The Plug-In Hybrid Electric Vehicle Routing Problem with Time Windows

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

There is an increasing interest in sustainability and a growing debate about environmental policy measures aiming at the reduction of green house gas emissions across different economic sectors worldwide. The transportation sector is one major greenhouse gas emitter which is heavily regulated to reduce its dependance on oil. These regulations along with the growing customer awareness about global warming has led vehicle manufacturers to seek different technologies to improve vehicle efficiencies and reduce the green house gases emissions while at the same time meeting customer's expectation of mobility and flexibility. Plug-in hybrid electric vehicles (PHEV) is one major promising solution for a smooth transition from oil dependent transportation sector to a clean electric based sector while not compromising the mobility and flexibility of the drivers.

In the medium term, plug-in hybrid electric vehicles (PHEV) can lead to significant reductions in transportation emissions. These vehicles are equipped with a larger battery than regular hybrid electric vehicles which can be recharged from the grid. For short trips, the PHEV can depend solely on the electric engine while for longer journeys the alternative fuel can assist the electric engine to achieve extended ranges. This is beneficial when the use pattern is mixed such that and short long distances needs to be covered. The plug-in hybrid electric vehicles are well-suited for logistics since they can avoid the possible disruption caused by charge depletion in case of all-electric vehicles with tight time schedules.

The use of electricity and fuel gives rise to a new variant of the classical vehicle routing with time windows which we call the plug-in hybrid electric vehicle routing problem with time windows (PHEVRPTW). The objective of the PHEVRPTW is to minimize the routing costs of a fleet of PHEVs by minimizing the time they run on gasoline while meeting the demand during the available time windows. As a result, the driver of the PHEV has two decisions to make at each node: (1) recharge the vehicle battery to achieve a longer range using electricity, or (2) continue to the next open time window with the option of using the alternative fuel. In this thesis, we present a mathematical formulation for the plug-in hybrid-electric vehicle routing problem with time windows. We solve this problem using a Lagrangian relaxation and we propose a new tabu search algorithm. We also present the first results for the full adapted Solomon instances.

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Dedication

This thesis would not have been possible without the support of my family, friends, and professors.

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Chapter 1

Introduction

Over the past 50 years, logistics has played a fundamental role in the economic development of societies. During this period, logistics was governed by a profit maximizing paradigm that ignored social and environmental costs. As the debate over sustainability increases, the main focus of logistics is shifting towards reducing the environmental impact of freight transport by improving the efficiency of the vehicles and reducing the dependance on oil.

The logistics sector faces a challenging future due to its crucial and growing role in the world energy use and green house gas (GHG) emissions. In 2004, the transport energy use amounted to 26% of the total world energy consumption and the transport sector was responsible for about 23% of world energy-related GHG emissions [Birol, 2007]. The growth rate of energy consumption in the transport sector in the past decade was highest among all the end-use sectors. Freight transport, among all sectors in the IEA-11 countries, showed the highest relative growth in CO_2 emissions percentage since 1973. Emissions increased as freight activity grew in line with GDP and with energy-intensive trucking taking a larger share of total ton-kilometers hauled [IEA, 2006]. Freight transport now consumes 35% of all transport energy, or 27 exajoules (out of 77 EJ total) with domestic freight gaining an increasing share of the market [WBCSD, 2004]. Domestic freight, in most of the developed countries, is dominated by road transport and is characterized by long traveling distances and overhauls Tester [2005]. The environmental impact of freight transport is thus determined by the efficiency of the road freight sector.

The transition to sustainable transportation is an essential step towards meeting the global emissions reduction targets. With the available breed of new technologies, this sector is radically changing the designs and components of the vehicles from fossil fuel based internal combustion engine (ICE) vehicles to electric and hybrid electric vehicles.

This technology shift will lead to the emergence of new technology strategies that will affect logistics sector traditional management approaches.

Proponents of renewable energy have created too many alternatives for the carbon intensive transportation; this bewildering collection of green mobility technology leaves a very simple question unanswered: Which technology is the best?

1.1 Available Technologies

Many technological improvements have been made to vehicles to reduce their environmental impact. Some of these advances have been imposed by environmental legislation, others have been incentivized by commercial pressure to improve energy efficiency and adhere to the growing environmental conscious customer base. We focus our discussion on four main categories: internal combustion engine vehicles using fossil fuels and biofuels, electric vehicles, and hybrid electric vehicles.

1.1.1 Internal Combustion Engine Vehicles

The internal combustion engine (ICE) vehicles have dominated the transportation market soon after the introduction of Henry Ford's T-model car in 1908 [Womack et al., 1991]. Since then, the ICE vehicles have reached maturity after receiving of almost a century of dedicated R&D support. Most of the current ICE vehicles run on petroleum fuel which results in high GHG emissions. Improving the efficiency of the ICEs can lead to substantial decrease in GHG emissions on the short term.

As the available fossil fuel resources grow thinner, the attention is shifting to focus on developing alternative fuels, to reduce the dependence on fossil fuels. We restrict our discussion to biofuels.

1.1.2 ICE Vehicles with Biofuels

These vehicles run on biofuels instead of fossil fuels. Biofuels are normally extracted from renewable plant materials and oils, and are mainly two types:

- Biodiesel

- Bioethanol

Biodiesel fuels (known as alkyl esters) are extracted from plant and animal oils through a process called "transesterification". The level and quality of the extracted oils vary highly from one country to another, depending on the local growing conditions. On the other hand, Bioethanol fuels are extracted from biological feedstock which contain sugar. Both fuels are usually mixed with existing fossil fuels to make them usable [Mckinnon et al., 2010].

As for their environmental effect, several research claim that significant decrease in the major GHG emissions can be achieved by using ICE vehicles with biofuels [Demirbas, 2007], [Demirbas, 2008]. However, an EPA report states that they were not able to identify an "unambiguous" difference in exhaust CO_2 emissions level [EPA, 2002].

The major criticism regarding the use of biofuels in transportation is the adverse change in the land use and deforestation which can negate any potential GHG emissions savings [Searchinger et al., 2008]. For small scale farming, biofuels can be grown on marginal agricultural lands without having any serious environmental impact. However, as the global demand increases, the biofuel farming lands might compete with agricultural lands causing a sever disruption in the global agricultural production. Hence, biofuel engine vehicles need to address the production cycle of biofuels and their land use impact before they can be claimed sustainable.

1.1.3 Electric Vehicles

The previous technologies mentioned depend on a single technology for power generation, the ICE. However, Electric vehicles (EVs) rely on a radically different electrical engine and control systems. The major advantage of electric vehicles lies in the high efficiency of the electric motors along with complete absence of tail pipe emissions which helps in improving the urban air quality. In addition, the EVs provide larger primary fuel flexibility, as it allows the integration of renewable energy in the electricity generation system.

Battery Electric Vehicles

Battery electric vehicles (BEVs), also known as all-electric vehicles, store the electricity acquired from the grid into on-board rechargeable batteries [Affanni et al., 2005].The environmental impact of BEVs is highly dependent on the generation portfolio of electricity while the economic attractiveness is highly dependent on the cost and performance of the energy storage batteries. The current battery technology is undergoing seminal changes but remains heavy, bulky and enable only limited distance ranges. In addition, the long charging time required by the BEVs are limiting their integration into supply chains as it leads to an increase in the overall haulage time.

Fuel Cell Vehicles

The fuel cell vehicles (FCVs) represent a shift from on-board electricity storage to on-board electricity generation using another energy carrier (mainly hydrogen). Like BEVs, the FCVs are propelled by efficient electric engines [Barbir, 1995].

Currently, the most promising fuel cell technology as a solution for sustainable transport is the polymer electrolyte membrane (PEM) technology [Ryan and Turton, 2007]. The relatively low weight and higher durability makes this technology well suited for transportation. However, the major drawback of this technology is that it requires an expensive catalyst (normally platinum) for the separation of protons and electrons [Bar-On et al., 2002].

Despite great progress in recent years, FCVs continue to face significant challenges particularly with durability. Fuel cells cannot achieve marketable durability levels without undesired reactions, corrosions, and degrading performance [Mench, 2008]. In addition, Fuel cells work most efficiently using hydrogen [Steele and Heinzel, 2001] which is scarce and difficult to store thus requiring FCVs to have large, heavy tanks for storage [Edwards et al., 2008].

FCVs are not yet ready for wide commercialization, not only due to the difficulties associated with fuel cell technology, but also because of the difficulties with storing, transporting, and distributing hydrogen fuel [King and Inderwildi, 2010].

1.1.4 Hybrid Electric Vehicles

The transportation technologies discussed so far can be divided into two main categories: ICE and electricity-based vehicles. While the first suffers from high GHG emissions (in case of fossil-based fuels), and land use disruption (in case of biofuels) the latter suffers from technological barriers. In order to overcome these disadvantages, plug-in hybrid electric vehicles (PHEVs) are gaining attention recently for their ability to alleviate the disadvantages of both technologies while pertaining to sustainable mobility.

PHEVs have two propulsion systems: an electric motor and an ICE. The electric power is supplied to the electric motor by the storage device (typically a battery). The storage device, however, receives its charge from two sources: the grid, or the ICE, which operates once the battery storage drops below a minimum threshold value. PHEVs normally include a highly sophisticated electronic control system that manages the coordination of the ICE, the battery and the electric motor [Ryan and Turton, 2007].

1.2 Technology Assessment

The path towards full electrification of the transportation sector requires significant long term investments infrastructure to accommodate charging spots and support the decarbonization of the electric grid. PHEVs provide room for incremental breakthroughs in renewable energy technology, storage systems and appropriate infrastructure. PHEVs represent a significant stride towards sustainable mobility as they combine the benefits of electrification while maintaining the flexibility of long range driving provided by ICE vehicles.

ICEs in the market currently have an efficiency of 20-30% [Grant, 2003], whereas diesel engines are 35-45% efficient [King and Inderwildi, 2010]. PEM fuel cells and electric motors are 40-60% [Campanari et al., 2009] and 90% [Rand et al., 2008] efficient respectively. However, the well-to-wheel efficiency (LCA of the efficiency of fuels used for road transportation) of the PHEV and FCV vary according to the source of energy utilized. For instance, a PHEV which recharges its battery from a typical electricity grid which runs on coal, natural gas, or fossil based fuel, has a well-to-wheel efficiency of 30%. On the other hand, when the electricity grid is powered by renewable energy sources the well-to-wheel efficiency can be more than 60% [Campanari et al., 2009]. As for FCV, the same report claims that the maximum efficiency that FCVs can achieve is roughly 22% when it runs on direct hydrogen electrolysis. In comparison, PHEVs are around 40% more efficient than ICE vehicles, thus they represent an attractive medium term plan toward commercial all-electric electric vehicles [Schäfer et al., 2009].

1.3 Current Practices

Several companies have already realized the potential benefits of introducing electric based trucks to their fleet. Last year, Walmart announced that the integration of hybrid vehicles into their logistics along with efficient operations has increased their fleet efficiency by more than 25 percent. They are currently working on doubling its fleet efficiency by 2015 [1]. For

[1] http://walmartstores.com/pressroom/news/8949.aspx

this reason, they are currently testing two new heavy-duty commercial hybrid trucks: a full-propulsion Arvin Meritor hybrid and a Peterbilt Model 386 heavy duty hybrid truck. Peterbilt claims that its model can achieve around 12% fuel economy savings over similar non-hybrid technologies which will help Walmart achieve its efficiency targets.

1.3.1 Lean Distribution Systems

The increasing appeal of lean agile production and service systems increased the attractiveness of just-in time deliveries. Just-in time (JIT) deliveries are one of the building pillars of the lean systems since they are essential to keeping zero inventory through smaller and frequent deliveries. Truck loads, as a result, decreased along with the trucks utilization leading to new distribution system known as less than a truckload (LTL) [Askin and Goldberg, 2007]. While many environmentalists argue that such practices are unsustainable due to their contribution to the increase in the haulage distances, JIT is an enabling lean strategy to minimize inventory [Rothenberg, 1999]. This empty space available in LTL distribution systems provides an opportunity for adding lithium ion batteries to trucks and shifting to lighter weight vehicles for freight transport.

The attraction of electric based vehicles for the freight industry is twofold: they can achieve zero tail-pipe emissions, and are quieter than conventional vehicles; hence, EVs and HEVs are suitable for city logistics. The recent advances in the battery technologies have increased the interest for van-based home deliveries and other van based operations. In 2007, TNT has recently bought 55 for trial in 22 depots[2], recently TNT and Dutch express starting expanding their electric fleet in China as well [3]. Smith Electric Vehicles (SEV) has been a leader in manufacturing small electric vehicle trucks where batteries are stored underside of the truck [Mckinnon et al., 2010].

1.4 The Plug-in Hybrid-Electric Vehicle Routing Problem

While many technologies which are currently available at hand represent potential solutions for sustainable transportation, these technologies are still in their incumbent phase where significant breakthroughs are still required to bring them to market standards. A report

[2]http://www.elogmag.com/magazine/44/parcels-carriers-the-world.shtml
[3]http://www.joc.com/logistics-economy/tnt-launches-fully-electric-vehicles-china

conducted by the University of Oxford on the future of mobility [King and Inderwildi, 2010] concluded that all electric drive vehicles will be the main source of transportation on the long term. However, it stated that road transport will continue to rely on the internal combustion engine, in an optimized, classic or hybrid set-up in the medium term.

Hybrid electric vehicles and in particular plug-in hybrid electric vehicles represent the missing cycle in the transition from carbon intensive transportation to sustainable transportation. These vehicles are equipped with large batteries which can be recharged from the electric grid. However, to gain the most GHG reduction benefit from the PHEVs, they need to be driven in such a way that the ICE is used as little as possible. This means that for short trips, the electric drive-train is used, while for longer trips, the ICE can be used to extend the range of the vehicles. This is beneficial when the use pattern is mixed, i.e. long and short distances have to be covered in a logistics environment. Nevertheless, even if biofuels managed to cross the chasm and become commercially viable, the high efficiency of the electric motors will require the hybridization of the energy source where biofuels can substitute the fossil based fuel in the hybrid electric vehicles. For this result, optimizing the vehicle routing of hybrid electric vehicles play an important role in reducing the GHG emissions in the logistics by ensuring that the routing and charging of the hybrid electric is optimized while demands are met on time; ultimately hybrid vehicle routing optimization can accelerate the commercialization of such vehicles.

This mixed type of fuel (electricity and fuel) gives rise to a novel vehicle routing problem which has not been investigated in the operations research literature. This problem is related to vehicle routing with time windows. However, the optimization of this hybrid electric and alternative fuel routing along with the charging requirements of the battery while meeting the tight time windows of deliver presents a challenge that can be tackled using the operations research tools.

