{X}^n$, yielding codewords $X^n(1), X^n(2), ..., X^n(M)$, the set of codewords is called the codebook.

3. A decoding function is as follows:

$$g : \mathcal{Y}^n \rightarrow 1, 2, ..., M$$

Conditional Probability of Error

Definition 2.1.11 *Let*

$$\lambda_i = Prob(g(Y^n) \neq i | X^n = X^n(i))$$

be the conditional probability of error is corresponding to the i-th message. The maximal probability of error $\lambda^{(n)} = \max_{i \in \{1,2,...,M\}} \lambda^i$

Rate

Definition 2.1.12 *The rate R of an (M, n) code is*

$$R = \frac{\log M}{n} \text{ bits per transmission}$$

Achievable Rate

Definition 2.1.13 *A rate R is said to be achievable if there exists a sequence of ($\lceil 2^{nR} \rceil$, n) codes such that the maximal probability of error $\lambda^{(n)}$ tends to 0 as $n \to \infty$. Later, we write (2^{nR}, n) codes to mean ($\lceil 2^{nR} \rceil$, n) codes. This will simplify the notation.*

Channel Capacity

Definition 2.1.14 *The capacity of a channel is the supremum of all achievable rates.*

$$C = \sup R$$

2.1.6 The Shannon Coding Theorem

From the above knowledge, we have known the definitions of channel capacity, entropy and mutual information, achievable rate, probability of error and several basic definitions. Now we can introduce a foundational and important theorem of information theory, it is the Shannon's second theorem, channel coding theorem, sometimes it is also called the noisy-channel coding theorem or Shannon's theorem. This theorem claimed that, although there are different levels of noise interferences in the ambient of a communication channel, it is possible to transmit messages in nearly zero possibility of error if the transmission rate is less than the channel capacity. It seems opposite to our intuition, since errors may still occur in the process of correcting an error, then the quantity of error could go to infinity. According to the outline of the proof Shannon stated in 1948, the theorem gives the maximum rate that can insure the information to be sent reliably in a noisy channel, and the maximum rate is up to channel capacity.

Conversely, if the transmission rate is larger than the channel capacity, then it is impossible to achieve an arbitrarily small probability of error. This means the information cannot be transmitted reliably across a channel when transmission rate is larger than the channel capacity. As the rate gets larger, each code will have a larger probability of error.

In 1948, Shannon only gave out a best possible efficiency of error detecting and correcting in a noisy channel, but he didn't prove it rigorously. It is Amiel Feinstein, in 1954, first gave an exact proof. In the proof of the converse theorem, Fano's Inequality will be used. The detailed proof can be found in [6, Section 7.7 and 7.9]

Theorem 2.1.15 *For a discrete memoryless channel (DMC), all rates below capacity C are achievable. Specifically, for every rate $R < C$, there exists a sequence of $(2^{nR}, n)$ codes with maximum probability of error $\lambda^{(n)} \to 0$.*

Conversely, any sequence of $(2^{nR}, n)$ codes with $\lambda^{(n)} \to 0$ must have $R \leq C$

2.2 Network Information Theory

Rather than one pair of sender and receiver, a system with many senders and receivers communicate with each other is called a communication network. Since other components exist besides the sender and its receiver during the communication, they may play a role as a cooperator, an interference, or a feedback. Thus network information theory is more complicated than basic information theory. In this section, we will introduce several typical channels in network communication, like multiple-access channels, broadcast channels, and relay channels which will be detailed discuss later.

2.2.1 AWGN Channel

Noise naturally exists in the general channels. Gaussian channel is time-discrete channel and it is the most important continuous alphabet channel which is illustrated by Figure 2.3. Assume at time i, X_i is the input signal, Z_i is the noise drawn i.i.d. with variance N from a Gaussian distribution, Y_i is the output signal which is the sum of input X_i and noise Z_i. This channel is a good model to simulate radio or satellite links.

$$Y_i = X_i + Z_i \quad Z_i \sim \mathcal{N}(0, N)$$

Definition 2.2.1 *The capacity of the Gaussian channel with power constraint P ($X(t) \sim \mathcal{N}(0, P)$) and noise variance N ($Z(t) \sim \mathcal{N}(0, N)$)is*

$$\begin{aligned} C &= \max_{f(x):E\ X^2 \leq P} I(X;Y) \\ &= \max_{f(x):E\ X^2 \leq P} (h(Y) - h(Y|X)) \\ &= \max_{f(x):E\ X^2 \leq P} (h(Y) - h(Z)) \end{aligned}$$

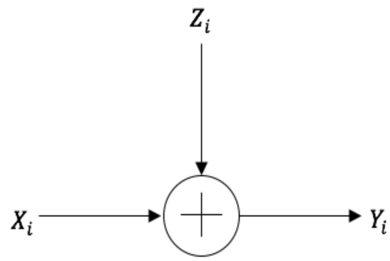

Figure 2.3: Gaussian Channel

$$= \frac{1}{2}\log(1+\frac{P}{N}) \quad \textit{bit per transmission}$$

Then the achievable rate of Gaussian channel is as follows,

$$R \ \leq \ C \ = \ \frac{1}{2}\log(1+\frac{P}{N})$$

2.2.2 Multiple-Access Channel (MAC)

We start from a simple channel, multiple-access channel (MAC), it is formed by several senders but only one receiver. Thus many sources transmit information to a collective destination. [6], the figure below Figure 2.4 illustrates such channel with two senders.

We use a two-sender channel model as an example which is easy to demonstrate the multiple-access channel.

W_1, and W_2 are the messages need to be transmitted, X_1, and X_2 are the codewords corresponding to W_1, and W_2 which transmitted by encoders, Y_n is the output, and $\hat{W}_1$, and $\hat{W}_2$ are the decoded messages.

Definition 2.2.2 *A discrete memoryless multiple-access channel consists of three alphabets $\mathcal{X}_1$, $\mathcal{X}_2$, and $\mathcal{Y}$, and a probability transition matrix $p(y|x_1, x_2)$.*

Definition 2.2.3 *A $((2^{nR_1}, 2^{nR_2}), n)$ code for the multiple-access channel consists of two sets of integers $\mathcal{W}_1$=⟨1, 2, ..., 2^{nR_1}}, and $\mathcal{W}_2$={1, 2, ..., 2^{nR_2}}, called the message*

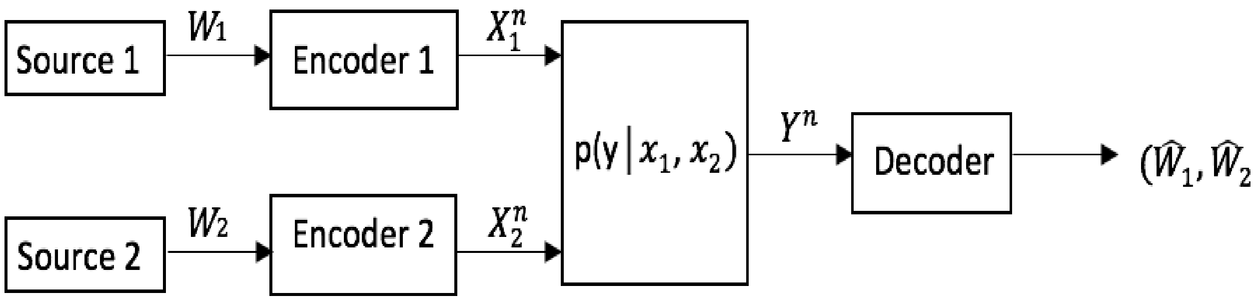

Figure 2.4: Multiple-Access Channel

sets, two encoding functions,

$$X_1 : \mathcal{W}_1 \to \mathcal{X}_1^n$$

and

$$X_2 : \mathcal{W}_2 \to \mathcal{X}_2^n$$

and a decoding function,

$$g : \mathcal{Y}^n \to \mathcal{W}_1 \times \mathcal{W}_2$$

Source 1 sends message to the destination, while encoder 1 choose one of the integer $\mathcal{W}_1$ from the message set $\{1, 2, ..., 2^{nR_2}\}$, then transmits the corresponding codeword $\mathcal{X}_1$. Source 2 and encoder 2 do the same thing. After $\mathcal{X}_1$ and $\mathcal{X}_2$ going through the probability transition matrix $p(y|x_1, x_2)$, we get $\mathcal{Y}$. With the decoding function g, we then get the decoded message $\hat{\mathcal{W}}_1$ and $\hat{\mathcal{W}}_2$.

Achievable Rate Pair

Definition 2.2.4 *A rate pair (R_1, R_2) is said to be achievable for the multiple access channel if there exists a sequence of $((2^{nR_1}, 2^{nR_2}), n)$ codes with $P_e^{(n)} \to 0$.*

Capacity Region of Multiple-Access Channel

Definition 2.2.5 *The capacity region of the multiple-access channel is the closure of the set of achievable (R_1, R_2) rate pairs.*

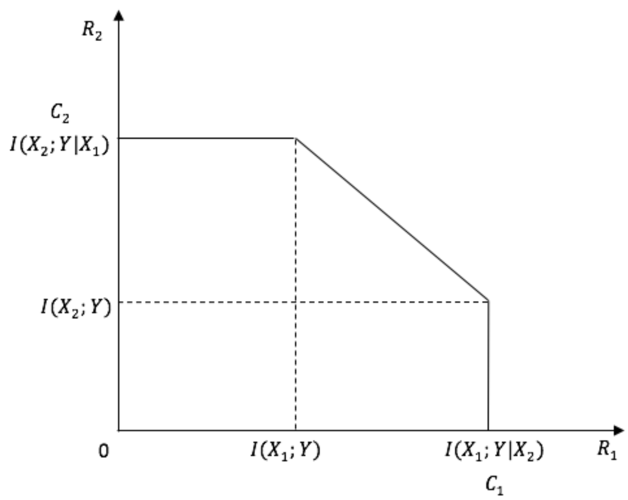

Figure 2.5: Capacity region for a Multiple-Access Channel and for fixed input distribution

The capacity region is illustrated by Figure 2.5 for a more obvious understanding.

