Applications of Game Theory and Microeconomics in Cognitive Radio and Femtocell Networks

by

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AUTHOR'S DECLARATION

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Cognitive radio networks have recently been proposed as a promising approach to overcome the serious problem of spectrum scarcity. Other emerging concept for innovative spectrum utilization is femtocells. Femtocells are low-power and short-range wireless access points installed by the end-user in residential or enterprise environments. A common feature of cognitive radio and femtocells is their two-tier nature involving primary and secondary users (PUs, SUs). While this new paradigm enables innovative alternatives to conventional spectrum management and utilization, it also brings its own technical challenges.

A main challenge in cognitive radio is the design of efficient resource (spectrum) trading methods. Game and microeconomics theories provide tools for studying the strategic interactions through rationality and economic benefits between PUs and SUs for effective resource allocation. In this thesis, we investigate some efficient game theoretic and microeconomic approaches to address spectrum trading in cognitive networks. We propose two auction frameworks for shared and exclusive use models. In the first auction mechanism, we consider the shared used model in cognitive radio networks and design a spectrum trading method to maximize the total satisfaction of the SUs and revenue of the Wireless Service Provider (WSP). In the second auction mechanism, we investigate spectrum trading via auction approach for exclusive usage spectrum access model in cognitive radio networks. We consider a realistic valuation function and propose an efficient concurrent Vickrey-Clarke-Grove (VCG) mechanism for non-identical channel allocation among r-minded bidders in two different cases.

The realization of cognitive radio networks in practice requires the development of effective spectrum sensing methods. A fundamental question is how much time to allocate for sensing purposes. In the literature on cognitive radio, it is commonly assumed that fixed

time durations are assigned for spectrum sensing and data transmission. It is however possible to improve the network performance by finding the best tradeoff between sensing time and throughput. In this thesis, we derive an expression for the total average throughput of the SUs over time-varying fading channels. Then we maximize the total average throughput in terms of sensing time and the number of SUs assigned to cooperatively sense each channel. For practical implementation, we propose a dynamical programming algorithm for joint optimization of sensing time and the number of cooperating SUs for sensing purpose. Simulation results demonstrate that significant improvement in the throughput of SUs is achieved in the case of joint optimization.

In the last part of the thesis, we further address the challenge of pricing in oligopoly market for open access femtocell networks. We propose dynamic pricing schemes based on microeconomic and game theoretic approaches such as market equilibrium, Bertrand game, multiple-leader-multiple-follower Stackelberg game. Based on our approaches, the per unit price of spectrum can be determined dynamically and mobile service providers can gain more revenue than fixed pricing scheme. Our proposed methods also provide residential customers more incentives and satisfaction to participate in open access model.

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List of Abbreviations

WRAN	Wireless regional area network
SU	Secondary user
PU	Primary user
MBS	Macrocell base station
FAP	Femtocell access point
MUE	Macrocell user equipment
WSP	Wireless service provider
DFC	Data fusion center
FUE	Femto user equipment
CA	Close access
OA	Open access
HA	Hybrid access
QoS	Quality of service
VCG	Vickrey-Clark-Grove
OFDMA	Orthogonal frequency division multiple access
CLT	Central limit theorem
FCC	Federal communication commissions
LP	Linear programming
BS	Base station
CDF	Cumulative distribution function
PDF	Probability density function
CU	Currency Unit
iid	Independent identical distribution
CSCG	Circularly symmetric complex Gaussian

PSD	Power spectral density
GPS	Global positioning system
MCKP	Multiple choice knapsack problem
RRRLP	Randomized rounding relax linear program
RR	Randomized rounding
RLP	Relax linear programming
HSPP	Homogenous spatial Poisson process
CSSP	Clustered spatial Poisson process
LTE	Long term evoultion
SNR	Signal to noise ratio
MLMF	Multiple leader multiple follower
NE	Nash equilibrium
QAM	Quadrature amplitude modulation

List of Notations

N	Number of SUs
P_f	Probability of false alarm
P_m	Probability of miss
P_u	Probability of correct detection when PU does not exist
P_d	Probability of detection
γ	Transition probability from idle to idle state in Markov model
α	Transition probability from idle to busy state in Markov model
$P_{idle}(j)$	Probability of the channel being in the idle state at the beginning of the jth time interval
$P_{idle}^{s}(j)$	Probability of the channel being in the idle state at the during of the jth time interval
x_i	Valuation of the ith bidder
b_i	Bid of the ith bidder
$h(x_i)$	Valuation of the ith bidder in the presence of interference
C	Channel Capacity
D_i	Data traffic of the ith bidder
Y_1	Highest order statistic
$(.)'$	Stands for the derivative operation
$\beta_R(x)$	Optimal bidding function
$m(x)$	Revenue of service provider
ξ	Threshold for the detector
$Q(.)$	Gaussian Q function
L	Number of SUs
K	Number of channels

H_0	Absence of PUs
H_1	Presence of PUs
$s(n)$	Rectangular M-QAM modulated signal
$u_i(n)$	Noise samples
$h_i(n)$	Fading coefficients
$h_i^{\mathrm{R}}(n)$	Real part of fading coeficinets
$h_i^{\mathrm{I}}(n)$	Imaginary part of fading coefficients
T	Length of frame
τ	Sensing time
$\Lambda(y_i)$	Sensing statistic
$\xrightarrow{d}$	Convergence in distribution
$R^i(.)$	Average throughput
f_d	Doppler frequnecy
τ_i^*	Optimum sensing time
M	Number of non-identical channels
Ω	Set of all possible ways in which M non-identical channels can be allocated to N SUs.
Ω_j	Particular channel allocation
K_i	Set of channels in the bundle allocated to the ith bidder
m	Number of channels in a bundle
$V_i(\mathrm{K}_i)$	Valuation function of the ith bidder
p_i	Payment of the ith bidder
$B_i(\mathrm{K}_i)$	Bid of the ith bidder for the channel allocation K_i
$SW(.)$	Social welfare function
D_i^d	Delay sensitive data traffic
λ_{d}	Mean of Poisson distribution of delay sensitive data

D_i^o	Delay insensitive data traffic
λ_o	Mean of Poisson distribution of delay sensitive data
$\mathbf{B}_i^1$	Bid vector of the ith bidder in case 1
$\mathbf{B}^1$	$N \times M$ Bid matrix in case 1
$b_{i,j}^1$	Bid of the ith bidder for the jth channel in case 1
$\mathbf{B}_i^2$	r -tuple vector of bidding of the ith bidder in case 2
$b_{i,j}^2$	Bid of ith bidder for j^{th} channel in case 2
$\mathbf{A}$	$N \times M$ matrix of assignments
$\mathbf{S}_{i,j}$	$r \times M$ matrix of the bids of the ith bidder
$\lvert \mathrm{K}_i \rvert$	Cardinality of set K_i
e_i	Hyper-edge in a hyper-graph
n_i	Norm of a hyper-edge
SWF_{gr}	Social Welfare Function of greedy algorithm
SWF_{opt}	Social Welfare Function of optimum algorithm
r	Number of submitted bids
H	Interior of a reference hexagonal macrocell
λ_C	Mean of MUE distribution
λ_f	Mean of FUE distribution
$S(p)$	Supply function
$D(p)$	Demand function
$\boldsymbol{b}$	Vector of shared spectrum from all MBSs
C_i	Fraction of spectrum that is used for FUEs
p_i	Unit price for spectrum from i^{th} MBS
A_i	Spectral efficiency for FUEs served by FAPs
d_i	Fixed coefficient
v	Substitutability coefficient

w_i	Total spectrum available for the ith MBS
M_i	Spectral efficiencies of MUEs served by the ith MBS
k_i	Spectral efficiency for MUEs served by the ith FAPs
p_i^*	Optimum price
$B_i(\boldsymbol{p}_{-i})$	Best response function of the ith MBS
I	Number of leaders

Chapter 1
Introduction

1.1 Motivation

The increasing usage of bandwidth-hungry wireless applications and services has fueled demands for radio spectrum. These resources are however fundamentally limited: First, the physical propagation mechanisms of radio waves restrict the range of usable frequencies. Second and perhaps more important reason is that virtually all usable radio frequencies have already been allocated to existing applications and services, leaving little spectrum for emerging and future wireless systems. This necessities innovative approaches for spectrum management and utilization.

Some studies in early 2000's [1] have pointed out underutilization for many parts of the licensed bands. In light of the imbalance between the spectrum scarcity and spectrum underutilization, the paradigm-shifting concept of "cognitive radio" was introduced by Mitola [2] as a promising solution for efficient spectrum utilization. By sensing and adapting to the wireless environment, a cognitive radio network is able to fill in the available spectrum holes of primary (licensed) users and serve its users without causing harmful interference to the licensed ones. An example for a cognitive network is IEEE 802.22 wireless regional area network (WRAN) standard published in July 2011 [3]. This standard aims to utilize the TV bands that remain largely unoccupied in many geographical areas.

Another emerging concept for innovative spectrum utilization is femtocells. Femtocells [4] are low-power and short-range wireless access points installed by the end-user in residential or enterprise environments. They operate in licensed spectrum to connect legacy wireless devices to the cellular operator's network through the end-user's broadband

connection. Due to the proximity between transmitter and receiver, femtocells can significantly lower the required transmit power enabling power savings and prolonging handset battery life. On the other hand, operators benefit from the femtocells as a method of offloading traffic from the macrocell base station. This will result in significant reductions in infrastructure and operational expenses. If indoor wireless usage can be absorbed into the IP backbone through femtocell deployment, the macrocell base station can further allocate its resources mainly to outdoor users resulting in a better overall user experience. Based on a report published in February 2013 by Small Cell Forum [5], [6], femtocell market as a win-win solution for both end-users and operators will experience a significant growth over the next few years

A common feature of cognitive radio and femtocells is their two-tier nature involving primary and secondary users (PUs, SUs). While this enables innovative alternatives to conventional spectrum management and utilization, it also brings its own technical challenges. A main challenge in cognitive radio is the design of efficient *resource trading* methods [7]. Based on the underlying technologies, resource can refer to frequency band, channel access time, transmission power etc. In this context, trading refers the process of selling and buying available resources in an incentive driven framework. In spectrum (resource) trading, the objective of PUs is to maximize their revenue by selling the available spectrum to SUs. On the other hand, the objective of SUs is to have access to the spectrum at a reasonable price while maximizing their satisfaction (i.e., maximizing the associated utility function). However, these objectives generally conflict with each other. Therefore, an optimal and stable solution in terms of price and allocated resources is required so that the revenue and utility are maximized, satisfying both the seller and the buyer [8].

Femtocell network comprises of operator-installed Macrocell Base Stations (MBS) underlaid with short-range consumer-installed Femtocell Access Points (FAPs). This two-tier

network structure with cross-tier (between femtocell tier and macrocell tier) and co-tier (among femtocell tiers) interference differ significantly from the conventional cellular architecture that relies on the careful network planning. Operating in the same licensed band, femtocell tier will inevitably impact macrocell tier. Resource allocation in such an interference-limited environment poses itself as a major technical challenge for the femtocell network design. In femtocell networks, macrocell user equipments (MUEs) can improve indoor reception with better voice service coverage and higher data throughput with connection to the FAPs and use their resources. Therefore innovative pricing models should be designed to tempt users to participate in these types of communication.

The problems of *spectrum trading in cognitive networks* and *pricing in femtocells* have attracted a growing attention in recent years. While some initial works rely on classical optimization [8], game theory and microeconomics theory provide some alternative mathematical tools for spectrum trading [8-9]. Game theory is a mathematical framework for the analysis of "conflict and cooperation between intelligent rational decision makers" [11]. It was originally introduced by Cournot in 1838 [12] and, since then, has been applied to various problems in a wide range of disciplines including economics, political science, philosophy, computer science, engineering, etc [11]. It therefore emerges as a natural design methodology for resource allocation in wireless networks where there are different rational entities with different types of demands and has been used in the past for various problems in wireless system and network design, see e.g., [12–15]. Microeconomics theory is a branch of economics that studies the behavior of how the agents in a market[1] make decisions to allocate limited resources[2] [16-17]. Its origins go back to Bernoulli's work in 1965 [19] and

[1] A market consists of sellers and buyers of commodities or services with an efficient pricing scheme to maintain the stability of market [7].

[2] A resource is any physical or virtual entity of limited availability that needs to be consumed to obtain a benefit from it.

has also been applied to some engineering problems such as in risk-return evaluation, conflicting interests, etc. [20].

In this thesis, we will investigate the strategic interactions between PUs and SUs through game theoretic and microeconomic approaches and propose solutions for spectrum trading, and pricing in two-tier networks such as IEEE 802.22 and femtocell networks with different access models.

1.2 Spectrum Access Models in Two-Tier Networks

In cognitive radio networks, spectrum management allows wireless systems to dynamically access/share the radio spectrum on a negotiated or an opportunistic basis. In *exclusive-usage model*, spectrum privileges are sold to commercial entities who have the right for exclusive usage under certain rules. Spectrum owner provides service to the PUs and sells the unused extra spectrum to the other wireless service providers (WSPs) for a specific period of time. If, within the leasing duration, the spectrum owner needs spectrum, it must wait until the end of lease period. Therefore in this model, cognitive systems do not need to sense or dynamically change the spectrum.

In *shared used model*, on the other hand, the spectrum owned by a licensee is simultaneously shared by a non-license holder. Such sharing takes place without the PUs being aware of SUs. Therefore, the transmissions of SUs are expected to have minimal impact on PUs devices. In practice, this requires that SUs should continuously sense the spectrum, find opportunities for transmission, and vacant the spectrum if any PU wants to enter the spectrum. Therefore, the practical implementation of cognitive radio networks requires the development of effective spectrum sensing methods [21]. Spectrum sensing can be either performed by a single SU or a number of cooperating SUs. In the latter one which is named as “cooperative sensing”, SUs cooperatively perform spectrum sensing [2] and send

the sensing results to a data fusion center (DFC). The DFC combines the results and makes a decision. In the literature on cognitive radio, it is commonly assumed [7-12] that fixed time durations are assigned for spectrum sensing and data transmission. If SUs spend more time on spectrum sensing, the probability of missing PUs will be decreased, but that also reduces the time for data transmission. On the other hand, if they spend more time on data transmission and less time on spectrum sensing, the probability of missing PUs and interfering with them will increase and therefore the data throughput will decrease. Therefore, there is a fundamental tradeoff between sensing time and throughput of cognitive radio networks which will be further discussed within this thesis.

In femtocell networks, both of the aforementioned models can be used. In exclusive usage model, MBS divides spectrums in two parts and allocates one part to the MUEs and the other to Femtocell User Equipments (FUEs). In this model, there is no interference between MUEs and FUEs. But inefficiency of spectrum utilization brings the fact that shared use model is more desirable in femtocell networks. In a femtocell network based on the shared use model, the spectrum is shared between the FUEs and MUEs. In this model, the FUEs can utilize spectrum with lower priority than MUEs. Therefore interference management becomes a critical design factor in the practice.

Another type of access in femtocell networks can be defined based on closeness or openness of FAPs. In *close access* (CA), limited number of users such as family members or friends can connect to the FAP. On the other hand, it is possible to have *open access* (OA) where all customers of the operator have the right to make use of any FAP. The use of OA in fact reduces the interference problems encountered in the case of CA. Indeed, all nearby MUEs would be authorized to connect to any FAP, reducing thus the negative impact of the femtocell tiers on the macrocell network. In this case, the MUEs are always connected to the strongest server (either macro or femto), avoiding cross-tier interference. As a result, the

overall throughput of the network increases. OA is therefore advantageous from the operator point of view.

From the user point of view, CA is obviously preferred who will have full control over the list of authorized users. However, some surveys indicate that OA might be an attractive business model for home market conditioned that competitive pricing is offered [4]. Hybrid access (HA) methods are also discussed to reach a compromise between the impact on the performance of subscribers and the level of access that is granted to non-subscribers. Therefore, the sharing of FAPs resources between subscribers and non-subscribers needs to be finely tuned. Otherwise, subscribers might feel that they are paying for a service that is to be exploited by others. The impact to subscribers must thus be minimized in terms of performance or via economic incentives. With a proper pricing model, deploying OA or HA model is more beneficial for network operators than CA.

1.3 Microeconomics Theory and Its Applications in Two-Tier Networks

Microeconomics theory provides some effective pricing schemes to maintain the stability of the market. There are two types of markets in terms of the number of sellers. Monopoly market is the simplest market structure when there is only one seller. Oligopoly market is the case when there are multiple sellers and multiple buyers in the market. The sellers compete with each other independently to achieve the highest revenue by controlling the quantity or the price of the supplied commodity. Spectrum trading in two-tier network can be modeled as a monopoly or oligopoly market based on the number of PUs. When there is a single PU as a seller and several SUs as buyers, monopoly market is used. Oligopoly market is used when there exist several PUs and several SUs.

An efficient pricing scheme derived from microeconomics theory should satisfy both buyers and sellers side. *Market equilibrium* and *auction theory* are the two popular pricing

schemes used for resource trading [9]. In recent studies [22]–[27], they have been applied to spectrum trading problem in cognitive radio networks and femtocells. The market equilibrium gives the spectrum price and allocated spectrum size for which spectrum demand equals to spectrum supply. At the market equilibrium, the profit of the seller and the satisfaction of the buyer(s) are maximized [22], [23]. An example of this approach for spectrum trading is presented in [28], where hierarchical bandwidth sharing in a cognitive radio network is considered. In [23], Dusit et.al proposed market equilibrium as a pricing scheme for spectrum sharing in cognitive radio network and compared this method with competitive and cooperative pricing in terms of revenue for service provider and satisfaction of buyers.

Auction theory provides another framework for resource trading problems. An auction is a process used to obtain the price of a commodity with an undetermined value [19]. Sellers use auctions to improve revenue by dynamic pricing based on buyer demands. Buyers benefit since auctions assign resources to buyers who value them the most. There are different kinds of auctions such as English auction, sealed bid first price auction, sealed bid second price auction (Vickery auction), double auction [29]. In the English auction, the minimum price is set by the auctioneer. Then, a bidder submits a bid higher than the minimum price to the auctioneer. Each bidder may observe the bids from other bidders and competes by increasing its bidding price. Thereafter, the bidding price is continuously increased until the bidder with the highest bidding price wins the auction. In a sealed-bid auction, all bidders submit sealed bids independently. The auctioneer opens the bids and determines the winning bidder whose bidding price is the highest. For the winning bidder, the price to pay the auctioneer becomes its bidding price (i.e., first-price auction) or the second highest bidding price (i.e., second-price auction or Vickrey auction). In a double auction, multiple buyers bid to buy commodities from multiple sellers.

In [24], Vickrey auction and English auction are used for allocating unused spectrum bands to SUs based on a guaranteed quality-of-service (QoS) assuming time-variant number of SUs and PUs. In [7], a hierarchical spectrum trading model is presented to analyze the interaction among the secondary service providers, TV broadcasters, and SUs. Furthermore, a double auction with a joint spectrum bidding and service pricing model is proposed among multiple TV broadcasters and secondary service providers who sell and buy the radio spectrum. In [25], spectrum auction mechanisms are investigated when multiple units of spectrum are available and the demand from SUs exceeds the available spectrum. Sequential and concurrent auctions are further studied. In the sequential auction, all the bidders submit their bids for all the channels simultaneously while in the concurrent auction the channels are auctioned one after another. In [26], the spectrum allocation is modeled as a sequential dynamic game and a pricing-based distributive collusion-resistant spectrum allocation is proposed.

In [30], user incentives for the adoption of femtocells and their resulting impact on network operator revenues are studied. The work in [29] models a monopolist network operator who offers two models of access (shared use and exclusive usage) to a population of users with linear valuations for the data throughput. It further compares the revenue from these two models and demonstrates that shared use yields revenue comparable or higher than that in exclusive usage. In another work on femtocells [31], the Dutch auction is used for the design of sub-channel and power allocation scheme when femtocell users co-exist with OFDMA macrocell users. In [32], Vickrey-Clark-Grove (VCG) auction is proposed as the pricing scheme for OA femtocells. In [33], a reverse auction (one-buyer-multiple-sellers) framework is proposed based on VCG mechanism for access permission trading between wireless service provider and private femtocell owners.

1.4 Game Theory and Its Applications in Two-Tier Networks

A classical optimization problem consists of a single objective under a set of constraints. while in a game model, several entities are involved with different interests [8]. The solutions of the game should satisfy all of the entities. A game has three fundamental components: A set of players, a set of strategies, and a set of payoffs for a given set of actions [17]. Players are the entities who make the decisions. Strategies are a set of rules to make a decision that defines an action for a player. Payoff (or revenue) of a player shows the satisfaction level of a player for a given strategy. The satisfaction of the players is usually shown by a utility function. In game theory, steady-state conditions are known as *Nash equilibrium.* Nash equilibrium is a set of strategies, one for each player, such that no player has incentive to unilaterally change his/her action.

Based on the cooperation between players, a game can be categorized in two groups as non-cooperative and cooperative games [9]. In a non-cooperative game, each player acts as an individual rational entity to maximize its payoff and make decisions independently. But in cooperative games, all players in a group act as a single entity. They do not have any competition between each other and do aim to maximize the total revenue of group. There are also other types of game that have been extensively used for resource allocation in cognitive radio networks such as Cournot game [34], Bertrand game [35], Stackelberge game, dynamic/repeated game [36], stochastic game [37], bargaining game [15], coalition game [38], game with learning [39].

In [30], the evolution and the dynamic behavior of SUs are modeled using the theory of dynamic game. Deterministic and stochastic models are used as dynamic evolutionary games. It is assumed in [29] that a SU does not have any intention to influence the decision of other SUs in the future and PU uses an iterative algorithm for strategy adaptation using the

local information of that PU and information available to that PU from the SUs. In [40], a solution for joint sub-channel assignment, adaptive modulation, and power control for a multi-cell multi-user OFDMA cognitive radio network is proposed using a distributed non-cooperative game. A virtual referee is introduced to improve the performance of the Nash equilibrium points. This referee can modify the rule of the resource competition game for efficient resource sharing. In [41], an optimum channel and power allocation scheme is proposed based on the Nash bargaining solution for an OFDMA cognitive radio network. This paper takes into account limits on the total interference for each PU as well as the minimum SNR requirement for SUs. In [23], three different pricing models (namely, market-equilibrium, competitive, and cooperative pricing models) are investigated for spectrum allocation in cognitive radio networks.

Game theory has been also applied to some femtocell design problems. In [36], a utility-based non-cooperative game is proposed for femtocell signal-to-interference-noise-ratio (SINR) adaptation. The adaptation forces stronger femtocell interferers to obtain their SINR equilibriums closer to their minimum SINR targets, while femtocells causing smaller cross-tier interference obtain higher SINR margins. In [42], a game theoretic approach is used to design decentralized resource allocation mechanisms for FAPs. In [43], a unique and fair Pareto[1] optimal operation is proposed for femtocell networks under certain minimum QoS requirements using Nash bargaining solution.

1.5 Sensing Throughput Trade-off in Cognitive Radio Network

As earlier noted in Section 1.2, the realization of cognitive radio networks in practice requires the development of effective spectrum sensing methods. A fundamental question is

[1] An outcome of a game is Pareto optimal if there is no other outcome that makes every player at least as well off and at least one player strictly better off. That is, a Pareto Optimal outcome cannot be improved upon without hurting at least one player.

how much time to allocate for sensing purposes. In fact, there exists a basic tradeoff between sensing time and throughput of cognitive radio networks which will be further discussed within this thesis. This tradeoff problem has been first investigated for non-cooperative spectrum sensing in [44] to find the optimal sensing time so as to maximize the total average throughput of a cognitive radio network over Rayleigh fading channels. In [45], [46], the optimization of the sensing time is pursued to maximize the total outage probability for the SUs, respectively, over Rayleigh and Nakagami fading channel. In [47], the optimum sensing time is derived to maximize the SUs throughput under the Markovian traffic assumption for SUs and limited interference for PUs.

This tradeoff problem is revisited in [44], [48]–[50] in the context of cooperative spectrum sensing. For sensing purposes, the spectrum can be divided into several channels and sensing can be performed separately for each channel. Particularly, in [44], Liang and Zheng assume a fixed number of SUs which sense each channel and send the results to the DFC for the final decision. Under this assumption, they derive the optimum sensing time. In [48], Peh *et al.* assume k-out-of-M rule with a fixed M (i.e., the number of SUs) and optimally calculate k (i.e., the minimum number of SUs required to decide about channel occupancy) and sensing time to maximize the throughput of SUs. In [50], Zhang *et al.* calculate the minimum required number of SUs in cooperative spectrum sensing to achieve a target error bound. In [49], Peh *et al.* propose a method to jointly optimize sensing time and power allocation to maximize the throughput of SUs. In [51], Zaheer *et al.* formulate the sensing throughput trade-off for distributed cognitive radio as a coalition formation game under probability of detection constraint. In [52], Wang *et al.* propose an evolutionary game to calculate the optimal time for decentralize spectrum sensing to maximize throughput of SUs.

The channel models in the above works [44], [48]–[50] are either quasi-static or symbol-by-symbol independent. Specifically, in [49], a quasi-static Rayleigh fading channel is considered and the channel coefficients are assumed constant over all the received signal samples. In [48], it is assumed that fading changes from one symbol to another independently. In [44], the sensing time frame (i.e., slot) is divided into several mini-slots, which each mini-slot consists of multiple samples and the samples are from different symbols. The channel coefficient is assumed to be constant for each mini-slot and varies from one mini-slot to the other one independently. In [50], the sensing channel is assumed as time-invariant during the sensing process. These are simplifying assumptions about channel coefficients for mathematical tractability, but not realistic for most mobile scenarios.

1.6 Outlines and contributions

The outlines and original contributions of our work in each chapter are as follows:

In Chapter 2, we address spectrum trading for cognitive radio networks with shared used model. Majority of the literature on spectrum trading have so far assumed the *exclusive-usage model*, see e.g. [7], [8], [24], [25]. Instead, we consider the shared used model in the context of IEEE 802.22 WRAN and aim to design a spectrum trading method via auction approach. In the shared used model, SUs perform the sensing of PU spectrum in order to detect the vacant spectrum. Spectrum sensing is a crucial function for such opportunistic spectrum access. Existing works on spectrum trading [8], [24], [26], [53] via auction theory typically assume that the environment is "risk"-free[1] [54], but this is highly unrealistic due to the non-ideality of spectrum sensing. However, to the best of our knowledge, there is no spectrum trading method specifically designed for the shared used model considering the effects of imperfect spectrum sensing.

[1] In auction theory, risk refers to the unexpected variability or volatility of returns.