The objective of the PHEVRP is to minimize the emissions of its hybrid electric vehicles by minimizing the time they run on gasoline while meeting the demand during the available time windows. As a result, the driver of the PHEV has two decisions to make at each node: (1) recharge the vehicle battery to achieve a longer range using electricity, or (2) continue to the next open time window with the option of using the alternative fuel. Another, variant of this problem is to consider minimizing the total cost incurred as running on electricity is cheaper than fuel, however this adds different dimension to the problem as the electricity cost changes from one node to another, and from one time window to another (electricity is cheaper during the night).

In this thesis, we present a novel mathematical formulation for the plug-in hybrid-electric vehicle routing problem with time windows. We solve this problem using a Lagrangian decomposition and a tabu search algorithm. The remainder of this thesis is organized as follows: chapter 2 presents the formulation of the plug-in hybrid-electric vehicle routing problem with time windows along with three different Lagrangian decomposition approach along with their numerical results, chapter 3 presents a tabu search algorithm based on λ-interchange neighborhood generations mechanism, and finally chapter 4 concludes with future research directions.

Chapter 2

Formulation and Lagrangian Relaxations

The increased globalization coupled with increased international trade volumes have lead companies to realize the potential competitive advantage of efficiently managing their logistics and supply chain activities. In 2012, the logistics costs in the U.S. have increased by 2.6% to reach 8.5% of the gross domestic product (GDP) which is equivalent to $1.28 trillion [FTA and PwC, 2012]. Compared to their US counterparts, the Canadian manufacturing, wholesale, and retail sectors suffer from higher logistics costs by an average 2%, 22% and 16% respectively [SCL, 2006].

Companies, nowadays, rely heavily on their logistics networks and supply chains to adopt reliable and cost efficient solutions for their products distribution and services. One of the most important decisions for any logistics or distribution network is the routing patterns of the fleet of vehicles and scheduling their deliveries. The relevance of these decisions and their respective trade-offs explain the richness and variety of research on vehicle routing problems and their extensions in the literature.

The classical vehicle routing problem (VRP) and its variants share common features; a distribution company with a depot is trying to decide on a set of routes to satisfy its geographically spread customers while minimizing the overall costs. Several variants exist related to the network characteristics (e.g multiple depot v.s. single depot), vehicle characteristics (e.g. homogenous v.s. heterogeneous vehicles), temporal characteristics (soft time windows v.s. hard time windows).

One of the most known extensions to the classical VRP is the consideration of time windows during which the customer should be served. This variant is called the vehicle routing problem with time windows (VRPTW). In general, there are two types of time windows characteristics that are studied in literature:

- Hard time windows: the customers should be served strictly within the given time window. If the vehicle arrives earlier then it should wait until the time window opens.
- Soft time windows: the customers can be served outside the time window but with a relative penalty cost incurred.

To the best of our knowledge, all the literature on VRP and its extensions consider vehicles with a single source of energy: either regular vehicles with internal combustion engines or, recently, fully electric vehicles. In this chapter, we introduce a novel variant of the vehicle routing problem with time windows where the fleet consists of homogenous plug-in hybrid electric vehicles. The introduction of plug-in hybrid electric vehicles (PHEVs) to the logistics fleet poses more complex technical and management challenges since these vehicles must carry tremendous weight, operate in near continuous use, make multiple stops and starts, and manage their charging and discharging patterns while meeting the customers demand on time. We call this problem the plug-in hybrid electric vehicle routing problem with time windows (PHEVRPTW).

We consider a service company with n geographically and temporally spread customers. Each customer has a deterministic demand which needs to be served on a daily basis denoted by h_i. The company's fleet consists of K plug-in hybrid vehicles equipped with batteries each with capacity $E\ kWh$. The company needs to decide on the optimal routing of its PHEV fleet while meeting its customers demand during their respective time windows.

Most of the models on routing problems have an objective which minimizes the distance traveled which translates into minimizing the routing costs. This explains why most algorithms used for these problems involve solving a variant of the shortest path problem. On the other hand, the objective of the PHEVRPTW is to minimize the routing costs of the PHEVRP by minimizing the time they run on gasoline and at the same time increasing the time they travel on charge. At each node, the driver of the PHEV has two decisions to make:

(1) recharge the vehicle's battery to drive for a longer range on an electric charge, or

(2) continue to the next open time window with the option of using the ICE engine when the charge has been fully depleted.

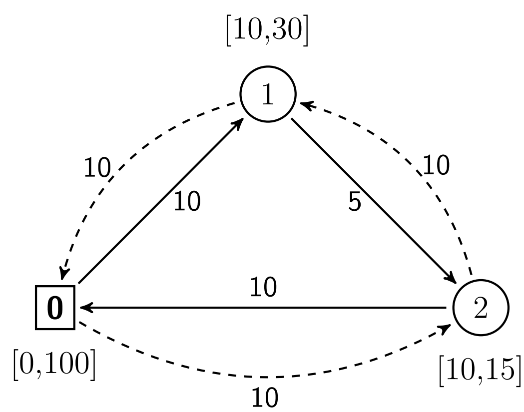

Figure 2.1: Illustrative Network

The key concept here is that the electricity cost is negligible when compared to the gasoline cost, hence, without loss of generality, in the rest of this chapter we ignore the cost of charging. Adding the charging cost can be easily incorporated into the model by subtracting the cost of charging from the savings of traveling on a charge. The difference in the PHEVRPTW is that minimizing the total distance traveled does not guarantee an optimal solution since additional cost savings can be achieved by traveling for a longer distance with more time available for recharging the battery. To illustrate more on the difference between the two problems a small example with 2 customers and a depot is shown in Figure 2.1.

In this example, the time window of each customer is shown in brackets (in *mins*) and the distance (in *mi*) is shown on the arc. Assume, that traveling time of 1 *mi* is 1 *min*. Clearly, if one was solving a classical VRPTW with a single vehicle, then the optimal solution would be to choose the shortest feasible path $(0-1-2-0)$ with a total distance of 25 *mi*. Now instead of a gasoline based vehicle, the logistics company is using a PHEV. Assume for simplicity that the battery capacity is 10 kWh, the charging rate is 1 kWh/min, the discharging rate is 1 kWh/mi, the vehicle starts with fully charged battery, and the charging cost is negligible while gasoline costs \$1. If the PHEV uses the shortest route, then it will go from the depot to customer 1 on charge hence depleting all its charge without incurring any gasoline costs. It will immediately leave customer 1 to reach customer 2 before its time window closes, but this time, since the charge is depleted, it will use gasoline and will incur a cost of \$5. It will service customer 2 and then charge the battery to full capacity and reach the depot on electric charge with no additional costs. Hence, the total cost of the shortest path is \$5. On the other hand, if the PHEV follows the dashed route $(0-2-1-0)$ then it can recharge its battery at customer 2 and customer 1 without the need for the use of gasoline and hence the overall cost is \$0 despite the fact that this route is 5 mi longer than the shortest path.

The rest of this chapter is organized as follows: section 2.1 presents the relevant literature review, section 2.2 presents describes the plug-in hybrid electric vehicle routing problem with time windows 2.3 introduces the mathematical model for the the PHEVRPTW, section 2.4 presents three different Lagrangian relaxation approaches, section 2.5 describes the heuristics used to find a feasible solution, and section 2.6 summarizes the computation results. Finally, section 2.7 concludes with future research directions.

2.1 Literature Review

The classical vehicle routing problem (VRP) was first proposed by Dantzig and Ramser [1959]. Over more than half a decade, the research on the VRP and its extensions has evolved rapidly in terms of efficient reformulations and solution methodologies including both exact and heuristics algorithms. For an in depth insight on the classical VRP and its extensions, readers are referred to [Ball et al., 1995], and for recent advances in vehicle routing problems readers are referred to [Golden et al., 2008].

The earliest discussion on the need for including the temporal aspects of the VRP, known as the VRPTW, was based on case studies presented by Pullen and Webb [1967] on routing mail delivery vans for the London district and by Knight and Hofer [1968] on routing for a contract transportation company also in the London district. However, the solutions were based on simple heuristic approaches presented as a computer software.

The classical VRP is a well known NP-hard problem [Lenstra and Kan, 1981]. The vehicle routing problem with time windows (VRPTW) is NP-hard in the strong sense as it generalizes both the VRP and the traveling salesman with time windows [Toth and Vigo, 2002b]. In fact, Savelsbergh [1985] showed that even finding a feasible solution to the VRPTW with a fixed number of vehicles is an NP-complete problem. Due to its computational complexity, the VRP and VRPTW have benefited from a growing literature on heuristics methods which are reviewed in Chapter 3. In parallel, there was also an interest in devising exact solution methods to provide optimal solutions efficiently. The most successful approach in the literature is based on the reformulation of the VRPTW to a set partitioning (SP) formulation, also known as Dantzig-Wolfe reformulation. The problem is then solved using a column generation mechanism to generate the sets/routes along with solving a pricing problem which is normally an elementary shortest part problem with time windows and resource constraints (ESPPTWRC) [Desrochers et al., 1992], [Desaulniers et al., 2008], and [Baldacci et al., 2011].

Over the past 20 years, the exact algorithm presented by Desrochers et al. [1992] has been the most famous approach to solve the VRPTW based on an SP reformulation. The success of Desrochers et al. [1992] is due to their proposed dynamic programming algorithm to solve a relaxed pricing problem which is an ESPPTWRC. Desrochers et al. [1992] realized the computational challenge of solving the ESPPTWRC, for this reason they proposed a state space relaxation based on [Christofides et al., 1981] and solved the shortest path problem with time windows and resource constraints (SPPTWRC) where negative cycles are allowed. They proposed a pseudo-polynomial time algorithm to solve the SPPTWRC along with a 2-cycle elimination process. Desrochers et al. [1992] were able to solve a total of 8 of Solomon's instances with 100 customers and short horizons (3 R1 and 5 C1 sets). Later, Ioachim et al. [1998] proposed new dominance rules to improve the efficiency of the label correcting algorithm. Recently, Feillet et al. [2004] and Chabrier [2006] have extended the label correcting algorithm to be able to solve the ESPPTWRC by adding path dominance rules without the need for the state space relaxation. Their approach lead to superior lower bounds and hence more efficient solutions.

Halse [1992] used a Lagrangian based approach based on variable splitting where the formulation is presented in terms of both three-index variables and 2-index variables. The constraint that relates these two sets of variables is relaxed which leads to two separate subproblems: 1) a trivial semi-assignment problem which can be solved by inspection and 2) the ESPPTWRC. Other variable splitting approaches have been proposed, see for example [Fisher et al., 1997]. Kohl and Madsen [1997] also uses the same variable splitting approach however they realized the slow convergence of the subgradient algorithm in solving the master problem and hence suggested a combination of a subgradient and bundle method to solve the master problem. The resulting algorithm was able to find exact solutions for all of the 8 C1 problems of Solomon's 100 customer data sets for the first time. Later, Kohl et al. [1999] introduced the k-path inequalities which is a generalization of the subtour elimination constraints and proposed an efficient separation algorithm for finding the violated inequalities. They used a Lagrangian decomposition method and they were able to solve 14 of Solomon's datasets with 100 customers and short horizons (3 R1, 8 C1, and 2 RC1 sets). Irnich and Villeneuve [2006] extended the 2-cycle elimination algorithm accompanies with the state-space relaxation of the ESPPTWRC to a k-cycle elimination ($k \geq 3$). The results were very promising as they were able to solve 7 of the yet unsolved Solomon's 100 customer data sets (1 R1, 3 RC1, 1 C2, and 2 RC2).

Kallehauge et al. [2006] was the first to propose a Lagrangian relaxation approach based on the cutting-plane algorithm by Kelley Jr [1960]. This approach led to solving an

ESPPTWRC subproblem which is identical to that obtained by Dantzig-Wolfe decomposition. However, the master problem obtained by the this approach is only the dual problem of the set partitioning reformulation as we will demonstrate later. They also used a number of algorithmic tweaks to accelerate their solution like boxstep stabilization, modified 2-path cuts, and parallel implementations. They were able to solve most of the solved problems that were solvable in the literature very efficiently. In addition, they were the first to report exact solutions for 7 of the extended datasets with 200 customers proposed by Gehring and Homberger [2001]. Also for the first time, they provided solutions to a dataset with 400 customers ($C1\_4\_1.100$) and another one with 1000 customers ($C110\_1.1000$) which is the largest to be solved at that time with an exact algorithm.

Desaulniers et al. [2008] implemented further refinements to the algorithms proposed by using a mix of tabu search to solve the subproblem in the beginning and adding both subset row inequalities as proposed by the Jepsen et al. [2008] and the k-path inequalities. Using this approach, they were able to close 5 of the 10 open Solomon's instances with 100 customers. Baldacci et al. [2008] proposed a very competitive solution framework for designing exact algorithms for the CVRP which can be easily adapted to solve a broader class of variants of the VRP and in particular the VRPTW [Baldacci et al., 2010]. Baldacci et al. [2011] proposed a new set of inequalities called ng-routes which is very effective in reducing the state-space graph of the sub-problem and proved very effective in solving the problems with wide time windows. They were able to solve 4 of the 5 unsolved 100 customers Solomon problems. To date, and after more than 25 years since they were first proposed by Solomon [1987] and despite the advances in algorithms and hardware the data set R208 with 100 customers is still open, to the best of our knowledge.

Recently, there has been an increasing research interest on sustainable operations research model in supply chain management [Benjaafar et al., 2009] and [Cachon, 2011]. However, contrary to this emerging research trend, it seems that the research on the sustainability in transportation and its study using operations research model is still lagging if not absent. However, it is worth mentioning that recently few articles have appeared in operations research journal which touches upon the idea of sustainable transportation. Bektaş and Laporte [2011] proposed a new variant for the VRP and VRPTW called the pollution routing problem (PRP) which accounts for the energy requirements and hence the resultant pollution of the fleet of vehicles based on the load size and speed among other factors. Mak et al. [2012] studied the operations of battery swapping in a two-stage decision process: the first stage involves strategically deciding on the location of the swapping station and the second stage involves deciding on the optimal inventory of batteries to satisfy a certain service level. In addition, Sioshansi [2012] studied the impact of different electricity tariffs on the charging and discharging decisions of customers with plug-in hybrid electric vehicle (PHEV).

As we have seen from the previous review, the operations research society and in particular research focusing on sustainability using transportation optimization models have not yet responded to the exciting new research challenges which arise from the use of the new fleet of electric vehicles. The common misconception among these researchers is that the new model are not very different from the classical models, and the idea of an electric charge can be accounted for as a capacity constraint and hence an additional resource which can be handled by many of the previously proposed algorithms. However, in this chapter, we will present a novel model called the plug-in hybrid electric vehicle routing problem with time windows and we will demonstrate the challenges and gaps in the current algorithms to solve such problems along with its potential variants.