Theorem 2.2.1 *The capacity of a multiple-access channel ($\mathcal{X}_1 \times \mathcal{X}_2$, $p(y|x_1, x_2)$, $\mathcal{Y}$) is the closure of the convex hull of all (R_1, R_2) satisfying*

$$
\begin{aligned}
R_1 &< I(X_1; Y|X_2) \\
R_2 &< I(X_2; Y|X_1) \\
R_1 + R_2 &< I(X_1, X_2; Y)
\end{aligned}
\tag{2.1}
$$

for some product distribution $p_1(x_1)p_2(x_2)$ on $\mathcal{X}_1 \times \mathcal{X}_2$. We can see the boundary more clearly on Figure 2.5.

To prove the achievability of the capacity region for the multiple-access channel, Theorem 2.2.2, we begin by fixing the product distribution $p(x_1, x_2) = p(x_1)p(x_2)$, and prove if the rate pair R_1 and R_2 which satisfy 2.1 are achievable. The brief outline is as follows. The detailed proof can be found in [6, Chapter 15.3.1].

1. *Codebook generation:*
    - Codewords $\{X_1^n(1), X_1^n(2), ..., X_1^n(2^{nR_1})\}$ are independent, generating these 2^{nR_1} elements identically independent distributed (i.i.d)$\sim \prod_{i=1}^{n} p_1(x_{1i})$, i$\in \{1, 2, ..., 2^{nR_1}\}$.

- Codewords $\{X_2^n(1), X_2^n(2), ..., X_2^n(2^{nR_2})\}$ are independent, generating these 2^{nR_2} elements identically independent distributed (i.i.d)$\sim \prod_{j=1}^{n} p_2(x_{1j})$, j$\in \{1, 2, ..., 2^{nR_2}\}$.

2. *Encoding:*

   - To send index i, encoder 1 transmit $X_1^n(i)$
   - To send index j, encoder 2 transmit $X_2^n(j)$

3. *Decoding:* Define $A_\epsilon^{(n)}$ the set of typical sequences, (x_1, x_2, y). Receiver Y^n finds the pair $(\hat{i}, \hat{j})$, such that $(\mathcal{X}_1^n(\hat{i}), \mathcal{X}_2^n(\hat{j}), y) \in A_\epsilon^{(n)}$. If such a pair$(\hat{i}, \hat{j})$ exist and unique, otherwise, an error is declared

4. *Analysis of the probability of error:* Assume (i, j)=(1, 1) was sent, i.e. $(X_1^n(1), X_2^n(1))$ was transmitted. An error occurs when the codewords and the received sequence are not jointly typical. To avoid an error, we need to show that *(1).Prob(*$X_1^n(1)$*,* $X_2^n(1)$*,* $Y^n \in A_\epsilon^n$*)* $\to$ *1, as* $n \to \infty$, and *(2).no other (i, j) to be jointly typical with* Y^n*.*

2.2.3 The Broadcast Channel

While a multiple-access channel has two or more senders and only one receiver, however a broadcast channel has one sender and two or more receivers. A simplest two-receiver model of broadcast channels is illustrated in Figure 2.6, a multiple-receiver model is defined similarly.

Broadcast channels are very common is our daily life, the most familiar example is a lecture in classroom. The lecturer is the only one sender who send out messages, the students in the classroom are the receivers, different people receive different levels of information. A radio or TV station is also a very common example, the station is the sender, and the audience who tune in are the receivers. But different people receive variance of sounds or picture qualities. So it is easy to see that the receivers are on different levels of receiving information. To ensure better receivers enjoy high-definition TV (HDTV) and the bad receivers acquire the regular TV. Then there arises many problems, such as on what rate can we used to send information in order to make sure of the worst receiver get at least the minimum information while the good receiver get some extra information. But a general achievable rate region for broadcast channels is still unknown.

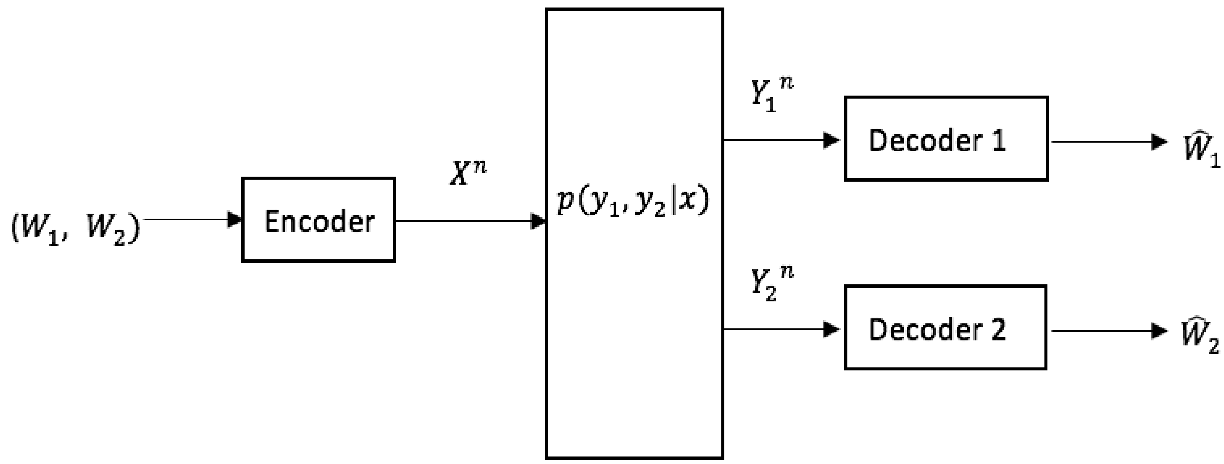

Figure 2.6: Broadcast Channel

Definition 2.2.6 *A broadcast channel consists of an input alphabet $\mathcal{X}$ and two output alphabets, $\mathcal{Y}_1$ and $\mathcal{Y}_2$, and a probability transition function $p(y_1, y_2|x)$. The broadcast channel will be said to be memoryless if $p(y_1^n, y_2^n|x^n) = \prod_{i=1}^n p(y_{1i}, y_{2i}|x_i)$*

We define codes, probability of error, achievability, and capacity regions for the broadcast channel as we did for the multiple-access channel. A $((2^{nR_1}, 2^{nR_2}), n)$ code for a broadcast channel with independent information consists of an encoder,

$$X \ : \ (\{1,\ 2,\ ...,\ 2^{nR_1}\} \ \times \ \{1,\ 2, ...,\ 2^{nR_2}\}) \ \rightarrow \ \mathcal{X}^n,$$

and two decoders,

$$g_1 \ : \ \mathcal{Y}_1^n \ \rightarrow \ \{1,\ 2,\ ...,\ 2^{nR_1}\}$$

and

$$g_2 \ : \ \mathcal{Y}_2^n \ \rightarrow \ \{1,\ 2,\ ...,\ 2^{nR_2}\}$$

We define the average probability of error as the probability that the decoded message is not equal to the transmitted message; that is,

$$P_e^{(n)} \ = \ P(g_1(Y_1^n) \neq W_1 \text{ or } g_2(Y_2^n) \neq W_2)$$

where (W_1, W_2) are assumed to be uniformly distributed over $2^{nR_1} \times 2^{nR_2}$.

Theorem 2.2.2 *The capacity region of a broadcast channel depends only on the conditional marginal distribution $p(y_1|x)$ and $p(y_2|x)$*

the marginal distribution are as follows [3]

$$p(y_1|x) = \sum_{y_2 \in Y_2} (y_1, y_2|x) p(y_2|x) = \sum_{y_1 \in Y_1} (y_1, y_2|x)$$

A capacity region of a broadcast channel is the closure set of achievable rates, but it is still unknown. The broadcast channel we discussed above is usually an theoretical model, receivers in practice often receive degraded signals [5].

Definition 2.2.7 *A broadcast channel is $p(y_1, y_2|x)$ is said to be degraded if there exists a distribution $\widetilde{p}(y_2|y_1)$* such that

$$p(y_2|x) = \sum_{y_1} p(y_1|x) \widetilde{p}(y_2|y_1)$$

Thomas Cover in 1971 first proposed the upper and lower bounds on capacity region of the degraded broadcast channels, he focused on several extreme classes of channels [3] to get those results. Two years later, Patric Bergmans gave a rigorous proof of the coding scheme for degraded broadcast channel [2]. Later Marton gave out a inner bound on the two-receiver general broadcast channels with no common message [12], then Gelfand and Pinsker figured out the case that broadcast channels with a common message [7]. After more than twenty years' work, although the best known inner bounds on general broadcast channels is Marton-Gelfand-Pinsker rigion[11], [8] .

2.2.4 The Slepian-Wolf Coding

Slepian-Wolf Coding Theorem

To encode one source X, we know that $R > H(X)$ is sufficient. To encode two sources $X, Y \sim p(x, y)$ together, a rate $R > H(X, Y)$ is sufficient. But if we want to encode X and Y separately for a later reconstruction, depicted in Figure 2.7 it is obvious that a rate $R = R_x + R_y > H(X) + H(Y)$ is sufficient. However on the contrary of our intuition, a paper by Slepian and Wolf showed that the sufficient rate of encoding two sources together can also be sufficient for separately encoding the two sources [16].