In light of these, the first specific contribution is to design a spectrum trading method to maximize the total satisfaction for the buyers (SUs) and revenue for the Wireless Service Provider (WSP) taking into account sensing errors. We assume that SUs do not have GPS and Internet to access to the online TV database for spectrum opportunities and need to perform sensing[1]. In our design, we consider the risk of imperfect spectrum sensing which causes the SUs miss the presence of licensed users and interfere with them. Taking into account this risk, we first propose a *multi-unit sequential sealed-bid first-price auction* to optimize the payoff of each SU. Then, we derive an expression for the total revenue of WSP and maximize it by optimizing the sensing time.

In Chapter 3, we return our attention on spectrum sensing which is a crucial mechanism for cognitive networks. Recall that Chapter 2 discusses tradeoff between sensing time and revenue and calculates the optimum sensing time to maximize the revenue of service provider. Chapter 3 discusses tradeoff between sensing time and throughput of SUs over time-selective channels. For time-selective fading channels, we derive an expression for the total average throughput of SUs each of which is equipped with energy detectors for cooperative sensing assuming different decision rules. In this derivation, we calculate the probability of detection and false alarm through a modified version of the central limit theorem (CLT) for correlated variables. Based on the derived throughput expression, we formulate an optimization problem in terms of sensing time and the number of SUs assigned to sense each channel. In terms of sensing time, it is a non-linear programming problem which can be solved using numerical methods. On the other hand, the problem in terms of the number of SUs is an integer programming problem. Based on this two dimensional

[1] These are defined as "sensing-only TV band devices" by the Federal Communication Commissions (FCC) [55].

optimization problem, we propose an algorithm to jointly optimize sensing time and the number of cooperating SUs.

Another challenge in cognitive radio networks is the spectrum trading problem for concurrent non-identical channel allocation which is pursued in Chapter 4. In the current literature on spectrum auctions [7], [25], [29], [56], [57], the valuation functions used for the bidders are somehow unrealistic. In this chapter, we propose *a realistic valuation function* which each channel has different values for different bidders; this leads to non-identical channels. In [25], [58], some methods for non-identical channel cases are investigated, but their proposed methods are not efficient. In all of them, single-minded bidders are assumed indicating that each bidder is willing to buy only one channel and other channels have zero values for the bidders and the losers do not have the chance of revisiting the remainder available channels. To address this issue, we consider ***r****-minded bidders* each of which can bid for $r>1$ bundles of channels in each round of auction, but is allowed to win at most one of these bundles. Another important issue that we want to address in this chapter is *truthfulness*[1] or incentive compatibility of combinatorial auction mechanism when the problem of determining auction outcomes is NP-hard.

In the light of above discussions, our main contributions in this chapter can be therefore summarized as follows: We first propose a novel realistic valuation function for the SUs which depends on delay-sensitive traffic (e.g., voice or video) and delay-insensitive traffic (e.g., e-mail or file transfer) as well as the capacity of each available channel. This function is proportional to the channel capacity and the weighted summation of data traffic types. Instead of the commonly assumed single-minded bidders, we assume that the SUs are r-minded bidders and design an efficient VCG-based auction mechanism. In our scheme, each

[1] If an auction mechanism has the property that each user should submit their true valuation if he/she wants to maximize his/her utility function, it is called truth-telling algorithm [59].

bidder can bid for r bundles of channels in each round of auction, but each bidder is allowed to win at most one of these bundles. Two cases are assumed: In case 1, the SUs are r-minded but they can submit bid only for single channels. In case 2, the SUs are r-minded and they can submit bid for bundles of channels. We show that the first case is solvable in polynomial time but in the other one, the problem of determining auction outcomes is NP-hard. We propose two sub-optimal methods for solving this problem, namely greedy algorithm and randomized rounding linear programming (LP) relaxation. Due to the sub-optimal nature of solutions in case 2, VCG mechanism is not truthful anymore and the SUs can lie to maximize their utilities. To address this, we further propose an auction mechanism with limited truthfulness property, based on an iterative greedy algorithm.

In Chapter 5, we return our attention on femtocell networks. The current market in femtocell network is mainly geared towards CA femtocells [60]. To enable the wide deployment of OA femtocells, innovative pricing models with incentives for residential femtocell users are required that will be pursued in this chapter. We consider oligopoly market and propose novel utility functions for the FAPs and MBSs which include requirements of OA femtocell networks. We take into account discounts for the FAPs which serve the other MUEs and dynamically set the price of spectrum different from earlier works assuming fixed pricing. Therefore, based on our defined utility function, FAPs have more incentives to participate in OA networks. We further propose four methods of pricing for oligopoly market based on market equilibrium, Bertrand game, multiple-leader-multiple-follower Stackelberg game and cooperative game. Among these four methods, we show that the approach on market equilibrium brings more revenue than the others for the FAPs. On the other hand, the pricing scheme with the cooperative game has the best revenue for the MBSs. We further provide comparisons with fixed pricing schemes [61] and demonstrate the superiority of our proposed schemes in terms of revenues.

In Chapter 6, we first provide the conclusion of research work done so far and then discuss some future directions.

Chapter 2
Spectrum Trading for Risky Environments in IEEE 802.22 Cognitive Networks

2.1 Introduction

In this chapter, we consider the *shared used model* and design a spectrum trading method to maximize the total satisfaction for the buyers (SUs) and revenue for the WSP taking into account sensing errors in TV bands. We first propose a *multi-unit sequential sealed-bid first-price auction* to optimize the payoff of each SU. Then, we derive an expression for the total revenue of WSP and maximize it by optimizing the sensing time. Our results demonstrate that the proposed auction-based spectrum trading method brings better revenue than its counterparts in [25], [29].

2.2 System Model

We consider a cognitive radio network which operates in TV bands and involves the point-to-multipoint communication between a Base Station (BS) and N SUs. Under the assumption of the shared used model, SUs and BS are responsible for sensing and finding opportunities in the spectrum and avoiding to make interferences with the PUs (i.e., TV channels).

There are two hypotheses for detection; namely H_0 and H_1 as the absence or the presence of the PUs, respectively. Due to the imperfect nature of spectrum sensing to identify the spectrum opportunities, we assume that there are some risks in the presence estimation of the PUs. Four types of probability associated with imperfect sensing can be defined:

1) Probability of false detection ($P_f = P(\mathrm{H}_1 \mid \mathrm{H}_0)$),

2) Probability of miss ($P_m = P(\mathrm{H}_0 \mid \mathrm{H}_1)$),

3) Probability of correct detection ($P_u = P(\mathrm{H}_0 \mid \mathrm{H}_0)$) when PU does not exist,

4) Probability of detection when PU exists ($P_d = P(\mathrm{H}_1 \mid \mathrm{H}_1)$).

We model the traffic of the SUs with a Markov process [62] as illustrated in Fig.2.1. The channel states are represented by 0 (busy) and 1 (idle). To reflect the effect of sensing, we consider four states, namely "idle-true", "idle-false", "busy-true", and "busy-false". For example, the state of "idle-true" indicates that the channel is decided to be idle when it is actually in idle state. If it is decided to idle when it is actually in busy state, this is named as "idle-false". State transitions are based on the transition probabilities γ and α in the two states Markov process and probabilities of detection and false alarm.

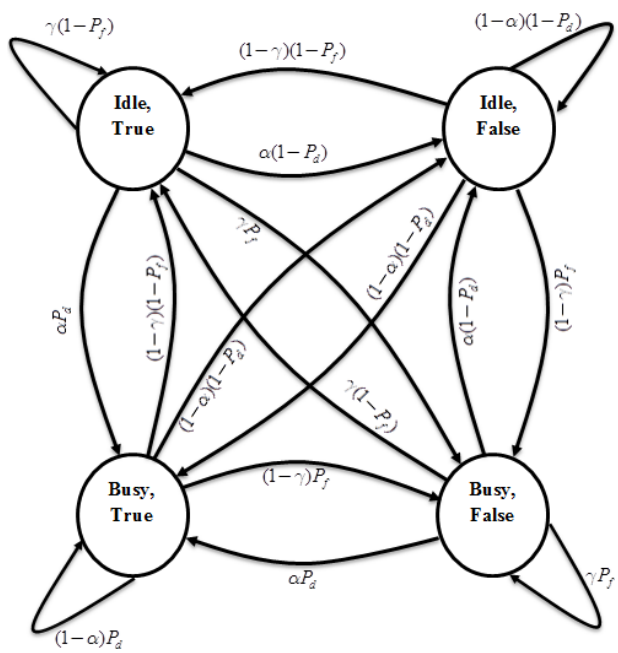


Figure 2.1 Markov chain model

Let $P_{idle}(j)$ denote the probability of the channel being in the idle state *at the beginning* of the jth time interval. Furthermore, let $P_{idle}^{s}(j)$ denote the probability of being in the idle state *during* the jth time interval. The latter takes into account sensing results acquired within the jth time interval. Based on whether the channel is sensed as idle or busy, we have

$$P_{idle}^{s}(j)=\begin{cases}\dfrac{(1-P_f)P_{idle}(j)}{(1-P_f)P_{idle}(j)+(1-P_{idle}(j))P_m} & \text{if channel is sensed as idle}\\[2ex] \dfrac{P_f P_{idle}(j)}{P_f P_{idle}(j)+(1-P_{idle}(j))P_d} & \text{if channel is sensed as busy}\end{cases} \tag{2.1}$$

where

$$P_{idle}(j)=P_{idle}^{s}(j-1)\gamma+\left(1-P_{idle}^{s}(j-1)\right)\alpha \qquad j\in\mathbb{N}=\{1,2,\ldots\}. \tag{2.2}$$

If the channel is sensed as idle, the auction will be held for that channel and risk related information $P_{idle}^{s}(j)$ is announced to the SUs for their biddings. On the other hand, if the channel is sensed as busy, no auction will be held.

2.3 Proposed Auction Mechanism

In this section, we propose a *sealed-bid first-price auction*[1] to optimize the payoff of each SU. In this auction type, the highest bidder wins and pays the amount he/she bids. This auction can be carried out either in concurrent or sequential version. Sequential version has better revenue for the auctioneer and bidders in the auctions with non-identical items [29]. Since the non-identical channels is considered, we can assume that channel conditions (such as noise, fading and interference) will change independently in each round of auction. Furthermore, we assume that the traffic of SUs will change independently from one round to

[1] Sealed-bid second-price auction is rarely used in practice because of the possibility of cheating by the seller [8], [29]. With the fear of cheating, a second price auction may become less profitable than a first price auction for non-cheating and fair seller.

another round. Therefore we assume that the SUs cannot use the history of the previous rounds of auction for their biddings in the future round of auction.

At the beginning of each time frame, the BS determines the channels and the amount of time that SUs should sense. After sensing period, each SU sends the sensing results to the BS using a single bit which represents the state of each channel (0 for busy and 1 for idle). The BS makes the final decision about the channel availability and calculates the associated risk for each channel based on (2.1) and (2.2). At the beginning of each round of auction, the BS announces the available channel and associated risk. SUs calculate their biddings and send them to the BS. The BS allocates the channel to the highest bidder. Since our proposed auction method is sequential, this process is iterated in multiple rounds. Considering the data rates supported in IEEE802.22, it can be shown that the required time for transmission of overhead information is negligible in comparison to the sensing time [3]. Since the bidding strategy and winner determination of the auction can be calculated in polynomial time, the proposed spectrum trading method has a linear computational complexity with $O(N)$ where N is the number of bidders.

In our proposed method, the bidder's payoff can be expressed as

$$\Pi_i = \begin{cases} x_i - b_i & \text{if } b_i > \max\limits_{i \neq j} b_j \ \& \text{ risk does not occur} \\ 0 & \text{if } b_i < \max\limits_{i \neq j} b_j \\ h(x_i) - b_i & \text{if } b_i > \max\limits_{i \neq j} b_j \ \& \text{ risk occur} \end{cases} \tag{2.3}$$

where x_i is the valuation of the ith bidder (i.e., the maximum amount that the jth bidder is willing to pay for the channel), b_i is the bid of the ith bidder, and $h(x_i)$ is the valuation of the ith bidder in the presence of interference.

We assume $x_i = C \cdot D_i \cdot (T - \tau)$ where C denotes the channel capacity, D_i is the data traffic of the ith bidder, T is the length of transmission frame (i.e., the total time for sensing

and data transmission), and τ is the sensing time[1]. The maximum channel capacity between SUs and BSs is denoted by $C_{\max}$ and the maximum data traffic of SUs is defined by $D_{\max}$. In the case of interference, the channel capacity for the SUs is smaller than channel capacity with no interference, therefore we have $h(x_i) < x_i$.

Fix a bidder, say the first one x_1 without losing generality, and let Y_1 denote the highest order statistics, i.e. $Y_1 = \max(x_2, ..., x_N)$, among $N-1$ remaining bidders $x_2, x_3, ..., x_N$. Clearly, for all $y, G(Y_1 \le y) = F(x_i \le y)^{N-1}$ where G (.) and $F(.)$ respectively denotes the cumulative distribution function (cdf) of Y_1 and x_i. Based on our assumptions for the distribution of parameters, the distribution of x is assumed[2] as $F(x)$. Therefore the probability density function (pdf) of Y_1 can be calculated as

$$g(x) = (N-1) f(x) \left[F(x) \right]^{N-1} \tag{2.4}$$

where $f(x)$ is the pdf of x. In our proposed method, the expected payoff for an SU is given by

$$m_R(x) = \left[P_{idle}^s(j)\left(x-b\right) + \left(1 - P_{idle}^s(j)\right)\left(h(x)-b\right) \right] \mathrm{G}(x). \tag{2.5}$$

The optimal bidding of bidders is a function of their valuations. Let $\beta_R(x)$ denote the optimal bidding function (strategy). It can be calculated by the maximizing $m_R(x)$ with respect to b, i.e.,

$$\frac{\mathrm{g}\left(\beta_R^{-1}(b)\right)}{\beta_R'\left(\beta_R^{-1}(b)\right)} \left[P_{idle}^s(j)\left(x-b\right) + \left(1 - P_{idle}^s(j)\right)\left(h(x)-b\right) \right] + \mathrm{G}\left(\beta_R^{-1}(b)\right) = 0 \tag{2.6}$$

where $(.)'$ stands for the derivative operation. After some mathematical manipulations, (2.6) can be rewritten as

[1] In this section, we assume that sensing time is fixed. In the next section, we further discuss its optimal choice to maximize the revenue.

[2] We ignore index *i* in the following for the sake of presentation simplicity.

$$\beta_R(x)g(x)+\beta_R'(x)G(x)=\left[P_{idle}^s(j)x+\left(1-P_{idle}^s(j)\right)h(x)\right]g(x). \tag{2.7}$$

Therefore, the optimal bidding function is obtained as

$$\beta_R(x)=\frac{1}{\mathrm{G}(x)}\int_0^x\left(P_{idle}^s(j)y+(1-P_{idle}^s(j))h(y)\right)\mathrm{g}(y)dy \tag{2.8}$$

2.4 Sensing-Revenue Trade-off

In the previous section, we have proposed an auction mechanism to optimize the payoff of each SU under the assumption of a given fixed sensing time. In this section, we will first calculate the total revenue of WSP and then maximize it by optimizing the sensing time.

Fixed time durations are typically assigned for spectrum sensing and data transmission. This is obviously not the optimal solution. If the SUs spend more time on spectrum sensing, the probability of missing PUs decreases, but this reduces the time for data transmission and therefore the payment of SUs to the WSP decreases. On the other hand, if they spend more time on data transmission and less time on spectrum sensing, the probability of missing PUs and interfering with PUs will increase. This will decrease the bidding and payment of SUs to the WSP. Hence, there is a trade-off between sensing time and revenue that we will discuss in the following.

SUs are mainly interested in the channels that are underutilized, such as channels with $P_{idle}\geq 0.5$. On the other hand, the total probability of detection (P_d) is usually more than 0.7 [44] and the total probability of false alarm (P_f) is usually small (<0.3). Under the assumption of these typical values, it is possible to ignore $(1-P_{idle}^s(j))h(y)$ in (2.8) and approximate it as

$$\beta_R(x)=\frac{1}{G(x)}\int_0^x P_{idle}^s(j)yg(y)dy. \tag{2.9}$$

The expected revenue of the WSP from an SU can be then calculated as

$$\mathrm{E}\left[m(x)\right]=\mathrm{E}\left[G(x)\beta_R(x)\right]=\mathrm{E}\left[\int_0^x P_{idle}^s(j)yg(y)dy\right] \tag{2.10}$$

where the expectation is with respect to the variable y. This yields

$$\mathrm{E}\left[m(x)\right]=\int_0^{C_{\max}D_{\max}(T-\tau)} P_{idle}^s(j)yg(y)(1-F(y))dy\,. \tag{2.11}$$

The total average revenue of auctioneer is the summation of the payments of N SUs to the auctioneer, therefore is given by $\mathrm{E}_{\mathrm{Total}}\left[m(x)\right]=N\mathrm{E}\left[m(x)\right]$. Here, we will maximize the total average revenue with respect to sensing time subject to adequate protection given to the PUs. Therefore, this problem can be formulated as

$$\begin{aligned}&\max_{\tau}\ \mathrm{E}_{\mathrm{Total}}\left[m(x)\right]\\ &\text{s.t}\quad \bar{P}_d\le P_d,\ \ 0\le\tau\le T\end{aligned} \tag{2.12}$$

where $\bar{P}_d$ is the minimum probability of detection that the BS needs to achieve to protect the PUs in the ith channel. For a given sensing time τ, if we have two probability of detection values, say $\bar{P}_d$ and $\bar{P}_d^1$ ($\bar{P}_d<\bar{P}_d^1$), it can be shown that $\mathrm{E}_{\mathrm{Total}}\left[m(x,\bar{P}_d^1)\right]<\mathrm{E}_{\mathrm{Total}}\left[m(x,\bar{P}_d)\right]$. Therefore we can conclude that the optimal solution of (2.12) occurs when constraint $\bar{P}_d\le P_d$ is at equality.

Modifying the constraint and replacing (2.4), (2.5) and (2.11) in (2.12), we can rewrite it as

$$\begin{aligned}&\max_{\tau}\ \frac{\left(1-P_f\right)P_{idle}\left(j\right)}{\left(1-P_f\right)P_{idle}\left(j\right)+\left(1-P_{idle}\left(j\right)\right)P_m}\int_0^{C_{\max}D_{\max}(T-\tau)} yg(y)(1-F(y))dy\,.\\ &\text{s.t}\quad P_d=\bar{P}_d,\ \ 0\le\tau\le T\end{aligned} \tag{2.13}$$

In (2.13), the probability of miss and the probability of false detection are respectively given by [55, Eqs. (13), (14)] $P_m=1-\bar{P}_d$ and $P_f(\xi,\tau)=Q\left(\left(Q^{-1}(\bar{P}_d)\sqrt{2\gamma+1}\right)+\gamma\sqrt{\tau f_s}\right)$, where ξ is the threshold for the detector, γ is the SNR for the received signal from PU, f_s is the sampling frequency and $Q(.)$ is Gaussian Q function. The above problem in (2.13) is a

nonlinear optimization problem and can be solved by numerical methods such as interior points method [8].

2.5 Numerical Results

In this section, we provide Monte-Carlo simulation results to demonstrate the effectiveness of the proposed auction model. In our simulations, we use the notation of "currency unit (CU)" instead of any particular currency. We assume that the number of available channels for bidding is 10 and the number of bidders is between 11 and 30. The channels will be sensed cooperatively by SUs and BS and each channel will be sensed by one SU. Also we assume that the sensing results are sent honestly to the BS by the SUs. PU traffic model is modeled by a two-state Markov model with $\gamma = 0.8$ and $\alpha = 0.2$. We assume that bandwidth of each channel is 6 MHz. The sampling frequency is the same as signal bandwidth. The fading channel coefficients between SUs and BS are Rayleigh distributed and noise has normal distribution with zero mean and $\sigma = 1$. The traffics of SUs are assumed to follow a Poisson distribution with a mean of 50 Kb/s. The length of transmission frame is 100 ms.

In Fig. 2.2, we present the performance (i.e., average payoff of bidders) for the proposed auction mechanism assuming different values of P_m. As a benchmark, we include the performance of the conventional auction method for the sealed-bid first-price auction. In the conventional method, the SUs select their bidding strategy without considering any uncertainties in the valuation of channels [25]. It is observed that the proposed bidding strategy outperforms the conventional one in a risky environment. The payoff of bidders by our proposed method is *at least* two times and, in the best case, five times more than that can be obtained from the conventional one. It is also observed that when the number of SUs

increases, average payoff of the SUs decreases. This decrease is due to the decreasing chance of winning for the bidders and also increases in the amount of bids.

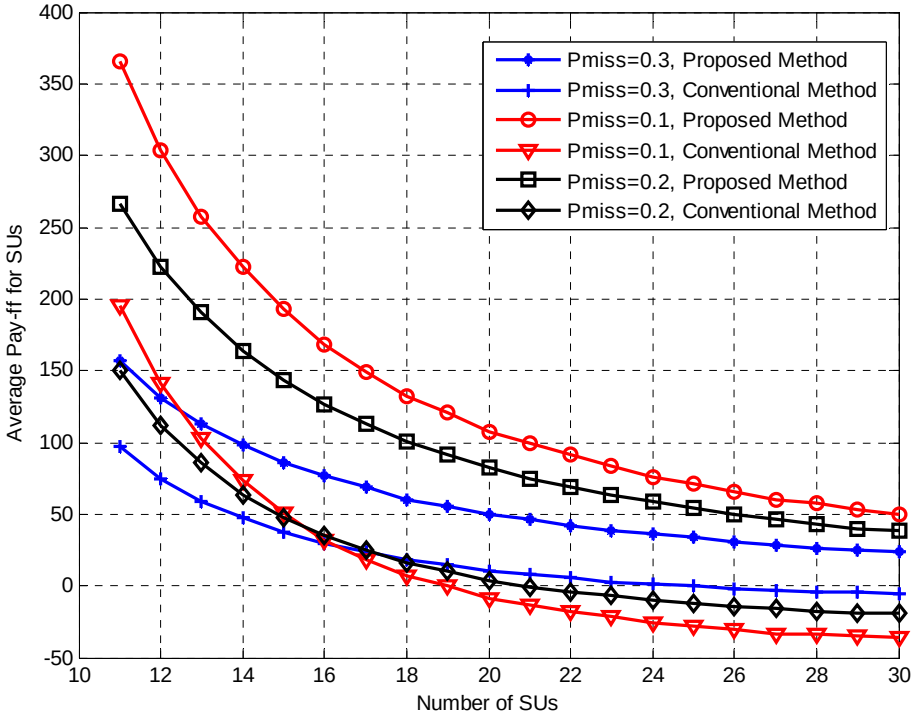


Figure 2.2 Average payoff for the SUs

Fig. 2.3 illustrates the revenue of the auctioneer for the proposed auction method in terms of sensing time. We assume $N = 15$ SUs and $P_m = 0.1$. It is observed that the revenue of auctioneer is a convex function of the sensing time and there is an optimal (in terms of revenue maximization) sensing duration. For the given numerical values, this is found to be 8.5 ms in our case. Fig. 2.4 illustrates the throughput of SUs in terms of sensing time. It is observed that the throughput is maximized for 33.5 ms. This clearly shows that the optimal

sensing time that maximize the revenue of auctioneer and throughput of SUs are different from each other.

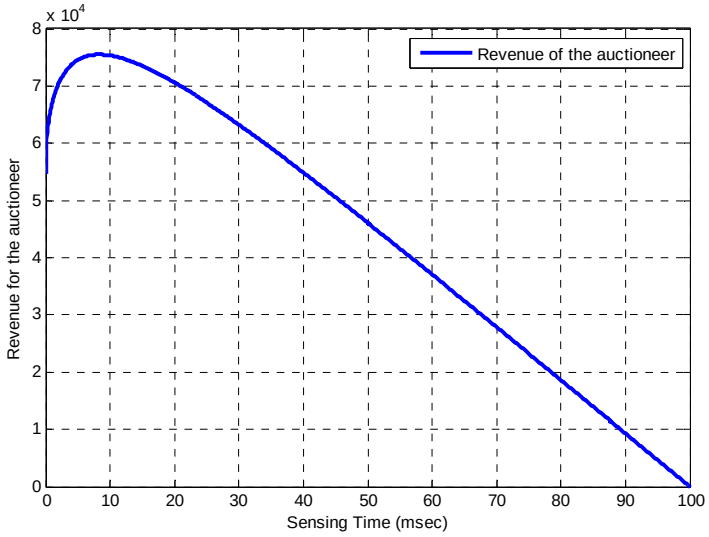


Figure 2.3 Revenue of the auctioneer in terms of sensing time for the proposed auction model.

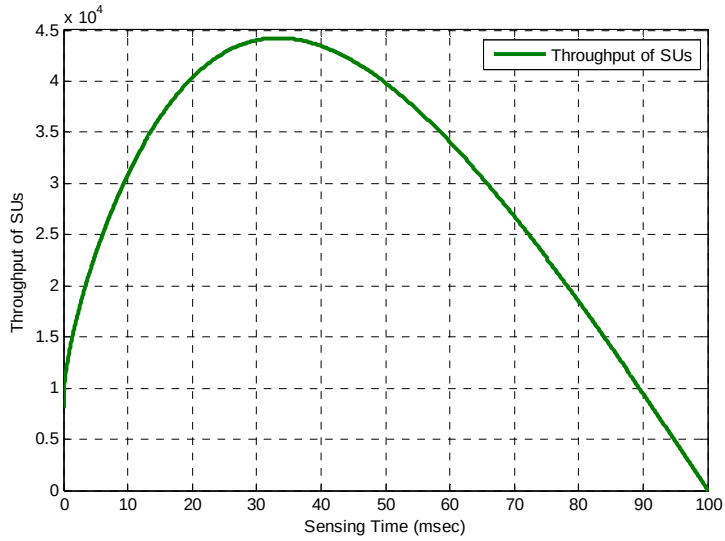


Figure 2.4 Throughput of SUs in terms of sensing time for the proposed auction model.

Fig. 2.5 illustrates the revenue of auctioneer in terms of the number of SUs for the proposed auction method assuming a) different values of fixed sensing time ($\tau = 10\,\text{ms}$ and $25\,\text{ms}$), b) optimized sensing time calculated from (2.13) to maximize the revenue, and c) optimized sensing time to maximize the throughput when the SUs are cooperating with each other. It is observed that the total revenue of auctioneer with optimized sensing time calculated from (2.13) outperforms the other two. The revenue of auctioneer in this case is respectively 5% and 25% more than that can be obtained from cases (a) and (c) while the throughput for the SUs is about 5% and 15% less. This indicates that maximizing throughput is not necessary to get the maximum revenue.