2.2 The Model

The description of the plug-in-hybrid electric vehicle routing problem with time windows (PHEVRPTW) is not very different from the classical VRPTW in the sense that you have a distribution company with a central depot with multiple homogenous vehicles with a capacity which need to serve multiple customers within predetermined time windows. However, the major distinction in the (PHEVRPTW) is that the vehicles used are plug-in-hybrid electric vehicles (PHEVs) which can run on both gasoline and electric change.

We consider a distribution problem having the following features:

1. m plug-in hybrid electric vehicles which have to serve a set of n customers exactly once during a period [0, T].
2. Each customer has to be served within a predetermined time window, $[a_i, b_i]$.
3. Each customer has a demand denoted by h_i.
4. All vehicles have identical load capacities C.

We consider a directed graph $G = (N, A)$ where $N = \{0, 1, \ldots, n\}$. Node 0 represents the central depot which is the starting and ending node on each route for each PHEV. We denote by Ω the set of all customers i.e. $\Omega = N \setminus \{0\}$. We associate with each arc (i, j) $\in A$, an arc distance $d_{ij} > 0$, a traveling time $t_i j$, and a cost of traveling on gasoline c_{ij}. Furthermore, we associate with each node i∈ N, a service time $s_i > 0$, a release time $a_i > 0$, and a due time $b_i > a_i$. We denote by $[a_i, b_i]$ as the time window for customer $i \in N$ with width $W = b_i - a_i$.

The PHEV battery has an energy storage capacity, denoted by E, in kWh. We define a charging rate η often denoted as C-rate to represent the ratio of capacity that can be recharged during an hour. Similarly, we define τ as the discharging ratio of the capacity per unit distance. We also denote by Q_i the total charge of the PHEV once node i is serviced. The PHEV is allowed to recharge the battery of the vehicle either before serving the node or after serving the node. For this reason we introduce two decision variables q_i^- and q_i^+ which represent the time spent for recharging the battery at node i before and after service respectively.

We define two functions $f()$ and $\phi()$ to represent the charging and discharging process respectively, such that:

$$f\left(q_i^{k+}\right) = \eta * q_i^{k+} \tag{2.1}$$

$$\phi\left(y_{ij}^k\right) = \tau * y_{ij}^k \tag{2.2}$$

2.3 Problem Formulation

In order to formulate the problem we further introduce the following decision variables:

Decision Variables

$$x_{ij}^k \triangleq \begin{cases} 1 & \text{if node } j \text{ is visited after node } i \text{ by vehicle } k \\ 0 & \text{otherwise} \end{cases}$$

$$y_{ij}^k \triangleq \text{total distance traveled on electric charge from node } i \text{ to node } j \text{ by vehicle } k$$

We can now formulate the problem as follows

2.3.1 Original Problem Formulation

$$[OP]: \quad \min \sum_{k\in K}\sum_{i\in N}\sum_{j\in N} c_{ij}d_{ij}x_{ij}^k - \sum_{k\in K}\sum_{i\in N}\sum_{j\in N} c_{ij}y_{ij}^k \tag{2.3}$$

subject to

$$\sum_{k\in K}\sum_{j\in N} x_{ij}^k = 1 \qquad \forall i \in \Omega \tag{2.4}$$

$$\sum_{j\in N} x_{0j}^k = 1 \qquad \forall k \in K \tag{2.5}$$

$$\sum_{i\in N} x_{il}^k - \sum_{j\in N} x_{lj}^k = 0 \qquad \forall l \in \Omega, \forall k \in K \tag{2.6}$$

$$\sum_{i\in \Omega} h_i \sum_{j\in N} x_{ij}^k \leq C \qquad \forall k \in K \tag{2.7}$$

$$a_i \leq s_i^k \leq b_i \qquad \forall i \in N, \forall k \in K \tag{2.8}$$

$$s_i^k + q_i^{k+} + t_{ij}^k + q_j^{k-} - s_j^k \leq M\left(1 - x_{ij}^k\right) \qquad \forall i \in N, \forall j \in \Omega, \forall k \in K \tag{2.9}$$

$$Q_i^k + f\left(q_i^{k+}\right) - \phi\left(y_{ij}^k\right) + f\left(q_j^{k-}\right) - Q_j^k \geq -M\left(1 - x_{ij}^k\right) \qquad \forall i \in N, \forall j \in \Omega, \forall k \in K \tag{2.10}$$

$$y_{ij}^k \leq d_{ij}x_{ij}^k \qquad \forall i,j \in N, \forall k \in K \tag{2.11}$$

$$\phi\left(y_{ij}^k\right) \leq Q_i^k + f\left(q_i^{k+}\right) \qquad \forall i,j \in N, \forall k \in K \tag{2.12}$$

$$Q_i^k + f\left(q_i^{k+}\right) \leq E \qquad \forall i \in N, \forall k \in K \tag{2.13}$$

$$q_i^{k+} \geq 0 \qquad \forall i \in N, k \in K \tag{2.14}$$

$$q_i^{k-} \geq 0 \qquad \forall i \in N, k \in K \tag{2.15}$$

$$y_{ij}^k \geq 0 \qquad \forall i,j \in N, k \in K \tag{2.16}$$

$$x_{ij}^k \in \{0,1\} \qquad \forall i \in N, k \in K \tag{2.17}$$

The objective function 2.3 minimizes the total cost of routing assuming that the cost of running on electric charge is negligible (it can be easily adjusted to include the cost of recharging). Constraint set 2.4 ensures that each node is visited exactly once. Constraint 2.5 ensures that the nodes are reached directly from the depot by only one vehicle. Constraint set 2.6 is the flow balance constraint where each node visited must also be departed. Constraint 2.7 sets the limit on the total demand served according to the vehicle load capacity. Constraint set 2.8 ensures that each node is served within its respective time window. Constraint set 2.9 is the newly proposed time windows constraint set which relates the servicing time between the visited nodes while accounting for the charging time. Note that while in the majority of the cases recharging both before and after servicing ($q_i^- > 0$ and $q_i^+ > 0$) is not justified from an operational perspective however there can be cases where there is some time available before the time window opens to recharge the battery partially and additional time after servicing to recharge it even it further. Constraint set 2.10 relates the battery charge between the visited nodes while accounting for the charging and discharging of the battery. Constraint set 2.11 ensures that one cannot travel from a node on electric charge unless the node is visited. Constraint set 2.12 sets a limit on the possible distance that can be traveled on electric charge based on the electric charge level of the battery. Constraint set 2.13 is the capacity constraint on the available electric charge at any given node. Constraint sets 2.14 - 2.17 are the non-negativity and binary constraints.

2.4 Lagrangian Relaxation

The Lagrangian relaxation approach is one of the classical decomposition approaches in operations research. It has been used extensively in the literature to solve a wide variety of problems by obtaining high quality lower bounds. In this part, we present three different relaxations for the PHEVRPTW in order to experiment with the quality of the lower bound and the computational requirement of each relaxation.

2.4.1 Relaxation 1 (LR1)

In this first relaxation, we follow the classical relaxation approach for the VRPTW where constraint set (2.4) is relaxed. However, the difference arises from the different structure of the subproblem is not an ESPPTWRC.

Lagrangian Dual

Let λ_i be the dual variables corresponding to the relaxed constraint set (2.4). The corresponding Lagrangian dual can be expressed as:

$$[LR1D] : z_{LR1D} = \max_{(\lambda)} \Bigg\{ \min \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} d_{ij} x_{ij}^k + \sum_{k \in K} \sum_{i \in \Omega} \sum_{j \in N} \lambda_i \left(1 - x_{ij}^k\right) + \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} y_{ij}^k \quad \text{subject to } (2.5) - (2.17) \Bigg\} \tag{2.18}$$

Lagrangian subproblem

The subproblem can be decomposed into $|K|$ identical subproblems of the form [SPKLR1].

$$[SPKLR1] : z_{SPKLR1}^k(\lambda) = \min \sum_{i \in N} \sum_{j \neq i \in N} \hat{c}_{ij} d_{ij} x_{ij} - \sum_{i \in N} \sum_{j \in N} c_{ij} y_{ij} \tag{2.19}$$

subject to

$$\sum_{j\in N} x_{0j} = 1 \tag{2.20}$$

$$\sum_{i\in N} x_{il} - \sum_{j\in N} x_{lj} = 0 \qquad \forall l \in \Omega \tag{2.21}$$

$$\sum_{i\in \omega} h_i \sum_{j\in N} x_{ij} \leq C \tag{2.22}$$

$$a_i \leq s_i \leq b_i \qquad \forall i \in N \tag{2.23}$$

$$s_i + q_i^+ + t_{ij} + q_j^- - s_j \leq M\left(1 - x_{ij}\right) \qquad \forall i \in N, \forall j \in \Omega \tag{2.24}$$

$$Q_i + f\left(q_i^+\right) - \phi\left(y_{ij}\right) + f\left(q_j^-\right) - Q_j \geq -M\left(1 - x_{ij}\right) \qquad \forall i, j \in N \tag{2.25}$$

$$\phi\left(y_{ij}\right) \leq Q_i + f\left(q_i^+\right) \qquad \forall i, j \in N \tag{2.26}$$

$$y_{ij} \leq d_{ij} x_{ij} \qquad \forall i, j \in N \tag{2.27}$$

$$Q_i + f\left(q_i^+\right) \leq E \qquad \forall i \in \Omega \tag{2.28}$$

$$q_i^+ \geq 0 \qquad \forall i \in N \tag{2.29}$$

$$q_i^- \geq 0 \qquad \forall i \in N \tag{2.30}$$

$$y_{ij} \geq 0 \qquad \forall i, j \in N \tag{2.31}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in N \tag{2.32}$$

Where $\hat{c}_{ij} = c_{ij} d_{ij} - \lambda_i \forall i \in \Omega, j \in N$ and $\hat{c}_{ij} = c_{ij} \forall i = 0, j \in N$

Lagrangian Master Problem

Now let P be the set of all feasible paths for $[SPKLR1]$ for each λ_i. We now replace all variables with the convex combination of the extreme points where we use the notation $x_{ijp}\ \forall\ p \in P$ to represent all the extreme points for x_{ij}. The same notation is used for the other variables. Then the Lagrangian dual can be characterized according to the generated paths as follows:

$$[LR1D] : z_{LR1D} = \max_{\lambda} |K| \left\{ \min_{P} \left(c_p - \sum_{i\in\Omega} a_{ip} \lambda_i \right) \right\} + \sum_{i\in\Omega} \lambda_i \tag{2.33}$$

$$\text{where } c_p = \sum_{(i,j)\in A} c_{ij} d_{ij} x_{ijp} - \sum_{(i,j)\in A} c_{ij} y_{ijp} \qquad \forall p \in P \tag{2.34}$$

$$a_{ip} = \sum_{j\in N : j\neq i} x_{ijp} \qquad \forall i \in N, \forall p \in P \tag{2.35}$$

The Lagrangian dual is a piecewise linear function which can be linearized to give the following Lagrangian master problem.

$$[LR1M] : z_{LR1M} = \max_{\lambda,\theta} |K|\,\theta + \sum_{i\in\Omega} \lambda_i \tag{2.36}$$

$$\text{subject to } \theta \leq c_p - \sum_{i\in\Omega} a_{ip}\lambda_i \qquad \forall p \in P \tag{2.37}$$

$$\theta \leq 0 \tag{2.38}$$

Dantzig-Wolfe Master Problem

The Dantzig-Wolfe master problem can be obtained either by applying a DW decomposition or taking the dual of [LR1M] where w_p is the dual variable for constraint set 2.37. The DW master problem is a set partitioning problem where P represent the set of feasible elementary paths for every vehicle. The master problem is then the following:

$$[DWM] : z_{DWM} = \min \sum_{p\in P} c_p w_p \tag{2.39}$$

$$\sum_{p\in P} a_{ip} w_p = 1 \qquad \forall i \in \Omega \tag{2.40}$$

$$\sum_{p\in P} w_p \leq |K| \tag{2.41}$$

$$w_p \geq 0 \qquad \forall p \in P \tag{2.42}$$

Lagrangian Lower Bound

For a fixed set of dual variable λ_i we obtain a Lagrangian lower bound denoted by z_{lag1} such that:

$$z_{OP} \geq z_{lag1}(\lambda) = |K|\,(z_{SPKLR1}) + \sum_{i\in\Omega} \lambda_i$$

2.4.2 Relaxation 2 (LR2)

Let λ_i and $\mu_{ij} \geq 0$ represent the dual variables of the relaxed set of constraints (2.4) and (2.9) respectively.