Definition 2.2.8 *A $((2^{nR_1}, 2^{nR_2}), n)$ distributed source code for the joint source (X, Y) consists of two encoder maps,*

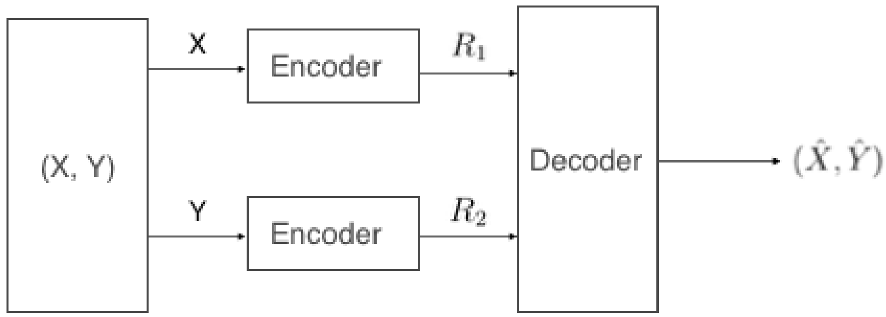

Figure 2.7: Slepian-Wolf coding

$$f_1 \ : \ \mathcal{X}^n \ \to \{1,\ 2,\ ...,\ 2^{nR_1}\},$$
$$f_2 \ : \ \mathcal{Y}^n \ \to \{1,\ 2,\ ...,\ 2^{nR_2}\},$$

and a decoder map,

$$g \ : \ \{1,\ 2,\ ...,\ 2^{nR_1}\} \times \{1,\ 2,\ ...,\ 2^{nR_2}\} \ \to \mathcal{X}^n \times \mathcal{Y}^n$$

$f_1(X^n)$ is the index corresponding to X^n, $f_2(Y^n)$ is the index corresponding to Y^n, (R_1, R_2) is the rate pair of the code.

Definition 2.2.9 *A rate pair (R_1, R_2) is said to be achievable for a distributed source if there exists a sequence of $((2^{nR_1}, 2^{nR_2}), n)$ distributed source code with probability of error $P_e^{(n)} \to 0$. The achievable rate region is the closure of the set of achievable rates.*

Theorem 2.2.3 *(Slepian-Wolf) For the distributed source coding problem for the source (X, Y) draw i.i.d.$\sim$ p(x, y), the achievable rate region is given by*

$$R_1 \geq H(X|Y),$$
$$R_2 \geq H(Y|X),$$
$$R_1 + R_2 \geq H(X,Y).$$

It is natural to ask the achievability of Slepian-Wolf Coding, a new coding method will be introduced to prove the achievability [6], which is random binning. This technique is widely used in amount of fields such as physics, biology and it is also used along through this thesis.

Random Binning

The procedure of random binning is outlined as follows, it is depicted in Figure 2.8.

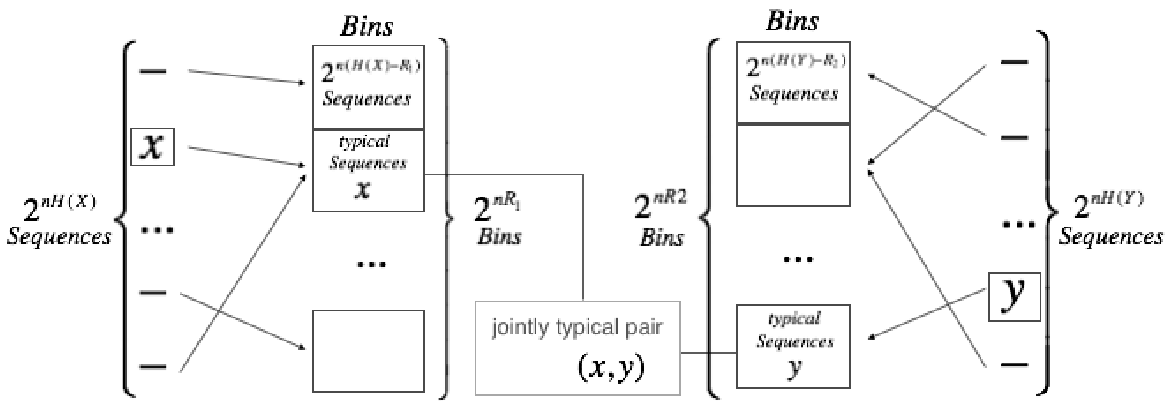

Figure 2.8: Random binning

First of all, random binning procedure index all sequences including typical sequences and untypical sequences, but we will discard the untypical sequences later. For each sequence X^n (same to Y^n), draw an index randomly from the sequence $\{1, 2, ..., 2^{nR}\}$ to a bin, i.e. throw X^n randomly to 2^{nR_1} bins, likewise, throw Y^n randomly to 2^{nR_2} bins. Each bin is formed by a set of sequence with the same index. We want to decode the source by the index of the bins, one and only one typical sequence X^n has to be found in a bin, if it exists, then we state it to be the estimated source sequence $\hat{X}^n$, otherwise, an error will be declared. Decoder cannot tell where a sequence is, it can only tell which bin it is in.

The proof of the achievability of Slepian-Wolf capacity region used the same idea of random binning procedure. First set up the bins similarly as the above introduced. In the decoding part, first given the received index pair (i_0, j_0), find a sequence pair (x, y) according to the encoder maps $f_1(x) = i_0, f_2(y) = j_0$, if that sequence pair (x, y) is the one and the only pair which is also a jointly typical pair, then we say $(\hat{x}, \hat{y}) = (x, y)$, otherwise an error will be declared.

2.3 Network Coding

Network coding plays an important role in wireless communication to prove the throughput and efficiency of a network. Instead of passing the data unaltered from one end to the other communication link, the intermediate nodes in the network transmit combined packets of information (sets of data), i.e. the incoming packets are combined into one or several outgoing packets, which the achievability was first proved by Ahlswede in 2000, [1]. In this paper, a butterfly network model Figure 2.9 was illustrated to show how linear network coding performs better than routing.

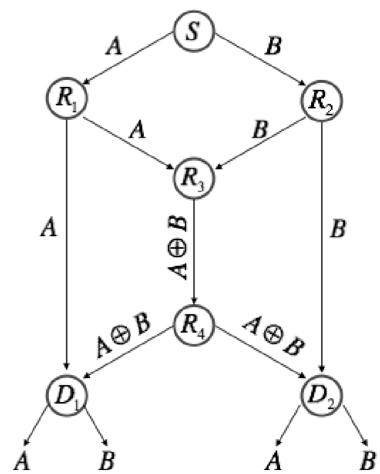

Figure 2.9: Network coding

A source node S contains two messages A and B which need to be transmitted to both two destinations D_1 and D_2, i.e. each of the destination node wants to know both messages A and B simultaneously. Then the central edge from R_3 to R_4 must be able to carry A and B, however an edge can only transmit one value at each time if Using routing, therewith either destination D_1 or D_2 only receive one message at a time. Now if we use network coding, the central link transmits the combination of A and B, note by $A \oplus B$. Then D_1 can receive A and $A \oplus B$, by calculating $A \oplus A \oplus B = B$, D_1 receives A and B simultaneously. Similar to D_2.

2.3.1 Relay Channel

Relay channel will be the main topic of this thesis, so we will discuss more in detail in a new section.

2.4 Relay Channel

2.4.1 Single-Relay Channel

The simplest relay channel as show in Figure 2.10

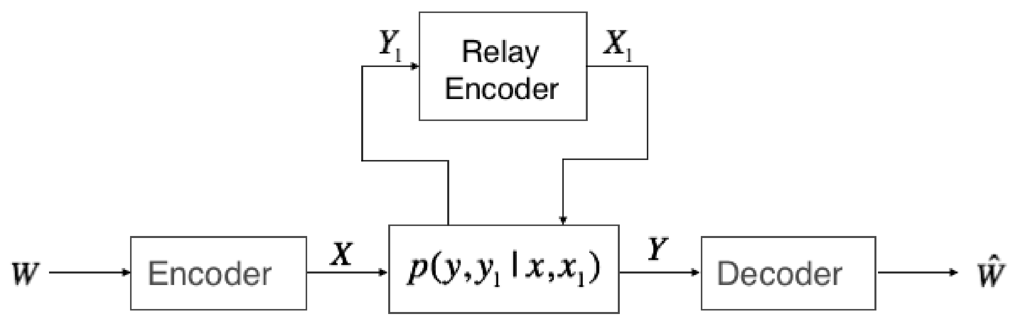

Figure 2.10: Relay channel

The main goal of a relay channel is to transmit message W$\in [1, 2^{nR}]$ reliably from X to Y with the help of the relay node (X_1, Y_1). It consists of a source node, a destination node, and a relay node. w is the message needs to be transmitted, x and y are the input and output of the channel respectively, y_1 is the relay's observation, x_1 is the relay input, and $\hat{w}$ is the estimate of the transmitted message. The channel is composed of four finite sets, $\mathcal{X}, \mathcal{X}_1, \mathcal{Y}, \mathcal{Y}_1$, and a collection of probability mass functions $(y, y_1|x, x_1)$ on $\mathcal{Y} \times \mathcal{Y}_1$, one for each $\mathcal{X} \times \mathcal{X}_1$, i.e. the received symbols which depend on the input are corresponding to $(y, y_1|x, x_1)$, thus the discrete memoryless relay channel is denoted by $(\mathcal{X} \times \mathcal{X}_1,\ p(y, y_1|x, x_1),\ \mathcal{Y} \times \mathcal{Y}_1)$, [4] [6].