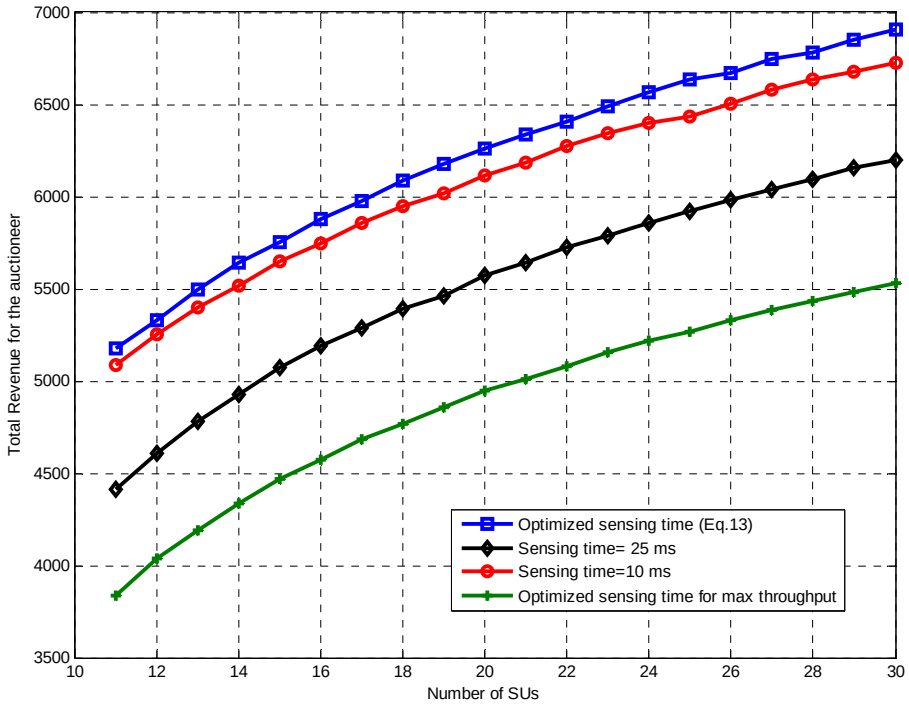


Figure 2.5 Total revenue of auctioneer in terms of the number of SUs for different sensing times.

Chapter 3

Sensing-Throughput Tradeoff in Cognitive Radio Networks with Cooperative Spectrum Sensing over Time-Varying Fading Channels

3.1 Introduction

In Chapter 2, we have addressed spectrum trading problem for shared use spectrum access model in IEEE 802.22. Specifically, we have considered the risk of imperfect spectrum sensing and proposed a sequential first price auction mechanism. In our proposed auction, the bidders calculate the optimum bidding to maximize their payoff given the risk of imperfect sensing. We have discussed about sensing-revenue tradeoff and calculated the optimum sensing time to maximize the revenue of auctioneer. As demonstrated, the optimal sensing time that maximizes the revenue of auctioneer and throughput of SUs are different from each other.

In Chapter 3, we now assume a fixed time for sensing and data transmission in shared use spectrum access model. In this chapter, we want to address sensing-throughput tradeoff in cognitive radio networks with cooperative spectrum sensing. The revenue of the service provider is out of scope of this chapter and our focus is on maximizing the throughput of SUs subject to limited interference with PUs. In this chapter, we consider a correlated fading channel model with the well-known Jakes model for power spectral density [55]. Under this channel model, we formulate the optimum tradeoff problem between sensing time and throughput. We further take into account the fact that each SU cannot sense all the channels and assume that each SU can be assigned for the sensing of a particular channel at a time and this assignment is done on a dynamic basis. Therefore, the number of SUs assigned for sensing of a particular channel will be determined dynamically based on the detection performance of PUs and the channel capacity of SUs in each channel.

3.2 System Model

We consider a cognitive radio network with L SUs and one DFC. There are K channels ($L > K$) to be sensed cooperatively by the SUs and the hard decision results will be sent to the DFC for the final decision. As the spectrum sensing is a time consuming task, we assume that each SU can sense only one channel during the sensing period. The assignment of SUs to each channel is made on a dynamical basis by the DFC. During sensing time in each time frame, the SUs are responsible to sense the assigned channels. The assigned channels to SUs can be changed from one time frame to the other one by the DFC.

There are two hypotheses for detection; namely H_0 and H_1 as the absence or the presence of the PUs, respectively. When the PU signal is present, the received sampled signal at the ith SU can be written as

$$y_i(n) = h_i(n)s(n) + u_i(n),\ n = 1, 2, ..., N \tag{3.1}$$

where $s(n)$ is a rectangular M-QAM modulated signal and N is the number of received signal samples. Signal samples are assumed to be independent identically distributed (i.i.d.) random variables with zero mean and variance of $\mathrm{E}\left[|s(n)|^2\right] = \sigma_s^2$ [44]. The noise samples, $u_i(n)$ are assumed to be circularly symmetric complex Gaussian (CSCG) i.i.d. random variables with zero mean and variance of $\mathrm{E}\left[|u_i(n)|^2\right] = \sigma_u^2$. In (3.1), $h_i(n)$ represents the fading coefficient and is modeled by a complex Gaussian random variable with zero mean and variance of $\mathrm{E}\left[|h_i(n)|^2\right] = \sigma_h^2$. Let $h_i^{\mathrm{R}}(n)$ and $h_i^{\mathrm{I}}(n)$ denote real and imaginary parts of the fading coefficient. $h_i(n)$ can be therefore written as

$$h_i(n) = \mathrm{Re}\left[h_i(n)\right] + j\,\mathrm{Im}\left[h_i(n)\right] = h_i^{\mathrm{R}}(n) + jh_i^{\mathrm{I}}(n)\,. \tag{3.2}$$

The autocorrelation function for the real/imaginary part of channel coefficients is defined as

$$\rho_i(l) \overset{\mathrm{def}}{=} \mathrm{E}\left[h_i^R(n)h_i^R(n+l)\right] = \mathrm{E}\left[h_i^I(n)h_i^I(n+l)\right]\ . \tag{3.3}$$

It should be further noted that $h_i^{\mathrm{R}}(n)$ and $h_i^{\mathrm{I}}(n)$ are independent of each other. The corresponding Doppler (PSD) is obtained taking the Fourier transform of the correlation function. For the Jakes model [55] under consideration, we have

$$\mathrm{F}\left(\rho_i(l)\right) = \mathrm{S}_i\left(f\right) = \frac{1}{f_d}\frac{1}{\sqrt{1-\left(f/f_d\right)^2}}, \quad |f| \le f_d \tag{3.4}$$

where f_d is the Doppler frequency.

3.3 Sensing Statistics

In cognitive radio networks, the frame structure consists of two main parts. The first part is for sensing and the second one is for data transmission. Let T denote the length of the frame (i.e., the total time required for sensing and transmission purposes) and assume that $\tau < T$ is allocated for sensing. We assume that each SU employs an energy detector to measure the received signal power during the sensing period. Let f_s denote the sampling frequency. The number of received samples is therefore given by $N = f_s\tau$. The decision test statistics for energy detector is expressed as

$$\Lambda(y_i) = \frac{1}{N}\sum_{n=1}^{N}|y_i(n)|^2 . \tag{3.5}$$

If N is large enough, the Probability Density Function (PDF) of $\Lambda(y_i)$ under hypothesis H_0 can be approximated with Gaussian distribution with mean $\mu_0 = \sigma_u^2$ and variance $\sigma_0^2 = \left(1/N\right)\left[\mathrm{E}\,|u_i(n)|^4 - \sigma_u^4\right]$ based on the CLT [44]. Since $u_i(n)$ is CSCG, it can be shown that $\mathrm{E}\left[|u_i(n)|^4\right] = 2\sigma_u^4$ and $\sigma_0^2 = \left(1/N\right)\sigma_u^4$. The probability of false alarm is then given by

$$P_f(\xi,\tau) = \Pr(\Lambda(y_i) > \xi \,|\, \mathrm{H}_0) = Q\left(\left(\frac{\xi}{\sigma_u^2} - 1\right)\sqrt{\tau f_s}\right) \tag{3.6}$$

where ξ denotes the threshold of the energy detector employed at the ith SU and $Q(.)$ is the complementary distribution of the standard Gaussian [63].

Under hypothesis H_1, since the channel coefficients $h_i(n)$ are correlated, $y_i(n)$ are correlated to each other, $n = 1, 2, ..., N$ and the CLT cannot be used in a straightforward manner. For the correlated case, we use a modified version of the CLT [64].

Theorem 3.1 (Central Limit theorem for Correlated Sequences) [64]**:** Let $\{x_n\}$ be a stationary and mixing[1] sequence of random variables satisfying a CLT condition such that

1) $\mathrm{E}\left[x_n\right] = \mu < \infty \quad , \quad \forall n \in \mathbb{N}$ (3.7)

2) $\mathrm{Var}\left[x_n\right] = \sigma^2 < \infty \quad , \quad \forall n \in \mathbb{N}$ (3.8)

3) $\lim_{n\to\infty} n\mathrm{Var}\left[\bar{x}_n\right] = \sigma^2 + 2\sum_{i=2}^{n}\mathrm{Cov}\left[x_1, x_i\right] = V < \infty \quad , \quad \forall n \in \mathbb{N}.$ (3.9)

Then, a central limit theorem applies to the sample mean $\bar{x}_n$

$$\sqrt{n}\left(\frac{\bar{x}_n - \mu}{\sqrt{V}}\right) \xrightarrow{d} Z \tag{3.10}$$

where Z is standard normal random variable and $\xrightarrow{d}$ indicates convergence in distribution.

This theorem indicates that the CLT holds for correlated random variables but with different variance than the independent case. In Appendix A, we prove that the received samples $y_i(n)$ satisfy the three conditions stated in (3.7)-(3.9) and therefore, following (3.10), $\Lambda(y_i)$ can be approximated as a normal random variable with mean $(\gamma + 1)\sigma_u^2$ and variance of

$$\sigma_1^2 = \frac{\sigma_u^4}{N}\left[\left(\frac{3\left(3\sqrt{M} - 7\right)}{5\left(\sqrt{M} - 1\right)} - 1\right)\gamma^2 + 2\gamma + 1\right] + \frac{2}{N}\sum_{i=1}^{N}\sigma_s^4\sigma_h^4\rho_i^2 \tag{3.11}$$

[1] The sequence is said to be "mixing" if the states are asymptotically independent, i.e., as the times between the measurements increase to infinity, the observed values of the measurements at those times become independent [64].

for large values of N. In the above, γ is the average received SNR at each SU and is given as $\sigma_h^2\sigma_s^2/\sigma_u^2$.

The probability of detection under hypothesis H_1 can be now calculated as

$$P_d(\xi,\tau) = \Pr(\Lambda(y_i) > \xi \mid \mathrm{H}_1)$$

$$= Q\left(\left(\frac{\xi}{\sigma_u^2} - \gamma - 1\right)\sqrt{\frac{\tau f_s}{\left(\frac{3\left(3\sqrt{M}-7\right)}{5\left(\sqrt{M}-1\right)} - 1 + 2\sum_{i=1}^{N}\rho_i^2\right)\gamma^2 + 2\gamma + 1}}\right). \tag{3.12}$$

The probability of false alarm can be written as

$$P_f(\xi,\tau) = Q\left(\left(Q^{-1}(\bar{P}_d)\sqrt{\left(\frac{3\left(3\sqrt{M}-7\right)}{5\left(\sqrt{M}-1\right)} - 1 + 2\sum_{i=1}^{N}\rho_i^2\right)\gamma^2 + 2\gamma + 1}\right) + \gamma\sqrt{\tau f_s}\right) \tag{3.13}$$

where $\bar{P}_d$ denotes the target probability of detection.

In cooperative sensing, after each SU makes its individual decision denoted by D_i, it sends its result to the DFC. Here, $D_i = 1$ means that there is a PU detected in the channel and $D_i = 0$ means that no PU is detected. The DFC can employ OR-rule, AND-rule or majority logic rule [21]. Let $P_d(\xi,\tau)$ and $P_f(\xi,\tau)$ respectively denote the probability of detection and false alarm for each SU. The detection and false alarm probability of cooperative spectrum sensing (i.e., $\mathbb{Q}_d(\xi,k,\tau)$ and $\mathbb{Q}_f(\xi,k,\tau)$, respectively) for different decision rules are calculated as

$$\mathbb{Q}_d(\xi,k,\tau) = \begin{cases} 1-\left(1-P_d(\xi,\tau)\right)^k, & \text{for OR rule} \\ \left(P_d(\xi,\tau)\right)^k, & \text{for AND rule} \\ \sum_{i=\lceil k/2\rceil}^{k}\binom{k}{i}\left(1-P_d(\xi,\tau)\right)^{k-i}\left(P_d(\xi,\tau)\right)^i, & \text{for majority logic rule} \end{cases}. \tag{3.14}$$

$$\mathbb{Q}_f(\xi,k,\tau)=\begin{cases}1-\left(1-P_f(\xi,\tau)\right)^k, & \text{for OR rule}\\ \left(P_f(\xi,\tau)\right)^k, & \text{for AND rule}\\ \sum_{i=\lceil k/2\rceil}^{k}\binom{k}{i}\left(1-P_f(\xi,\tau)\right)^{k-i}\left(P_f(\xi,\tau)\right)^i, & \text{for majority logic rule}\end{cases}. \tag{3.15}$$

3.4 Problem Formulation for Sensing-Throughput Tradeoff

The total average throughput is given by

$$R_T=\sum_{i=1}^{K}R^i\left(\tau_i,k_i,\xi_i\right) \tag{3.16}$$

where $R^i\left(\tau_i,k_i,\xi_i\right)$ is the average throughput for the ith channel , $i=1,2,...,K$. For the calculation of $R^i(.)$, we need to consider two cases based on whether DFC makes a correct decision or not. In the first case, the DFC truly detects the absence of PUs. The second case is that the DFC misses the presence of PUs. Let $R_0^i\left(\tau_i,k_i,\xi_i\right)$ and $R_1^i\left(\tau_i,k_i,\xi_i\right)$ denote the corresponding throughput for the ith channel. They are given by

$$R_0^i\left(\tau_i,k_i,\xi_i\right)=C_0^iP_i(\mathrm{H}_0)T\left(1-\frac{\tau_i}{T}\right)\left(1-\mathbb{Q}_f\left(\tau_i,k_i,\xi_i\right)\right) \tag{3.17}$$

$$R_1^i\left(\tau_i,k_i,\xi_i\right)=C_1^iP_i(\mathrm{H}_1)T\left(1-\frac{\tau_i}{T}\right)\left(1-\mathbb{Q}_d\left(\tau_i,k_i,\xi_i\right)\right) \tag{3.18}$$

where C_0^i and C_1^i denote the channel capacity of the ith channel under hypothesis H_0 and H_1, respectively. $P_i(\mathrm{H}_0)$ and $P_i(\mathrm{H}_1)$ are the probabilities of the PU being absent or present in the ith channel, respectively. Therefore the average throughput of the SUs from the ith channel is

$$R^i\left(\tau_i,k_i,\xi_i\right)=R_0^i\left(\tau_i,k_i,\xi_i\right)+R_1^i\left(\tau_i,k_i,\xi_i\right). \tag{3.19}$$

SUs are mainly interested in the channels that are underutilized, such as channels with $P_i(\mathrm{H}_0)\geq 0.5$. On the other hand, the probability of detection for cooperative spectrum

sensing ($\mathbb{Q}_d$) is usually more than 0.7 and $\left(1-\mathbb{Q}_d\right)<0.3$ [44]. But the probability of false alarm for cooperative spectrum sensing ($\mathbb{Q}_f$) is usually small (<0.3) and therefore $1-\mathbb{Q}_f>0.7$. Thus it is possible to ignore $R_1^i\left(\tau_i,k_i,\xi_i\right)$ in (3.19) and approximate (3.19) as $R^i\left(\tau_i,k_i,\xi_i\right)\cong R_0^i\left(\tau_i,k_i,\xi_i\right)$. This will yield

$$R_T\cong\sum_{i=1}^{K}R_0^i\left(\tau_i,k_i,\xi_i\right). \tag{3.20}$$

In our work, we want to maximize the average total throughput of SUs with respect to the optimization variables, namely sensing time (τ_i), the number of SUs (k_i) and detection threshold (ξ_i). Therefore we can write this optimization problem as

$$\max_{\tau_i,k_i,\xi_i}\sum_{i=1}^{K}R^i\left(\tau_i,k_i,\xi_i\right) \tag{3.21}$$

s.t. i) $\mathbb{Q}_d\left(\tau_i,k_i,\xi_i\right)\geq\overline{P}_d^i$,ii) $0\leq\tau_i\leq T$, iii) $1\leq k_i\leq K$, iv) $\sum_{i=1}^{K}k_i=L$

where $\overline{P}_d^i$ is the minimum probability of detection that the DFC needs to achieve to protect the PUs in the ith channel. $\overline{P}_d^i$ can take different values in each channel based on the activity, power and sensitivity of PUs.

Lemma: Optimal solution of (3.21) occurs when the first constraint is at equality for all decision rules.

Proof: For a given k_i and τ_i, we may choose a detection threshold ξ_0^i which satisfies $\mathbb{Q}_d(\xi_0^i)=\overline{P}_d^i$. We may also choose a detection threshold $\xi_1^i<\xi_0^i$ such that $\mathbb{Q}_d(\xi_1^i)<\mathbb{Q}_d(\xi_0^i)$. Obviously from (3.13), we can conclude that $\mathbb{Q}_f\left(\xi_0^i\right)<\mathbb{Q}_f\left(\xi_1^i\right)$. However, we observe from (3.17) that $R_0^i\left(\xi_1^i\right)\leq R_0^i\left(\xi_0^i\right)$ and, therefore, $R^i\left(\xi_1^i\right)\leq R^i\left(\xi_0^i\right)$. Hence, it proves that $R^i(.)$ will be maximized only if $\mathbb{Q}_d(\tau_i,k_i,\xi_i)=\overline{P}_d^i$. When the first constraint in (3.21) is an equality in all decision rules for any given pair of τ_i, k_i, we are able to determine a threshold from (3.22) to satisfy $\mathbb{Q}_d\left(\tau_i,k_i,\xi_i\right)=\overline{P}_d^i$ which is given by

$$\xi_i = \sigma_u^2 \left(Q^{-1}\left(1-\left(1-\overline{P}_d^i\right)^{\frac{1}{k_i}}\right)\sqrt{\frac{\left(\frac{3\left(3\sqrt{M}-7\right)}{5\left(\sqrt{M}-1\right)}-1+2\sum_{i=1}^{N}\rho_i^2\right)\gamma^2+2\gamma+1}{\tau f_s}}+\left(\gamma+1\right)\right). \tag{3.22}$$

Using the Lemma, the problem formulation in (3.21) can be rewritten as

$$\begin{aligned} &\max_{\tau_i,k_i} \sum_{i=1}^{K} \hat{R}^i\left(\tau_i,k_i\right) \\ &\text{s.t.} \quad \text{i) } 0 \le \tau_i \le T,\ \text{ii) } 1 \le k_i \le K,\ \text{iii) } \sum_{i=1}^{K} k_i = L \end{aligned} \tag{3.23}$$

where $\hat{R}^i(\tau_i,k_i)$ denotes the value of $R^i\left(\tau_i,k_i,\xi_i\right)$ with the threshold ξ_i chosen by (3.22). The resulting problem is a two dimensional optimization problem whose solution will be pursued in the following section.

3.5 Proposed Solution

In the first step, we will optimize the sensing time for a given $k_i = \tilde{k}_i$. For each channel, we find the optimal value of sensing time (τ_i^*) that maximizes the throughput of SUs in the ith channel. The optimization problem is given as

$$\begin{aligned} &\max_{\tau_i} \tilde{R}^i(\tau_i) \triangleq \hat{R}^i\left(\tau_i,k_i\right)\Big|_{k_i=\tilde{k}_i} = C_0^i P^i(\mathrm{H}_0) T\left(1-\frac{\tau_i}{T}\right)\left(1-\mathbb{Q}_f(\tau_i)\right) \\ &\qquad \text{s.t} \quad 0 \le \tau_i \le T \end{aligned} \tag{3.24}$$

where $\mathbb{Q}_f(\tau_i)$ is the $\mathbb{Q}_f\left(\tau_i,\tilde{k}_i\right)$. We will prove that $\tilde{R}^i(\tau_i)$ is a unimodal function in the range of $0 \le \tau_i \le T$ for a given $k_i = \tilde{k}_i$. If $\tilde{R}^i(\tau_i)$ is a unimodal function, then $\tilde{R}^i(\tau_i)$ monotonically increases within the range of $0 \le \tau_i < \tau_i^*$ while it decreases within the range of $\tau_i^* < \tau_i \le T$. Hence $\tilde{R}^i\left(\tau_i^*\right)$ is the only local maximum in the range of $0 \le \tau_i \le T$.

Theorem 3.2 (Unimodal Function): If $\tilde{R}^i(\tau_i)$ satisfies the following three conditions in the range of $0 \le \tau_i \le T$ for a given $k_i = \tilde{k}_i$ then $\tilde{R}^i(\tau_i)$ is a unimodal function.

1) $\partial \tilde{R}^i(0) / \partial \tau_i > 0, \ k_i = \tilde{k}_i$. (3.25)

2) $\partial \tilde{R}^i(T) / \partial \tau_i < 0, \ k_i = \tilde{k}_i$. (3.26)

3) $\exists$ unique τ_i^*, which $\partial \tilde{R}^i(\tau_i^*) / \partial \tau_i = 0, \ k_i = \tilde{k}_i$. (3.27)

In Appendix B, we prove that $\tilde{R}^i(\tau_i)$ satisfies (3.25)-(3.27) for majority logic rule, also this proof can be extended for OR-rule and AND-rule.

Each channel should be sensed by at least one SU, therefore there are $L-K$ optimization problems to find τ_i^* given different values of SUs for each channel. These optimization problems can be solved by bisection search method, or Newton's method [65]. In total, we should solve $(L-K)K$ optimization problem to find all τ_i^*. The second step of our solution involves finding the optimal allocation of SUs to sense the channels with optimum τ_i^* (that we have calculated in the previous step). Therefore the optimization problem in the second step is given by

$$\begin{aligned} &\max_{k_i} \sum_{i=1}^{K} C_0^i P^i(\mathrm{H}_0) T \left(1 - \frac{\tau_i^*(k_i)}{T}\right) \left(1 - \mathbb{Q}_f(\tau_i^*(k_i), k_i)\right) \\ &\text{s.t} \ \sum_{i=1}^{K} k_i = L \end{aligned} . \tag{3.28}$$

Let $\tau_i^*(k_i)$ denote the optimum sensing time when k_i SUs sense the ith channel. There are $(L-1)!/((K-1)!(L-K)!)$ cases to search and find the optimum allocation of SUs. When L and K are small numbers, it is possible to carry out exhaustive search. But it is infeasible for large values as the total number of cases increase exponentially. For example, for $K = 10$ and $L = 40$, there are more than 2×10^6 cases.

As an alternative to exhaustive search, we model (3.28) as a *multiple choice knapsack problem* (MCKP) [66] (commonly studied in combinatorial optimization) and propose a

solution with low computational complexity for this step. Given a set of items each with a weight and a value, the original knapsack problem determines the count of each item to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible [67]. The difference of MCKP from the original knapsack problem is that in MCKP, there are multiple classes of items and exactly one item must be taken from each class. We will use dynamic programming for the solution of (3.28). The main idea is first to break down a complex problem into simpler sub-problems, then find solutions for the sub-problems and finally combine these solutions to reach the overall solution.

In our case, we want to assign SUs to the channels as to maximize the total average throughput. In an analogy to the MCKP, K different sets represent K different channels and the total weight of knapsack represents the total number of SUs (i.e., L). The weight of the ith item in the jth set is the number of SUs assigned to sense the jth channel and the value of the item is the throughput calculated from (3.24). In the first step of our proposed solution, we calculate the throughput for any possible allocation of SUs to each channel; therefore we know all the weights and all the values for the set of items. In the second step, we will solve the MCKP using dynamic programming.

Eq. (3.28) can be rewritten in the form of MCKP as

$$\max_{k_i} \sum_{i=1}^{K} \sum_{j=1}^{L-K} \tilde{R}_{i,j} a_{i,j}$$

$$\text{s.t} \begin{cases} \text{i)} \sum_{i=1}^{K} \sum_{j=1}^{L-K} j a_{i,j} = L \\ \text{ii)} \sum_{j=1}^{L-K} a_{i,j} = 1, \ i = 1, 2, ..., K \\ \text{iii)} \ a_{i,j} \in \{0,1\}, i = 1, 2, ..., K, \ j = 1, 2, ..., L-K \end{cases} \tag{3.29}$$

where $\tilde{R}_{i,j}$ is the throughput of the SUs in the ith channel when j SUs are assigned for sensing of the channel. $a_{i,j}$ will be 1 if j SUs assigned for the sensing of the ith channel, otherwise set to 0. The second constraint $\sum_{i=1}^{L-K} a_{i,j} = 1$ means that at least one SU should be assigned for the sensing of each channel. The first constraint $\sum_{i=1}^{K}\sum_{j=1}^{L-K} ja_{i,j} = L$ indicates that the total number of SUs for the sensing of the channels is L.

Based on (3.29), we propose a dynamic programming algorithm for solving our MCKP. The steps of the algorithm are provided in Table 3.1. In the first step, we initialize three matrices, namely V (valuation), a (assignment) and $Index$ with the size of $K \times (L-K)$. In first step, we solve the knapsack problem for $i = 1$ (i.e., the number of sets) and different number of SUs (i.e., the weight in knapsack problem) for $1 \le j \le L-K$. In the second step, we use $\max\left(V\left(i-1,1:j\right) + \tilde{R}\left(i,j:-1:1\right)\right) > V\left(i,j-1\right)$ to find $V(i,j)$ and $Index(i,j)$ for each i and j. Therefore we find $V(K, L-K)$ (which is the optimum value of (3.29)) and $Index(K, L-K)$ in this step. In the third step, we use backward induction to find $a(i,j)$ for each channel. Specifically, we start from $Index(K, L-K)$; if $Index(K, L-K) > 0$ then $a(K, index(K, L-K)) = 1$ which means that the Kth channel should be sensed by a number of SUs i.e., $index(K, L-K)$. After that we use $j = j - Index(i,j)$; $i = i-1$; to update the parameters i, j and find the number of sensing SUs for other channels.