Lagrangian Dual

The Lagrangian dual can be written as follows:

$$[LR2D] : z_{LR2D} = \max_{\mu \geq 0, \lambda} \left\{ \min \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} d_{ij} x_{ij}^k - \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} y_{ij}^k \right.$$
$$+ \sum_{i \in \Omega} \lambda_i \left(1 - \sum_{k \in K} \sum_{j \in N} x_{ij}^k \right)$$
$$+ \sum_{k \in K} \sum_{i \in N} \sum_{j \in \Omega} \mu_{ijk} \left(s_i^k + q_i^{k+} + t_{ij}^k + q_j^{k-} - s_j^k - M + M x_{ij}^k \right)$$
$$\left. \text{subject to } (2.5) - (2.8), (2.10) - (2.17) \right\} \tag{2.43}$$

Lagrangian subproblem

The subproblem can be decomposed into $|K|$ identical subproblems of the form $[SPKLR2]$.

$$[SPKLR2] : z_{SPKLR2}^k(\lambda, \mu) = \min \left\{ \sum_{i \in N} \sum_{j \in N} \hat{c}_{ij} x_{ij} + \sum_{i \in N} \sum_{j \in N} c_{ij} y_{ij} + \sum_{i \in N} \sum_{j \in \Omega} \mu_{ij} q_i^+ \right.$$
$$\left. + \sum_{i \in N} \sum_{j \in \Omega} \mu_{ij} q_j^- + \sum_{i \in N} \grave{c}_i s_i + \sum_{i \in N} \tilde{c}_i Q_i \right\} \tag{2.44}$$

subject to

$$\sum_{j \in N} x_{0j} = 1 \tag{2.45}$$

$$\sum_{i \in N} x_{il} - \sum_{j \in N} x_{lj} = 0 \qquad \forall l \in \Omega \tag{2.46}$$

$$\sum_{i \in \omega} h_i \sum_{j \in N} x_{ij} \leq C \tag{2.47}$$

$$a_i \leq s_i \leq b_i \qquad \forall i \in N \tag{2.48}$$

$$Q_i + f\left(q_i^+\right) - \phi\left(y_{ij}\right) + f\left(q_j^-\right) - Q_j \geq -M_2\left(1 - x_{ij}\right) \qquad \forall i, j \in N \tag{2.49}$$

$$\phi\left(y_{ij}\right) \leq Q_i + f\left(q_i^+\right) \qquad \forall i, j \in N \tag{2.50}$$

$$y_{ij} \leq d_{ij} x_{ij} \qquad \forall i, j \in N \tag{2.51}$$

$$Q_i + f\left(q_i^+\right) \leq E \qquad \forall i \in \Omega \tag{2.52}$$

$$q_i^+ \geq 0 \qquad \forall i \in N \tag{2.53}$$

$$q_i^- \geq 0 \qquad \forall i \in N \tag{2.54}$$

$$y_{ij} \geq 0 \qquad \forall i, j \in N \tag{2.55}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in N \tag{2.56}$$

where

$$\hat{c} = \begin{cases} c_{ij} d_{ij} - \lambda_i + M\mu_{ij} & \forall i \in \Omega, \forall j \in \Omega \\ c_{ij} d_{ij} - \lambda_i & \forall i \in \Omega, j = 0 \\ c_{ij} d_{ij} + M\mu_{ij} & i = 0, \forall j \in \Omega \end{cases} \tag{2.57}$$

$$\grave{c} = \begin{cases} \sum_{j \in \Omega} \mu_{ij} - \sum_{j \in N} \mu_{ji} & \forall i \in \Omega \\ \sum_{j \in \Omega} \mu_{ij} & \forall i = 0 \end{cases} \tag{2.58}$$

$$\tilde{c} = \begin{cases} -\sum_{j \in \Omega} \mu_{ij} + \sum_{j \in N} \mu_{ji} & \forall i \in \Omega \\ -\sum_{j \in \Omega} \mu_{ij} & \forall i = 0 \end{cases} \tag{2.59}$$

Lagrangian Master Problem

Notice that $[SPKLR2]$ returns $|K|$ identical paths. As a result the Lagrangian dual problems can be expressed as:

$$[LR2D] : z_{LR2D} = \max_{\mu \geq 0, \lambda} \Bigg\{ \min |K| \Bigg(\sum_{i \in N} \sum_{j \in N} \hat{c}_{ij} x_{ij} + \sum_{i \in N} \sum_{j \in N} -c_{ij} y_{ij} + \sum_{i \in N} \sum_{j \in \Omega} \mu_{ij} q_i^+$$

$$+ \sum_{i \in N} \sum_{j \in \Omega} \mu_{ij} q_j^- + \sum_{i \in N} \grave{c}_i s_i + \sum_{i \in N} \tilde{c}_i Q_i$$

$$+ \sum_{i \in \Omega} \sum_{j \in N} (\mu_{ij} t_{ij} - \mu_{ij} M) \Bigg) + \sum_{i \in \Omega} \lambda_i$$

$$\text{subject to } (2.45) - (2.56) \Bigg\} \tag{2.60}$$

Now let P be the set of all feasible paths for $[SPKLR2]$ for each λ_i and $\mu_i j$. We now replace all variables with the convex combination of the extreme points where we use the notation x_{ijp} $\forall$ $p \in P$ to represent all the extreme points for x_{ij}. The same notation is used for the other variables. This yields the following Lagrangian Dual representation:

$$[LR2D] : z_{LR2D} = \max_{\mu \geq 0, \lambda} \Bigg\{ \min_P |K| \Bigg(c_p - \sum_{i \in \Omega} a_{ip} \lambda_i + \sum_{i \in \Omega} \sum_{j \in N} M \mu_{ij} x_{ijp} + \sum_{i \in N} \sum_{j \in \Omega} \mu_{ij} q_{ip}^+$$

$$+ \sum_{i \in N} \sum_{j \in \Omega} \mu_{ij} q_{ip}^- + \sum_{i \in N} \sum_{j \in \Omega} \mu_{ij} s_{ip} - \sum_{i \in N} \sum_{j \in \Omega} \mu_{ij} s_{jp} - \sum_{i \in N} \sum_{j \in \Omega} \mu_{ij} Q_{ip}$$

$$+ \sum_{i \in N} \sum_{j \in \Omega} \mu_{ij} Q_{jp} + \sum_{i \in \Omega} \sum_{j \in N} (\mu_{ij} t_{ij} - \mu_{ij} M) \Bigg) + \sum_{i \in \Omega} \lambda_i \Bigg\} \tag{2.61}$$

where

$$c_p = \sum_{(i,j) \in A} c_{ij} d_{ij} x_{ijp} - \sum_{(i,j) \in A} c_{ij} y_{ijp} \qquad \forall p \in P \tag{2.62}$$

$$a_{ip} \sum_{j \in N} x_{ijp} \qquad \forall i \in N, \forall p \in P \tag{2.63}$$

After rearranging the terms with respect to the dual variables, we get the following expression for the Lagrangian dual problem:

$$[LR2D] : z_{LR2D} = \max_{\mu \geq 0, \lambda} \left\{ \min_{P} |K| \left(c_p - \sum_{i \in \Omega} a_{ip} \lambda_i \right. \right.$$

$$\left. \left. \sum_{i \in N} + \sum_{j \in \Omega} \left(s_{ip} + q_{ip}^{+} + t_{ij} + q_{jp}^{-} - s_{jp} - M + M x_{ijp} \right) \mu_{ij} \right) + \sum_{i \in \Omega} \lambda_i \right\}$$

Let $\theta = z_{SPKLR2}$

Then the master problem can be written as:

$$[LR2M] : z_{LR2M} = \max_{\mu \geq 0, \lambda} |K| \, \theta + \sum_{i \in N} \sum_{j \in \Omega} |K| \left(\mu_{ij} t_{ij} - \mu_{ij} M \right) + \sum_{i \in \Omega} \lambda_i \tag{2.64}$$

subject to

$$\theta \leq c_p - \sum_{i \in \Omega} a_{ip} \lambda_i$$

$$+ \sum_{i \in N} \sum_{j \in \Omega} \left(s_{ip} + q_{ip}^{+} + q_{jp}^{-} - s_{jp} + M x_{ijp} \right) \mu_{ij} \quad \forall p \in P \tag{2.65}$$

$$\theta \leq 0 \tag{2.66}$$

$$\mu_{ij} \geq 0 \qquad \forall i, j \tag{2.67}$$

Lagrangian Lower Bound

For a fixed set of dual variables (λ, μ) we obtain a Lagrangian lower bound denoted by z_{lag2} such that:

$$z_{OP} \geq z_{lag2}(\lambda, \mu) = |K| \left(z_{SPKLR2} + \sum_{i \in N} \sum_{j \in \Omega} \left(\mu_{ij} t_{ij} - \mu_{ij} M \right) \right) + \sum_{i \in \Omega} \lambda_i$$

2.4.3 Relaxation 3 (LR3)

The idea behind this relaxation is to separate the charging problem from the routing problem so as to get a subproblem which is an ESPPTWRC. Let λ_i and $\{\mu_{ij}, \nu_{ij}, \varpi_{ij}\} = \Lambda \geq 0$ represent the dual variables of the relaxed set of constraints (2.4), (2.9), (2.10), and (2.11) respectively.

Lagrangian Dual

$$
\begin{aligned}
[LR3D]: z_{LR3D} = \max_{\Lambda \geq 0, \lambda} \Bigg\{ & \min \sum_{k\in K}\sum_{i\in N}\sum_{j\in N} c_{ij} d_{ij} x_{ij}^k - \sum_{k\in K}\sum_{i\in N}\sum_{j\in N} c_{ij} y_{ij}^k \\
& + \sum_{i\in\Omega} \lambda_i \left(1 - \sum_{k\in K}\sum_{j\in N} x_{ij}^k\right) \\
& + \sum_{k\in K}\sum_{i\in N}\sum_{j\in\Omega} \mu_{ijk} \left(s_i^k + q_i^{k+} + t_{ij}^k + q_j^{k-} - s_j^k - M + M x_{ij}^k\right) \\
& + \sum_{k\in K}\sum_{i\in N}\sum_{j\in\Omega} \nu_{ijk} \left(-M + M x_{ij}^k - Q_i^k - f\left(q_i^{k+}\right) + \phi\left(y_{ij}^k\right) - f\left(q_j^{k-}\right) + Q_j^k\right) \\
& + \sum_{k\in K}\sum_{i\in N}\sum_{j\in N} \varpi_{ijk} \left(y_{ij}^k - d_{ij} x_{ij}^k\right) \\
& \text{subject to } (2.5) - (2.8), (2.12) - (2.17) \Bigg\}
\end{aligned}
\tag{2.68}
$$

Lagrangian subproblem

The subproblem can be decomposed into two classes of subproblems:

- $[SP1KLR2]$ the routing problem which is an ESPPRTWRC modeled as an MIP.
- $[SP2KLR2]$ the charging problem which is an LP.

Each of these subproblems can be decomposed into $|K|$ identical subproblems independent of k.

$$
[SP1KLR3]: z_{SP1KLR3}^k(\lambda, \Lambda) = \min \sum_{i\in N}\sum_{j\in N} \hat{c}_{ij} x_{ij} \tag{2.69}
$$

subject to

$$\sum_{j\in N} x_{0j} = 1 \tag{2.70}$$

$$\sum_{i\in N} x_{il} - \sum_{j\in N} x_{lj} = 0 \qquad \forall l \in \Omega \tag{2.71}$$

$$\sum_{i\in \omega} h_i \sum_{j\in N} x_{ij} \leq C \tag{2.72}$$

$$a_i \leq s_i \leq b_i \qquad \forall i \in N \tag{2.73}$$

$$s_i + t_{ij} - s_j \leq M\left(1 - x_{ij}\right) \qquad \forall i \in \Omega, \forall j \in N \tag{2.74}$$

$$x_{ij} \in \{0, 1\} \qquad \forall i \in N \tag{2.75}$$

where

$$\hat{c} = \begin{cases} c_{ij}d_{ij} - \lambda_i + M\mu_{ij} + M\nu_{ij} - \varpi_{ij}d_{ij} & \forall i \in \Omega, \forall j \in \Omega \\ c_{ij}d_{ij} - \lambda_i - \varpi_{ij}d_{ij} & \forall i \in \Omega, j = 0 \\ c_{ij}d_{ij} + M\mu_{ij} + M\nu_{ij} - \varpi_{ij}d_{ij} & i = 0, \forall j \in \Omega \end{cases} \tag{2.76}$$

$$[SP2KLR3] : z_{SP2KLR3}(\lambda, \Lambda) = \min \sum_{i\in N}\sum_{j\in N} -\check{c}_{ij}y_{ij} + \sum_{i\in N} \breve{c}_i q_i^+ + \sum_{j\in N} \acute{c}_j q_j^- + \sum_{i\in N} \grave{c}_i s_i + \sum_{i\in N} \tilde{c}_i Q_i \tag{2.77}$$

subject to

$$\phi\left(y_{ij}\right) \leq Q_i + f\left(q_i^+\right) \qquad \forall i, j \in N \tag{2.78}$$

$$y_{ij} \leq d_{ij} \qquad \forall i, j \in N \tag{2.79}$$

$$Q_i + f\left(q_i^+\right) \leq E \qquad \forall i \in N \tag{2.80}$$

$$y_{ij}, q_i^+, q_i^- \geq 0 \qquad \forall i, j \in N \tag{2.81}$$

$$s_i, Q_i \geq 0 \qquad \forall i \in N \tag{2.82}$$

where

$$\check{c} = \begin{cases} c_{ij} - \nu_{ij}\tau - \varpi_{ij} & \forall i \in N, \forall j \in \Omega \\ c_{ij} - \varpi_{ij} & \forall i \in N, j = 0 \end{cases} \tag{2.83}$$

$$\breve{c} = \sum_{j\in\Omega} \mu_{ij} - \sum_{j\in\Omega} \nu_{ij}\eta \quad \forall i \in N \tag{2.84}$$

$$\acute{c} = \begin{cases} \sum_{i\in N} \mu_{ij} - \sum_{i\in N} \nu_{ij}\eta & \forall j \in \Omega \\ 0 & j = 0 \end{cases} \tag{2.85}$$

$$\grave{c} = \begin{cases} \sum_{j\in\Omega} \mu_{ij} - \sum_{j\in N} \mu_{ji} & \forall i \in \Omega \\ \sum_{j\in\Omega} \mu_{ij} & \forall i = 0 \end{cases} \tag{2.86}$$

$$\tilde{c} = \begin{cases} -\sum_{j\in\Omega} \mu_{ij} + \sum_{j\in N} \mu_{ji} & \forall i \in \Omega \\ -\sum_{j\in\Omega} \mu_{ij} & \forall i = 0 \end{cases} \tag{2.87}$$

Lagrangian Master Problem

Notice that $[SP1KLR3]$ and $[SP2KLR3]$ returns $|K|$ identical paths. As a result the Lagrangian dual problems can be expressed as:

$$[LR3D] : z_{LR3D} = \max_{(\Lambda)\geq 0,\lambda} \Bigg\{ \min |K| \Bigg(\sum_{i\in N}\sum_{j\in N} \hat{c}_{ij}x_{ij} + \sum_{i\in N}\sum_{j\in N} -\check{c}_{ij}y_{ij} + \sum_{i\in N} \breve{c}_i q_i^+ + \sum_{j\in N} \acute{c}_j q_j^- + \sum_{i\in N} \grave{c}_i s_i + \sum_{i\in N} \tilde{c}_i Q_i + \sum_{i\in\Omega}\sum_{j\in N} (\mu_{ij}t_{ij} - \mu_{ij}M - \nu_{ij}M) \Bigg) + \sum_{i\in\Omega} \lambda_i \text{ subject to } (2.70) - (2.75), (2.78) - (2.82) \Bigg\} \tag{2.88}$$