Definition 2.4.1 *A* $(2^{nR},\ n)$ *code for a relay channel consists of a set of integers* $\mathcal{W} = \{1, 2, ..., 2^{nR}\}$*, an encoding function*

$$X \ : \ \{1, 2, ..., 2^{nR}\} \ \rightarrow \ \mathcal{X}^n,$$

a set of relay functions $\{f_i\}_{i=1}^n$ *such that*

$$x_{1i} = f_i(Y_{11}, Y_{12}, ..., Y_{1i-1}), \quad 1 \leq i \leq n,$$

and a decoding function,

$$g \ : \ \mathcal{Y}^n \rightarrow \{1, 2, ..., 2^{nR}\}.$$

Note that we can also say it is an (M, n) code by replacing 2^{nR} to M. From 2.4.1, the relay input x_{1i} only depends on the past observation of relay $y_1^i = (y_{11}, y_{12}, ..., y_{1i-1})$, the definition was introduced by Van Der Meulen [13].

With knowledge of the formal definition of a relay channel, it is natural to ask the achievable rates and the capacity region of a relay channel. The exact capacity region of general relay channels is still unknown.

Theorem 2.4.1 *For any relay channel* ($\mathcal{X}_1 \times \mathcal{X}_2, \ p(y, y_1|x, x_1), \ \mathcal{Y} \times \mathcal{Y}_1$)*, the capacity* C *is bounded above by*

$$C \ \leq \ sup_{p(x,x_1)} \, min\{I(X, X_1; Y), I(X; Y, Y_1|X_1)\} \tag{2.2}$$

We will discuss about the channel capacity of relay channels in the following part.

Decode-and-Forward (DF) Relay Channel

Decode-and-Forward (DF) relay channel, [4, Theorem 1]. The responsibility of the relay is to decode user messages and then forward the re-encoded messages in the following blocks to the destination. The achievable rate of a Decode-and-Forward relay channels is as follows,

$$\max_{p(x_1,x_2)} \min(I(X_1; Y_1|X_2), I(X_1, X_2; Y))$$

Rankov and Wittneben, in [18], proposed a decode-and-forward coding scheme and applied it to a two-way relay channel. The DF scheme provided an achievable rate region on that scheme that using block Markov superposition coding, the two source nodes use partial block Markov encoding, while the codeword transmitted by the relay is a superposition of the two decoded message.

Xie, in [20], proposed the achievable rate region on a different decode-and-forward scheme by using network coding and random binning. the codeword transmitted by the relay is a random binning of the two decoded message.

Compress-and-Forward (C-F) Relay Channel

Compress-and-Forward (C-F) relay channel, [4, Theorem 6]. The responsibility of relay is to quantize the received messages and then transmits the encoded version of the quantized symbols in the following block to the destination. [10] The achievable rate of Compress-and-Forward relay channel is as follows,

$$\max_{p(x_1)p(\hat{y}_1|y_1)p(x_2)} I(X_1; Y_1; Y|X_2) \quad \textit{subject to } I(X_2; Y) \geq I(Y_1; \hat{Y}_1|Y)$$

The first coding scheme on compress-and-forward channels is Wyner-Ziv coding, it was introduced in [19] by Wyner and Ziv. A relay node compresses the symbol it received and forward the bin index by using Wyner-Ziv coding.

Chapter 3

A Single-Relay Channel To A Two-Relay Channel

The relay channels we will discuss later are decode-and-forward relay channels, we use block Markov coding, and the relay receiver is better than the ultimate receiver. We will start from the simplest channel scheme in section 3.1, which is the one-way single-relay channel. We will introduce its channel model and achievable rate region. Then we will get closer to our research and begin to discuss the channel model and achievable region for the two-way single-relay channel.

3.1 Single-Relay Channel

We have introduced the formal definition of the single-relay channel in 2.3.1. Now we will talk about the achievable rates of single-relay channels, including the one-way single-relay channel and the two-way single-relay channel. In this thesis, we will only concern about the family of the relay channels which the relay receiver is better than the ultimate receiver.

3.1.1 One-way Single-relay Channel

An One-way single-relay Channel consists of a source node 1, a relay node 2, and a destination node 3. We omit the detailed relay observation y_2 and relay input x_2 for this and future figures for simplicity, since we have offered the figure and introduction about this in 2.4.1.

Note that we only talk about the case that the relay node helps a transmission process, then the achievable region is larger when messages transmitted by the relay node 2 rather than transmitted directly from source 1 to destination 3. The achievable rates of this channel are as follows,

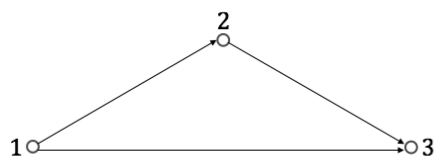

Figure 3.1: One-way single-relay channel

$$R < I(X_1; Y_2|X_2) \tag{3.1}$$

$$R < I(X_1, X_2; Y_3|X_3) \tag{3.2}$$

(3.1) means for relay node 2, it receives X_1 with the knowledge of X_2.

(3.2) means for node 3, it receives X_1 and X_2 with the knowledge of X_3.

3.1.2 Two-way Single-relay Channel

The Two-way Single-relay channel consists of two source nodes and one relay node. This relay model is illustrated in Figure 3.2. Node 2 is the relay node for both ways. For one way, node 1 is the source node while node 3 is the destination node; for another way, node 3 becomes the source node and node 1 is the destination node. Then for one way, the information flows from 1 to 2 to 3, $(1 \to 2 \to 3)$, and for the other way, the information flows form 3 to 2 to 1, $(3 \to 2 \to 1)$. The information flow in one way towards an opposite direction with the information flow of the other way. The simultaneous achievable rates (R_1, R_3) of this scheme is as follows,

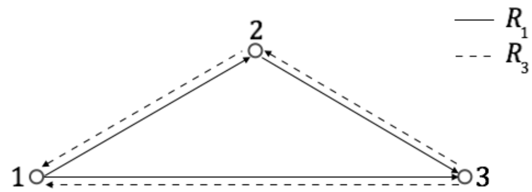

Figure 3.2: Two-way single-relay channel

$$R_1 < I(X_1, X_2; Y_3|X_3) \tag{1}$$

$$R_3 < I(X_3, X_2; Y_1|X_1) \tag{2}$$

and

$$R_1 < I(X_1; Y_2|X_2, X_3) \tag{3}$$

$$R_3 < I(X_3; Y_2|X_2, X_1) \tag{4}$$

$$R_1 + R_3 < I(X_1, X_3; Y_2|X_2) \tag{5}$$

In [20], there is a simply interpretation that if (1)-(5) are achievable rates of a single-relay channel, then (1) and (2) are the cut-set bounds without beamforming, and (3)-(5) are the multiple-access region if the relay can fully decodes both sources.

3.2 Two-Relay Channel

In two-relay channels, what we are interested in is two-way two-relay channels. It is a network consists of four nodes, node 1 and 4 are source nodes, while node 2 and 3 are relay nodes. Thus it is a discrete memoryless relay channel deonoted by

$$(\mathcal{X}_1 \times \mathcal{X}_2 \times \mathcal{X}_3 \times \mathcal{X}_4,\ p(y_1, y_2, y_3, y_4|x_1, x_2, x_3, x_4),\ \mathcal{Y}_1 \times \mathcal{Y}_2 \times \mathcal{Y}_3 \times \mathcal{Y}_4)$$

At time t=1,2,..., for i=1, 2, 3, 4, the inputs $x_{i,t}$ is simultaneously transmitted by node i, and it can be chosen according to the past received outputs $(y_{i,t-1}, y_{i,t-2}, ..., y_{i,1})$. The received symbols $y_{i,t}$ only depend on the inputs respectively corresponding to $p(y_1, y_2, y_3, y_4|x_1, x_2, x_3, x_4)$.

3.2.1 Channel scheme 1

According to the route that two sources communicate. The first channel scheme that comes into our mind must be the most straightforward one, illustrated in Figure 3.3. In this scheme, one way transmits messages the same route with the other way, but in opposite direction. It is this channel scheme that inspired us maybe we can use a new method to generate an (N+1)-relay. The main idea of the method is to separate an (N+1)-relay channel to two parts: an N-relay channel and a new node attached on it. It will be much easier for us to get the achievable rates of a new (N+1)-relay channel scheme by using this way, since after separating the scheme into two part, we only need to add several new constrains to the previous achievable rates of N-relay channel. It is always easier to get the achievable rate region of a channel with less relay nodes. Thus later we will talk about if we can obtain the achievable rate region of most of the multiple-relay channel by adding nodes one by one to a single-relay channel.