Table 3.1 Proposed algorithm for MCKP

Step1: (Initialization)

$V(K, L-K) \leftarrow 0$, $Index(K, L-K) \leftarrow 0$, $a(K, L-K) \leftarrow 0$ $i \leftarrow 1$, $j \leftarrow 1$

For $j = 1 : L-K$;

$V(1, j) = \max\left(\tilde{R}(i, 1:j)\right)$; $Index(1, j) = \arg\max\left(\tilde{R}(i, 1:j)\right)$;

end

Step2: (Solve MCKP)

For $i = 2 : K$

For $j = 1 : L-K$

If $\max\left(V(i-1, 1:j) + \tilde{R}(i, j:-1:1)\right) > V(i, j-1)$;

$V(i, j) = \max\left(V(i-1, 1:j-1) + \tilde{R}(i, j-1:-1:1)\right)$;

$Index(i, j) = \arg\max\left(V(i-1, 1:j-1) + \tilde{R}(i, j-1:-1:1)\right)$;

else

$V(i, j) = \tilde{R}(i, j-1)$;

$Index(i, j) = 0$;

end

end

end

Step3: (Backward Induction)

$i = K$; $j = L-K$;

when($i \neq 0$)

If $Index(i, j) > 0$

$a(i, Index(i, j)) = 1$; $j = j - Index(i, j)$; $i = i - 1$;

else

$j = j - 1$;

end

end

3.6 Numerical Results

In our simulation study, we assume a cognitive radio network with different number of SUs and channels for sensing. The bandwidth of each channel is 6 MHz. The sampling frequency is the same as signal bandwidth. The received SNR from PUs signals varies in each channel and its mean takes values within the range of -25 dB and 0 dB. The inactivity

probability of PUs is $P(H_0) = 0.5$. The target probability of detection for each channel is $\overline{P}_d^i = 0.99$. The frame length is assumed to be $100\,\text{ms}$. Our simulation study consists of two parts. In the first part, we focus only on *one channel* and illustrate the results for sensing-throughput tradeoff over time-selective channels. In the second part of our study, we demonstrate the results for jointly optimizing the number of assigned SUs and the sensing time for *all channels*.

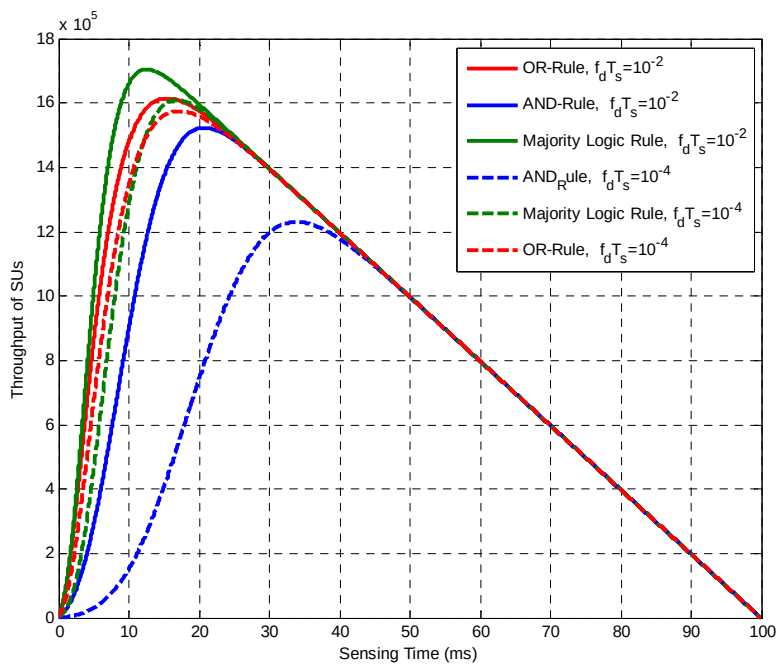


Figure 3.1 Throughput of SUs in terms of sensing time

Fig. 3.1 illustrates the throughput of SUs for a single channel in terms of sensing time assuming OR-rule, AND-rule and majority logic rule. We consider $L = 10$ SUs and normalized Doppler values of $f_d T_s = 10^{-2}$ and $f_d T_s = 10^{-4}$. It is observed that the throughput of SUs is a unimodal function of the sensing time and there is an optimal (in terms of

throughput maximization) sensing duration which can be calculated from (3.17). It is further observed that the optimal sensing time increases when the normalized Doppler value decreases. The SUs need more time for sensing because the channel coefficients are more correlated with decreasing Doppler value.

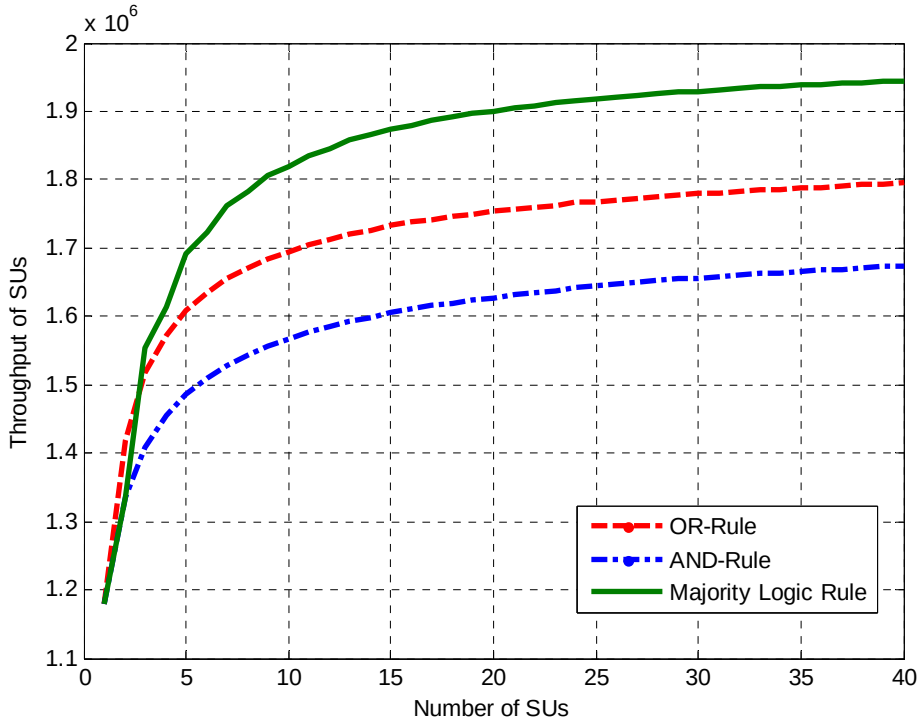


Figure 3.2 Throughput of SUs in terms of the number of SUs

Fig. 3.2 demonstrates the relationship between the throughput and the number of SUs under the assumption that optimal sensing duration is used. In this simulation, we have $1 \le L \le 40$, $f_d T_s = 10^{-2}$ and use (3.17) to optimize the sensing time. We observe that the throughput increases with the increasing number of SUs as expected since the probability of false alarm decreases.

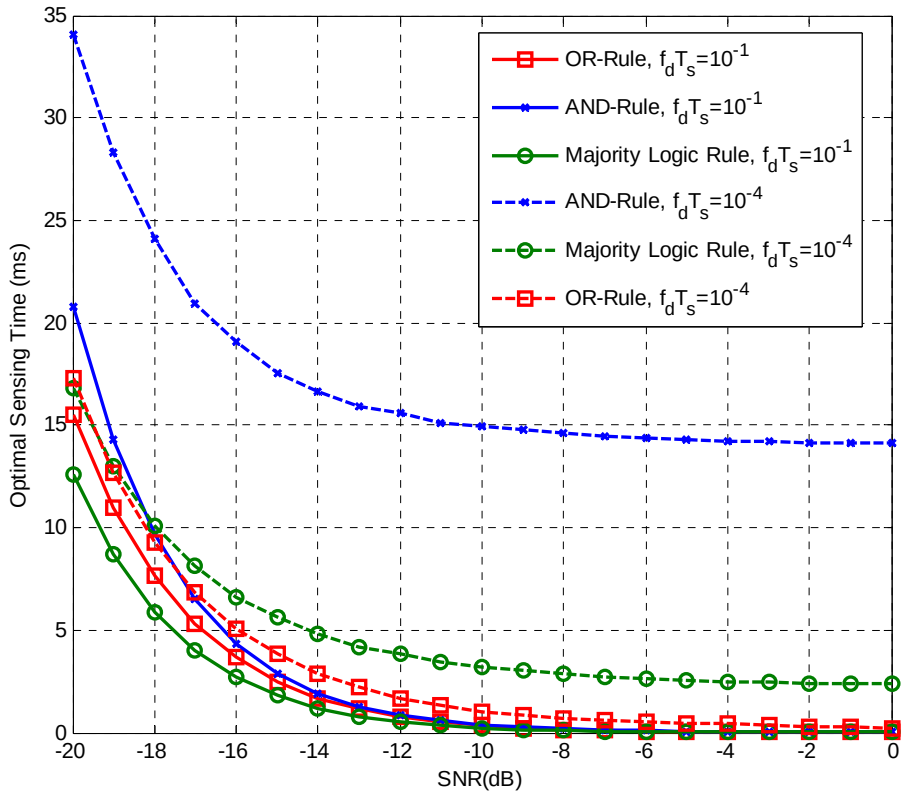


Figure 3.3 Optimum sensing time of SUs over single channel in terms of SNR

In Fig. 3.3, we demonstrate the optimum sensing time versus the SNR of SUs. We consider $L = 10$ SUs and normalized Doppler values of $f_d T_s = 10^{-2}$ and $f_d T_s = 10^{-4}$. The optimum sensing time decreases with the increasing SNR of SUs as a result of the improvement in detection performance of SUs due to this SNR increase. With increase in Doppler value, the channel coefficients are less dependent on each other and the detection performance is improved, therefore there is less time required to maximize the throughput in the same SNR. It can be also observed that OR-rule is more resistant to the channel selectivity in comparison to the other two methods.

In Fig. 3.4, we demonstrate the throughput of SUs versus the SNR of SUs over a single channel. We consider $L = 10$ SUs and normalized Doppler values of $f_d T_s = 10^{-2}$ and $f_d T_s = 10^{-4}$. The throughput of SUs increases with the increasing SNR of SUs similar to what is observed for the sensing time in Fig.3.3. With increase in Doppler value, the channel coefficients are less correlated and there is more temporal diversity available for the detection of PUs. This therefore results in an increase in the throughput of SUs.

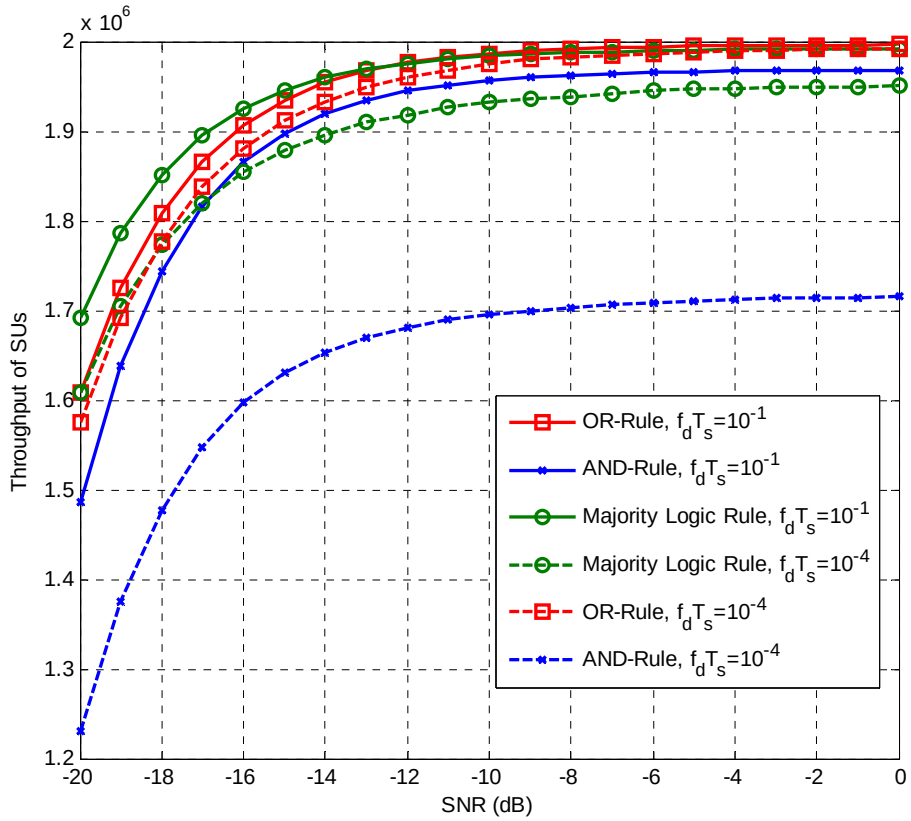


Figure 3.4 Optimum Throughput for SUs over single channel in terms of SNR

In the second part of our simulation, we assume a cognitive radio with 40 SUs and 10 channels. We jointly optimize the number of assigned SUs and the sensing time based on (3.23) to maximize the total throughput of the network. In Fig. 3.5, we investigate the effect of time selectivity on the throughput of SUs assuming majority logic rule. We demonstrate the results for time varying Rayleigh fading channels with Doppler values of 10^{-1}, 10^{-2}, 10^{-3} and further include the results for symbol-by-symbol independent Rayleigh fading channel as a benchmark [48]. For $f_d T_s = 10^{-3}$, the channels coefficients are highly dependent on each other. Among the Doppler values considered, the worst detection performance and throughput of SUs is observed in this case. When the Doppler value increases to $f_d T_s = 10^{-2}$, the dependency of the channel coefficients decreases. This indicates a decrease in the variance of decision statistic $\Lambda(y_i)$, therefore the detection performance is improved in comparison to $f_d T_s = 10^{-3}$. For $f_d T_s = 10^{-1}$, the channel coefficients are less correlated to each other and it is observed that the throughput of SUs in this case converges to the throughput of SUs over symbol-by-symbol independent Rayleigh channel.

In Fig 3.5, we have jointly optimized sensing time (τ_i) and the number of SUs (k_i) based on (3.23). Now, in Fig. 3.6 we demonstrate that the joint optimization of these two variables will indeed outperform when we optimize only one of the variables in each part and fix the other one. We assume $f_d T_s = 10^{-2}$ and consider the following two semi-optimal benchmarking methods:

- **Method A (Fixed SU number and optimized sensing time):** We assume that the number of SUs for each channel is fixed and identical and only the sensing time is optimized.
- **Method B (Fixed sensing time and optimized SU number):** We assume a fixed sensing time and optimize the number of SUs.

We observe that the joint optimization yields a throughput gain around 10% for some SNRs in comparison with method A. In method A, each channel will be sensed by 4 SUs. For Method B, we assume that sensing time occupies 25% and 10% of total time frame. It can be seen that our proposed method outperforms Method B. We observe that the joint optimization yields a throughput gain of around 15% and 25% related to method B which the sensing time is 10% and 25% of total time frame, respectively.

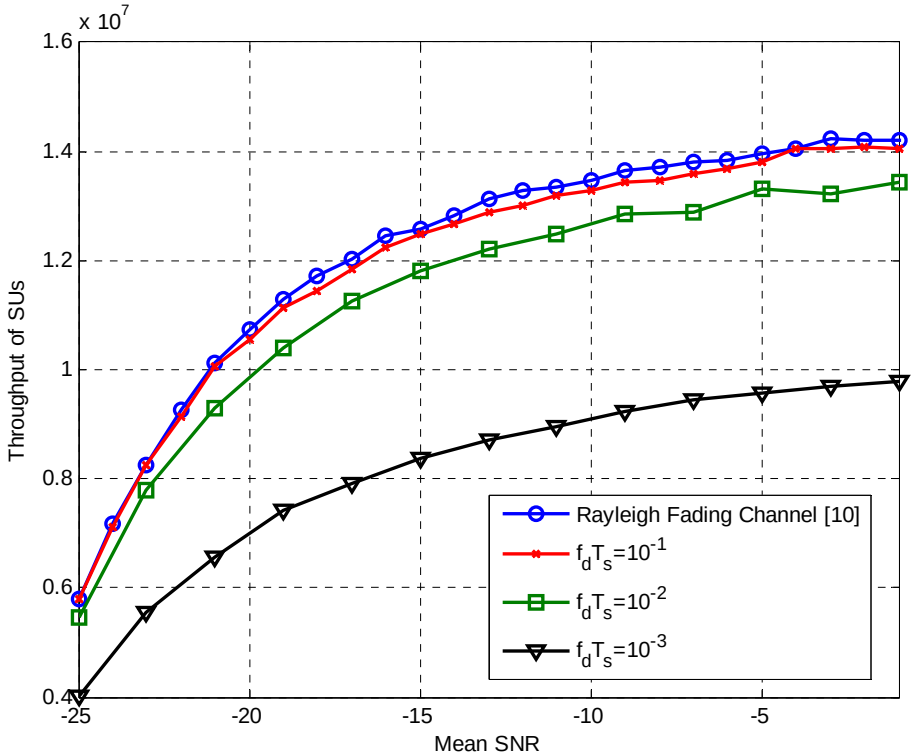


Figure 3.5 Throughput of SUs over time varying channels with different Doppler values.

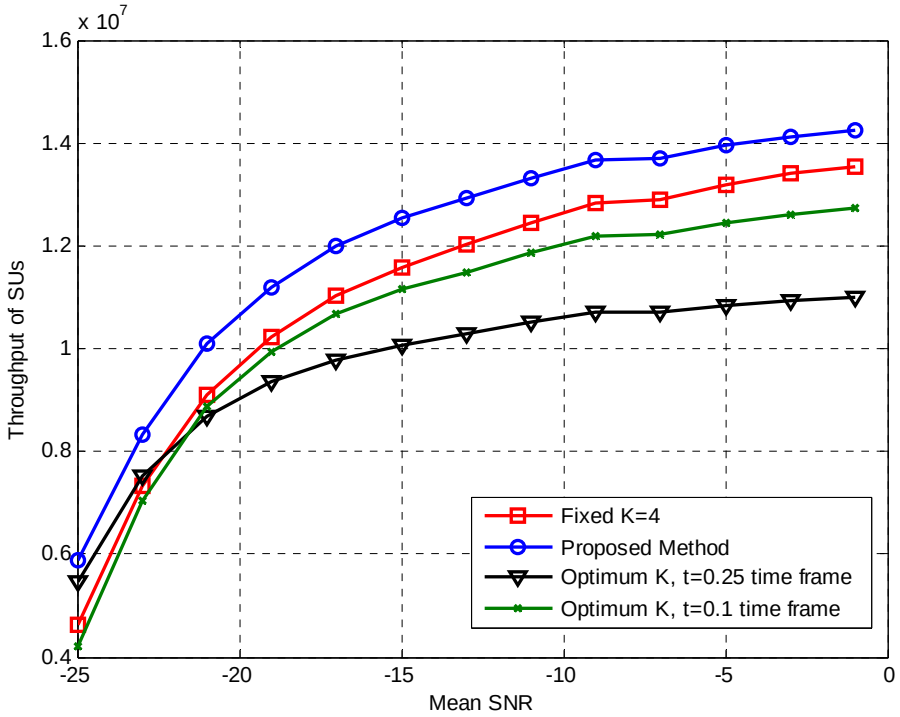


Figure 3.6 Throughput of SUs for the proposed method

Chapter 4
Spectrum Trading for Concurrent Non-Identical Channel Allocation in Cognitive Radio Networks

4.1 Introduction

In Chapter 2, we have proposed the spectrum trading using a *multi-unit sequential sealed-bid first-price auction* for the shared used access model in cognitive radio. We have assumed that the channels are non-identical and the value of each channel can be determined by the channel capacity, data demand of SUs, and data transmission time. In sequential auction, the channels are auctioned one after each other and each SU can submit one bid at the time, while in concurrent auction the bidders submit their bids simultaneously for the channels. Therefore concurrent auction needs less time and information overhead for holding auction than the sequential auction. In this chapter, we aim to design a spectrum trading for non-identical channel allocation using concurrent auction mechanism with exclusive usage spectrum access model.

An important question faced by a licensee is how to allocate the spectrum rights to the SUs in an optimal manner, i.e., to ensure maximum revenue for the PUs and maximum satisfaction for SUs. Different SUs have different estimation about the value of available channels. Since the license holder does not know the values that bidders attach to the channels, auctions [7], [8], [25], [29], [56], [57], [68] provide an efficient mechanism for the licensee to get higher revenue than that is obtainable via static pricing. Auctions are also

beneficial for the bidders, since, in general, they assign commodities to the bidders who value them most.

In the current literature on spectrum auctions [7], [29], [56], [57], the valuation functions used for the bidders are somehow unrealistic. In [56] and [25], Kasbekar *et al*. and Sengupta *et al*. have considered a uniform distribution for the valuation function of the bidders. However, in practical systems, the channel conditions and quality of service (QoS) requirements of each user are different and therefore the channels have different and non-identical values for SUs. In [68], a non-uniform valuation function has been proposed which takes into account the channel capacity; however only one type of data traffic is considered in this function ignoring the possibly different QoS requirements. In this chapter, we will consider *a realistic valuation function* based on not only channel capacity, but also delay sensitive and delay-insensitive data traffics of SUs.

Another common assumption in the current literature on spectrum auctions is that all bidders are single-minded. It means that the bidders have non-zero valuation function for only one bundle of channels. Moreover, the losers do not have the chance of revisiting the remaining available channels. Such an assumption about bidders for non-identical channels is not efficient, since the chance of winning for the bidders will decrease despite of the existing unused channels for sale. From the auctioneer side, there are unused channels causing inefficiency in channel utilization and, therefore, revenue degradation. To address this issue, we consider ***r****-minded bidders* each of which can bid for $r>1$ bundles of channels in each round of auction, but is allowed to win at most one of these bundles.

Another important issue that we want to address in this chapter is *truthfulness*[1] or incentive compatibility of combinatorial auction mechanism. To date, Vickrey-Clarke-Grove

[1] If an auction mechanism has the property that each user should submit their true valuation if he/she wants to maximize his/her utility function, it is called truth-telling algorithm [59].

(VCG) mechanism is the only general method in which bidders should reveal their true valuation to maximize their utility function or social welfare[1] function. VCG mechanisms are however required to compute the optimum outcomes of auction to be truthful. When there are r-minded bidders with bid submission for more than one bundle of channels or when there are single minded bidders with more than two channels in each bundle, the problem of determining auction outcomes is NP-hard and there is not an optimum solution for these cases. This problem should be solved by sub-optimal solutions therefore the VCG mechanism is not truthful anymore. One of the open problems is to find sub-optimal solutions which are truthful when the problem of determining auction outcomes is NP-hard. In [59], Nissan et al. have proposed two VCG-based[2] truthful sub-optimal solutions for single-minded bidders using the so-called second chance mechanism. In [69], Lavi et al. have introduced a sub-optimal solution for NP-hard VCG mechanism for single minded bidders which is truthful in expectation[3]. In [70], Lehman et al. have demonstrated that there is a simple truthful greedy algorithm for single-minded bidders and further proven that there is not any payment scheme for the greedy algorithm with r-minded bidders to make it truthful in general. To address this problem, we impose some restrictions for the bidders and propose a *weaker concept of truthfulness for r-minded bidders.*

4.2 System Model and Problem Formulation

We consider a cognitive radio network with N SUs. Assume the availability of M ($< N$) non-identical channels that the primary network wants to sell to the SUs. Let $\Omega = \left\{\Omega_1, \Omega_2, ..., \Omega_{M^N}\right\}$ be the set of all possible ways in which the M non-identical channels

[1] The social welfare is the sum of utilities of all bidders for the specific channel allocation.
[2] VCG-based mechanism is a mechanism that uses the suboptimal algorithm for determining the outcomes and calculates the payment of bidders based on VCG.
[3] Truthfulness in expectation is a weaker concept than deterministic truthfulness [69].

can be allocated to N SUs. Let $\Omega_j = \{\mathrm{K}_1, \mathrm{K}_2, ..., \mathrm{K}_N\}$, $j = 1, 2, ..., M^N$, denote a subset of Ω and each of its elements corresponds to a particular channel allocation. Therefore, each element therein $\mathrm{K}_i = \{k_1, k_2, ..., k_m\}$, $i = 1, 2, ..., N$, represents the set of channels in the bundle allocated to the ith bidder and m is the number of channels in a bundle.

We denote $V_i(\mathrm{K}_i)$ as the valuation function or utility of the ith SU for the channel allocation $\mathrm{K}_i \in \Omega_j$ (i.e., the value that it conjectures or expects to derive from the allocation when it submits the bids). Note that the valuation function of the ith SU is simply expressed as $V_i(k)$, $1 \le k \le M$ if the channel allocation K_i consists of only one channel. The profit of the ith SU is given by $u_i(\mathrm{K}_i, p_i, V_i) = V_i(\mathrm{K}_i) - p_i$ where p_i is the payment that the ith SU makes to the auctioneer for the channel allocation K_i. Let $B_i(\mathrm{K}_i)$ denote the bid of the ith SU for the channel allocation K_i, i.e., the amount of money that the ith SU is willing to pay if the allocation $\mathrm{K}_i \in \Omega_j$ is chosen. The auctioneer determines the channel allocation and the payment p_i.

The design of an auction can be formulated as an optimization problem to maximize either revenue or social welfare. The social welfare [10] is the sum of the utilities of all bidders from the allocation and therefore defined as $SW = \sum_{i=1}^{N} V_i(\mathrm{K}_i)$. Furthermore, let K_i^* denote the channel allocation that maximizes the revenue of the auctioneer, given the bids $B_i(.)$, $i = 1, \ldots, N$. That is, K_i^* satisfies

$$\sum_{i=1}^{N} B_i(\mathrm{K}_i^*) \ge \sum_{i=1}^{N} B_i(\mathrm{K}_i) \quad \forall \mathrm{K}_i \in \Omega . \tag{4.1}$$

For a truthful auction mechanism, the net utility of the ith bidder for any possible bidding is maximized when it sets $B_i(\mathrm{K}_i) = V_i(\mathrm{K}_i)$, $\forall \mathrm{K}_i \in \Omega$ [58]. Therefore, in a truthful auction mechanism such as VCG, since $B_i(.)$ is the announced valuation of bidders to the auctioneer, K_i^* obtained from (4.1) will also maximize the social welfare of the bidders. Let

K^*_{-j} denote the allocation that would have maximized the social welfare if the jth bidder did not participate in the auction. That is, K^*_{-j} satisfies

$$\sum_{i=1,i\neq j}^{N} B_i(\mathrm{K}^*_{-j}) \geq \sum_{i=1,i\neq j}^{N} B_i(\mathrm{K}^*_i) \quad \forall \mathrm{K}_i \in \Omega \,. \tag{4.2}$$

Under the VCG mechanism, the payment made by SU to the auctioneer is given by

$$p_j = \sum_{i=1,i\neq j}^{N} B_i(\mathrm{K}^*_{-j}) - \sum_{j=1,i\neq j}^{N} B_i(\mathrm{K}^*_i) \quad . \tag{4.3}$$

The implementation of the VCG mechanism requires the calculation of K^*_i and K^*_{-j}, $j = 1,...,N$. K^*_i can be found by using a combinatorial optimization algorithm for the channel allocation problem stated in (4.1), and K^*_{-j} can be found by running the same algorithm on the set of bidders ignoring the jth bidder.