Now let P be the set of all feasible paths for $[SP1KLR3]$ and $[SP2KLR3]$ for each λ_i and Λ. We now replace all variables with the convex combination of the extreme points where we use the notation $x_{ijp}\ \forall\ p \in P$ to represent all the extreme points for x_{ij}. The same notation is used for the other variables. This yields the following Lagrangian Dual representation:

$$[LR3D]: z_{LR3D} = \max_{\Lambda\geq 0,\lambda} \min_{P} |K| \Bigg(c_p - \sum_{i\in\Omega} a_{ip}\lambda_i + \sum_{i\in\Omega}\sum_{j\in N} M(\mu_{ij}+\nu_{ij})x_{ijp} - \sum_{i\in N}\sum_{j\in N} \varpi_{ij} d_{ij} x_{ijp}$$
$$+ \sum_{i\in N}\sum_{j\in\Omega} \nu_{ij}\tau y_{ijp} + \sum_{i\in N}\sum_{j\in N} \varpi_{ij} y_{ijp} + \sum_{i\in N}\sum_{j\in\Omega}(\mu_{ij} - \nu_{ij\tau})q^{+}_{ip}$$
$$+ \sum_{i\in N}\sum_{j\in\Omega}(\mu_{ij} - \nu_{ij\tau})q^{-}_{ip} + \sum_{i\in N}\sum_{j\in\Omega} \mu_{ij} s_{ip} - \sum_{i\in N}\sum_{j\in\Omega} \mu_{ij} s_{jp} - \sum_{i\in N}\sum_{j\in\Omega} \mu_{ij} Q_{ip}$$
$$+ \sum_{i\in N}\sum_{j\in\Omega} \mu_{ij} Q_{jp} + \sum_{i\in\Omega}\sum_{j\in N} (\mu_{ij} t_{ij} - \mu_{ij} M - \nu_{ij} M) \Bigg) + \sum_{i\in\Omega}\lambda_i \qquad (2.89)$$

where

$$c_p = c^k_p = \sum_{(i,j)\in A} c^k_{ij} d^k_{ij} x^k_{ij} - \sum_{(i,j)\in A} c^k_{ij} y^k_{ij} \qquad \forall k\in K, \forall p \in P^k \qquad (2.90)$$

$$a_{ip} = a^k_{ip} = \sum_{j\in N:j} x_{ijp} \qquad \forall k\in K, \forall i\in N, \forall p\in P^k \qquad (2.91)$$

After rearranging the terms with respect to the dual variables we get the following representation of the Lagrangian Dual problem:

$$[LR3D]: z_{LR3D} = \max_{\Lambda\geq 0,\lambda} \min_{P} |K| \Bigg(c_p - \sum_{i\in\Omega} a_{ip}\lambda_i$$
$$\sum_{i\in N}\sum_{j\in\Omega} \left(s_{ip} + q^{+}_{ip} + t_{ij} + q^{-}_{jp} - s_{jp} - M + M x_{ijp}\right)\mu_{ij} +$$
$$\sum_{i\in N}\sum_{j\in\Omega} \left(-M + M x_{ijp} - Q_{ip} - f\left(q^{+}_{ip}\right) + \phi\left(y_{ijp}\right) - f\left(q^{-}_{jp}\right) + Q_{jp}\right)\nu_{ij} +$$
$$\sum_{i\in N}\sum_{j\in N} \left(y_{ijp} - d_{ij} x_{ijp}\right)\varpi_{ij} \Bigg) + \sum_{i\in\Omega}\lambda_i \qquad (2.92)$$

Let $\theta = z_{SP1KLR3} + z_{SP2KLR3}$

Then the master problem can be written as:

$$[LR3M] : z_{LR3M} = \max_{\lambda,\mu,\nu,\varpi,\theta} |K|\,\theta + \sum_{i\in N}\sum_{j\in\Omega} |K|\left(\mu_{ij}t_{ij} - \mu_{ij}M - \nu_{ij}M\right) + \sum_{i\in\Omega}\lambda_i \tag{2.93}$$

subject to

$$\begin{aligned}
\theta \leq\ & c_p - \sum_{i\in\Omega} a_{ip}\lambda_i \\
& + \sum_{i\in N}\sum_{j\in\Omega}\left(s_{ip} + q^+_{ip} + q^-_{jp} - s_{jp} + Mx_{ijp}\right)\mu_{ij} \\
& + \sum_{i\in N}\sum_{j\in\Omega}\left(Mx_{ijp} - Q_{ip} - f\left(q^+_{ip}\right) + \phi\left(y_{ijp}\right) - f\left(q^-_{jp}\right) + Q_{jp}\right)\nu_{ij} \\
& + \sum_{i\in N}\sum_{j\in N}\left(y_{ijp} - d_{ij}x_{ijp}\right)\varpi_{ij} \quad \forall p\in P
\end{aligned} \tag{2.94}$$

$$\theta \leq 0 \tag{2.95}$$

$$\mu_{ij},\ \nu_{ij},\ \varpi_{ij} \geq 0 \qquad \forall i,j \tag{2.96}$$

Lagrangian Lower Bound

For a fixed set of dual variables we obtain a Lagrangian lower bound denoted by z_{lag3} such that:

$$z_{OP} \geq z_{lag3}(\lambda,\Lambda) = |K|\left(z_{SP1KLR2} + z_{SP2KLR2} + \sum_{i\in N}\sum_{j\in\Omega}\left(\mu_{ij}t_{ij} - \mu_{ij}M - \nu_{ij}M\right)\right) + \sum_{i\in\Omega}\lambda_i$$

2.5 Heuristics for Obtaining an Upper Bound

We use Algorithm (1) in order to find a feasible upper bound to the problem. The heuristic starts once the best Lagrangian bound is obtained. Using the DW master problem we collect the paths for which $\omega \geq 1$. If some nodes are not visited we look for the paths that visits only a subset of the nodes that are not visited. If still we do not obtain a feasible path, we solve the subproblem over a subgraph that consists only of the depot and the remaining nodes which are not visited. Once done, a feasible solution is reported if the collected paths are less than the available number of vehicles.

Require: Lagrangian bound is found
$collectedpaths \leftarrow \{\}$
$visitednodes \leftarrow \{\}$
$notvisitednodes \leftarrow \Omega$
if $\omega \geq 0.9$ **then**
    $collectedpaths \leftarrow collectedpaths \cup nodes_p$
    $visitednodes \leftarrow notvisitednodes \cap nodes_p$
    $notvisitednodes \leftarrow notvisitednodes \backslash nodes_p$
end if
while *notvisited* is not empty **do**
    use lagragian subproblem to generate a path over the subgraph containing *notvisitednodes* only in addition to the depot
    $collectedpaths \leftarrow collectedpaths \cup nodes_p$
    $visitednodes \leftarrow notvisitednodes \cap nodes_p$
    $notvisitednodes \leftarrow notvisitednodes \backslash nodes_p$
end while
if $|collectedpaths| \leq |K|$ **then**
    a feasible solution in found
end if

Algorithm 1: Heuristic to find a feasible solution

2.6 Computational Results

In this section, we report 3 sets of computational results. The first set compares the quality of the lower bounds obtained by the three different Lagrangian relaxations. The second set compares the quality of the lower bound obtained by Lagrangian relaxation 1 (LR1) with optimal solution obtained by Cplex. Finally, we report the solution of the data sets which LR1 was able to find a Lagrangian bound along with optimal solution when available. In these results, we use a subset of nodes in particular the first 16, 25, and 30 nodes of Solomon's instances. These results will be used later in Chapter 3 to evaluate the quality of the tabu search solutions. The algorithms were implemented using Matlab and run on a Lenovo workstation with 64-bit windows 7, 2.3 GHZ CPU processor, and 48 GB of RAM.

2.6.1 Solomon's Instances

Solomon's well-known instances are the classical benchmark data sets which have been used extensively in the literature on routing problems. The instances were first proposed by Solomon [1987] based on some data sets presented by Christofides et al. [1979]. Solomon [1987] proposed a total of 56 data sets each consisting of 100 customers with their coordinates, a central depot, number of vehicles, capacity limits, time windows, and service times. However, in the literature it is common to use a subset of the 100 customers like first 25 or 50 customers of the set. There is a total of 56 datasets classified into 6 classes as follows:

- $R1$xx consists of 12 datasets and contains customers which are located randomly using a random uniform distribution with short time horizons.
- $R2$xx consists of 11 datasets and contains customers which are located randomly using a random uniform distribution with long time horizons.
- $C1$xx consists of 9 datasets and contains customers which are clustered with short time horizons.
- $C2$xx consists of 8 datasets and contains customers which are clustered with long time horizons.
- $RC1$xx consists of 8 datasets and contains a mix of clustered and randomly located customers with short time horizons.
- $RC2$xx consists of 8 datasets and contains a mix of clustered and randomly located customers with long time horizons.

2.6.2 PHEVRPTW additional Parameters

We use realistic values to characterize the properties of the plug-in hybrid vehicles used. We assume all vehicles have a battery with a capacity of 20 kWh, charging rate of 5 kWh/hr and a charge depletion rate of 1 KWh/mi. We assume that the coordinates in Solomon's instances are given in miles and the time in minutes. We assume the gasoline cost to be $\$3.5/gallon$ while the charging cost to be negligible. All vehicles start at the depot on fully charged batteries and the time window for coming back to the depot to be infinite. We set the travel time (in $mins$) between the nodes to be equal to the distance between the nodes (in mi). Finally, we ignore the service times at each node to allow for more charging and discharging time. We also assume that there is enough number of vehicles ($K = 50$). Clearly, these assumptions can be made with out loss of generality.

2.6.3 Lower Bound Comparison

We first compare the quality of the lower bounds of the Lagrangian relaxation 1 (LR1), Lagrangian relaxation 2 (LR2), and Lagrangian relaxation 3 (LR3). We use the dataset R101 as a benchmark using small sets of customers (max 16 customers) of Solomon instances. The results are summarized in Table 2.1.

Table 2.1: Comparison of the Lagrangian bounds obtained by 3 relaxations

	LR1			LR2			LR3		
	Iterations	time (s)	z_{lag}	Iterations	time (s)	z_{lag}	Iterations	time (s)	z_{lag}
R101_4 nodes	8	0.55	3.43	14	0.55	1.77	91	3.52	0.00
R101_8 nodes	23	2.02	7.82	51	1.92	3.51	1104	153.60	0.00
R101_10 nodes	32	3.54	14.92	80	3.25	6.32	2787	1779.09	0.00
R101_16 nodes	62	19.79	40.65	57	3.22	10.62	3170	34410.00	N/A

We can observe that the LR3 performs very badly even for very small datasets. The Lagrangian bounds obtained from LR3 are the worse and of potentially no use from an algorithmic effort since it is very far away from LR1. In addition, as the the number of customers increase the convergence of the cutting plane becomes very slow as the cuts obtained from the ESPPRCTW are very weak. As for LR2, we can see that the Lagrangian bound is better than LR3 but is still very far from LR1. Clearly, LR1 has the best Lagrangian bound but we notice that even for small data sets Cplex struggles with the subproblem.

2.6.4 LR1 v.s. Cplex

In this section, we summarize the computational results of our algorithm implemented on 6 of the famous Solomon Instances. The instances were reduced from 100 nodes and 25 vehicles to 16 nodes and 5 vehicles. The algorithm was implemented using Matlab and were solved on a Dell Latitude E6400 with 4 GB RAM running at 2.4 GHz. We summarize our results in Table 2.2.

Despite its simplicity, the proposed heuristics to get a feasible solution provides the optimal solution in many of the instances. Of course, more sophisticated algorithms can be proposed which can improve this heuristics even further. In terms of the quality of the solution, we notice the Lagrangian bound is extremely tight as it is within 1% of the optimal value provided by Cplex.

Table 2.2: Computational Results using 16 nodes of some Solomon instances

	Lagrangian Relaxation				Heuristic			Cplex		
Instances	zlag	time (s)	SP time (s)	Iterations	Feasobj	final time (s)	Gap	sol	time(s)	Gap
R101 16	36.786	7.416	6.956	49	36.992	7.445	1%	36.992	904.712	1%
R201 16	9.068	52.809	51.971	81	9.068	52.846	0%	9.068	4.368	0%
C101 16	2.972	9.507	8.970	65	3.259	9.908	10%	2.972	9.594	0%
C201 16	4.949	8.337	7.802	76	4.949	10.277	0%	4.949	2.512	0%
RC101 16	28.798	99.626	99.086	64	33.832	103.171	17%	29.004	3353.800	1%
RC201 16	14.069	80.024	78.606	112	14.069	80.057	0%	14.069	4.602	0%

2.6.5 LR1 Complete Results

The LR1 Lagrangian relaxation was applied to all of the Solomon's instances (R1, R2, C1, C2, RC1, and RC2) using the first 16, 25, and 30 customers and a cutting-plane iterative approach. The subproblem was attacked aggressively by all possible cuts in Cplex. In addition, the subproblem algorithm was set to return a pool of negative reduced cost solutions (max of 20 solutions) instead of a single optimal solution only. The time limit on the subproblem was set to $600s$ and the overall problem to $6500s$. The results are summarized in Table 2.3.

We notice from the table, that the proposed Lagrangian relaxation is very efficient in computing very tight lower bounds however the subproblem is very heavy and cannot be solved efficiently using Cplex. In addition, due to the continuous nature of the charge level, this subproblem cannot be solved using the known label correcting algorithms used for the classical VRPTW.

2.7 Conclusion

As the technology of batteries advances and their cost decreases, plug-in hybrid vehicles (PHEVs) have the potential to become the ubiquitous transportation technology in supply chains due to their potential savings on gasoline costs and the flexibility provided by operating on both internal combustion engines and electric charge. In this chapter, we presented a novel formulation for the Plug-in-Hybrid electric vehicle routing problem with time windows (PHEVRPTW). The problem is an extension of the classical vehicle routing problem with time windows (VRPTW) where the vehicles used are plug-in hybrid.