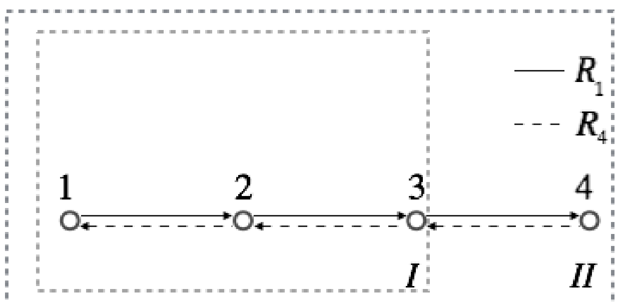

Figure 3.3: Two-way Two-relay channel

1. Move node 4 to node 3

This scheme is shown in Figure 4.1, it can be separated into two parts, I and node 4. After moving node 4 to node 3, it becomes a superposed node, the information which should be sent from node 4 is sent by node 3. We want to show a transfer of source information, so we use the word move rather than remove. the channel becomes a two-way single-relay channel, Figure 3.3-I, and the achievable rates are as follows,

$$R_1 < I(X_1, X_2; Y_3|X_3) \quad (3.3)$$
$$R_4 < I(X_3, X_2; Y_1|X_1) \quad (3.4)$$

and

$$R_1 < I(X_1; Y_2|X_2, X_3) \quad (3.5)$$
$$R_4 < I(X_3; Y_2|X_2, X_1) \quad (3.6)$$
$$R_1 + R_4 < I(X_1, X_3; Y_2|X_2) \quad (3.7)$$

2. Combine node 4 into part [I]

Move node 4 back to its original place, shown in Figure 3.3-II. The channel was recovered to a two-way two-relay channel. Now, node 4 becomes the source node rather than node 3, while node 3 becomes a relay node. Then we need to add two parts to the previous scheme: (1). message transmits from node 4 to node 3, then goes to node 2 and node 1. (2). signal sent by source node 1 need to be forward from node 3 to node 4. Thus the following constrains need to be added to the previous achievable rate region.

$$R_1 < I(X_1, X_2, X_3; Y_4|X_4) \quad (3.8)$$
$$R_4 < I(X_4; Y_3|X_1, X_2, X_3) \quad (3.9)$$
$$+ I(X_4; Y_2|X_1, X_2, X_3) \quad (3.10)$$
$$+ I(X_4; Y_1|X_1, X_2, X_3) \quad (3.11)$$

3. Integrate 1 and 2

Add up the achievable rates which have the same decoder. Add (3.10) to (3.6) and (3.7),

$$R_4 < (X_4; Y_2|X_1, X_2, X_3) + I(X_3; Y_2|X_2, X_1) = I(X_4, X_3; Y_2|X_2, X_1),$$

add (3.11) to (3.4), we get,

$$R_4 < I(X_4; Y_1|X_1, X_2, X_3) + I(X_3, X_2; Y_1|X_1) = I(X_4, X_3, X_2; Y_1|X_1),$$

which is one of the cut-set bounds of the achievable rate region of this channel, and the other one is (3.8). After adding a new source node, node 3 become relay nodes rather than a source node, therefore both node 2 and node 3 will hold a multiple-access region. We already have the region of node 2, (3.5)-(3.7). (3.9) is an achievable rate in the multiple-access region of relay node 3. we can also add one more constrain to set the boundary of $R_1 + R_4$ in the region of node 3,

$$R_1 + R_4 < I(X_1, X_4, X_3; Y_2|X_2)$$

It is obvious that node 4 is now the interference of (3.3) and (3.5), can we use some way to get rid of the interference? The answer is yes. We will show the method after introducing the second channel scheme in 3.2.2.

The new achievable rates are as follows,

$$\begin{aligned} R_1 &< I(X_1, X_2, X_3; Y_4|X_4) && (3.12) \\ R_4 &< I(X_4, X_3, X_2; Y_1|X_1) && (3.13) \end{aligned}$$

and

$$\begin{aligned} R_1 &< I(X_1; Y_2|X_2, X_3, X_4) && (3.14) \\ R_4 &< I(X_4, X_3; Y_2|X_2, X_1) && (3.15) \\ R_1 + R_4 &< I(X_1, X_3, X_4; Y_2|X_2) && (3.16) \end{aligned}$$

and

$$\begin{aligned} R_1 &< I(X_1, X_2; Y_3|X_3, X_4) && (3.17) \\ R_4 &< I(X_4; Y_3|X_1, X_2, X_3) && (3.18) \\ R_1 + R_4 &< I(X_1, X_2, X_4; Y_3|X_3) && (3.19) \end{aligned}$$

3.2.2 Channel scheme 2

The scheme we talked about in 3.2.1 is a channel model that two sources communicate with each other in path but different directions. What about the schemes that two information flows different routes.

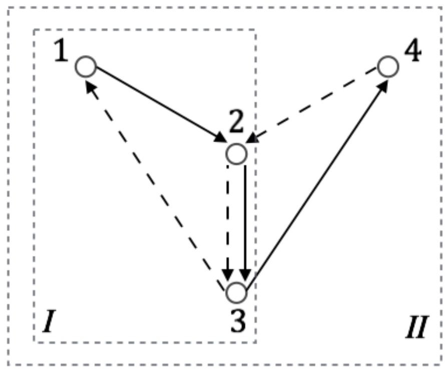

Figure 3.4: Two-way Two-relay channel-2

By using the same method in 3.2.1, the achievable rate region of scheme 2 is as follows,

$$R_1 < I(X_1, X_2, X_3; Y_4|X_4) \tag{3.20}$$
$$R_4 < I(X_2, X_3, X_4; Y_1|X_1) \tag{3.21}$$

and

$$R_1 < I(X_1; Y_2|X_2, X_3) \tag{3.22}$$
$$R_4 < I(X_4; Y_2|X_1, X_2, X_3) \tag{3.23}$$
$$R_1 + R_1 < I(X_1, X_4; Y_2|X_2, X_3) \tag{3.24}$$

and

$$R_1 < I(X_1, X_2; Y_3|X_3) \tag{3.25}$$
$$R_4 < I(X_2, X_4; Y_3|X_1, X_3) \tag{3.26}$$
$$R_1 + R_4 < I(X_1, X_2, X_4; Y_3|X_3) \tag{3.27}$$

3.2.3 Analysis of scheme 1 and scheme 2

Take scheme 1 as an example. This is the achievable rate region we want to obtain, we note it *region 1*:

$\{(3.12)-(3.13),\ and\ (R_1 < I(X_1; Y_2|X_2, X_3, X_4))-(3.16),\ and\ (R_1 < I(X_1, X_2; Y_3|X_3, X_4))-(3.19)\}$

But there exists a "deadlock" problem obtaining that region, which is identified in [21]. The "deadlock" is a conflict of achieving the two multiple-access region at the same time. That is, if we want to achieve ($R_1 < I(X_1;Y_2|X_2,X_3,X_4)$)-(3.16), then node 3 needs to decode message before node 2 in order to help, but if we want to achieve ($R_1 < I(X_1,X_2;Y_3|X_3,X_4)$-(3.19), node 2 needs to decode before node 3. But later on, Ponniah and Xie proved in [14] that we can obtain *region 1* but we need to add one more constrain to ensure that some relay can decode at least one of the sources before the other relay, or some relay can decode both sources before the other relay.

The following theorem is the result of Ponniah and Xie proposed in [14].

Theorem 3.1 *For the two-way two-relay, any rates* (R_1, R_4) *satisfying region 1 are simultaneously achievable provided that at least one of the following constrants hold:*

$$R_1 < I(X_1;Y_2|X_2,X_3) \tag{3.28}$$

$$R_4 < I(X_4;Y_3|X_2,X_3) \tag{3.29}$$

$$R_1 + R_4 < \max\{I(X_1,X_4;Y_2|X_2,X_3), I(X_1,X_4;Y_3|X_2,X_3) \tag{3.30}$$

The added constrains are symmetric, we only need to consider two mutually exclusive case in (3.28)-(3.30). For one case, either (3.28) or (3.29) hold, it is the case that some relay decodes one source before the other relay. For the other case, the first part of (3.30) holds, this is the case that one relay decodes both source before the other relay. Only one case is needed to be satisfied.

We want to prove that our schemes can fit this result.

The outline of proof is as follows,

First return to our scheme in 3.2.1. Observe that in (3.14) and (3.17), node 4 is an interference. If we take this as case 1. (1). With the constrain (3.29) added to *region 1*, it means R_4 is as low as node 3 can decode the message even though node 1 is an interference. Then This is the case that node 3 can decode before node 2. Then ($R_1 < I(X_1,X_2;Y_3|X_3,X_4)$ can be achieved.(2). As for (3.14), (3.16) can be implied by (3.14) and (3.15), and(3.16) can be decomposed as follows,

$$\begin{aligned} R_1 + R_4 &< I(X_1,X_3,X_4;Y_2|X_2) \\ &= \underset{(3.14)}{I(X_1;Y_2|X_2,X_3)} + \underset{(3.15)}{I(X_3,X_4;Y_2|X_2,X_1)} \end{aligned}$$

If $R_1 < I(X_1; Y_2|X_2, X_3)$, then node 2 can decode the message and $R_1 < I(X_1; Y_2|X_2, X_3, X_4)$ is achieved, that is what we will choose. (*) If $R_1 > I(X_1; Y_2|X_2, X_3)$, then (3.13) is satisfied. We will note here for a later use.

Secondly return to our scheme in 3.2.2. Observe that there exists the same "deadlock" problem. The achievable rate region we want to obtain is as follows, we note it *region 2*,

$$\{(3.20)-(3.21),\ and\ (R_1 < I(X_1; Y_2|X_2, X_3, X_4))-(3.24),\ and\ (R_1 < I(X_1, X_2; Y_3|X_3, X_4))-(3.27)\}$$

We want to achieve *region 2*, but notice that the "deadlock" problem exists as well. If we take this as case 2. With the first part of (3.30) added into *region 2*, it means that node 2 decode both source before node 3. Since we take this case as a complementary of case 1, with the first part of (3.30), these three constrains will show as below,

$$\begin{aligned} R_1 &> I(X_1; Y_2|X_2, X_3) && (*) \\ R_4 &> I(X_4; Y_3|X_2, X_3) && (**)\ (opposite\ of\ (3.29)) \\ R_1 + R_4 &< I(X_1, X_4; Y_2|X_2, X_3) && (***) \end{aligned}$$

region 2 plus the first part of (3.30) is verified as follows. (1). (3.23) can be implied by (*) and (***), then $R_1 < I(X_1; Y_2|X_2, X_3, X_4)$ can be achieved. (2)$R_1 < I(X_1; Y_2|X_2, X_3, X_4)$ can be implied by (**) and (3.27).

Thus we demonstrated that by using our schemes, we can obtain the same result as the result has been proved by other method. So we can say that we are able get a valid achievable rate region by using the method of separating a four-terminal channel into three-terminal channel + a node.