We consider two cases in which r-minded SUs can submit bid either only for single channels or for bundles of channels. For case 1, let $\mathbf{B}_i = \left[b_{i,1}, b_{i,2}, ..., b_{i,M} \right]$ be the bidding vector (with r non-zero elements) of the ith bidder where $b_{i,j}$ is its bid for the jth channel. For case 2, let $\mathbf{B}^1_i = \left[b^1_{i,1}, b^1_{i,2}, ..., b^1_{i,r} \right]$ be the r-tuple bidding vector of the ith bidder where $b^1_{i,j}$ denotes its bid for the jth bundle of channels. Furthermore, let $\mathbf{S}_{i,j} = \left(s_{i,1}, s_{i,2}, ..., s_{i,M} \right)$ be an M-tuple vector including the channels for which the ith bidder submits bid as its jth bid.

The SUs have their own valuation for the channels and this valuation mainly depends on the capacity of the channel and traffic of SUs. The ergodic capacity of the jth channel for the ith SU is given by [8]

$$C_{i,j} = \omega_j \, ln\left(1 + G_{i,j} \, | h_{i,j} |^2 \, P_{i,j} \big/ \sigma^2_{i,j} \right) \tag{4.4}$$

where ω_j is the bandwidth of the jth channel and $P_{i,j}$ is the signal power of the ith SU in the jth channel. In (4.4), $G_{i,j}$ and $h_{i,j}$ represent, respectively, the path loss and the fading coefficient of the jth channel between the ith SU and Base Station. The fading coefficients

are modeled as complex Gaussian with zero mean and unit variance, leading to Rayleigh fading model. The additive noise is modeled by a complex zero-mean Gaussian random variable with a variance of $\sigma_{i,j}^2$ which follows a uniform distribution between $\left[\sigma_{\min}^2, \sigma_{\max}^2\right]$ for each SU. Besides channel capacity, the valuation of a channel for SUs depends on their data traffics. We assume that data packets arrive to the SUs following a Poisson distribution with mean λ_{d} for delay-sensitive traffic and λ_{o} for delay-insensitive traffic ($\lambda_d < \lambda_o$) [29].

In this case, r-minded bidders can bid for single channels. Based on the channel capacity and traffic of SUs, we propose a valuation function for the single-item bidding as

$$V_i(k) = \begin{cases} \left(Q_1 D_i^d + Q_2 D_i^o\right) C_{i,k} & \text{if } \left(D_i^d + D_i^o\right) \geq C_{i,k} \\ Q_1 D_i^d C_{i,k} & \text{if } D_i^d \leq C_{i,k} < \left(D_i^d + D_i^o\right) \\ 0 & \text{if } C_{i,k} < D_i^d \end{cases} \tag{4.5}$$

where D_i^d and D_i^o denote the required data rates of the SUs for transmitting , respectively, delay-sensitive and delay-insensitive data traffic. We assume that data packets arrive to the SUs following a Poisson distribution with mean λ_{d} for delay-sensitive traffic and λ_{o} for delay-insensitive traffic ($\lambda_d < \lambda_o$) [56]. Therefore, D_i^d and D_i^o are respectively functions of λ_{d} and λ_{o} . Furthermore, let Q_1 and Q_2 $\left(Q_1 > Q_2\right)$ denote the valuation coefficients of, respectively, delay-sensitive and delay-insensitive data. $Q_1 > Q_2$ shows that delay-sensitive data has higher valuation than delay-insensitive data for the SUs. Based on this difference in the valuation, the SUs should pay more to the auctioneer if they want to transmit delay-sensitive data. We assume that two traffic types are independent from each other and also independent from channel capacity.

In this case, the auctioneer asks bidders to submit at most r bids for the available single channels, therefore each bidder submits an r-pair bid vector. As discussed above, we use VCG as the auction mechanism. Since VCG is truthful when it is solved optimally, each user

should submit its true valuation to maximize its social welfare. If there are more than r non-zero channel valuations for a bidder, the bidder submits the r most valuable channels to maximize its valuation function since each bidder can submit only r bids.

Define the matrix $\mathbf{B} = [\mathbf{B}_1;\ \mathbf{B}_2;\ ...;\mathbf{B}_N]$ of size $N \times M$ which contains all the biddings. The ith row of this matrix represents the bidding of the ith bidder and the jth column represents all the biddings for the jth channel. Let $\mathbf{A}_{M\times N}$ denote an assignment matrix with each of its element $a_{i,j}$ being 0 or 1. If the ith bidder wins the jth channel, $a_{i,j}$ is set to one, otherwise zero. The channel allocation can be now stated as an optimization problem as

$$\begin{aligned} &\arg\max_{\mathbf{A}} \sum_{i=1}^{N}\sum_{j=1}^{M} a_{i,j} b_{i,j} \\ &\text{s.t. i) } a_{i,j} \in \{0,1\}, \quad \text{ii) } \sum_{i=1}^{N} a_{i,j} \in \{0,1\}, \quad \text{iii) } \sum_{j=1}^{M} a_{i,j} \in \{0,1\} \end{aligned} . \tag{4.6}$$

In the above, the second constraint means that the jth channel can be allocated to at most one bidder. The third constraint indicates that each bidder can win at most one single channel. This problem can be modeled as a bipartite maximum weighted matching (BMWM)[1] problem [71], [72] and be efficiently solved by sequential algorithms such as Hungarian algorithm [73].

4.2.1 Case 2: r-minded bidders with bundle channel auction

In the previous sub-section, we have assumed that if the channel capacity is less than SU's delay-sensitive data traffic demand (i.e., $C_{i,k} < D_i^d$), the bidder assigns zero value to the channel. In this sub-section, we assume that bidders have the bundling capability of two or more channels. Each of these channels is supposed to have lower capacity than the data demand of bidders, but their aggregate capacities are equal or more than the demand. The

[1] A weighted bipartite graph (or bi-graph) is a graph whose vertices can be divided into two disjoint sets such that every edge connects a vertex in one set to another set and the edges are weighted [71].

maximum number of the channels that can be bundled in a bid is denoted by m and is determined by the auctioneer, ($1 \le m \le M$).

The valuation function for a bundle of channels is given by

$$V_i(\mathrm{K}_i) = \left(Q_1 D_i^d + Q_2 D_i^o\right) \sum_{j=1}^{|\mathrm{K}_i|} C_{i,j} \qquad \text{if} \quad \max_{j \in \{1,2,..,|\mathrm{K}_i|\}} (C_{i,j}) < D_i^d + D_i^o < \sum_{j=1}^{|\mathrm{K}_i|} C_{i,j} \tag{4.7}$$
$$\mathrm{K}_i \in \Omega, \quad |\mathrm{K}_i| \le m$$

where $V_i(\mathrm{K}_i)$ is the valuation of the ith bidder for the bundle of channels K_i. Each bidder can place r bids and each bid consists of at most m channels. The cardinality of K_i is less than or equal to m (i.e., $|\mathrm{K}_i| \le m$).

In this case, bidders make their valuations for a single channel based on (4.5) and for a bundle of channels on (4.7). As there is no budget constraint in our problem formulation, the bidders can submit bids for m most valuable bids. Let $\mathrm{K}_{i,j}$ denote the subset of the channels as the jth bid of the ith bidder. The elements of $\mathbf{S}_{i,j}$ are given by

$$s_{i,j} = \begin{cases} \dfrac{1}{|\mathrm{K}_{i,j}|} & \text{if } j \in \mathrm{K}_{i,j} \\ 0 & \text{o.w} \end{cases} \tag{4.8}$$

where $|\mathrm{K}_{i,j}|$ is the number of channels in the bid. For example if there are 5 channels for sale and the first bidder wants to submit a bundled bid as its first bid consists of channels 1,2 and 4, the $\mathbf{S}_{1,1} = [1/3, 1/3, 0, 1/3, 0]$. The matrix $\mathbf{S}_i = \left[\mathbf{S}_{i,1}; \mathbf{S}_{i,2}; \ldots ; \mathbf{S}_{i,r}\right]$ of size $r \times M$ represents all channels that the ith bidder wants to submit bid for them. Let $\mathbf{A}_i$ denote an M-tuple assignment vector where each element $a_{i,j}$ is either 0 or 1. When the ith bidder wins the jth channel, $a_{i,j}$ is set to one, otherwise zero. The channel allocation problem in case 2 can be now formulated as

$$\begin{aligned} &\arg\max_{\mathbf{A}_i} \sum_{i=1}^{N} \mathbf{A}_i \mathbf{S}_i^T (\mathbf{B}_i^1)^T \\ &\text{s.t} \quad \text{i)} \ \sum_{i=1}^{N} a_{i,j} \in \{0,1\}, \ \ \text{ii)} \ \mathbf{A}_i \mathbf{S}_i^T \in \{0,1\}, \ \ \text{iii)} \ \mathbf{A}_i \mathbf{S}_i^T \mathbf{I}_{r\times 1} \in \{0,1\}, \ \ \text{iv)} \ a_{i,j} \in \{0,1\} \end{aligned} \quad . \tag{4.9}$$

Here, the first constraint means that the jth channel can be allocated to at most one bidder. The second constraint involves $\mathbf{A}_i \mathbf{S}_i^T$ which is an r-tuple vector whose elements take the value of either 0 or 1. It imposes that the ith bidder can only win the channels for which it has submitted bidding. The third constraint $\mathbf{A}_i \mathbf{S}_i^T I_{r\times 1} \in \{0,1\}$ checks that each bidder can only win at most one bundle.

The problem in (4.9) is, in general, NP-hard and not solvable in polynomial time. There have been some efforts in the literature for a solution in some special cases. Particularly, in [58], for $r = 1$, $m = 2$, it was solved by maximum weighted matching in a graph. In [74], we have considered $r \geq 2$, $m = 2$, modeled the problem as a 3-uniform hyper-graph[1] and proposed a greedy algorithm for finding a suboptimal channel allocation to the bidders. For $r \geq 2$, $m \geq 2$ under consideration, the problem in (4.9) is NP-hard and can be modeled by maximum weighted matching in hyper-graphs. This so-called "set packing" problem has been studied in the context of combinatorial optimization and is one of the Karp's NP-complete problems [58].

Theorem 4.1: The allocation problem among r-minded bidders is NP-hard or the decision problem of whether the optimal allocation has social welfare of at least q (q is a non-negative real number) is NP-complete

Proof: See Appendix C.

[1] A hyper-graph is a generalization of a graph, where an edge can connect any number of vertices.

4.3 Sub-Optimal Solutions for Case 2

In the following subsections, we propose two approaches based on *greedy algorithm* [70] and *randomized rounding of LP relaxation algorithm* [69] to find sub-optimal solutions for our NP-hard problem. The prominent characteristics of these two approaches are good approximation factor and being near-optimal despite of their computational simplicity.

4.3.1 Greedy Algorithm

Greedy algorithm [70] is a heuristic approach which involves finding the local optimum at each stage which can be an approximation of the global optimum. Here, we propose a simple greedy algorithm to find a sub-optimal solution for case 2 with an approximation factor of $\max\left(1/m, 1/\sqrt{M}\right)$. We model our problem as a weighted hyper-graph and the solution is to find the maximum weighted matching in this hyper-graph. A norm of a hyper-edge e_i is denoted by $n(e_i) = w_i \big/ \left(v_i - 1\right)^l$ and is called "average bid per channel" for the ith bid. Here, w_i is the weight of ith hyper-edge which is equal to the amount of submitted bid for K_i, v_i is the number of vertices of ith hyper-edge which is equal to $|\mathrm{K}_i|+1$, and l is a constant factor ($l \geq 0$).

Our proposed greedy algorithm selects the highest norm of hyper-edge in each step and deletes all vertices inside the hyper-edge and other hyper-edges that have joint vertices with the selected hyper-edge until there is not any non-zero hyper-edge left. The proposed greedy algorithm steps can be then summarized as follows:

Step 1: Compute the norm of all hyper-edges and sort them in descending order. Then go to step 2.

Step 2: Select the hyper-edge with the highest norm and allocate items inside this hyper-edge to the bidder who placed this bid. Then go to step 3.

Step 3: Delete all hyper-edges that have any joint vertices with the selected hyper-edge in the step 2. Delete all vertices belonging to the selected hyper-edge. Then go to step 4.

Step 4: If there is any hyper-edge with non-zero norm, go to step 2, otherwise end.

The proposed algorithm can be solved in $O(n\log(n))$ where n is the total number of bids.

Theorem 4.2: The approximation factor of the proposed greedy algorithm with norm $l = 1/\sqrt{2}$ is $\max\left(1/m, 1/\sqrt{M}\right)$.

Proof: See Appendix D

Since our auction mechanism is VCG-based, the payment of each winner bidder should be calculated based on (4.3). The key part to calculate the payment of the jth bidder (i.e., p_j) is to find $\sum_{j=1,i\neq j}^{N} B_i(\mathrm{K}_i^*)$ and $\sum_{j=1,i\neq j}^{N} B_i(\mathrm{K}_{-j}^*)$. K_i^* can be found via the greedy algorithm for the channel allocation problem stated in (4.1), and K_{-j}^* can be found by running the same algorithm on the set of the bidders ignoring the jth bidder. Therefore the payment of the jth bidder can be calculated as $p_j = \sum_{j=1,i\neq j}^{N} B_i(\mathrm{K}_{-j}^*) - \sum_{j=1,i\neq j}^{N} B_i(\mathrm{K}_i^*)$.

4.3.2 Randomized Rounding Relaxed LP (RRRLP) Algorithm

In the relaxed version of problem formulation in (4.9), each element of matrix $\mathbf{A}$ can be in the interval $[0,1]$. Therefore we can reformulate (4.9) as

$$\begin{aligned} &\arg\max_{\mathbf{A}_i} \sum_{i=1}^{N} \mathbf{A}_i \mathbf{S}_i^T \mathbf{B}_i^T \\ &\text{s.t} \quad \text{i)} \sum_{i=1}^{N} a_{i,j} \leq 1, \quad \text{ii)}\, \mathbf{A}_i \mathbf{S}_i^T \leq 1, \quad \text{iii)}\, \mathbf{A}_i \mathbf{S}_i^T I_{r\times 1} \leq 1, \quad \text{iv)} 0 \leq a_{i,j} \leq 1 \end{aligned} \quad . \tag{4.10}$$

The re-formulated problem given by (4.10) is a LP problem that can be solved by simplex method or interior points [75]. The output of the relaxed LP problem provides an upper bound for the total social welfare and is not necessarily integral; therefore rounding the

fractional solution can be used to obtain an integral feasible solution. Different methods of rounding have been proposed in the literature [75]. In this section, we will use randomized rounding (RR). The basic idea of this method is to use the probabilistic method to convert an optimal LP solution to a valid solution of the original problem. As the value of each assignment is less than 1, a probability equal to this number can be assigned to each fractional solution, i.e., if this value for the jth bid of the ith bidder is $a_{i,j}$, the channel will be allocated to the jth bidder with the probability $P(a_{i,j}) = a_{i,j}$. Since in RR method, all fractional solutions of (4.10) have the chance of rounding, this method is fairer related to the bidders than the other rounding algorithms [69].

The steps of our proposed RRRLP can be therefore summarized as follows:

Step 1: Relax the integrality constraints in (4.9) and solve the relaxed LP version of the problem in (4.10). Then go to step 2.

Step 2: Sort all the bids in decreasing order based on their fractional solution $a_{i,j}$. Then go to step 3.

Step 3: Allocate the desired bundle to the bidder with the highest level in step 2 based on the probability value that comes from the relaxed LP solution. Then go to step 4.

Step 4: Delete all the bids of winner bidder and all the bids that have joint item with the allocated channel. Then go to step 5.

Step 5: If there is any non-zero bid or any channel is left, go to step 3. Otherwise end.

Because of randomized nature of this algorithm, it is possible that the integral solution of this algorithm might be far from the LP fractional solution. However, based on the probability distribution of this technique, the probability of being far from the optimal solution is so small and it can be shown that the performance of this algorithm is near optimal.

For the payment calculation of the RRRLP algorithm, similar to the greedy algorithm discussed in the previous sub-section, first we need to find K_i^* and K_{-j}^*. For the RRRLP method, K_i^* can be found by running the RRRLP algorithm to solve the outcome determination problem of auction stated in (4.1), and K_{-j}^* can be found by running the same algorithm on the set of bidders ignoring the jth bidder. Finally, p_j can be calculated using (4.3).

4.4 Truthful Iterated Greedy Algorithm for Case 2

The main challenge about VCG mechanism is that it should be calculated optimally to be truthful. Since our problem is NP-hard for case 2, it cannot be optimally solved. In the previous section, two sub-optimal methods have been proposed to solve this problem but these methods ruin the truthfulness property of the VCG payment.

In [70], Lehman et al. have identified four properties for an auction mechanism to be truthful. Based on these properties, they have designed a new truthful payment method for single-minded bidders that satisfies these properties. They have further shown that there is not any payment method to make an auction truthful for r-minded bidders [70]. To alleviate this problem, we will use a weaker concept of truthfulness for bidders with some restrictions. Particularly, we will consider myopic bidders[1] and show that it is possible to design a truthful auction for myopic complex bidders.

When the bidders are myopic, they cannot gain any information from the previous rounds of auction. Such an assumption can be easily justified in our case: Since the bidders are SUs and the items for sale are channels, we can assume that channel conditions (such as noise, fading and interference) will change independently in each round of auction. Therefore, bidders are not able to use the information from previous rounds. Furthermore, we

[1] A bidder who disregards future auctions in a sequential setting is called myopic bidder.

assume that traffic of SUs will change independently from one round to another round; therefore SUs do not have any estimation about the demand of other SUs.

The complexity of the algorithms for determining auction outcomes can be decreased through the employment of some iterative techniques. Another advantage for iterative outcome determination algorithm is that our problem can be reduced from r-minded bidders to single minded bidders. In [76], it has been shown that if the output of an iterative auction is computed optimally in each iteration, it will be truthful. On the other hand, if the outcome determination will be greedy in each iteration but satisfies monotonicity and participation property it will be truthful for myopic bidders [70].

Definition: An outcome determination algorithm has monotonicity property if a bidder i is granted a bundle with bid K_i, it is also granted each bundle $\mathrm{K}'_i \subseteq \mathrm{K}_i$ with norm $n'_i \geq n_i$.

Participation property ensures that the bidder will not lose by participating in the auction. It indicates that the winning bidder's payment should not be more than its submitted bid. In our outcome determination algorithm, the bids of all bidders are sorted based on their average bid per item or norm of each bid. In each iteration of the algorithm, a most valuable bid of each bidder is selected for use in the current iteration. After each iteration, all bids used in this iteration, all bids of winning bidders, and all bids that have joint item with the winning bids are deleted. Iterations continue until there is no item or no bid left. The steps of our proposed truthful iterated greedy algorithm can be therefore summarized as follows:

Step 1: Sort r bids of each bidder based on their norms in descending order. Then go to step 2.

Step 2: Pick the most valuable bid of each bidder. Then go to step 3.

Step 3: Apply the proposed greedy algorithm in sub-section 4.3.1 and determine the winning bidders. Then go to step 4.

Step 4: Payment of each bidder should be determined based on the employed payment method (See below for further discussions) Then go to step 5.

Step 5: Delete all the bids of winner bidder and all bids of other bidders that have joint items with the winner bid. If there is any channel or any bid left, go to step 2, else end.

For the payment, we use a modified version of the payment method in [70]. In this method, we find the most valuable bid that has at least one joint item with the winning bids. Let $n_L(j)$ denote this bid. If the number of items in winning bid is $|\mathrm{K}_j|$, the winning bidder should pay $|\mathrm{K}_j| \times n_L(j)$. In the original payment method of [70], all bids are considered in payment determination. In our modified version, the payment of each bidder will be determined by the participant bids in each iteration, not all bids. Therefore, the payment of each bidder is affected by at most one bid of other bidders used in each iteration. With this method of payment and under the assumption of myopic bidders, we can model r-minded bidders as single minded bidders.

Proposition: The proposed iterated greedy algorithm for the myopic bidders is truthful.

Proof: The proposed modified payment method guarantees that the payment of the bidders is always less than the submitted bid. In our proposed iterated greedy method, the winners in each iteration will be determined based on the most valuable bids with the highest norms. For the payment calculation of the winner bidder in each iteration, first we find the most valuable bid that has at least one joint item with the winning bid of bidder in the same iteration. It is clear that the selected bid (which has joint item with the winning bid) has less norm than the norm of winning bid; otherwise, this bid should have been selected as the winning bid. Therefore, we can conclude that our proposed payment method satisfies the participation property.

Based on the monotonicity definition, an outcome determination algorithm has the monotonicity property if a bidder i is granted a bundle with the bid K_i and the norm n_i, it

will be granted each bundle $\mathrm{K}_i' \subseteq \mathrm{K}_i$ with the norm $n_i' \geq n_i$. Therefore for the monotonicity proof, it is sufficient to show that if this bidder submits bundle K_i' (i.e., $\mathrm{K}_i' \subseteq \mathrm{K}_i$) with $n_i' \geq n_i$ instead of the bundle K_i, it should also win bundle K_i'. The greedy algorithm works based on the highest norms. The bidder i was winner with norm n_i for the bundle K_i, therefore it means that the bundle K_i with norm n_i was among optimal bids with the highest norms. If the bidder i submits a bid for bundle K_i' with norm n_i', since its norm n_i' is bigger or equal than n_i, therefore we can conclude that n_i' is among optimal bids with the highest norms and also it has no conflicts with bids with higher norms since the bidder i was winner with the bid K_i. Therefore the bidder i is also the winner for the bundle K_i'. Since the iterated greedy algorithm satisfies the monotonicity and the participation property for the myopic bidders, the proposed algorithm is truthful.

In addition to the truthfulness property, there are two advantages of our proposed algorithm in comparison with the VCG auction. First, the complexity of our problem is less than that of VCG mechanism. Second, when the VCG mechanism is solved by a sub-optimal algorithm, rationality of the bidders may not be satisfied but in our proposed algorithm, the payment of the bidders are always less than the bidders submission and therefore their rationality are satisfied.

4.5 Simulation Results

In this section, we provide Monte-Carlo simulation results to demonstrate the effectiveness of the proposed auction framework and compare it with previous methods [57], [68] in terms of total social welfare, revenue of the auctioneer, and profit of bidders. In our simulations, we use the notation of “currency unit (CU)” instead of any particular currency.

We assume that all channels have a bandwidth of 6 MHz. For data traffic of SUs, we assume $\lambda_o = 1$ Mbps and $\lambda_d = 0.5$ Mbps.

In Fig. 4.1, we illustrate the average auctioneer revenue versus the number of bids r for case 1. We assume that the number of channels is M =10 and the number of bidders are $11 \leq N \leq 16$. We see that the average revenue increases when the number of bids increases. This indicates that the efficiency of the spectrum utilization is better for $r > 1$, because there is not any unused channel. It is also observed that the auctioneer's revenue saturates for $r > 7$. On the other hand, the overhead information caused by bidder's submissions increases with the increasing number of submitted bids. Therefore we can prevent the increase in overhead information by fixing r at the saturation point which is $r = 7$ in our simulation. It is also observed from Fig. 4.1 that the auctioneer's revenue increases as the number of bidders increases from 11 to 16. This increase is a result of the increasing competition between bidders and increasing in amount of their bids. Since the outcome determination of auction is optimum in case 1, the auction mechanism in case 1 is truthful.

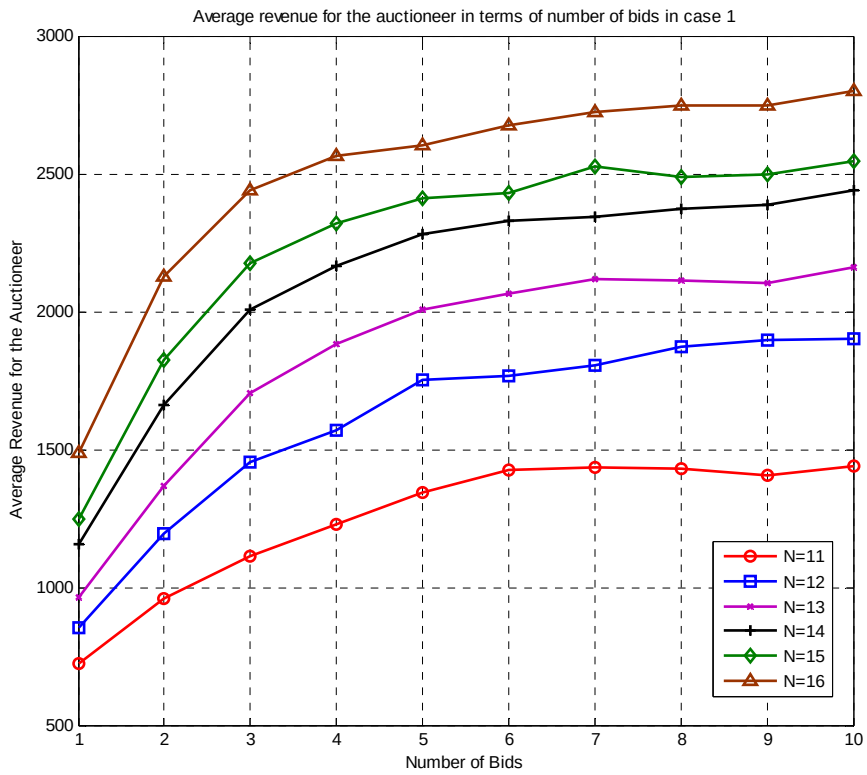


Figure 4.1 Average revenue of the auctioneer in case 1, i.e., the SUs are r-minded but they can submit bid only for single channels.