Due to the different type of vehicles, the classical shortest path thinking paradigm

Table 2.3: LR1 Results on Solomon instances 16-30 nodes

Dataset	Lag time (s)	Heuristic time (s)	Total Time (s)	Z_{master}	Z_{lag}	OPT	Gap
R101_16nodes	7.16	0.01	7.18	26.27	26.27	26.27	0%
R101_25nodes	23.93	0.17	24.10	43.62	43.62	43.75	0%
R101_30nodes	61.17	0.00	61.18	45.25	45.25	45.25	0%
R102_16nodes	5217	0.01	5217	19.97	19.97	19.97	0%
R105_16nodes	49	0.00	49	25.23	25.23	25.23	0%
R105_25nodes	1583	0.00	1583	41.16	41.16	41.16	0%
R105_30nodes	5581	0.00	5581	42.66	42.66	42.66	0%
R109_16nodes	2054	0.09	2054	23.93	23.93	24.17	1%
R201_16nodes	96	0.00	96	9.09	9.09	9.09	0%
R202_16nodes	2486	0.00	2486	4.95	4.95	4.95	0%
R203_16nodes	2349	0.00	2349	3.02	3.02	3.02	0%
R209_16nodes	5906	0.07	5907	3.78	3.78	-	-
R210_16nodes	5022	0.00	5022	3.02	3.02	3.02	0%
C101_16nodes	38	0.08	39	2.97	2.97	2.97	0%
C101_25nodes	306	0.20	306	3.16	3.16	-	-
C101_30nodes	1080	0.60	1081	3.16	3.16	-	-
C105_16nodes	87	0.24	87	1.80	1.80	-	-
C105_25nodes	2800	0.11	2801	2.54	2.54	2.54	0%
C105_30nodes	5828	0.47	5828	2.54	2.54	2.54	0%
C106_16nodes	43	0.07	43	2.97	2.97	3.02	2%
C106_25nodes	458	0.21	458	3.12	3.12	-	-
C106_30nodes	1449	0.44	1450	3.12	3.12	-	-
C107_16nodes	112	0.07	112	0.54	0.54	-	-
C108_16nodes	79	0.07	79	0.00	0.00	0.00	0%
C109_16nodes	52	0.07	52	0.00	0.00	-	-
C109_25nodes	2703	0.21	2703	0.00	0.00	-	-
C201_16nodes	46	0.00	46	5.52	5.52	5.52	0%
C201_25nodes	103	0.00	103	1.17	1.17	1.17	0%
C201_30nodes	130	0.31	130	1.17	1.17	-	-
C202_16nodes	106	0.00	106	3.64	3.64	3.64	0%
C202_25nodes	2473	0.05	2473	0.00	0.00	1.91	0%
C203_16nodes	435	3.05	438	3.64	3.64	-	-
C204_16nodes	332	0.00	332	2.59	2.59	2.59	0%
C205_16nodes	127	0.51	128	4.49	4.49	-	-
C205_25nodes	797	0.04	798	0.71	0.71	-	-
C205_30nodes	4040	0.34	4040	0.71	0.70	-	-
C206_16nodes	131	0.00	131	4.49	4.49	4.49	0%
C206_25nodes	4397	0.04	4397	0.71	0.70	-	-
C207_16nodes	118	0.00	118	3.64	3.64	3.64	0%
C208_16nodes	180	0.00	180	3.87	3.87	3.87	0%
RC101_16nodes	505	0.07	505	29.71	29.71	-	-
RC101_25nodes	4279	0.44	4280	50.08	50.07	-	-
RC101_30nodes	5765	0.18	5765	69.62	69.62	-	-
RC201_16nodes	66	0.00	66	14.07	14.07	14.07	0%
RC201_25nodes	3044	0.00	3044	22.26	22.09	22.26	0%

is not optimal anymore. Hence, the decision process is more complicated, as additional consideration should be given to the charging and discharging patterns of the battery while meeting the time windows of each customer. Since this is a new problem and has not been studied before, we experiment with three different Lagrangian relaxations to study the quality of lower bounds obtained. The computational results show that the best Lagrangian bound is obtained by using similar relaxation approaches to the classical VRPTW. However the resultant subproblem is different from the classical elementary shortest path problem with resource constraints and the classical label correcting algorithms available in the literature are not suitable for solving this subproblem.

The Lagrangian relaxation approach was implemented using the cutting plane algorithm. Solving the subproblem generates a valid cut to be added to the master problem which, in return, updates the Lagrangian parameters of the subproblem. The master problem is an LP while the subproblem is an MIP which were both solved using Cplex. We reported our results using Solomon's instances which are classical for any routing problem. The PHEVRPTW proves to be a hard problem to be solved where only a subset of Solomon's instances with customers ranging between 16 and 30 were solved. The maximum we were able to solve, is only two datasets with 50 customers given the time limit (6500s).

The scope of this chapter was not to provide the most efficient algorithm for PHEVRPTW. However, it is the first step towards a novel set of problems. Any extension of the classical PHEVRPTW can be considered a potential extension of the PHEVRPTW. Our approach was focused on studying different possible Lagrangian relaxations and identifying research challenges and gaps related to this new set of problems. In the next chapter, we present a tabu search algorithm specific to the PHEVRPTW and we report the first results on Solomon's datasets with 100 customers.

Chapter 3

Tabu Search Algorithm

The traditional vehicle routing problem with time windows has been studied intensively using both exact and heuristics algorithm. While Lagrangian relaxation and column generation are the common approaches for exact algorithms, there is a wide range of different proposed heuristics in the literature. Over the past years, meta-heuristics have been successfully used to provide optimal or near optimal solutions to a wide range of combinatorial optimization problems. Some of the most successful and widely used meta-heuristics are: simulated annealing [Kirkpatrick et al., 1983], [Aarts and Korst, 1988], [van Laarhoven and Aarts, 1987], genetic algorithms [Holland and Reitman, 1977], [Goldberg and Holland, 1988], [Davis, 1991], [Dorigo and Stützle, 2010], particle swarm [Kennedy and Eberhart, 1995], [Clerc, 2006], memetic algorithms [Moscato, 1989], [Moscato and Cotta, 2003], and tabu search [Glover, 1989], [Glover, 1990], [Glover and Laguna, 1998].

In this chapter, we focus on tabu search algorithms due to their renowned success for vehicle routing problems. In fact, one can argue that the reputation of the tabu search algorithms is due to their widespread success in providing high quality solutions for different variants of routing problems. Tabu search is considered a local search meta-heuristic where a local neighborhood of a current solution is evaluated based on a devised cost evaluation function to decide on the next solution. Most tabu search algorithms use an elitist strategy to choose the next best solution however some attempt to diversify their choices. One of the most distinctive features of the tabu search algorithms is the tabu list which prevents cycling. The tabu list helps to escape local optimal solutions. The basic idea is to keep a list of attributes of previous moves in the form of a tabu list to restrict these moves until a certain number of iterations have passed. Some exceptions exist for cases when these restricted moves can improve the best incumbent solution. This approach has been first formalized by Cordeau et al. [1997] and is commonly referred to as the aspiration criteria. This search process is continued until a certain stopping criteria is reached.

3.1 Literature Review

The literature on heuristics and their different applications is vast however we will focus on the literature related to vehicle routing problems. Most of the heuristics proposed for the vehicle routing problems with time windows have first been successfully applied to the traveling salesman problem (TSP) and the traditional vehicle routing problem (VRP) and have been later adapted to account for time windows requirements by using a simple feasibility check or applying a large penalty for violating the time windows restrictions.

The earliest forms of heuristics for routing problems are generally referred to as classical heuristics as denoted by Laporte and Semet [2002] and Cordeau et al. [2006]. These classical heuristics algorithms can be further classified as: (1) construction heuristics, (2) two-phase heuristics, and (3) route improvement heuristics.

The most famous construction heuristic is the savings algorithm proposed by Clarke and Wright [1964] for the VRP. The algorithm merges two separate routes into a single feasible route if a certain savings criteria is satisfied. In particular two routes $r_1 = \{0, \cdots, n_i, 0\}$ and $r_2 = \{0, n_j, \cdots, 0\}$ are merged to a single route $r'_{12} = \{0, \cdots, n_i, n_j, \cdots, 0\}$ if $s_{ij} = c(n_i, 0) + c(0, n_j) - c(i, j)$. Several other variants for the savings algorithm with additional savings weights and different merging processes have been proposed, for example see [Gaskell, 1967], [Yellow, 1970], [Golden et al., 1977], [Paessens, 1988], [Nelson et al., 1985]... It is worth mentioning that the first attempt to adapt the savings algorithm for the VRPTW was made by Solomon [1987] by accounting for the route orientations in the merging process and time feasibility. However, the results were not promising as the average deviation from the best known solution for R1 sets and C1 sets was 22% and 17% compared to an average of 0% on both data sets for another insertion heuristic denoted by I_1 which we will discuss later.

The two-phase heuristics, also known as "cluster-first route-second" splits the routing problem into two well known optimization problems. The first is partitioning the customers into separate clusters where each cluster represents a separate route served by a separate vehicle. This is normally modeled as a generalized assignment problem. The second is a TSP problem to determine the scheduling of the customers in each cluster. One of the most famous algorithms which belongs to this category is the sweep algorithm which was popularized by Gillett and Miller [1974] but was first introduced by Wren and Holliday

[1972]. The idea is to cluster the products based on their polar coordinates with respect to the depot. Gillett and Miller [1974] described two versions of the algorithm called forward and backward sweep algorithms. In the forward algorithm, the customers are sorted based on the smallest polar coordinates first. Next, customers are inserted sequentially to the seeded route until the capacity is full where a new route is seeded by the next customer in the list. In the backward sweep algorithm, the customers are sorted based on the largest polar coordinates first. Further, intra-routes improvements were also described in their paper. Different variants of the two-phase heuristics have been proposed by Foster and Ryan [1976], Ryan et al. [1993], Fisher and Jaikumar [1981], and Bramel and Simchi-Levi [1995]. Solomon [1987] also presented an extension of the sweep algorithm for the VRPTW. The proposed algorithm was superior to the savings algorithm yet it was dominated by the I_1 insertion heuristic.

The push forward (PF) heuristic was first described in Solomon [1987] as a sequential method to account for the adjustments needed to the service time of the subsequent customers due to the insertion of a new customer. In addition, Solomon [1987] proposed several extensions for the savings and sweep heuristics to account for the temporal aspect of the problem. Most importantly, Solomon proposed 3 different insertion heuristics; I_1 which accounts for the savings as a weighted sum of the distance and the unnecessary waiting due to hard time windows; I_2 which accounts for the savings as a weighted sum of the distance and total traveling time; I_3 is similar to I_1 but the width of the time window of the newly inserted customer is added to the weighted average savings. In terms of total transportation costs, I_1 insertions outperformed the other heuristics. In addition to the insertion heuristic, Solomon [1987] described two initialization procedures to start a route seed, (a) use the farthest unrouted customers and (b) use the unrouted customer with the earliest deadline. Later, Thangiah et al. [1994] coined the term push forward insertion heuristic (PFIH) to represent Solomon's PF mechanism along with Solomon's I_1 insertion heuristic. However, for initializing the route seed they also add a weight to the clustering approach similar to that proposed in the sweep algorithm. A detailed description of the PFIH is presented in section 3.2.2.

In addition to these construction heuristics, several authors proposed several route improvement heuristics which are implemented on top of the construction heuristics. These improvement heuristics can be classified into two categories (1) intra-route heuristics and (2) inter-route heuristics. As the name suggests, intra-route heuristics are heuristics focus on the improvement of individual routes separately by various operations like swapping, removing, and adding customers to a single route. Most intra-route heuristics have been first developed to solve the TSP problem since by definition the TSP is a single Hamiltonian cycle/route. The most famous intra-route heuristics are the k-opt exchanges Lin [1965] and Or-opt by Or [1976]. On the other hand, inter-route heuristics focus on improvements across several routes mainly through interchange heuristics. The most famous inter-route heuristics are the ejection chains Glover [1992], k-opt* Potvin and Rousseau [1995], the λ-interchange by Osman [1993], and CROSS-exchange Taillard et al. [1997]. These routes improvements provide the basis of the neighborhood generation mechanism in the best tabu search algorithms in the literature for different routing problem variants.

Another track of heuristics used in combinatorial optimization is "meta-heuristics". Meta-heuristics has been defined by Osman and Laporte [1996] as:

> "... a class of *approximate method* ... designed to attack complex optimization problems where classical heuristics and optimization methods have failed to be effective and efficient. A **meta-heuristic** is formally defined as an *iterative generation process* which guides a subordinate heuristic by combining intelligently different concepts for exploring and exploiting the search space, learning strategies are used to structure information in order to find efficiently *near-optimal solutions*."

Although it is hard to characterize these meta-heuristics into well structured categories, it is common to classify meta-heuristic algorithms by two search strategies (1) local search and (2) population search.

Most local search algorithms move sequentially from one solution to another in an approach similar to decent algorithms while implementing different strategies to escape locally optimal solutions. The most famous local search meta-heuristics include: tabu search (TS) [Glover, 1986], simulated annealing (SA) [Kirkpatrick et al., 1983], and variable neighborhood search (VNS) [Mladenović and Hansen, 1997].

On the other hand, population based heuristics commonly iterate between different population of solutions. The most common strategies are classified as evolutionary algorithms such as genetic algorithms (GA) [Goldberg and Holland, 1988], and learning algorithms known as swarm intelligence (SI) such as ant colony optimization (ACO) and particle swarm optimization (PSO). Several hybrid algorithms have been proposed in literature which integrates both local and population search strategies and several different heuristics combined [Thangiah, 1998]. A very well known example is memetic algorithms proposed by Moscato [1989]. It is noted that Nagata et al. [2010] has recently proposed a very efficient memetic algorithm which was able to improve 184 best-known solutions out of the 356 available instances in the literature.

Out of all these meta-heuristics, tabu search algorithms have been well known for their success in vehicle routing problems and this explains the abundance of tabu search algorithms proposed for various vehicle routing problems [Cordeau and Laporte, 2005]. We will limit ourselves to discussing the literature of tabu search, however readers interested in other meta-heuristics are referred to [Gendreau and Potvin, 2010].

Tabu search was first introduced by Glover [1986] based on ideas developed in [Glover, 1977]. The first implementation to solve the classical VRP is attributed to Willard [1989] who used a simple k-opt neighborhood generation mechanism. They merged the multiple routes into a single TSP route by replicating the depot as a connecting node. Later, Osman [1993] proposed a new neighborhood generation mechanism called λ-interchange to solve the classical VRP problem. The λ-interchange allows for up to λ nodes to be exchanged between a pair of routes. Taillard [1993] presented a parallelized tabu search based on λ-interchange. They defined a cost evaluation function which allows for further diversification of the solution pool by adding a penalty cost for the frequency of the move leading to this solution. They also did a periodic optimization of the the routes using an exact TSP algorithm. Another successful implementation for tabu search is the Tabouroute by Gendreau et al. [1994] which uses the GENIUS insertion heuristic proposed for TSP by Gendreau et al. [1992]. This algorithm integrates a generalized insertion heuristic (GENI) and a post optimization mechanism based on unstringing and stringing (US) of nodes. Another interesting implementation for tabu search is the adaptive memory procedure (AMP) which was proposed by Rochat and Taillard [1995]. They were able to demonstrate the benefits of diversification and intensification strategies. The AMP keeps track of the best solutions during a tabu search run, then uses a probabilistic approach to select one of these solutions to restart the tabu search as an initial solution. Taillard et al. [1997] adapted this approach to solve the VRP with soft time windows. Similar diversification strategies have been implemented successfully by Chiang and Russell [1997] and Schulze and Fahle [1999]. Other strategies for tabu search are based on the ejection chains neighborhood definitions have been implemented by Xu and Kelly [1996], Rego and Roucairol [1996], and Rego [1998].

3.2 Algorithm Description

In this section, we present a Tabu search algorithm to solve the plug-in hybrid electric vehicle routing problem with time windows (PHEVRPTW).