Chapter 4

A Three-Terminal Relay Channel to A Four-Terminal Relay Channel

4.1 All possible schemes of three-terminal relay channels

Can we use this "attaching a node" method to obtain all of the two-way relay schemes. We will show our testing method and discuss in detail. First, we need to know all the schemes of three terminal relay channels, which we regard as our basic channel.

In this section, we are going to show all the possible channel schemes of three-terminal channel and collect them in two classes. The first class consists of the channels which have separate and different source nodes. The second class will show the case when the two source nodes overlap and two different information will be sent from one node. Among these six channel schemes, only (a) and (e) are single-relay channels, since there exists only one relay node and it is the common relay node.

4.1.1 Two source nodes non-overlapping

If we fix two nodes as the source nodes, then there are a total of four kinds of different schemes according to the message transmission route. we say node 1 and node 3 as the source nodes respectively for each way. The schemes are illustrated in the figure below.

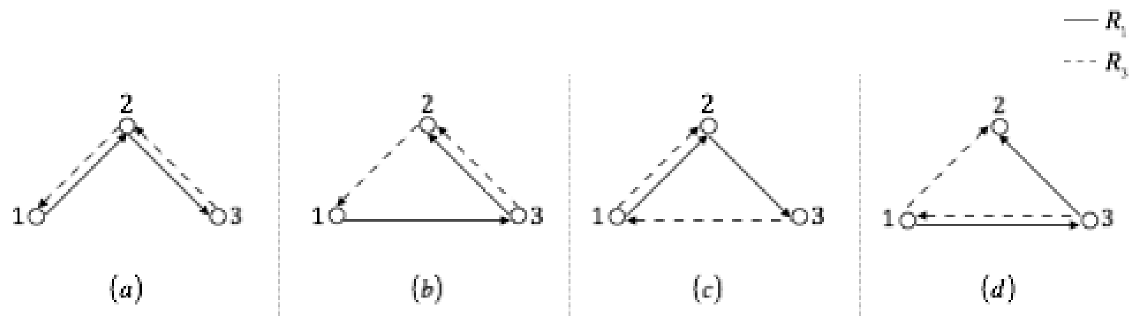

Figure 4.1: All the cases when fixing node 1 and 3 as source nodes

For (a), the source nodes are also the destination nodes of the other way, and both messages transmitted from their source nodes directly go to the common relay node 2 and then arrive their destination respectively. By using binning technique, a node can decode a message only if it knows both of the information sent from the two sources. For this case, node 2 can decode the two sources, and then messages can be transmitted to the next node. By receiving the message of source one and knowing source two originally, node 3 can decode the two sources. Similar to node 1. Intuitively, there should be no interference in the achievable rates, but theoretically, the achievable rate region is as follows,

$$R_1 < I(X_1; Y_3|X_3, X_2) \qquad (i)$$

$$R_3 < I(X_3; Y_2|X_2) \qquad (ii)$$

$$R_1 + R_3 < (X_1, X_3; Y_2|X_2) \qquad (iii)$$

Observe that (i) has interference from node 3, and (ii) has interference from node 1. This is different from our intuition. So we will see if we can find out any hidden constraints which can eliminate that interference. (iii) can be expanded like this,

$$\begin{aligned} R_1 + R_3 &< I(X_1, X_3; Y_2|X_2) \\ &= I(X_1; Y_2|X_2, X_3) + I(X_3; Y_2|X_2) \end{aligned}$$

there are two cases to make this inequality satisfy.

Case 1: If $R_1 > I(X_1; Y_2|X_2, X_3)$, then $R_3 < I(X_3; Y_2|X_2)$ must be true, which is (ii).

Case 2: If $R_1 < I(X_1; Y_2|X_2, X_3)$, Node 2 can decode node 1, and node 1 is not an interference of (ii). Then $R_3 < I(X_3; Y_2|X_2, X_1)$ can be achieved, therewith the interference from node 1 in (ii) disappear.

Thus if we add one more constraint, $R_1 < I(X_1; Y_2|X_2, X_3)$, then the achievable rate region of case (1) is as follows without interference,

$$\begin{aligned} R_1 &< I(X_1, X_2; Y_3|X_3) \\ R_3 &< I(X_3, X_2; Y_1|X_1) \end{aligned}$$

and

$$\begin{aligned} R_1 &< I(X_1; Y_2|X_2, X_3) \\ R_3 &< I(X_3, Y_2|X_2, X_1) \\ R_1 + R_3 &< (X_1, X_3; Y_2|X_2) \end{aligned}$$

(b) and (c) are symmetric with the two sources, so we will only discuss (b) here. For one way, the message transmits from source node 3 to relay node 2 then goes to destination node 1; for the other way, the previous source node 3 becomes its relay node, so the message transmit from its source node 1 to node 3 and then goes to its destination node 2, which is the relay of the other way. Node 3 can decode two sources since it knows source two itself and it knows source one by receiving the message sent by node 1. Node 2 can decode the two sources since the two sources both transmit from node 3 to node 2. Because of the successful decoding at node 2, the messages carrying the information of source two can be transmit to node 1, where source two can be decoded. (c) is the case that interchange node 1 and node 3.

But theoretically, the multiple-access region is as follows,

$$R_1 < I(X_1, X_3; Y_2|X_2)$$

$$R_3 < I(X_3; Y_2|X_2)$$

$$R_1 + R_3 < I(X_1, X_3; Y_2|X_2)$$

Observe that the rate of node 3 has interference from node 1. Using the same method as in (1). If $R_1 > I(X_1; Y_2|X_2, X_3)$, then $R_3 < I(X_3; Y_2|X_2)$; if $R_1 < I(X_1; Y_2|X_2, X_3)$, then node 2 can decode node 1, therefore $R_3 < I(X_3; Y_2|X_2, X_1)$ without the interference of node 1. Since we want the achievable rates witoute interference, so we add one constrain $R_1 < I(X_1; Y_2|X_2, X_3)$, and obtain the achievable rate region of (b) as follows,

$$\begin{aligned} R_1 &< I(X_1; Y_3|X_2, X_3) \\ R_3 &< I(X_3, X_2; Y_1|X_1) \end{aligned}$$

and

$$\begin{aligned} R_1 &< I(X_1; Y_2|X_2, X_3) \\ R_3 &< I(X_3; Y_2|X_2, X_1) \\ R_1 + R_3 &< I(X_1, X_3; Y_2|X_2) \end{aligned}$$

As for (d), the source node of one way is the relay node of the other way, so for each way, the message sent by its source goes to the relay which is the source node of the other way, then transmit to the common destination node 2. In this scheme, node 1 knows both of the source information from itself and the message sent from source node 3, then node 1 can decode both of the sources, and then transmit the message carrying source two to node 2. Similar to node 3, it transmit the message carrying source one to node 2. Then node 2 which is the destination node for both way can decode both of the sources and finish this transmission. The achievable rate region of (d) is as follows,

$$\begin{aligned} R_1 &< I(X_1, X_3; Y_2|X_2) \\ R_3 &< I(X_3; Y_1|X_1, X_2) \end{aligned}$$

and

$$\begin{aligned} R_1 &< I(X_1; Y_3|X_2, X_3) \\ R_3 &< I(X_3, X_1; Y_2|X_2) \\ R_1 + R_3 &< I(X_1, X_3; Y_2|X_2) \end{aligned}$$

4.1.2 Two source nodes overlap

When the two source nodes overlap each other, it looks like there is only one node containing two sources. It happens when two information need to be sent out from a same place. So except the 4 kinds of schemes in 3.3.1, the following two channel schemes are also the component of all possible three-terminal relay channel.

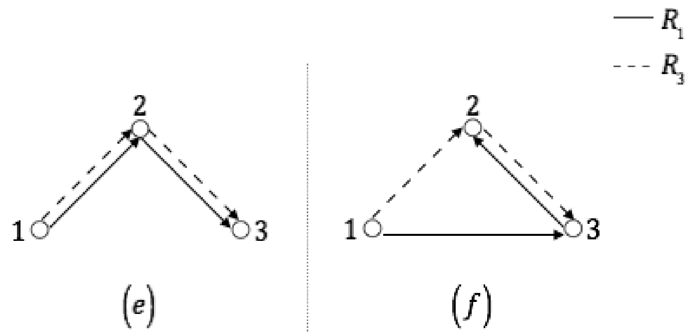

Figure 4.2: All the cases when source node 1 and 3 are overlapped

The achievable rate region of (e) is as follows,

$$\begin{aligned} R_1 &< I(X_1;Y_2|X_2,X_3) \\ R_3 &< I(X_1;Y_2|X_2,X_3) \\ R_1+R_3 &< I(X_1;Y_2|X_2,X_3) \end{aligned}$$

and

$$\begin{aligned} R_1 &< I(X_1,X_2;Y_3|X_3) \\ R_3 &< I(X_1,X_2;Y_3|X_3) \\ R_1+R_3 &< I(X_1,X_2;Y_3|X_3) \end{aligned}$$

The achievable rate region of (f) is as follows,

$$\begin{aligned} R_1 &< I(X_1;Y_2|X_2,X_3) \\ R_3 &< (X_1;X_3|X_3,X_2) \\ R_1+R_3 &< (X_1,X_3;Y_2|X_2) \end{aligned}$$

and

$$\begin{aligned} R_1 &< I(X_1,X_2;Y_3|X_3) \\ R_3 &< (X_1,X_3;Y_2|X_2) \\ R_1+R_3 &< (X_1,X_2;Y_3|X_3) \end{aligned}$$

4.2 From three-terminal to four-terminal channel

We have showed all of the possible channel schemes and their achievable rates for three-terminal relay channels. Then for four-node channel, we only interested in the case which the sources are in two separate nodes. Still fix two nodes as sources, there will be $(3 * 2 * 1)^2 = 36$ kinds of channel schemes for two-way relay channel, we will carefully test each scheme to see if all of them can fit the *attaching a node* method to obtain the achievable rate region. Firstly, we say the source nodes 1 and 4 for each way respectively. Secondly, see if the channel can be separated into two parts consisting of a three-terminal channel we mentioned in 3.3.1 and 3.3.2, and a single node that can be attached on. Thirdly, if the second part is true, figure out the achievable rate region of the two-way four-node relay channel.