In Fig. 4.2, we illustrate the average auctioneer revenue versus the number of bids r for case 2 and compare the performance of proposed greedy, RRRLP and truthful greedy algorithms (See Section 4.3 and 4.4). We assume that the number of channels is M =10 and the number of bidders is N = 19. As discussed in Section 4.4, the RLP algorithm provides an upper bound for the revenue of auctioneer and is included here as a benchmark. We further include the performance of the greedy algorithm proposed in [70] as a competing scheme. It can be observed that the RRRLP method outperforms greedy and truthful greedy method in

terms of auctioneer's revenue. The performance of our RRRLP and greedy algorithms are also better than the greedy algorithm in [70]. It can be also observed that the greedy algorithm in [70] outperforms the truthful greedy algorithm in terms of auctioneer revenue. Specifically, the approximation factors for the RRRLP, the greedy algorithm, the truthful greedy algorithm are 75%, 65% and 30%, respectively, while the approximation factor for the greedy algorithm in [70] is 55%.

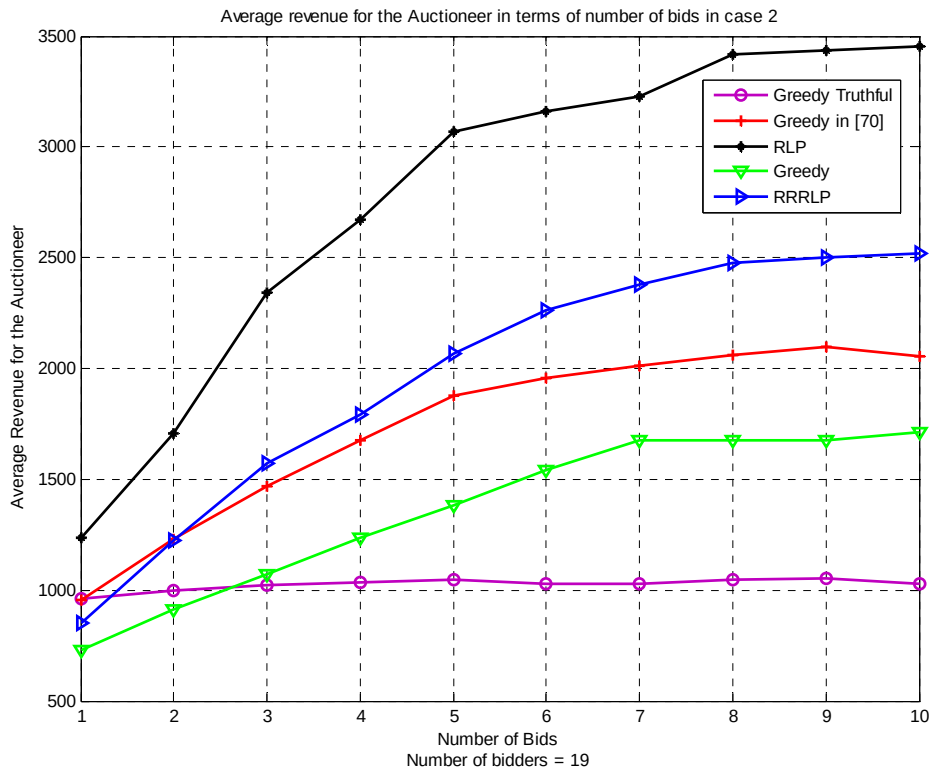


Figure 4.2 Average revenue of auctioneer in case 2, i.e., the SUs are r-minded and they can submit bid for bundles of channels.

In Fig. 4.3, we illustrate the total social welfare for case 2. We assume that the number of channels is M =10 and the number of bidders are N = 19. Similar to Fig. 4.2, RLP provides an upper bound on the performance of methods under consideration. RRRLP performs the best followed by the proposed greedy algorithm, greedy algorithm in [70] and the proposed truthful iterated greedy algorithm.

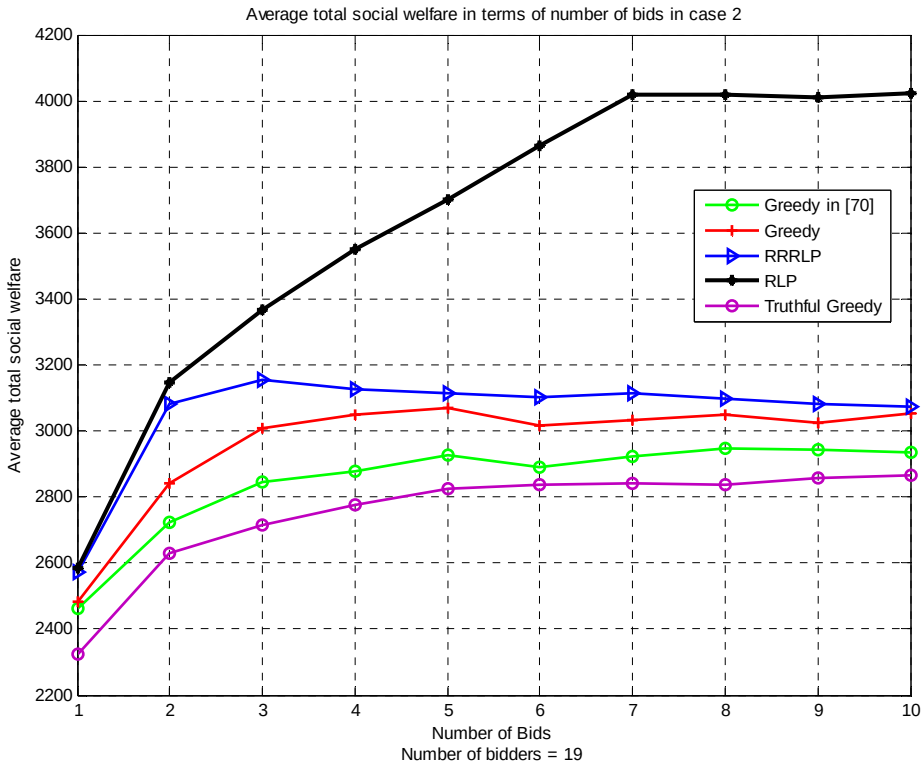


Figure 4.3 Average total social welfare in case 2.

In Fig. 4.4, we provide the average profit of bidders in case 1. It can be observed when the number of bidders are 11, 12, 13, SUs have profit improvement in parallel to the increase in r, since all the channels are not fully utilized. The profit of the SUs saturates when r is

close to the number of channels, i.e., 10 in our simulation. When the number of bidders are 14, 15 and 16, the average profit of bidders degrades in parallel to the increase in r. Because, the spectrum is fully utilized therefore with increasing r, the payment of bidders increases due to more competitors for each channel and the profit of them (that is the difference between valuation and payment) decreases.

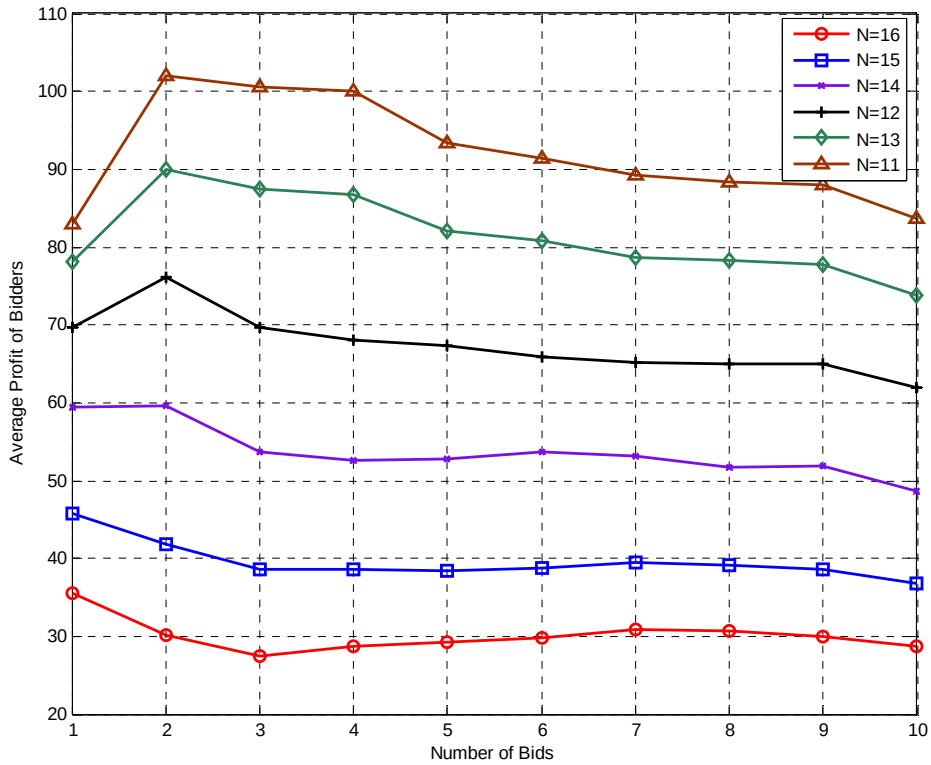


Figure 4.4 Average revenue of bidders in case 1.

In Fig. 4.5, we provide the average profit of bidders in case 2. We assume that the number of channels is M =10 and the number of bidders are N = 19. Similar to case 1, the

average profit of bidders degrades with the increase of number of bids. We also observe that the truthful greedy algorithm outperforms RRRLP and greedy algorithms in terms of profit for the bidders. Since in truthful greedy algorithm, bidders submit their true valuation to maximize their profit, the bidder gains more revenue than two other methods. It should be also noted that the complexity of truthful iterated greedy algorithm is less than greedy and RRRLP algorithm.

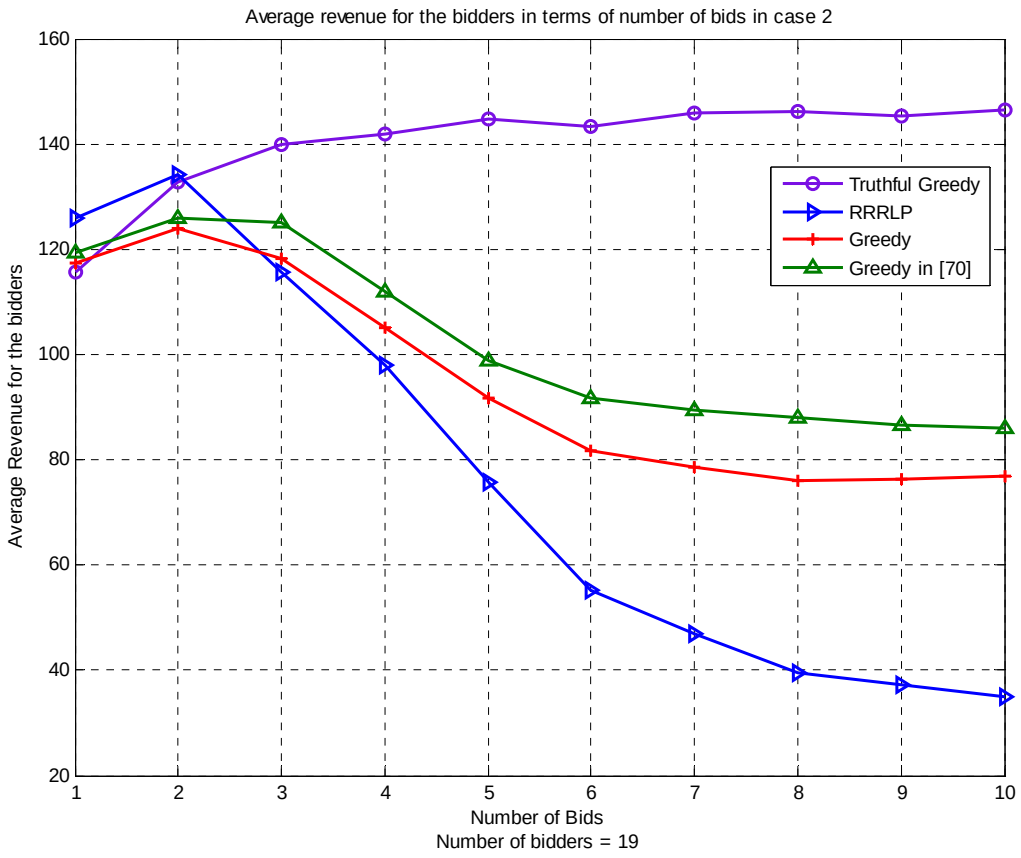


Figure 4.5 Average revenue of bidders in case 2.

Chapter 5
Pricing for Open Access Oligopoly-Market Femtocell Networks

5.1 Introduction

The current market in femtocell networks is mainly geared towards close access (CA) femtocells [60]. To enable the wide deployment of open access (OA) femtocells, innovative pricing models with incentives for residential femtocell users are required that will be pursued in this chapter. In [30], Shetty at al. study the impacts of user incentives on the revenue of a femtocell operator in CA model. In [61], the economic aspects of openness of femtocells compared with close femtocells is studied. This paper considered the monopoly market which is the simplest market structure when there is only one seller in the system. Some papers [5], [61] propose fixed pricing strategy for open access femtocells, but fixed pricing has some drawbacks such as the lack of economic incentives for femtocells and revenue degradation for mobile service provider. In [77], Duan et al. investigate the economic incentive for the cellular operator to add femtocell service on the top of its existing macrocell service. They model the interaction between cellular operator in a monopoly market and users as a Stackelberg game. They showed that the operator choose to only provide femtocell service if femtocell service has full spatial coverage. In [32], VCG auction is proposed as the pricing scheme for open access femtocells. In [33], a reverse auction (one-buyer-multiple-sellers) framework is proposed based on VCG mechanism for access permission trading between wireless service provider and private femtocell owners.

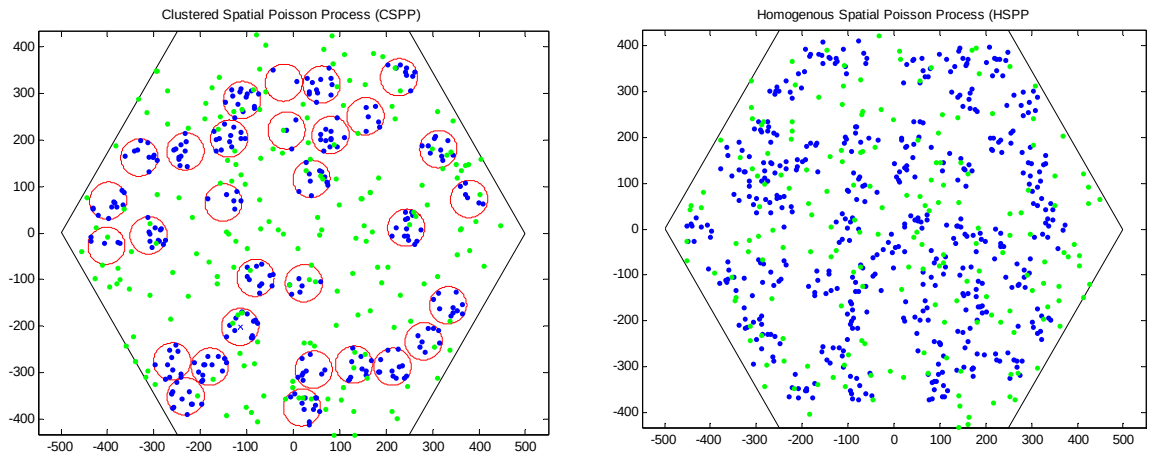
In our work, we assume oligopoly market [8] where there exist multiple sellers and multiple buyers. The sellers compete with each other independently to achieve the highest revenue by controlling the quantity or the price of the supplied commodity. The oligopoly market is more realistic to use in femtocell networks because there are different mobile

service providers in practice. The providers should involve residential customers to share their spectrum by innovative policies in pricing and QoS warranties. In this regard, interaction of users and mobile service providers play a vital role in the OA femtocell networks. From the user point of view, CA is obviously preferred by residential customers who will have full control over the list of authorized users. However, some surveys indicate that OA might be an attractive business model for home market conditioned that competitive pricing is offered [4], [78]. Hybrid access (HA) methods are also discussed to reach a compromise between the impact on the performance of subscribers and the level of access that is granted to non-subscribers [78]. Therefore, the sharing of femtocell resources between subscribers and non-subscribers needs to be finely tuned. Otherwise, subscribers might feel that they are paying for a service that is to be exploited by others. The impact to subscribers must thus be minimized in terms of performance or via economic incentives. With a proper pricing model, deploying of an OA or HA model is more beneficial for network operators than CA model by providing an inexpensive way to expand their network capacities by leveraging third-party backhaul for free.

5.2 System Model and Problem Formulation

Consider a wireless cellular network consisting of N macro base stations (MBSs) and several femtocell access points (FAP). The macro user equipments (MUEs) are served by N MBSs from different mobile service providers. In the literature, Homogenous Spatial Poisson Process (HSPP) is widely used to statistically model the locations of FAPs and MUEs [79]. In our work, we consider Clustered Spatial Poisson Process (CSPP) for FAPs and MUEs (See Fig.5.1.a) which is more realistic in practical scenarios. In Fig.5.1, the blue points are FAPs and the green points are MUEs. In Fig.5.1.a, the FAPs with CSPP distribution form a

cluster represented by the red circle. It is assumed that the FAPs inside each cluster can be controlled by a controller.



a. Clustered Spatial Poisson Process (CSPP)

b. Homogenous Spatial Poisson Process (HSPP)

Figure 5.1 Geographical distribution of FAPs under HSPP and CSPP assumptions.

In CSPP model, we have some clusters of femtocells in macrocell layer with HSPP distribution and there is a random number of femtocells distributed identically within each cluster area. This assumption is particularly realistic for urban residential areas in which several FAPs exist clustered in apartment buildings. Denote $H \subset R^2$ as the interior of a reference hexagonal macrocell C of radius R_C. The cellular users (i.e., MUEs) are distributed on R^2 according to a CSPP Ω_C of intensity λ_C. On the other hand, locations of FAPs are assumed to form a CSPP Ω_f with intensity λ_f. Each femtocell includes a Poisson distributed population of actively transmitting femtocell user equipments (FUEs) with mean U_f in a circular coverage area of radius R_f, $R_f \ll R_C$ [79].

Since the distribution of FAPs follows CSPP, they can be grouped for cooperation in a macrocell layer and the available channels can be divided among FAPs of each group. The

same channels can be used by another group of the FAPs in a macrocell with far enough distances between each other. Consider a two-tier LTE system and assume that FAPs can be grouped with respect to their intra-channel coefficients and geographical location to each other (i.e., FAPs with high channel gain between each other and distances less than a certain threshold can be categorized in the same group). Resource allocation, scheduling and other signaling of FAPs with MBSs can be carried out via the controller. We also assume that MBS's antenna is sectorized, therefore the available channels for FAPs in each sector would be different from those in other sectors (See Fig.5.2).

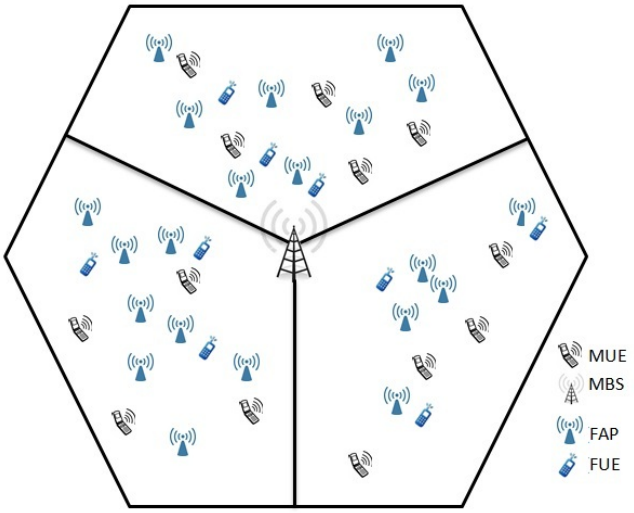


Figure 5.2 Cellular network underlaid with femtocell network

To determine the available channels, we assume that FAPs are equipped with spectrum sensing ability and perform cooperative sensing. This method avoids the overhead required for periodical MBS announcements if spectrum sensing is carried out by MBSs. Therefore

FAPs find their suitable channels based on their requirements and send their requests to the controller of group. The controller collects and sends their request to the MBSs.

5.3 Pricing Schemes

In our work, we assume OA model in which any user can access to any femtocell. The use of OA FAPs at home in fact reduces the interference problems caused by CA FAPs. Indeed, all nearby users would be authorized to connect to any femtocell, reducing thus the negative impact of the femtocell tier on the macrocell network. In this case, the MUEs are always connected to the strongest server (either macrocell or femtocell), avoiding cross-tier interference. As a result, the overall throughput of the network increases. OA is therefore advantageous from the operator point of view.

In OA femtocell networks, the unit price of spectrum should be determined dynamically because of different demand and supply from femtocell and macrocell sides. In this case, the femtocell users have good incentives to participate in OA model and the service providers can maximize their revenue. In previous works [61], fixed pricing is suggested for OA that is not desirable by residential users and they do not have any tendency to use this model.

As we mentioned in the first chapter, the design of the pricing model can be considered as a spectrum trading problem and microeconomics and game theory provide a set of powerful mathematical tools for the analysis of spectrum trading problem. In the following sub-sections, we propose pricing models for OA femtocells based on market equilibrium, Bertrand game, multiple leader multiple follower Stackelberg game and Cooperative game.

5.3.1 Market Equilibrium

In market equilibrium approach used in microeconomics, the profit of the seller and the satisfaction of the buyer(s) are maximized. The amount of commodity that the seller is

willing to supply to the market is indicated by a supply function. The supplied quantity is a function of price p and denoted by $S(p)$ [8]. On the buyer side, the amount of commodity that the buyer is willing to buy from the market is defined by a demand function denoted by $D(p)$. In general, the amount of supply from the seller is an increasing function of price while the demand for a commodity in the market is a decreasing function of price. Given the demand and supply functions in a market, the market-equilibrium price is given by the price for which the supply equals the demand, i.e., $S(p) = D(p)$.

In our case, the negotiation on price and the size of allocated spectrum between the FAPs and MBS is performed through a controller (it could be a server which has control interfaces to both MBSs and FAPs). We categorize FAPs in groups with respect to their channel gains and geographical locations. All FAPs of a specific group submit their spectrum requirements to the controller. The controller checks the spectrum availability and unifies all demands into one demand and submits them to MBSs. MBS determines the price for per unit of spectrum based on its supply function and returns this price to the controller. Then the controller allocates demanded spectrum to each FAP. We assume that the maximum demand of a FAP is limited and each FAP can access to a limited number of channels. Furthermore, it is assumed that the sum of demands of FAPs for spectrum is less than a fraction of the available spectrum because MBS should also serve other MUEs that request services but not serviced by the FAPs. Spectrum demand of FAPs can be determined using their utility function. The utility function of all FAPs can defined as [80].

$$u(\boldsymbol{b}) = \sum_{i=1}^{N} C_i A_i b_i - C_i b_i p_i + (1 - C_i) d_i p_i b_i - \frac{1}{2}\left[\sum_{i=1}^{N} b_i^2 + 2v \sum_{i \neq j} b_i b_j\right] \tag{5.1}$$

where $\boldsymbol{b}$ is the vector of shared spectrum from all MBSs with FAPs, i.e., $\boldsymbol{b} = [b_1, b_2, ..., b_N]$, N is the number of MBSs, and C_i is the fraction of spectrum that is used for FUEs. The

reason for the power 2 for b_i in (5.1) is that it makes the utility function concave. $(1-C_i)$ is the fraction of spectrum that is used for MUEs. p_i is the unit price for spectrum from i^{th} MBS. A_i is the spectral efficiency for FUEs served by FAPs and is a function of signal-to-noise ratio (SNR) and targeted error rate performance. MBS pays to FAPs $d_i p_i$ (as the unit price of spectrum for serving of MUEs by FAPs) where d_i is a fixed coefficient and $0 \le v \le 1$ is another coefficient [81] which shows the substitutability. Specifically, if $v = 0$, the FAPs cannot switch among frequency spectra, while if $v = 1$, the FAPs can switch among frequency spectra freely.

The demand function for spectrum which belongs to the ith MBS can be calculated by

$$\frac{\partial U(\boldsymbol{b})}{\partial b_i} = C_i A_i - C_i p_i + (1-C_i) d_i p_i - b_i - v \sum_{j \neq i} b_j, \quad i \in \{1,2,...,N\} \ . \tag{5.2}$$

Solving this linear system with N equations, we find

$$\begin{aligned} D_i(\boldsymbol{p}) = & \frac{1+(N-2)v}{(1-v)(1+(N-1)v)} \left(C_i A_i - C_i p_i + (1-C_i) d_i p_i\right) \\ & - \sum_{j \neq i} \frac{v}{(1+(N-1)v)(1-v)} \left(C_j A_j - C_j p_j + (1-C_j) d_j p_j\right) \end{aligned} \ . \tag{5.3}$$

This shows the optimum amount of spectrum that MBSs want to sell with price vector $\boldsymbol{p}$. The demand function for the ith service provider can be separated in two parts; one part that is dependent on p_i and the other part that is dependent on the price of others $\boldsymbol{p}_{-i} = [... \ p_j ...]$, $j \neq i$. The demand function can be therefore written as

$$D_i(\boldsymbol{p}) = X_i p_i + \sum_{j \neq i} Y_j p_j + l_i \tag{5.4}$$

where X_i, Y_j and l_i are some constant coefficients. The revenue of the ith MBS from selling spectrum to the FAPs can be calculated by [22]

$$R_i(p) = C_i p_i b_i - (1-C_i) d_i p_i b_i + M_i (w_i - b_i) + k_i (1-C_i) b_i - \left[(k_i - M_i)(w_i - b_i)\right]^2 \tag{5.5}$$

where w_i is the total spectrum available for the ith MBS to be allocated to the MUEs and FAPs. Power coefficient in the last term of (5.5) makes the utility function concave. M_i, $i = 1, 2, ..., N$ are the spectral efficiencies of MUEs served by the ith MBS. k_i is the spectral efficiency for MUEs inside the group area served by FAPs. $(1 - C_i)b_i k_i$ is the revenue of MBSs from MUEs which are serviced by the FAPs. $C_i p_i b_i$ is the payment of FAPs to the MBSs based on their usage from spectrums. $(1 - C_i)d_i p_i b_i$ is the amount of discounts that MBSs should give to the FAPs for providing services to MUEs. Since the spectrum efficiency for the MUEs served by FAPs is more than that of MUEs served by MBS ($k_i > M_i$) therefore the term $\left[(k_i - M_i)(w_i - b_i)\right]^2$ represents the performance degradation of MUEs users served by MBS. The supply function can be calculated from

$$\frac{\partial R_i(\boldsymbol{p})}{\partial b_i} = C_i p_i - (1 - C_i)d_i p_i - M_i + k_i(1 - C_i) - 2(k_i - M_i)^2 (w_i - b_i) = 0 \quad . \tag{5.6}$$

By solving (5.6), $S_i(\boldsymbol{p})$ can be calculated as

$$S_i(\boldsymbol{p}) = \frac{C_i p_i - (1 - C_i)d_i p_i - M_i + k_i(1 - C_i) - 2(k_i - M_i)^2 w_i}{2(k_i - M_i)^2} \quad . \tag{5.7}$$

The supply function for the ith service provider can be separated in two parts; one part that is dependent on p_i and the other part is a constant. Therefore it can be rewritten as

$$S_i(\boldsymbol{p}) = V_i p_i + q_i \tag{5.8}$$

where s_i and q_i are constant coefficients. The market-equilibrium solution will be then defined as the price p_i^* at which spectrum supply equals spectrum demand given by $S_i(\boldsymbol{p}^*) = D_i(\boldsymbol{p}^*)$. To calculate the prices for the market equilibrium, we should solve following set of linear equation system using (5.4) and (5.8), i.e,

$$X_i p_i + \sum_{j \neq i} Y_j p_j + l_i = V_i p_i + q_i \ , \quad i, j \in \{1, 2, ..., N\} \ . \tag{5.9}$$

p_i^* can be calculated as (See Appendix E)

$$p_i^* = \left[\frac{1}{X_i - V_i - Y_i} - \frac{1}{1 + \sum_{i=1}^{N} \frac{Y_i}{X_i - V_i - Y_i}} \left(\frac{Y_i}{(X_i - V_i - Y_i)^2} \right) \right] (l_i - q_i) \\ - \sum_{i \neq j} \frac{1}{1 + \sum_{k=1}^{N} \frac{Y_k}{X_k - V_k - Y_k}} \left(\frac{Y_j}{(X_i - V_i - Y_i)(X_j - V_j - Y_j)} \right) (l_j - q_j) \quad . \tag{5.10}$$

To have a market equilibrium, for all i, we should have $p_i^* > 0$.