3.2.1 Overall Framework for the Tabu Search

The tabu search algorithm integrates all the previous heuristics to search for a near optimal solution in a systemic way while implementing a tabu list to avoid cycling. The algorithm is initiated using the push forward insertion heuristic with an LP post optimization algorithm to decide on the charging and discharging patterns. A neighborhood is generated based on a 2-interchange mechanism where only the best solution of all possible interchanges for each pair of routes is kept in the neighborhood of the solution. The algorithm explores the neighborhood to find the best non-tabu move. Then an acceptance criteria based on aspiration criteria and a simulated annealing procedure is used to the next move. The solution quality is estimated based on the cost insertion function 3.2. The algorithm is stopped once the best solution does not improve for 10 consecutive iterations. The general framework for the implemented tabu search is summarized in Algorithm 2.

Require: Feasible Problem Set

$TabuList \leftarrow \phi$
$IncumbentSolution \leftarrow \infty$
$k \leftarrow numberofvehicles$
$C \leftarrow Capacity$
$S \leftarrow PFIH$
$S \leftarrow 2opt(PFIH)$
$S \leftarrow half(S)$
while Not Stopping Criteria **do**
    Generate Neighborhood $N_\lambda(S)$
    $S' \leftarrow MinCost(N_\lambda(S))$
    if $S' \in TabuList$ **then**
        if $Cost(S') < IncumberSolution$ **then**
            $S \leftarrow S'$
        end if
    else
        if $Cost(S') < Cost(S)$ **then**
            $S \leftarrow S'$
            $TabuList \leftarrow TabuList \cup Move$
        end if
    else
        Simulated Annealing Criteria
    end if
end while

Require: Stopping Criteria

$BestSolution \leftarrow LPoptimized(Incumbent)$
Restart using $BestSolution$

Algorithm 2: Tabu Search Framework

3.2.2 Initialization

In order to initialize the tabu search algorithm we use the well know push forward insertion heuristic (PFIH) which was first introduced by Solomon [1987]. The PFIH gives priority to route customers which are farthest away from the depot. These are normally harder to assign to a route in a later stage due to the time and capacity restrictions. The customers are sorted in ascending order according to the following function:

$$C_j = -\alpha d_{0j} + \beta b_j + \gamma(\frac{\theta_j d_{0j}}{360}) \tag{3.1}$$

where θ_j is the polar coordinate angle between customer j and the depot.

The unrouted customer with the lowest value is first assigned to a route. This is repeated for every unrouted customer every time a new route needs to be constructed. This classification tends to favor the customers that are farthest from the depot and who has the earliest time window deadline. After experimentation, the values of (α, β, γ) are chosen to be $(1, 0.1, 0.7)$.

After the first customer is assigned, the PFIH then iteratively chooses the unrouted customer with the cheapest feasible insertion cost and adds it to the route. This is repeated until either all customers are routed or the capacity of the vehicle is exceeded or non of the possible insertions is feasible. If there are still unrouted customers, a new route is started based on 3.1 and the cheapest insertion is continued. The PFIH does not have any restriction on the number of vehicles available but since its priority is to fill the vehicles up to the maximum possible capacity, then the algorithm normally uses the minimum possible number of vehicles. The algorithm for PFIH is summarized in Algorithm 3.

```
collectedroutes ← {}
currentpath ← {}
unroutedcustomers ← Ω
while unroutedcustomers is not empty do
  Start a new path
  Sort unroutedcustomers based on 3.1
  Pick first sortedcustomer
  while collecteddemand + demand(sortedcustomer(1)) <= vehiclecapacity do
    find cheapest insertion location
    if istimefeasible then
      currentpath ← currentpath ∪ sortedcustomer(1) at the cheapest location
      index
      collecteddemand ← collecteddemand + demand (sortedcustomer(1))
      unroutedcustomers ← unroutedcustomers\ {sortedcustomer(1)}
    end if
    Pick first sortedcustomer
  end while
  collectedroutes ← collectedroutes ∪ currentpath
end while
if |collectedroutes| ≤ |K| then
  a feasible solution in found
  LP optimize the charges on the given fixed routes
  Bestroutes ← postLProute
end if
```

Algorithm 3: Push First Insertion Heuristic

However, to intensify the diversification we break down the routes by half to increase the number of possible routes. We use a crossover operator to split each route at the midpoint to two separate routes. The rational behind this approach is that the design of the λ-interchange allows closing certain routes however it does not allow opening new routes when needed. For this reason, breaking down the routes by half, allows for a larger diversification of the solution space. Our experience with this approach is that it has been very effective in improving the solution quality.

3.2.3 Neighborhood Generation

The tabu search algorithm is a local search algorithm which success depends on the definition of the neighborhood and the efficient evaluation of the best moves in this neighborhood. There has been a number of successful neighborhood generation mechanisms of which we note:

- k-opt exchanges by Lin [1965] which was first used to generate solutions for the traveling salesman problem. The k-opt exchanges is an intra-route approach which allows for k edges to be removed from a route and replaced by k new edges while preserving the feasibility of the route. However, this procedure is not well suited for routing problems with time windows since it does not preserve the orientation of the routes.

- OR-opt* is an intra-route operator which was first introduced by Or [1976] also for the traveling salesman problem. The idea is to relocate a chain of consecutive nodes from one position to another while preserving the orientation of the route.

- k-opt* exchange is an inter-route operator which was first introduced by Potvin and Rousseau [1995]. This approach adapts the k-opt exchanges to routing problems with time windows by preserving the orientation of the routes. The basic idea is similar to a single crossover approach implemented in genetic algorithm. Two routes are improved by removing edge $(i,\ i+1)$ from the first route and $(k,\ k+1)$ from the second route and then connecting $(i,\ k+1)$ and $(k,\ i+1)$.
- CROSS-exchange is another inter-route operator which was first introduced by Taillard et al. [1997]. As its name suggests, it is a generalization of the 2-opt* exchange where it resembles a 2-point cross-over in the genetic algorithm context. The idea is to remove two edges $(i-1,\ i)$ and $(j,\ j+1)$ from the first route and $(l-1,\ l)$ and $(k,\ k+1)$ from the second route. Then the whole link $i-j$ is exchanged with link $l-k$ by connecting edges $(i-1,\ l)$ and $(j,\ k+1)$ for the updated first route and edges $(l-1,\ i)$ and $(k,\ j+1)$ for the updated second route.

Other commonly used neighborhood search mechanisms involve ejection chains by Glover [1992] and GENI-exchange by Gendreau et al. [1992]

λ - Interchange

In our tabu search algorithm, we adopt the λ- interchange mechanism which has been first proposed by Osman [1993]. The idea is to interchange up to λ consecutive nodes between two distinct routes. The most commonly used λ values is either 1 or 2. For λ=2 the set of possible moves are

- $(0, 1)$: remove one node from the second route and add it to the first route.
- $(0, 2)$: remove two nodes from the second route and add them to the first route.
- $(1, 0)$: remove one node from the first route and add it to the second route.
- $(2, 0)$: remove two nodes from the first route and them to the second route.
- $(1, 2)$: interchange two nodes from second route with one node from the first route.
- $(2, 1)$: interchange two nodes from the first route with one node from the second route.
- $(2, 2)$: interchange two nodes from the first route with two nodes from the second route.

For each possible λ-combination (for example $(1, 2)$) the interchanges are investigated at each possible position in the routes. Hence each combination produces one alternative pair of routes. The new set of routes which involves the newly modified routes is considered as a single possible neighbor of the current solution. To generate the full neighborhood, each possible λ combination is investigated and each possible pair or routes undergo these λ-interchanges. The set of routes are defined as an array of routes from and to the depot denoting the sequence of nodes visited.

3.2.4 Cost Evaluation Function

The tabu search is guided by a cost evaluation function to compare the solution quality of the neighborhood. In this section, we present a new cost evaluation function which is specific to the PHEVRPTW. We borrow from Gendreau et al. [1994] the idea of adding a penalty cost in the objective function for violating the capacity constraints and the time windows constraint. However, Gendreau et al. [1994] uses the penalty function along with the traditional routing cost function $\sum_{ij} c_{ij} d_{ij}$ which is not suitable for the PHEVRP due to the potential savings of routing on a charge. Before discussing the cost evaluation function let us define some additional terminology. let $S_l := \{\cup_{i=1}^{m} R_i : \quad m < k\}$ be a solution with m distinct routes such that $R_i := \{0, n_{i_1}, n_{i_2}, \cdots, 0\}$ and $R_i \cap R_{j \neq i} = \phi \ \ \forall R_i,\, R_j \subset S_l$. We denote by $(n_l, n_k) \in R_i$ as two consecutive customers on the route R_i.

The cost evaluation function is defined as follows:

$$C(R_l) = \bar{C}(R_l) + \rho_1 \sum_{(i,j) \in R_l} [h_i - C]^+ + \rho_2 \sum_{(i,j) \in R_l} \left[s_i + q_i^+ + t_{ij} + q_j^- - s_j^k\right]^+ \tag{3.2}$$

Where $\bar{C}(R_l)$ is calculated using a dynamic program which approximates the optimal cost of a route R_l. We note that in the traditional VRPTW, $\bar{C}(R_l)$ can be computed easily as it is simply the sum of the arc costs on a specific route. However, in the case of the PHEVRPTW this task is more complicated as one needs to account for the continuous variables related to the charging and discharging decisions and the resulting savings. One way of doing that is to calculate the costs on the arcs using the traditional way and then fix the the integer decision variables and solve an LP problem to decide on the optimal charging and discharging patterns. However, this approach is time consuming due to the time required to generate the constraints and call Cplex to solve the resulting LP problem for each possible interchange of each pair of routes in each neighborhood which proves to be very inefficient. For this reason, we approximate this cost using equation 3.2 and algorithm 4. For each pair of routes in a given solution, we return neighborhood solution with the lowest possible cost for every possible lambda. However, only the returned solution is then optimized using an LP program. Our experience with the dynamic program is that in most of the cases it is equal to the returned LP optimal value yet in some cases it mildly underestimates the savings from routing on a charge.

Require: Feasible Solution

$previousDepartCharge(depot) \leftarrow E$
$accumCharge \leftarrow 0$
$elapsedTime \leftarrow 0$
$totalDistanceCost \leftarrow 0$
$totalChargeSavings \leftarrow 0$
for all $(n_i, n_{i+1}) \in R(l)$ **do**
  $arrivalCharge(n_i) \leftarrow$
  $max(0, previousdepartCharge(n_i) - distance(n_i, n_{i+1})/dischargerate)$
  $preCharge(n_i) \leftarrow min(E - arrivalCharge(n_i), availChargeTime(n_i))$
  $chargeAccum(n_i) \leftarrow arrivalCharge(n_i) + preCharge(n_i)$
  $elapsedTime \leftarrow elapsedTime + travelTime(n_i, n_{i+1})$
  $totalDistanceCost \leftarrow totalDistanceCost + distanceCost(n_i, n_{i+1})$
  $distanceOnCharge \leftarrow previousDepartCharge(n_i) - arrivalCharge(n_i, n_{i+1})$
  $chargeSavings \leftarrow chargeSavings - distanceOnCharge$
  if $earliestDueTime(n_{i+1}) > elapsedTime$ **then**
    $availWaitTime(n_{i+1}) \leftarrow earliestDueTime(n_{i+1}) - elapsedTime$
    $postCharge(n_i) \leftarrow$
    $min(E - chargeAccum(n_i), availWaitTime(n_{i+1}) * chargingRate)$
    $elapsedTime \leftarrow earliestDueTime(n_{i+1})$
  else
    $availWaitTime(n_{i+1}) \leftarrow 0$
    $postCharge(n_i) \leftarrow 0$
  end if
  $previousDepartCharge(n_i) \leftarrow chargeAccum(n_i) + postCharge(n_i)$
  $availWaitTime(n_{i+1}) \leftarrow availWaitTime(n_{i+1}) - postCharge(n_i)$
end for

Algorithm 4: Dynamic program to calculate $\bar{C}(R_l)$

If an interchange operator is applied to pairs R_l and R_k to generate a new routes $R_{l'}$ and $R_{k'}$ then the cost difference is defined according to the following function:

$$\Delta = C(R_l) + C(R_k) - C(R_{l'}) - C(R_{k'}) \tag{3.3}$$

Hence, one solution in the neighborhood is generated by choosing the new routes leading to the minimum Δ generated by all the possible interchanges on the given pair. The overall neighborhood is defined as the set of minimum cost possible interchanges for each possible pair of routes and is denoted by $N(S)$.

3.2.5 The Tabu List and Aspiration Criteria

The tabu search uses a short term memory of prohibited moves denoted by a tabu list. The tabu list prevents potential cycling between solutions hence allowing the algorithm to escape any potential local optima. Using the tabu list, implies that a move represented by moving customer i from route l to route k at iteration t is not allowed until iteration $t+\theta$ where θ is the size of the tabu list. In our algorithm, θ was set to 15.

The aspiration criteria is very important in the VRP since re-insertion of the customer i can lead to a better solution due to the update in the solution routes even before θ iterations have passed. For this reason, the aspiration criteria is used in particular cases to allow for tabu moves granted that they lead to a better incumbent solution.

3.2.6 Diversification Strategy

In order to diversify the search and prevent the homogeneity of the neighborhoods, a diversification strategy is required to force the tabu search to explore new solutions that are not discovered yet. Most tabu search algorithms diversify the search space by penalizing recurrent routes using a long term memory[Cordeau and Laporte, 2005]. This idea of tracking the frequency of the routes has been first proposed by Glover [1989]. However, in our proposed algorithm we adopt the idea of incorporating a hybrid simulated annealing into the tabu search proposed by Thangiah [1999] to accept moves which are not the fittest and hence avoid being trapped in local optimality. This approach helps to diversify the search area by giving a higher probability for solutions with lower costs while avoiding myopic elitism.

Simulated annealing is a one of the simplest meta-heuristics strategy which was first proposed to solve optimization problems by Kirkpatrick et al. [1983]. Simulated Annealing (SA) uses a stochastic approach to direct the search and provides better structured randomization procedure for diversification. The idea is very simple; when the best move does not lead to a better incumbent solution or any improvement over the current solution ($\Delta > 0$) then a solution is accepted with probability $e^{(-\frac{\Delta}{T})}$, where T is a parameter called the temperature. We start with $T = 20$ which decreases using a cooling factor (0.01). The simulated annealing diversification heuristic is summarized in Algorithm 5.