There are two classes of four-node relay channels. One is two-way two-relay channel we talked about, which two nodes in the channel are source nodes, and the other two are relay nodes. The other one is at least in one way, the relay is the source node of the other way. We will now discuss the second class.

4.2.1 Feasible Schemes

For two-way two-relay channel, if we fix two nodes as the sources for both way, then there will be $(3 * 2 * 1)^2 = 36$ kinds of channel schemes with different routes for two information transmitting to their destinations.

Since we are interested in the method if every two-relay channel scheme can be generated from a single-relay channel, then we will detailed focus on each channel scheme or each family of channel schemes to see if there exist any law of generation that can be summarized. we make a classification of the 36 schemes.

1. Channels generated from (a)

Here are the schemes with the component part: single-relay channel (a).

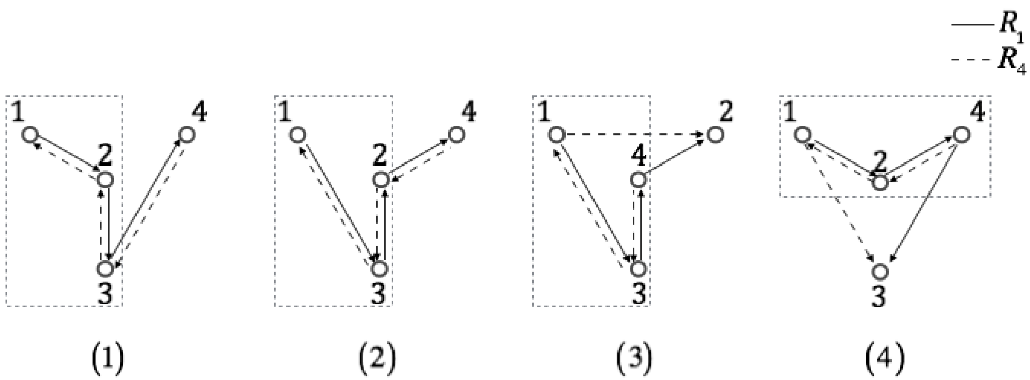

Figure 4.3: schemes can be generated from (a)

We can say (1) and (2) are the same scheme. The only difference between (1) and (2) are the order of node 2 and 3, because the two relay nodes 2 and 3 have the same function, (1) and (2) will be completely the same if we interchange the number of the two relay node 2 and 3. We have detailed talked about the achievable rate region of (1) in section 3.4.1.

(3) and (4) are also the same if we interchange node 3 and 2. In each of this two channels, these two node are both the destination node of the scheme, so they have the same function to the channel.

Thus there are two kinds of achievable rate region need to be figured out.

2. Channels generated from (b) and (c)

Here are the schemes with the component part: single-relay channel (b) and (c) since they are symmetric.

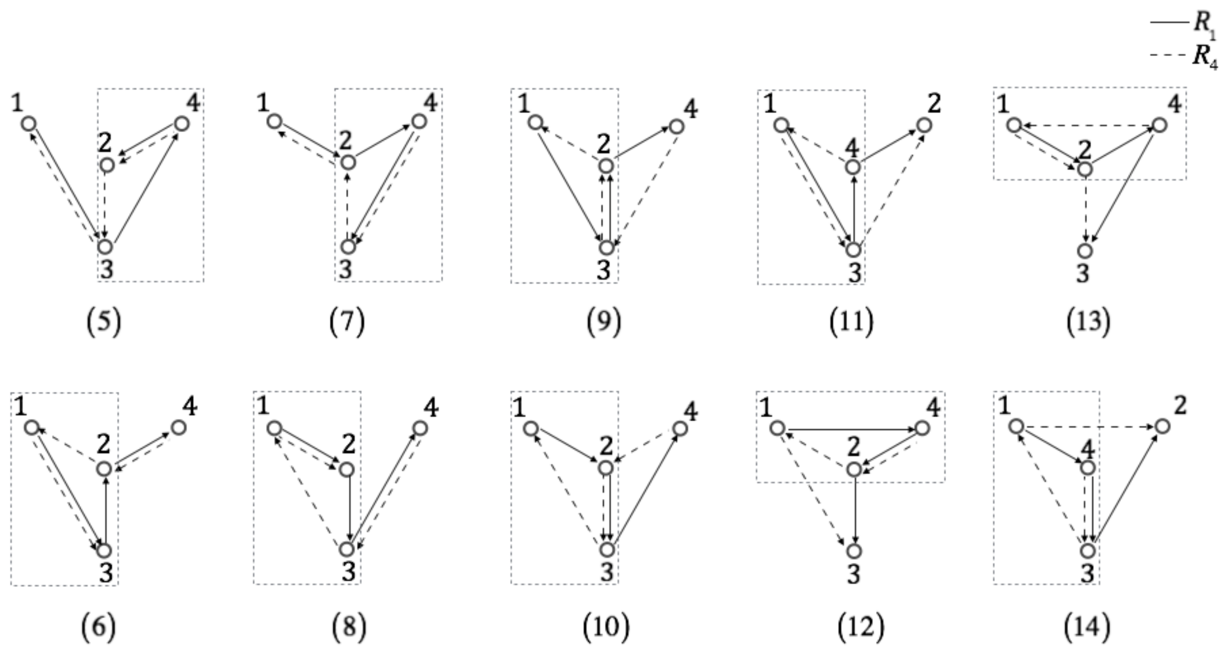

Figure 4.4: schemes can be generated from (b) and (c)

(5) and (6) are the same if we exchange node 2 and 3 because of the symmetry, also node 2 and 3 are both the relay nodes of the channel so they equal. So do (7) and (8).

(5) and (7) will be the same if interchange node 1 and 4, since source nodes are equal, i.e. having the same resiponsibility to the channel. If we change over the name of two source nodes in (7), then the scheme will be the same with (5).

(6) and (8) are in the same situation with the pair (5) and (7), the only difference in between is an interchange of two source nodes.

(9) and (10) can be interconverted if we exchange node 2 and 3.

Note that because of the connections among (5) to (8), the four schemes will be treated as a group. Another thing is that from (5) to (10), the node that can separately add on the single-relay model is a source node itself.

The following two pairs, (11) and (12), can be interconverted to (13) and (14) respectively if switching node 2 and 3. Like the group (5)-(8), the two pairs (11) and (12), (13) and (14) also have links in between, the two schemes in each pair can be interchanged by switching source nodes 1 and 4. So (11) to (14) is another group.

Note that (11) to (14) is another group[1]. And the node can be separately added on to the single-relay channel in each scheme is a destination nodes for both ways

3. Channels generated from (d)

Here are the schemes with the component part: single-relay channel (b) and (c) since they are symmetric.

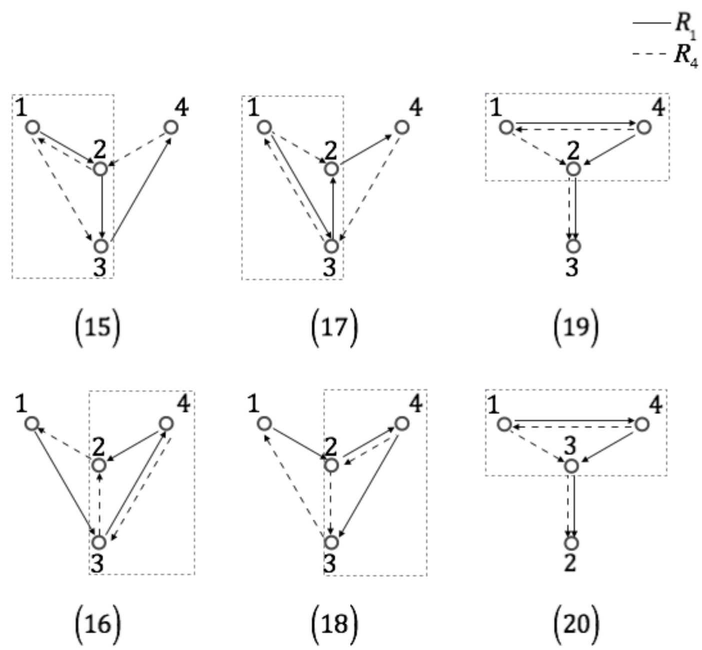

Figure 4.5: schemes can be generated from (d)

(15) to (18), (19) and (20) are two groups.

[1]For simplicity in the following paragraph, according to the corresponding figure, we say schemes buildup a group if they can transform to each other by switching source node 1 and 4, or interchange node 2 and 3, or both.