5.3.2 Bertrand Game

In the market equilibrium pricing scheme, there is no competition between players or agents. By using a competition scheme between players, it is possible to increase the revenues with respect to the previous scheme. In our model, players are different MBSs which provide service to both FAPs and MUEs and aim to maximize their revenue. In Bertrand game, all MBSs first choose their prices for the spectrum and announce them to the customers (i.e., FAPs). The FAPs send their demands to the MBSs using their controllers. The best strategy for each MBS is maximizing its revenue given the spectrum price of other MBSs. In a Bertrand game, the solution depends mainly on the substitutability of the spectrum. If the spectrum from the different MBSs are identical (i.e., the homogeneous case), then they are said to be fully substitutable. On the other hand, if the commodities are different, the spectrum may be partly substitutable or may be completely unsubstitutable.

In the homogenous case, since the FAPs can buy spectrum from any one of MBSs, therefore FAPs will always select the MBS with the lowest price. In this case, it can be proven that there is a unique Nash equilibrium in which the prices charged by all MBSs are identical. The revenue of a MBS can be calculated by the utility function of a MBS in (5.5) that comes from profit of selling spectrum to FAPs and MUEs. This game can be solved by

Nash equilibrium using the best response function. The best response function of the ith MBS given the spectrum price of other MBSs is defined as

$$B_i(\boldsymbol{p}_{-i}) = \arg\max_{p_i} R_i(p_i, \boldsymbol{p}_i) . \tag{5.11}$$

The Nash equilibrium of this game is denoted by the vector $\boldsymbol{p}^* = [... \ p_i^* ...]$ where $p_i^* = B_i(\boldsymbol{p}_i^*)$. $\boldsymbol{p}_i^*$ is the vector of the best response of other player except i. For calculation of the Nash equilibrium, we should solve the set of equations $\partial R_i / \partial p_i = 0$ for all i. The size of shared bandwidth in (5.5) is replaced by the demand of femtocells $D_i(\boldsymbol{p})$. The revenue function of ith MBS can defined as

$$\begin{aligned} R_i(\boldsymbol{p}) &= C_i p_i D_i(\boldsymbol{p}) - (1-C_i) d_i p_i D_i(\boldsymbol{p}) \\ &\quad + M_i(w_i - D_i(\boldsymbol{p})) + k_i(1-C_i) D_i(\boldsymbol{p}) - \left[\left(k_i - M_i\right)\left(w_i - D_i(\boldsymbol{p})\right)\right]^2 . \end{aligned} \tag{5.12}$$

Solving $\partial R_i / \partial p_i = 0$ yields the Nash equilibrium. The Nash equilibrium can be achieved by solving following linear equations:

$$\begin{aligned} \frac{\partial R_i(\boldsymbol{p})}{\partial p_i} &= (C_i - (1-C_i) d_i) D_i(\boldsymbol{p}) \\ &\quad + \left[(C_i - (1-C_i) d_i) p_i + k_i(1-C_i) - M_i\right] X_i + 2\left(k_i - M_i\right)^2 X_i D_i(\boldsymbol{p}) \end{aligned} \tag{5.13}$$

(5.13) can be rewritten as

$$A_i p_i + \sum_{i \neq j} Z_j p_j + e_i = 0, \quad i, j \in \{1, 2, ..., N\} \tag{5.14}$$

where A_i, Z_j and e_i are respectively given by

$$A_i = 2(C_i - (1-C_i) d_i) X_i + 2\left(k_i - M_i\right)^2 X_i^2 . \tag{5.15}$$

$$Z_j = \sum_{i \neq j} \left((C_i - (1-C_i) d_i) + 2\left(k_i - M_i\right)^2 X_i\right) Y_j . \tag{5.16}$$

$$e_i = \left((C_i - (1-C_i) d_i) + 2\left(k_i - M_i\right)^2 X_i\right) l_i + \left(k_i(1-C_i) - M_i\right) X_i . \tag{5.17}$$

Since there are N service providers, we have N linear equations to solve and calculate p_i, $i \in \{1, 2, ..., N\}$. Based on the lemma in the Appendix E, we can solve (5.14) as

$$p_i = \left[\frac{1}{A_i - Z_i} - \frac{1}{1 + \sum_{i=1}^{N} \frac{Z_i}{A_i - Z_i}} \left(\frac{Z_i}{(X_i - Z_i)^2} \right) \right] (e_i) - \sum_{i \neq j} \frac{1}{1 + \sum_{i=1}^{N} \frac{Z_i}{A_i - Z_i}} \left(\frac{Z_j}{(X_i - Z_i)(X_j - Z_j)} \right) (e_j)$$

.(5.18)

To have Nash equilibrium for the bargaining game for all i, we should have $p_i > 0$.

5.3.3 Multiple Leader Multiple Follower Stackelberg Game

In multiple leader multiple follower (MLMF) Stackelberg game, there are some service providers (called as "leaders") who enter the market sooner than the other service providers, therefore they determine the prices of spectrum sooner than the others. Therefore the other service providers (called as "followers") should determine their prices after the leaders determine the price of spectrum. In the Stackelberg game, since the leaders will make the decision before the followers, the followers will choose their optimal strategy based on the observed strategy chosen by the leader. Consequently, the solution of this game is a set of strategies where the profit of the leader is maximized and the followers choose their best responses.

In a Stackelberg game, the solution which maximizes the profit of the leader is defined as the Stackelberg equilibrium. The Stackelberg equilibrium can be obtained by backward induction. In backward induction, the best response of the follower is first obtained. Then, from this best response of the follower, the leader optimizes its strategy to achieve the highest profit. We assume that there are I leaders and $N - I$ followers in the system. In our case, MBSs $i = \{1, 2, ..., N - I\}$ are followers and MBSs $i = \{N - I + 1, ..., N\}$ are leaders. Based on the backward induction, first we should calculate the price of followers in terms of price of leaders. Mathematically speaking, we need to calculate $\partial R_i / \partial p_i = 0$ for

$i=\{1,2,...,N-I\}$ and calculate $[p_1, p_2,..., p_{N-I}]$ in terms of the price of leaders, i.e., $[p_{N-I+1},..., p_N]$. Then we can calculate price of leaders by $\partial R_i/\partial p_i = 0$ for $i=\{N-I+1,...,N\}$. Therefore, based on the Stackelberg equilibrium, leaders will have more flexibility to determine the price of spectrum and therefore will gain more revenue than the followers.

To calculate price of followers in terms of price of leaders, first we should use (5.11) to calculate price of service providers $\{I+1,...,N\}$. Therefore if we use same method in the previous sub-section, the linear equations for calculating price of followers are given by

$$A_i p_i + \sum_{i\neq j} Z_j p_j + e_i + \sum_{k=1}^{I} Z_k p_k = 0, \quad k\in\{1,2,...,I\},\ i,j\in\{I+1,I+2,...,N\} \tag{5.19}$$

where A_i, Z_j and e_i are respectively given by

$$A_i = 2(C_i-(1-C_i)d_i)X_i + 2(k_i-M_i)^2 X_i^2 \ . \tag{5.20}$$

$$Z_j = \sum_{i\neq j}\left((C_i-(1-C_i)d_i)+2(k_i-M_i)^2 X_i\right)Y_j \ . \tag{5.21}$$

$$e_i = \left((C_i-(1-C_i)d_i)+2(k_i-M_i)^2 X_i\right)l_i + \left(k_i(1-C_i)-M_i\right)X_i \ . \tag{5.22}$$

In (5.19), p_k $, k\in\{1,...,I\}$ are the price of leaders and considered to be constant coefficients. Therefore, the price of followers in terms of the price of leaders can be calculated as

$$\begin{aligned} p_i = &\left[\frac{1}{A_i-Z_i} - \frac{1}{1+\sum_{i=1}^{N}\frac{Z_i}{A_i-Z_i}}\left(\frac{Z_i}{(X_i-Z_i)^2}\right)\right]\left(e_i+\sum_{k=1}^{I} Z_k p_k\right) \\ &-\sum_{i\neq j}\frac{1}{1+\sum_{i=1}^{N}\frac{Z_i}{A_i-Z_i}}\left(\frac{Z_j}{(X_i-Z_i)(X_j-Z_j)}\right)\left(e_j+\sum_{k=1}^{I} Z_k p_k\right) \qquad ,i\in\{I+1,...,N\} \end{aligned} \tag{5.23}$$

We can rewrite the price of followers as

$$p_i = e_i' + \sum_{k=1}^{I} Z_k' p_k, \qquad i \in \{I+1,...,N\}, k \in \{1,2,...,I\} \tag{5.24}$$

where e_i' and Z_i' are respectively given by

$$\begin{aligned} e_i' = & \left[\frac{1}{A_i - Z_i} - \frac{1}{1+\sum_{i=1}^{N} \frac{Z_i}{A_i - Z_i}} \left(\frac{Z_i}{(X_i - Z_i)^2} \right) \right] e_i \\ & - \sum_{i \neq j} \frac{1}{1+\sum_{i=1}^{N} \frac{Z_i}{A_i - Z_i}} \left(\frac{Z_j}{(X_i - Z_i)(X_j - Z_j)} \right) e_i \end{aligned} \tag{5.25}$$

$$\begin{aligned} Z_i' = & \left[\frac{1}{A_i - Z_i} - \frac{1}{1+\sum_{i=1}^{N} \frac{Z_i}{A_i - Z_i}} \left(\frac{Z_i}{(X_i - Z_i)^2} \right) \right] Z_i \\ & - \sum_{i \neq j} \frac{1}{1+\sum_{i=1}^{N} \frac{Z_i}{A_i - Z_i}} \left(\frac{Z_j}{(X_i - Z_i)(X_j - Z_j)} \right) Z_i \end{aligned} . \tag{5.26}$$

Then we can calculate the price of spectrum for the leaders with the following linear equations:

$$(A_i + Z_i') p_i + \sum_{i \neq j} (Z_j + Z_j') p_j + e_i + e_i' = 0, \quad i, j \in \{I+1, I+2, ..., N\} \tag{5.27}$$

where we have defined the parameters in (5.19) and (5.24). Therefore the price for the leader can be calculated as

$$p_i = \left[\frac{1}{A_i + Z_i' - Z_i} - \frac{1}{1 + \sum_{i=1}^{N} \frac{Z_i + Z_i'}{A_i - Z_i}} \left(\frac{Z_i}{(X_i - Z_i)^2} \right) \right] (e_j + e_j') \\ - \sum_{i \neq j} \frac{1}{1 + \sum_{i=1}^{N} \frac{Z_i + Z_i'}{A_i - Z_i}} \left(\frac{Z_j + Z_j'}{(X_i - Z_i - Z_i')(X_j - Z_j - Z_j')} \right) (e_j + e_j') \quad , i, j \in \{1, 2, ..., I\} \quad . \tag{5.28}$$

To have NE for the MLMF Stackelberg game for all i, we should have $p_i > 0$.

5.3.4 Cooperative Game

In the Cooperative Game, the service providers try to cooperate with each other and maximize the total profit of all service providers. Therefore the optimization problem can be stated as

$$\begin{aligned} &\max_{\boldsymbol{p}} \sum_{i=1}^{N} R_i(\boldsymbol{p}) \\ &\text{subject to}: \ 0 \le b_i \le w_i \\ &\qquad\qquad p_i \ge 0 \end{aligned} \tag{5.29}$$

where the total revenue of all service providers can be represented by $\sum_{i=1}^{N} R_i(\boldsymbol{p})$. Also $0 \le b_i \le w_i$ can be rewritten as $0 \le D_i(\boldsymbol{p}) \le w_i$. Therefore (5.29) can be solved using Lagrangian multiplier method. The Lagrangian can be expressed as

$$\Im(\boldsymbol{p}) = \sum_{i=1}^{N} R_i(\boldsymbol{p}) - \sum_{j=1}^{N} \alpha_j p_j - \sum_{k=1}^{N} \beta_k (D_k(\boldsymbol{p}) - w_k) - \sum_{l=1}^{N} \gamma_l (-D_l(\boldsymbol{p})) \tag{5.30}$$

where α_j, β_k and γ_l are the Lagrangian multipliers for the constraints $p_i \ge 0$ and $0 \le b_i \le w_i$, respectively.

5.4 Information Exchange Protocol and Price Determination

In this section, we study the information exchange protocol among service providers, controllers and FAPs. The full information about the system such as spectrum price of other service providers or demand function of FAPs may not be available for the service providers; therefore there are some limitations for the price determination in practice. In a cellular network, the service providers may not know about the type of demand function of FAPs or not be aware of parameters within the demand function of FAPs. Furthermore, in Bertrand game, the service providers may not be able to see the price and profit of other service providers. In MLMF Stackelberg game, the leaders may not be aware of supply function of the followers. In cooperative game, the service providers may only reveal their profit to the others and not the details of their profit (e.g., the number of FAPs that they have served). Therefore the service providers need to learn about the behavior of other players through the history of network and also need to use learning to gradually reach the solution for the system.

Since the FAPs will not be directly involved in the process of price determination, they should send their requirements to the controllers. Therefore in each cluster, the FAPs submit their demands to the controller based on the required bandwidth of their users and the required bandwidth of MUEs served by the FAPs. The controllers unify the demand of different clusters together. Some solutions have been proposed for this problem in [82], [83], [84].

For the market equilibrium method, the stable price in the market is determined when the amount supply and demand become equal to each other. Therefore the price determination in the market can be carried out gradually through multiple steps to minimize the difference between spectrum supply and demand. In this case, at the initial step, each service provider

determines the price of spectrum i.e. $p_i[0]$ and announces it to the controllers. Based on their demand function and the announced price for the spectrum, the controller determines the amount of spectrum that the FAPs want to buy from each service provider i.e., $D_i(\boldsymbol{p}[0])$ and announces it to the service providers. Based on the difference between demand and supplied spectrum, the service providers determine the price of next step and announce it to the controllers. It is given by

$$p_i[t] = p_i[t-1] + \alpha(D_i(\boldsymbol{p}[t-1]) - S_i(p_i[t-1])) \tag{5.31}$$

where α is a learning coefficient. This process will be continued until the difference between supply and demand function will be zero.

In the Bertrand game, the service providers do not have access to the price of other service providers and they can only use local information and spectrum demands of FAPs. Therefore, in this case, each service provider initializes the price of spectrum at $p_i[0]$ and announces it to the FAPs. FAPs respond to the service providers with their required spectrum. The service providers should estimate the marginal profit function and, based on the received demand from FAPs, determine the price for the next step. Therefore, the price for the nest step can be determined as

$$p_i[t] = p_i[t-1] + \alpha\left(\frac{\partial R_i(\boldsymbol{p})}{\partial p_i}\right). \tag{5.32}$$

To calculate $\partial R_i(\boldsymbol{p})/\partial p_i$, service providers should track the change in demand of FAPs for small change in price. Therefore $\partial R_i(\boldsymbol{p})/\partial p_i$ can be calculated as

$$\frac{\partial R_i(\boldsymbol{p}[t])}{\partial p_i[t]} = \frac{R_i(..., p_i[t]+\varepsilon, ...) - R_i(..., p_i[t]-\varepsilon, ...)}{2\varepsilon} \tag{5.33}$$

where ε is a small number such as 10^{-3}.

In cooperative game, the total profit of all service providers should be maximized. In this case, the initial price is announced by each service provider to the controllers and they respond to these prices based on the demand of FAPs. Then the service providers communicate between each other to estimate the total marginal profit function and use this together with demands of FAPs to determine the price for the next step. Therefore the price in next step can be expressed as

$$p_i[t+1] = p_i[t] + \alpha \left(\frac{\partial \sum_{i=1}^{N} R_i(\boldsymbol{p}[t])}{\partial p_i[t]} \right). \tag{5.34}$$

To estimate $\partial \sum_{i=1}^{N} R_i(\boldsymbol{p}[t]) \big/ \partial p_i[t]$, similar to the Bertrand game, the service providers can observe the marginal total profit for a small variation in price i.e. ε.

$$\frac{\partial \sum_{i=1}^{N} R_i(\boldsymbol{p}[t])}{\partial p_i[t]} \approx \frac{\sum_{i=1}^{N} R_i(..., p_i[t]+\varepsilon, ...) - \sum_{i=1}^{N} R_i(..., p_i[t]-\varepsilon, ...)}{2\varepsilon}. \tag{5.35}$$

In MLMF Stackelberg game, there are two groups of service providers. The first group determine the price of spectrum sooner than the other group. In this case, the initial price $p_i[0]$ is announced by the leaders to the FAPs and then FAPs respond to the leaders with their demands. After this process, the followers announce their initial price to the controllers and get their responses for the spectrum demand. The price determination for the next step is the same as that in Bertrand Game with the difference that we have two steps in each round on information exchange. First, the leaders send their prices to the FAPs, get their responses, estimate the individual marginal function and select their prices for the next step, and then the followers do the same things as the leaders.

5.5 Simulation and Numerical Results

In the simulations, we consider a macrocell system with two MBSs from two different service providers. We assume that both of these MBSs are located at the center of a hexagonal cell. The total frequency of each MBS is 30 MHZ. The distribution of FAPs and MUEs are assumed to be CSPP with $\lambda_f = 10$, $\lambda_C = 5$, respectively. The FAPs are categorized in the groups based on their distances from each other. The spectrum efficiency values of both service providers M_1, M_2 are assumed between 2 to 5 bit/sec/Hz. The spectrum efficiency values of FUEs and MUEs A_i, k_i served by FAPs are assumed between 2 to 10 bit/sec/Hz. The required bandwidth of all FAP should be less than 80% of total bandwidth of each service provider. C_1, C_2 are assumed 0.7 and 0.8, respectively, which means that 0.7 and 0.8 of total selling bandwidth to FAPs are used for serving FUEs by FAPs. The substitutability coefficient for FAPs is assumed $v = 0.7$.

In Fig.5.3 we show the supply and demand function with respect to the offered price from the first service provider. We observe that the demand function is a decreasing function of offered price and the spectrum supply is an increasing function of offered price. The market equilibrium price is the point that supply and demand are equal together. It can be also shown that market equilibrium price exists or does not exist for some FAPs or MBS requirements.

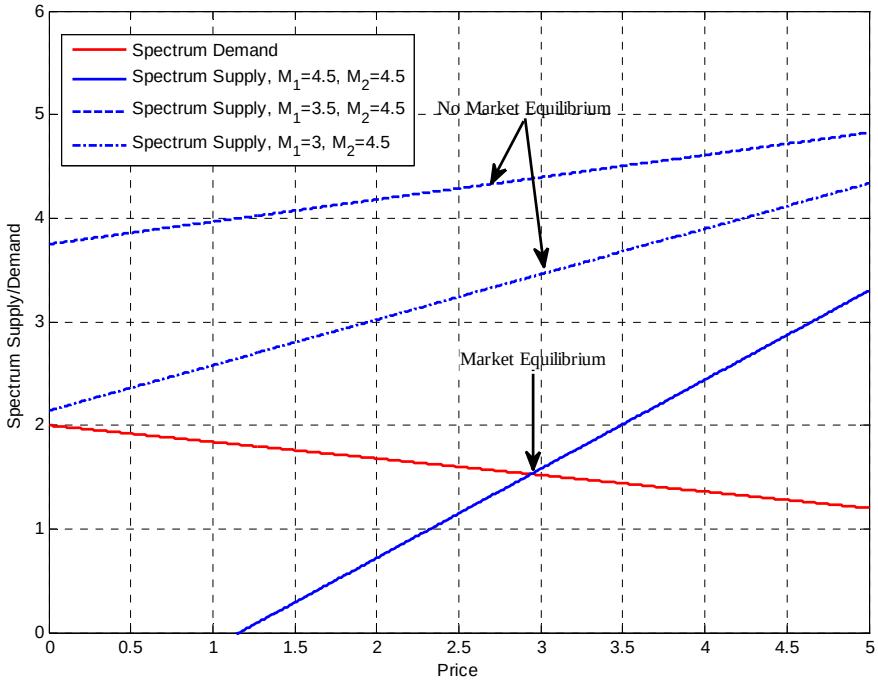


Figure 5.3 Supply and demand functions in terms of price for service provider 1.

Fig.5.4 shows the best response of the service providers in the case of Bertrand game and MLMF Stackelberg game. The intersection of best responses curves shows the Nash equilibrium for Bertrand game. The location of Nash equilibrium is dependent on the requirements of FAPs and MUEs as well as their demands. In the MLMF Stackelberg game, it is assumed that the first service provider is the leader and sets the price of spectrum sooner than the other second service provider. It can be observed that the Stackelberg equilibriums for two service providers are different from Nash equilibrium in Bertrand game. In the Stackelberg game, competition between service providers is less than Bertrand game and leaders tend to set higher prices than the Bertrand game, since the followers have to set their

prices after the leaders. Therefore, it can be seen that the prices in MLMF Stackelberg game are more than those in Bertrand game.

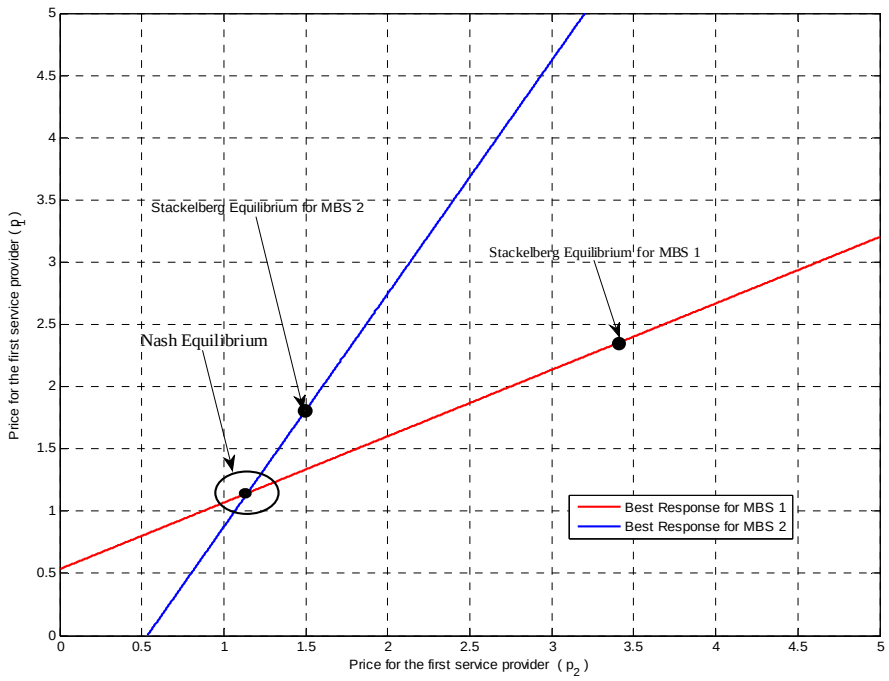


Figure 5.4 Bertrand Game and MLMF Stackelberg game

Fig.5.5 shows the best response of the service providers in the case of cooperative game. In a cooperative game, all the service providers cooperate with each other to maximize the total revenue; therefore the price for the spectrum is more that all previous cases. In application, to implement this type of pricing, service providers should communicate with each other to reach Nash equilibrium.