Require: $\Delta > 0$
$T \leftarrow MaxTemperature$
while $T > 0$ **do**
  if $e^{(-\frac{\Delta}{T})} > rand(0, 1)$ **then**
    Accept new solution
    $T \leftarrow \frac{T}{(1+CoolingFactor*T)}$
    if $CurrentSolution < IncumbentSolution$ **then**
      $BestTemperature \leftarrow T$
    end if
  else
    $ResetTemperature \leftarrow Max(BestTemperature, \frac{ResetTemperature}{2})$
    $T \leftarrow ResetTemperature$
  end if
end while

Algorithm 5: Simulated Annealing Heuristic

In addition to the simulated annealing, we use a long term memory to keep track of the best solution. We restart the solution once using this best found solution after breaking down the routes into half.

3.3 Computational Results

In this section we present the computational results of the proposed tabu search algorithm for the well known Solomon's instances (R1, R2, C1, C2, RC1, and RC2). The PHEVRPTW is a novel problem so there is no available implementations to compare our tabu search with. For this reason, we compare the tabu search to the Lagrangian lower bounds presented in Chapter 2. All of these instances are subsets of Solomon's instances which include 100 customers. However, due to the complexity of the problem, the proposed Lagrangian relaxation was able to provide optimal bounds for subsets of 16, 20, and 30 nodes in addition to the depot (Some exceptions exist particularly for R101). We first show the results of the tabu search on these instances in order to evaluate the solution quality of the proposed tabu search algorithm. We then present the tabu search results for all of the Solomon's instances with 100 customers.

The instances were solved on a Lenovo workstation with 64-bit windows 7, 2.3 GHZ CPU processor, and 48 GB of RAM.

3.3.1 Comparison Tabu vs. Lagrangian Relaxation

The tabu search, just like all other heuristics algorithms, is myopic in terms of optimality. It provides no guarantee for reaching optimal solution nor an optimality gap to assess the quality of the solutions. For this reason, we first compare the solutions obtained from our proposed tabu search algorithms to the Lagrangian lower bounds presented in Chapter 2. The comparison is summarized in Table 3.1

The gap in Table 3.1 corresponds to the gap between the tabu search solution (z_{tabu}) and the Lagrangian lower bound (z_{lag}). The column titled "Vehicles" represents the number of vehicles used in the best tabu search solution. The proposed tabu search proves to be very effective as it was able to find solutions within less than 5% gap from the Lagrangian lower bound for 23 out of the 45 datasets. The average gap on the given data sets is 16% and the maximum resultant gap is 70%. However, the large gaps are realized only for data sets with small objective values for which any slight deviation from the optimal solution corresponds to a large gap.

3.3.2 Tabu Search Results for the 100 Customers Datasets

In this part, we present the tabu search result for the full datasets by Solomon with 100 customers. These are the first results in the literature so it is not possible to assess the quality of these solutions. However, given the promising results on the smaller datasets we expect the results on these larger datasets to be equally competitive at least for some instances. These results can be used as a benchmark for future research on either exact or heuristics algorithms. The results are summarized in Tables 3.2-3.7 classified according to Solomon's datasets types.

3.4 Conclusion

Heuristics have played an important role in the advancement of the research on vehicle routing problems and their extensions. While the exact algorithms continue to evolve and stretch our ability to find optimal solutions efficiently, heuristics are extending our capability to solve larger datasets even farther. The problem with heuristics however is that they are myopic in terms of the quality of the solutions obtained. A common approach in heuristics is to compare the results to the best known exact algorithms in order evaluate the quality of the solutions generated by these heuristics.

Table 3.1: Comparison between the Lagrangian relaxation bound and the tabu search

	Lag Relaxation		Tabu Search				
Dataset	Time (s)	z_{lag}	Iter	Time (s)	Vehicles	z_{tabu}	Gap
R101_16nodes	7.33	26.27	23	31.7	10	26.27	0%
R101_25nodes	30.27	43.62	54	105.9	13	44.03	1%
R101_30nodes	102.10	45.25	64	159.4	14	45.25	0%
R102_16nodes	5216.76	19.97	24	29.6	9	23.29	17%
R105_16nodes	49.17	25.23	21	27.9	10	25.23	0%
R105_25nodes	1582.79	41.16	54	89.2	15	41.43	1%
R105_30nodes	5580.73	42.66	64	150.7	15	43.07	1%
R109_16nodes	2054.39	23.93	36	26	10	24.22	1%
R201_16nodes	96.28	9.09	36	21.4	5	9.09	0%
R202_16nodes	2486.10	4.95	36	23.3	6	6.36	28%
R203_16nodes	2349.17	3.02	36	21.4	6	4.88	62%
R209_16nodes	5906.51	3.78	31	17.1	4	6.41	70%
R210_16nodes	5022.33	3.02	36	23.4	6	4.88	62%
C101_16nodes	38.52	2.97	36	25.2	8	2.97	0%
C101_25nodes	306.08	3.16	54	56.1	6	3.49	10%
C101_30nodes	1080.64	3.16	64	104.6	11	4.16	32%
C105_16nodes	86.94	1.80	36	25.1	8	2.02	12%
C105_25nodes	2800.53	2.54	54	71	9	3.21	26%
C105_30nodes	5828.48	2.54	64	113.3	14	2.54	0%
C106_16nodes	42.86	2.97	36	22.5	8	2.97	0%
C106_25nodes	458.34	3.12	54	64.4	9	3.42	10%
C106_30nodes	1449.69	3.12	64	115.1	10	3.46	11%
C107_16nodes	111.82	0.54	36	22.1	6	0.54	0%
C108_16nodes	78.98	0.00	41	24.4	6	1.46	N/A
C109_16nodes	52.08	0.00	36	16.5	4	0	0%
C109_25nodes	2703.10	0.00	44	46.5	9	2.36	N/A
C201_16nodes	45.93	5.52	36	13.5	4	5.52	0%
C201_25nodes	103.30	1.17	47	42.2	5	1.17	0%
C201_30nodes	129.99	1.17	52	64.8	5	1.17	0%
C202_16nodes	106.27	3.64	25	12.1	5	4.67	28%
C202_25nodes	2472.99	0.00	50	43.6	3	0	0%
C203_16nodes	437.89	3.64	31	14	4	4.16	14%
C204_16nodes	332.29	2.59	29	11.6	3	2.59	0%
C205_16nodes	127.66	4.49	35	12.8	4	4.49	0%
C205_25nodes	797.52	0.71	54	46.9	4	1.17	65%
C205_30nodes	4040.03	0.70	43	49.3	5	1.17	67%
C206_16nodes	130.61	4.49	36	13.3	3	4.49	0%
C206_25nodes	4397.20	0.70	39	31	4	1.07	53%
C207_16nodes	117.56	3.64	36	14	3	4.95	36%
C208_16nodes	179.72	3.87	34	13.5	3	4.95	28%
RC101_16nodes	504.59	29.71	29	14.5	4	32.13	8%
RC101_25nodes	4279.89	50.07	71	61	5	52.04	4%
RC101_30nodes	5764.78	69.62	51	63.3	6	72.19	4%
RC201_16nodes	66.18	14.07	36	15.3	3	19.67	40%
RC201_25nodes	3043.98	22.09	54	44.3	3	22.26	1%

Table 3.2: Tabu search results for R1xx datasets with 100 nodes

Dataset	Iter	Time (s)	Distance Cost	Charge Savings	z_{tabu}	Vehicles
R101_100nodes	150	11899.8	472.8	-351.7	121.1	34
R102_100nodes	130	8823.8	476.8	-364.7	112.2	38
R103_100nodes	137	6610.3	451.4	-353.1	98.2	38
R104_100nodes	106	8691.2	412.8	-320.7	92.1	31
R105_100nodes	111	19086.8	451.1	-340.7	110.3	36
R106_100nodes	117	6308.9	462.7	-358.7	103.9	36
R107_100nodes	125	10100.2	400.9	-304.9	96.0	31
R108_100nodes	117	13182.3	382.1	-290.3	91.8	31
R109_100nodes	106	18330.9	427.5	-322.6	104.9	33
R110_100nodes	130	25577.8	386.5	-290.8	95.7	30
R111_100nodes	101	23034.9	394.1	-298.6	95.6	31
R112_100nodes	113	3145.8	354.4	-261.4	93.0	28

Table 3.3: Tabu search results for R2xx datasets with 100 nodes

Dataset	Iter	Time (s)	Distance cost	Charge Savings	z_{tabu}	Vehicles
R201_100nodes	127	5436.4	369.4	-353.2	16.3	19
R202_100nodes	111	7832.9	396.8	-380.7	16.1	22
R203_100nodes	107	8420.7	353.6	-336.8	16.8	19
R204_100nodes	148	11714.6	342.3	-335.2	7.2	19
R205_100nodes	141	2845.1	367.7	-356.5	11.2	22
R206_100nodes	137	2929.4	319.6	-304.9	14.7	17
R207_100nodes	104	2912.3	329.8	-310.5	19.3	18
R208_100nodes	106	3247.5	284.4	-277.9	6.4	14
R209_100nodes	126	5720	333.9	-326.1	7.8	19
R210_100nodes	165	12771.5	358.1	-345.6	12.5	21
R211_100nodes	84	3865.2	258.2	-242.4	15.9	12

Table 3.4: Tabu search results for C1xx datasets with 100 nodes

Dataset	Iter	Time(s)	Distance Cost	Charge Savings	z_{tabu}	Vehicles
C101_100nodes	154	21115.2	347.2	-318.7	28.5	19
C102_100nodes	123	23467.1	347.4	-321.3	26.1	20
C103_100nodes	139	3595.8	331.1	-311	20.1	18
C104_100nodes	120	5013.1	311.8	-286.8	25.0	18
C105_100nodes	106	8053.2	321.6	-295.2	26.4	19
C106_100nodes	104	11726.6	340.6	-317.3	23.3	20
C107_100nodes	113	15214	351.7	-333.8	17.9	22
C108_100nodes	128	18443.7	306.4	-286.5	20.0	17
C109_100nodes	145	3871.3	330.7	-307.6	23.2	16

Table 3.5: Tabu search results for C2xx datasets with 100 nodes

Dataset	Iter	Time(s)	Distance Cost	Charge Savings	z_{tabu}	Vehicles
C201_100nodes	127	4135.3	304	-301.5	2.5	14
C202_100nodes	121	5257.5	327.1	-323.4	3.7	14
C203_100nodes	137	2452.7	311	-306.1	4.8	13
C204_100nodes	79	1700.6	302.5	-297.7	4.8	14
C205_100nodes	99	2547.3	292.8	-291.5	1.4	13
C206_100nodes	129	3786.9	278.6	-269.1	9.6	10
C207_100nodes	119	5745.9	326.5	-322.3	4.3	15
C208_100nodes	83	3719.8	282.5	-275.4	7.1	11

Table 3.6: Tabu search results for RC1xx datasets with 100 nodes

Dataset	Iter	Time (s)	Distance Cost	Charge Savings	z_{tabu}	Vehicles
RC101_100nodes	115	5733.3	497.3	-317.4	179.9	33
RC102_100nodes	144	11761.6	411.7	-250.4	161.3	26
RC103_100nodes	129	19304.3	411	-270.6	140.4	29
RC104_100nodes	111	3387.7	389.1	-263.1	126.0	29
RC105_100nodes	103	5049.6	472.8	-316	156.9	33
RC106_100nodes	108	6373.8	368	-218.4	149.7	21
RC107_100nodes	114	11172.5	352.2	-223.9	128.3	21
RC108_100nodes	118	8370.4	356.3	-237.2	119.1	26

Table 3.7: Tabu search results for RC2xx datasets with 100 nodes

Dataset	Iter	Time (s)	Distance Cost	Charge Savings	z_{tabu}	Vehicles
RC201_100nodes	128	3484.7	417.9	-354.8	63.1	20
RC202_100nodes	110	3953.8	382.4	-322.1	60.3	18
RC203_100nodes	103	4158.1	381.8	-322.7	59.1	18
RC204_100nodes	102	3319	286	-253.2	32.8	12
RC205_100nodes	139	11457.2	414.2	-349.5	64.7	18
RC206_100nodes	115	9355.9	395.8	-327.3	68.6	17
RC207_100nodes	132	10064.2	347	-303	44.0	16
RC208_100nodes	117	8452	307.4	-256.9	50.5	16

In this chapter, we have developed a tabu search algorithm for the plug-in hybrid electric vehicle routing with time windows. We used the push first insertion heuristic method proposed first by Solomon [1987] to initialize our tabu search algorithm. We defined our neighborhood search using the well known λ-interchange method ($\lambda = 2$) proposed first by Osman [1993]. In order to evaluate the cost of the neighborhood solutions we proposed a new cost evaluation function which is specific to the PHEVRPTW. As for the diversification strategy, we used a mix of simulated annealing approach and restarting the tabu search using the best found solution and applying some further modifications of the routes obtained. The quality of the solutions, in terms of both computation efficiency and optimality gap, was competitive when compared to the Lagrangian relaxation lower bounds. The tabu search was able to find solutions within less than 5% gap from the Lagrangian lower bound for 23 out of the 45 datasets. The average gap on the given data sets was 16%. In addition, we reported solutions for all the adapted Solomon instances with 100 customers for the first time in the literature.

Chapter 4

Conclusion

Plug-in hybrid electric vehicles (PHEV) is a promising solution for a smooth transition from oil dependent transportation sector to a clean electric based sector while not compromising the mobility and flexibility of the drivers. The diffusion of PHEVs to the logistics fleets gives rise to a new vehicle routing problem which we called the plug-in hybrid electric vehicle routing problem with time windows (PHEVRPTW).

In this thesis, we have presented a new mathematical model for the PHEVRPTW. We have also compared three different Lagrangian relaxation however the subproblems are hard to be solve by Cplex. We were only able to solve few subsets of Solomon datasets of up to 30 nodes using Lagrangian relaxation. For this reason, we have also developed a tabu search algorithm based on λ-interchange neighborhood generation mechanism. The solution results were first compared to the lower bound obtained by the proposed Lagrangian relaxation algorithm. The tabu search was able to find solutions within less than 5% gap from the Lagrangian lower bound for 23 out of the 45 datasets. The average gap on the given data sets was 16% In addition, we have provided the first set of solutions for the adapted 100-nodes Solomon's datasets.

The most important challenge in the proposed framework is that the resultant subproblem of the proposed Lagrangian relaxation is hard to be solved even by a powerful commercial solver like Cplex. This requires further work on developing new algorithms to solve this subproblem. In parallel with the algorithmic research, we believe this work opens up a new field of studies for new plug-in hybrid vehicle routing problems. Basically, any extension for the classical vehicle routing problem can be studied as an extension for the plug-in hybrid vehicle routing problems. Nevertheless, new variants can be proposed like the study of a heterogeneous fleet of both plug-in hybrid electric vehicles and regular internal combustion engines. Another interesting extension is studying the impact of a vehicle-to-grid (V2G) systems on the routing problem where the vehicle can sell the stored charge to the grid.

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