4. Channels generated from (e)

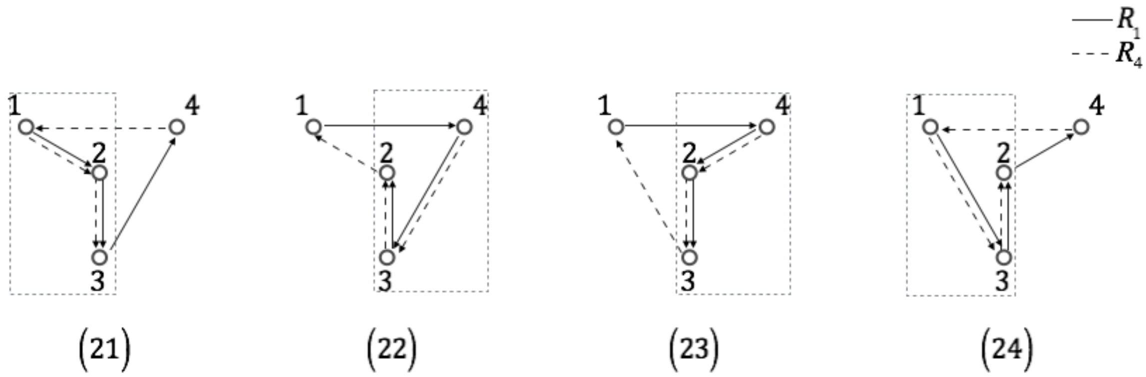

Figure 4.6: schemes can be generated from (e)

(21) to (24) are a group.

5. Channels generated from (f)

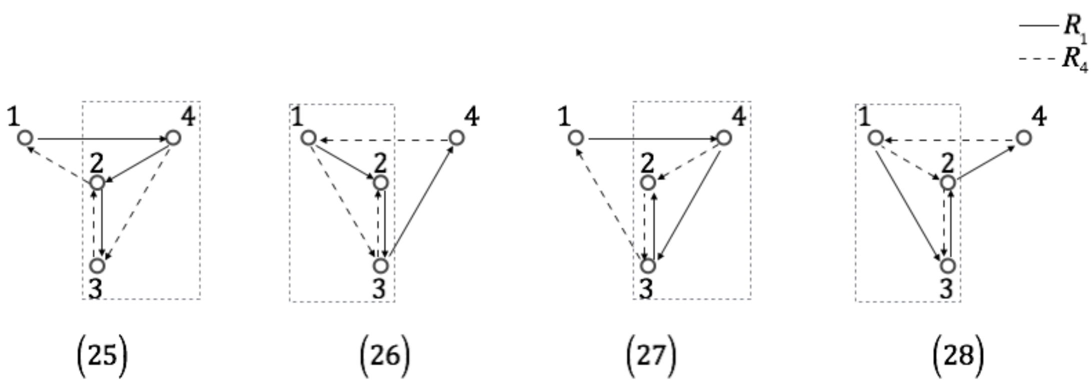

Figure 4.7: schemes can be generated from (f)

(25) to (28) are a group.

4.2.2 Infeasible Schemes

These schemes cannot be generated from a three-node channel since we cannot separate them into a three-node channel and a node can be added on.

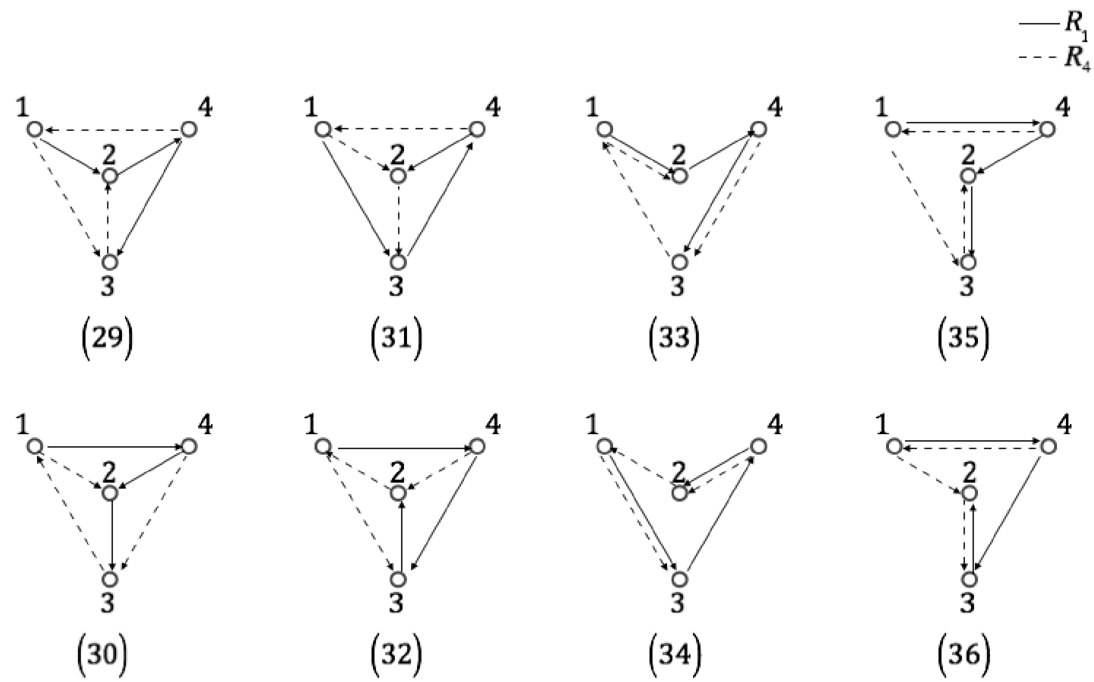

Figure 4.8: infeasible schemes

Observe from figure 3.12 that (29) to (32) are a group, and (33) to (36) are another group.

Since none of they cannot be generated from a three-node channel, we need to figure out if they still have their achievable rate regions. We will take (29) as an example. However during the process we obtain its achievable rates, we found that the scheme itself is infeasible. Here is the reason.

For node 1, it knows both information of two sources since it is source one itself and source node 4 transmits its message to 1. Then node 1 can decode the sources and transmits the signal to the next node. Since we use random binning technique, each node needs to know both source information in order to decode. Thus for node 2, it knows the information of source one by receiving message one directly sent from node 1, the way it gets to know the other source information is through the signal sent by node 3. Then we need node 3 to decode the message first. But the thing is node 3 gets into the same situation node 2 encountered. Now node 3 knows the information of source four by receiving the signal sent by node 1, if it wants to know source one, it needs the signal sent from which requires node 2 to decode the message before node 3. Each node 2 and 3 needs the other node to decode the message before itself. So this scheme is infeasible for message transmission.

For group (33)-(36), the situation is similar to group (29)-(32), take (33) as an example. We will consider this problem node by node. For node 1, it knows source one, but if it wants to know source four, it need the help of node 3 to transmit to node 1. For node 3, it will know the both sources if node 4 knows source one then it can decode and transmit the signal to node 3. However, the way to let node 4 knows source one is by the help of node 2. Node 2 can decode only if node i can decode. The problem comes back to the original point.

From the analysis of each group, the reason cause their infeasibility is there exists two nodes that each of them need to decode before the other in order to help, thus no one can decode.

Corollary 4.1 *In a feasible channel, it consists of two kinds of nodes:*
1. A kind nodes send out a signal consisting of two source messages to the next node.
2. A kind nodes send out two signals to two different nodes respectively. Each of the signal consists one source message, besides each of the corresponding receiver has previous knowledge about the other source

Chapter 5

Conclusion and Future Work

5.1 Conclusion

We reviewed the development of wireless communication, and introduced the rise of relay channels. The current situation of a much more wide use of relay channels nowadays arouse people's passion of exploring the unknown parts of relay channels. To get a more general achievable rate region and to get rid of the interference are both common projects to do research on.

With a throughly study of the basic information theory and the network information theory. we built a basic framework of what is the information theory and what kind of fields can they apply to. Then we focused on relay channels, with the help of several papers, we learned about the development of relay channels and the different fields we can particularly pay attention to.

We proposed a three-step method to study a two-way two-relay channel. First separate the two-way two-relay channel into two parts, one is a three-terminal relay channel, and the other one is a common end node. Then we add the corresponding constrains of the end node to the achievable rates of a three-terminal relay channel. Finally, we collect and simplify the new achievable rates. All of the possible schemes of two-way two-relay channels are examined, we talked about this in 3.2. With the verification of the result proved before, we showed that we can get the valid achievable rate regions by using that three-step method.

In addition, we want to consider all of the possible two-way relay channel schemes when we fix two different nodes as the source nodes. Not only we want to study the communication between the two source nodes, we also consider the other schemes which the destination of some way is not the source node of the other way. Then we found that there are several schemes that we cannot separate them into a three-terminal relay channel and a common node can be added to. Then we showed they are the schemes that infeasible, attached with our demonstration.

5.2 Future Work

During the review of relay channels, we found that a correction of the previous achievable rate region is proposed by adding one constrain to fix a conflicting problem. We only know that there is a way to get a general achievable rate region for limited several two-way relay schemes. But the region for a general relay channel is still unknown. Maybe the method we introduced in this thesis can do some help for future work.

By separating the four-terminal channel into a three-terminal channel + a node, one possible generation of the this work is to extend this method to n+1-terminal relay channels study. Then use this way to obtain a relatively achievable rate region especially for the general two-way relay channels.

In addition, we need to study relay channels more comprehensively. Only if we are practiced in all the coding schemes and coding strategies, can we have a clear mind to recognize what to use when coming into different problems, and we could arise some innovative ideas. The most optimal scheme for a relay channel is still unknown.

The achievable rate region for general relay channels has been unknown for more than 40 years, while huge of new methods have been applied to it. If I would keep on doing the research about the achievable rate region for relay channels, I would begin to analysis all the existence achievable rate regions of all different relay channels in reality and simulations. Then focus on the theoretical studies, maybe there would be some new and better results for the general achievable rate region.

Also, the most difficulty problem are always built up by the simplest ones, we need to study information theory, probability theory and statistical analysis more thoroughly. To master more fundamental knowledge is a significant preparation for working on complicated problems and those basic information is the original inspiration for us to decompose the puzzles into pieces that can be solved.

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