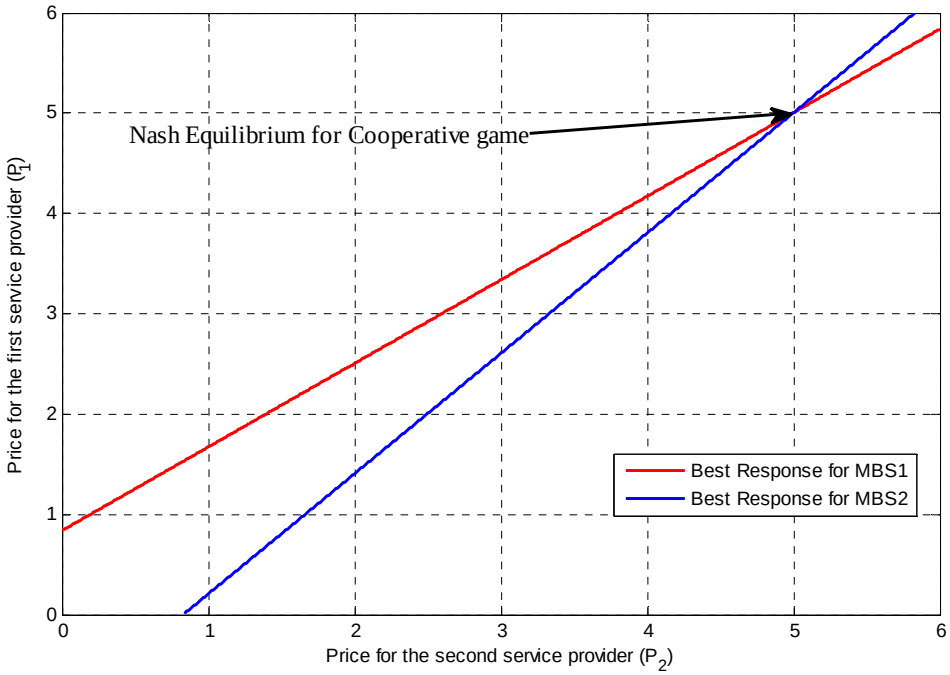


Figure 5.5 Nash Equilibrium for Cooperative game

Fig.5.6 shows the total revenue of service providers from FAPs based on fixed pricing [61], market equilibrium, Bertrand game, MLMF Stackelberg game and cooperative game for OA. It can be observed that our proposed methods outperform fixed pricing in terms of revenue for service provider. It can be seen that the total revenue of service providers in cooperative game is more than that in other methods. The total revenue of service providers in market equilibrium is less than other three methods in OA, since there is neither competition nor cooperation between service providers. In the market equilibrium scheme, the service providers are not aware of existence of each other and try to satisfy the demand from FAPs. In this scheme, the service providers try to maximize the satisfaction of FAPs. In the case of Bertrand game and Stackelberg game, the service providers are aware of the

competition between each other and try to set the unit price for spectrum to maximize their revenue. But in Stackelberg game, the per unit price is more than the Bertrand game, therefore the revenue of service providers are more than the Bertrand game.

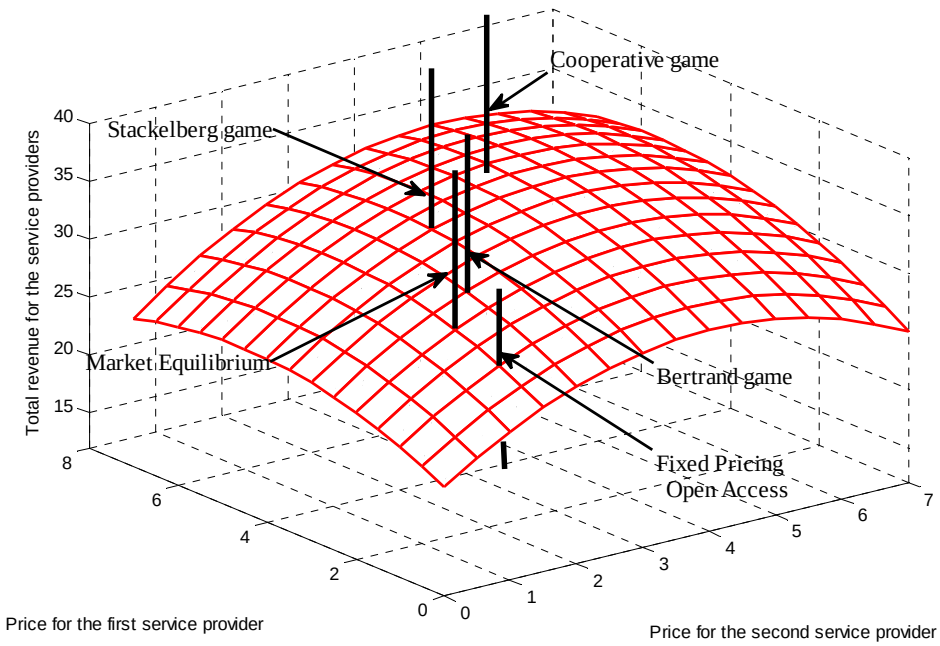


Figure 5.6 Total profit of service providers from different methods

In Fig.5.7, we investigate effect of spectrum substitutability (v) on the spectrum price. We set all specifications of two service providers the same as each other. For $v = 0$, the FAPs cannot change its operating spectrum but when $v = 1$, the FAPs can switch between service providers freely. It can be observed that with increasing v, the spectrum price for the first service provider decreases in all four proposed methods. It is also observed that the spectrum price has the highest value for all values of v under cooperative game and has the lowest price for the market equilibrium. For $v = 0$, the Bertrand game and MLMF

Stackelberg game have the same price as the cooperative game, since the FAPs cannot switch between service providers, therefore there is no competition between service providers. For $v = 1$, the level of competition between service providers are high and service providers should decrease the spectrum prices to attract more FAPs.

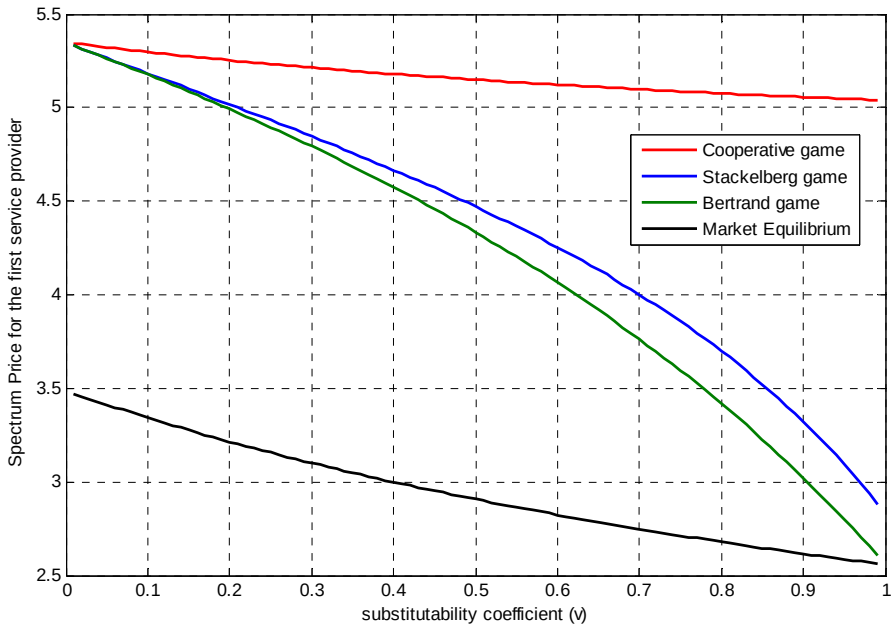


Figure 5.7 Spectrum price in terms of spectrum substitutability (v)

Chapter 6
Conclusions and Future Work

In this chapter, we first provide the conclusion of research work done so far, then provide some future research directions.

6.1 Conclusions

Cognitive radio and femtocell networks are two emerging concepts for innovative spectrum utilization. A common feature of cognitive radio and femtocells is their two-tier nature involving primary and secondary users. Two main challenges in two-tier networks is the design of efficient resource allocation and pricing methods which we have addressed in this thesis.

In Chapter 2, we have considered the spectrum trading problem for shared used model in cognitive radio networks. We have designed a multi-unit sequential sealed-bid first-price auction taking into accounts risks due to imperfect sensing. We have derived an expression for the total revenue of WSP and maximized it by optimizing the sensing time. Our numerical results have demonstrated that the proposed bidding strategy outperforms the conventional one (i.e., designed ignoring any risk) in a risky environment. For some typical values of probability of miss and number of SUs, it was found out that the payoff of bidders by our proposed method is *at least* two times and in best case, five times more than that can be obtained from the conventional one. Our results have further revealed that maximization of throughput is not necessary to get the maximum revenue.

In Chapter 3, we have investigated the sensing-throughput tradeoff in a cognitive radio network with cooperative sensing over time varying fading channels. First, we have derived an expression for the total average throughput of the SUs each of which is equipped with energy detectors. Then we have formulated a throughput optimization problem and solved

this based on a two-step algorithm. The first step involves the calculation of the optimal sensing time based on a non-linear programming problem. The second step involves finding the optimum number of SUs for each channel to maximize the total throughput. The numerical results demonstrate that our proposed method based on joint optimization of the sensing time and the number of sensing SUs for each channel over conventional cases than in which either the sensing time is fixed or the number of sensing SUs is fixed.

In Chapter 4, we have considered the spectrum trading problem for non-identical channels under the assumption of exclusive usage model in a cognitive radio network. We have proposed a realistic valuation function for the SUs which depends on delay-sensitive/-insensitive traffic as well as the capacity of each available channel. Instead of the commonly assumed single-minded bidders, we have assumed r-minded SU bidders and designed an efficient VCG-based auction mechanism. In our scheme, each bidder can bid for r bundles of channels in each round of auction, but each bidder is allowed to win at most one of these bundles. We have investigated two cases: In the first case, the SUs are r-minded but they can submit bid only for single channels. In the second case, the SUs are r-minded and they can submit bid for bundles of channels. We have shown that the first case is solvable in polynomial time but in the other one, the problem of determining auction outcomes is NP-hard. We have proposed two sub-optimal methods for solving this problem, namely greedy algorithm and randomized rounding linear programming (LP) relaxation. In the numerical results, we have demonstrated that the revenue of auctioneer increases with the increasing number of bidders and submitted bids. The average profit of bidders also increases with the increasing number of submitted bids when the number of bidders is around the number of channels. When the number of bidders is two times or more than number of channels, the average profit of bidders degrades with the increase in the number of submitted bids. Due to the sub-optimal nature of solutions in the second case, VCG mechanism is not truthful

anymore and the SUs can lie to maximize their utilities. To address this, we have proposed an auction mechanism with limited truthfulness property, based on an iterative greedy algorithm.

In Chapter 5, we have introduced pricing schemes for OA femtocell networks. Specifically, we have proposed four dynamic pricing schemes based on market equilibrium, Bertrand game, multiple-leader-multiple-follower Stackelberg game and cooperative game. In the first scheme, service providers are not aware of each other and try to satisfy the demand of FAPs. In this scheme, satisfaction and incentive of FAPs to participate in OA model are maximized. In Bertrand game, service providers compete with each other to determine the price of spectrum. In this scheme, the revenue of service providers are more and the satisfaction of FAPs are less than previous scheme. In multiple-leader-multiple-follower Stackelberg game, there are some service providers who enter femtocell market sooner that the others and they can set the price of spectrum before the others which they called leaders and other service providers called followers should set the spectrum price after them. In cooperative game, the service providers try to maximize the total revenue of all of them. We have compared the performance of our schemes and shown that the cooperative game outperforms the other methods in terms of revenue for service providers. On the other hand, we have demonstrated that the pricing scheme based on market equilibrium outperform other methods in terms of FAP satisfaction.

6.2 Future Works

Based on the discussion in Chapter 2, we have assumed that all available channels have the same risks, therefore same risks are taken account for all of them in optimal bidding calculation. In cases where the detection and false alarm probabilities are different for

underlying channels, these derivations need to be revisited. On the other hand, we have not considered the chance of PUs who enter the channels during the SUs transmission. This can be further considered as another practical constraint.

In auction mechanism, the SUs are selfish and they aim to maximize their profit. They may cheat or collude in the spectrum auction if profitable, which ruins the auction and therefore the auction mechanisms should be resistant to bidders collusion. In Chapter 3, we have proposed truthful auction mechanism for non-identical channel allocation. In truthful auction mechanism, the bidders prevent from lying about their valuation, but they can collude together to maximize their profit. Therefore it would a good idea to extend the auction mechanism in chapter 3 to a collusion resistant and truthful auction for non-identical channel allocation.

To ensure minimal impact on the performance of the existing MBSs, femtocell network needs to be designed with smart resource allocation and interference management strategies. As we mentioned in chapter 1, towards this overall purpose, game theory can be used as a natural design methodology for resource allocation in such an interference-limited environment where there are different rational entities with different types of demands. Two different game-theoretic approaches can be considered for different scenarios based on the availability of information exchange between two tiers in femtocell networks. Non-cooperative game-theoretic approach can be used for a deployment scenario where there is no information exchange between femtocell and macrocell tiers, but information exchange is allowed between nearby FAPs. Also a bargaining game formulation can be used in situations where we can assume some information exchange between femtocell and macrocell tiers along with the femtocell-to-femtocell information exchange.

Appendix A
Proof of Theorem 3.1

In this Appendix, we will show that three conditions in (3.7)-(3.9) are satisfied in our case. In the following, without loss of generality, we ignore index i for simplicity.

1) Condition in (2.7): For this condition, we need to show that the mean value of $|y(n)|^2$ takes finite values. Replacing (3.1) within, we have

$$\mathrm{E}\left[|y(n)|^2\right] = \mathrm{E}\left[|h(n)s(n)+u(n)|^2\right] = (\gamma+1)\sigma_u^2 \tag{A.1}$$

σ_h^2, σ_s^2 and σ_u^2 have limited values, therefore (2.7) is satisfied.

2) Condition in (3.8): For (3.8), we need to show that the variance of $|y(n)|^2$ takes finite values. Replacing (3.1) within, we have

$$\sigma_{|y|^2}^2 = \mathrm{Var}\left[|y(n)|^2\right] = \mathrm{E}\left[|h(n)|^4\right]\mathrm{E}\left[|s(n)|^4\right] + \mathrm{E}\left[|u(n)|^4\right] - (\sigma_h^2\sigma_s^2+\sigma_u^2)^2 . \tag{A.2}$$

In our case, $s_i(n)$ is a rectangular M-QAM modulation signal with zero mean and variance σ_s^2. Therefore, we have

$$\mathrm{E}\left[|s_i(n)|^4\right] = \frac{3}{10}\left(\frac{3\sqrt{M}-7}{M-1}\right)\sigma_s^4 .$$

(A.3)

Inserting $\mathrm{E}\left[|h_i(n)|^4\right] = 2\sigma_h^4$, $\mathrm{E}\left[|u_i(n)|^4\right] = 2\sigma_u^4$ and (A.3), we have

$$\sigma_{|y|^2}^2 = \sigma_u^4\left[\left(\frac{3\left(3\sqrt{M}-7\right)}{5\left(\sqrt{M}-1\right)}-1\right)\gamma^2+2\gamma+1\right]. \tag{A.4}$$

Since the SNR and noise power have limited values, it can be concluded that $\sigma_{|y|^2}^2 < \infty$. Therefore the second condition given by (2.8) is satisfied. It should be noted that this condition can be satisfied with any type of limited energy signal and limited noise power.

3) Condition in (3.9): We need to show that

$$\lim_{N\to\infty} N\mathrm{Var}[\Lambda(y)] = \sigma^2_{|y|^2} + 2\sum_{i=2}^{N}\mathrm{Cov}\left[\,|\,y(1)\,|^2, |\,y(i)\,|^2\right] = V \tag{A.5}$$

takes finite values. Since we have already shown that $\sigma^2_{|y|^2}$ is limited, it is sufficient to prove that the second summation term is finite. We first focus on a single term inside the summation, i.e.,

$$\begin{aligned}\mathrm{Cov}\left[\,|\,y(i)\,|^2, |\,y(j)\,|^2\right] &= \mathrm{E}\left[\,|\,h(i)s(i)+u(i)\,|^2 |\,h(j)s(j)+u(j)\,|^2\right] \\ &\quad - \mathrm{E}\left[\,|\,h(i)s(i)+u(i)\,|^2\right]\mathrm{E}\left[\,|\,h(j)s(j)+u(j)\,|^2\right]\end{aligned} \quad . \tag{A.6}$$

Since $s(n)$, $u(n)$, and $h(n)$ are assumed independent of each other and have zero mean, it is straightforward to show that (A.6) can be simplified as

$$\mathrm{Cov}\left[\,|\,y(i)\,|^2, |\,y(j)\,|^2\right] = \mathrm{E}\left[\,|\,s(i)\,|^2 |\,s(j)\,|^2\right]\left(\mathrm{E}\left[\,|\,h(i)\,|^2 |\,h(j)\,|^2\right] - \mathrm{E}\left[\,|\,h(i)\,|^2\right]\mathrm{E}\left[\,|\,h(j)\,|^2\right]\right). \tag{A.7}$$

Replacing $h(i) = h^R(i) + jh^I(i)$ in (A.7) and noting that $h^R(i), h^I(i)$ are independent from each other, (A.7) can be rewritten as

$$\mathrm{Cov}\left[\,|\,y(i)\,|^2, |\,y(j)\,|^2\right] = \sigma_s^4\sigma_h^4\rho^2_{(j-i)} \tag{A.8}$$

where $\rho_{(j-i)} = \mathrm{E}\left[h^R(i), h^R(j)\right] = \mathrm{E}\left[h^I(i), h^I(j)\right]$. Based on Parseval equality, we have $\sum_{n=1}^{\infty}\rho^2(n) = \int_{-\infty}^{\infty}|\,\mathrm{S}(f)\,|^2 df$, where $\mathrm{S}(f)$ is Doppler PSD function. The integration of Doppler PSD under Jakes model within the range of $\left[-f_d, f_d\right]$ yields infinity. However, in simulations, a truncated Jakes model is commonly used to avoid the singularities at the edges [101], [102]. Through truncation of the channel impulse response, we have

$$\int_{-f_d+\varepsilon}^{f_d-\varepsilon}\frac{1}{f_d}\frac{1}{\sqrt{1-\left(f/f_d\right)^2}}df < \infty, \quad \forall\varepsilon > 0\,. \tag{A.9}$$

Therefore, we can conclude that we have $\lim_{N\to\infty}\sum_{i=1}^{N}\sigma_s^4\sigma_h^4\rho_i^2 < \infty$ for the truncated version of Jakes model.

Appendix B
Proof of Theorem 3.2

In this Appendix, we show that three conditions in (3.25)-(3.27) are satisfied for majority logic rule, therefore $\tilde{R}^i(\tau_i)$ is unimodal in the range of $0 \le \tau_i \le T$. In the following, without loss of generality, we ignore index i in the following equations for simplicity.

1) Condition in (3.25): First we need to obtain $\partial\tilde{R}(\tau)/\partial\tau$ which is given as:

$$\partial\tilde{R}(\tau)/\partial\tau = -C_0 P(\mathrm{H}_0)\left(1-\mathbb{Q}_f(\tau)+(T-\tau)\left(1-\frac{\partial\mathbb{Q}_f(\tau)}{\partial\tau}\right)\right) \tag{B.1}$$

where $\partial\mathbb{Q}_f(\tau)/\partial\tau$ can be calculated as

$$\frac{\partial\mathbb{Q}_f(\tau)}{\partial\tau} = \lceil\tilde{k}/2\rceil\binom{\tilde{k}}{\lceil\tilde{k}/2\rceil}\frac{\partial P_f(\tau)}{\partial\tau}(1-P_f(\tau))^{\tilde{k}-\lceil\tilde{k}/2\rceil}\left(P_f(\tau)\right)^{\lceil\tilde{k}/2\rceil-1} \tag{B.2}$$

where

$$\partial P_f(\tau)/\partial\tau = -\frac{\beta}{\sqrt{8\pi\tau}}\exp\left(-\frac{\left(\alpha+\beta\sqrt{\tau}\right)^2}{2}\right) \tag{B.3}$$

$\beta = \gamma\sqrt{f_s}$ and,

$$\alpha = Q^{-1}(\bar{P}_d)\sqrt{\left(\frac{3\left(3\sqrt{M}-7\right)}{5\left(\sqrt{M}-1\right)}-1+2\sum_{i=1}^{N}\rho_i^2\right)\gamma^2+2\gamma+1}\,. \tag{B.4}$$

At $\tau = 0$, $\partial P_f(\tau)/\partial\tau = -\infty$, and hence $\partial\mathbb{Q}_f(\tau)/\partial\tau = -\infty$ for majority logic rule. Therefore $\partial\tilde{R}(\tau)/\partial\tau = 0$ and the first condition is satisfied.

2) Condition in (3.26): At $\tau = T$, we obtain $\partial\tilde{R}(\tau)/\partial\tau = -C_0 P(\mathrm{H}_0)\left(1-\mathbb{Q}_f(\tau)\right)$. For $0 \le P_f(\tau) \le 1$ $\mathbb{Q}_f(\tau) < 1$, therefore $\partial\tilde{R}(\tau)/\partial\tau < 0$ and the second condition is satisfied.

3) Condition in (3.27): Set $\partial\tilde{R}(\tau)/\partial\tau = 0$ and after simplifications we have

$$-2\ln\left(\left(\frac{\sqrt{8\pi\tau}}{\beta\tilde{k}\binom{\tilde{k}}{\tilde{k}/2}(T-\tau)}\right)^{\lceil k/2\rceil-1}\sum_{i=0}^{\lceil k/2\rceil-1}\binom{\tilde{k}}{i}(1-P_f(\tau))^{i-\lceil\tilde{k}/2\rceil+1}\left(P_f(\xi,\tau)\right)^{\lceil k/2\rceil-i}\right)=\left(\alpha+\beta\sqrt{\tau}\right)^2 \qquad \text{(B.5)}$$

Eq. (3.27) is satisfied if the right hand side (RHS) and the left hand side (LFS) of (B.4) intersect each other only once in the range of $0 \le \tau \le T$. It can be shown that RHS is monotonically decreasing for $\mathbb{R}_L = \left\{\tau \mid \left(\alpha+\beta\sqrt{\tau}\right) \le 0, 0 \le \tau \le T\right\}$ and monotonically increasing for $\mathbb{R}_H = \left\{\tau \mid \left(\alpha+\beta\sqrt{\tau}\right) > 0, 0 \le \tau \le T\right\}$. It can be further shown that LHS is monotonically decreasing for $0 \le \tau \le T$ and decreasing rate of LHS is faster than the RHS.

If RHS and LFS intersect in the $\mathbb{R}_L$ region, it is impossible for them to intersect more than once in $\mathbb{R}_L$ region because LHS is decreasing at a faster rate than RHS. Since RHS is increasing in $\mathbb{R}_H$ region, RHS and LHS cannot intersect with each other if they have intersected in $\mathbb{R}_L$ region.

If RHS and LHS do not intersect in the $\mathbb{R}_L$ region, they must intersect with each other in $\mathbb{R}_H$ region. Since RHS is monotonically increasing and LHS is monotonically decreasing in this region, therefore they can intersect only once with each other. Hence, we show that if LHS and RHS intersect with each other in either $\mathbb{R}_L$ or $\mathbb{R}_H$, they can intersect only once with each other and therefore (3.27) is satisfied.

Appendix C
Proof of Theorem 4.1

This appendix provides the proof of Theorem 4.1. Assume a hyper-graph $G = \left((B,W),E\right)$ that consists of two disjoint sets of vertices denoted by B and W. Here, B is the set of bidders that each bidder is represented by a vertex. W is a set of channels each of which is represented by a vertex. When a bidder wants to bid for a bundle of channels, we take into account an edge consisting the vertex of the bidder in set B and vertices of the channels in set W. For example, assume that there are 4 bidders and 3 channels for sale and each bidder can submit 2 bids, the bidding of bidders can be represented as illustrated in Fig. C.1. Determining the optimal allocation among bidders involves finding a collection of disjoint sets that has the maximum total weight. Now assume a set Z of winners with disjoint elements, the social welfare obtained by Z is exactly the size of this set. It follows that an independent set with the weight of at least q exists if and only if the social welfare of the optimal allocation is at least q. This concludes NP-hardness proof.

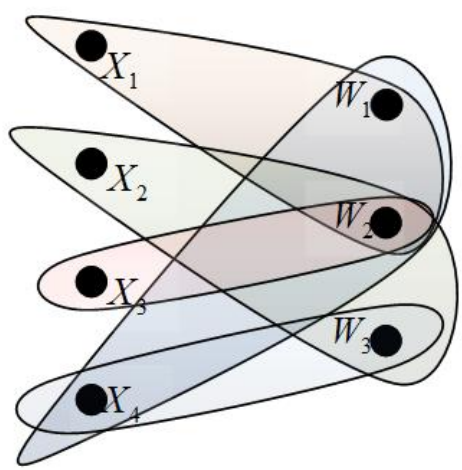


Fig. C.1 Modeling r-minded bidders in case 2

Appendix D
Proof of Theorem 4.2

This appendix provides the proof of Theorem 4.2. Assume that we model the auction mechanism with a hyper-graph consisting of $M+N$ vertices which they can be divided in two separate sets. The number of hyper-edges is rN and the maximum cardinality of each hyper-edge is m. Let W_i and $s_i = (|\mathrm{K}_i|-1)$ denote weight and the number of items in each hyper-edge. The norm of each hyper-edge is given by $n_i = W_i \big/ \sqrt{s_i}$. Assume that the optimum solution for this problem is $SWF_{opt} = \sum_{i \in opt} W_i$ and the solution of greedy algorithm is $SWF_{gr} = \sum_{i \in gr} W_i$. We need to show that the following inequality holds:

$$SWF_{opt} \le SWF_{gr} \min\left(m, \sqrt{M}\right) \quad . \tag{D.1}$$

First, we will prove $SWF_{opt} \le SWF_{gr}\sqrt{M}$, then show that $SWF_{opt} \le SWF_{gr} m$ holds. Without loss of generality, we assume that there is no common hyper-edge in optimum and greedy solutions. If there are any hyper-edges in common in two solutions, the common hyper-edge and vertices can be removed and the new problem becomes similar to the original one. By doing some algebraic computation for greedy solution, we have

$$SWF_{gr} \ge \sqrt{\sum_{i \in gr} W_i^2} = \sqrt{\sum_{i \in gr} n_i^2 s_i} \quad . \tag{D.2}$$

By applying the Cauchy-Schwarz inequality for optimum solution, we obtain

$$SWF_{opt} = \sum_{i \in opt} W_i = \sum_{i \in opt} n_i \sqrt{s_i} \le \sqrt{\sum_{i \in opt} n_i^2} \sqrt{\sum_{i \in opt} s_i} \quad . \tag{D.3}$$

Noting that $\sum_{i \in opt} S_i$ is the total number of items allocated optimally to the bidders and therefore it is bounded by M, we can conclude

$$SWF_{opt} \le \sqrt{\sum_{i \in opt} n_i^2} \sqrt{\sum_{i \in opt} s_i} \quad . \tag{D.4}$$

Now, we need to show that $\sum_{i \in opt} n_i^2 \le \sum_{i \in gr} n_i^2 s_i$. We have assumed that there is no joint bid between optimum and greedy algorithm. This means that if a hyper-edge or a bid i from optimal solution is considered within the execution of greedy algorithm, it cannot be entered in the partial allocation already built. This implies that there is an item that has been already allocated in the partial greedy solution. Therefore there is a bid j in greedy algorithm with $n_j \ge n_i$ and that item also belongs to bid j. In optimal solution, there are at most s_i different bids associated with bid j of greedy algorithm, since the sets of items requested by two different bids of optimal solution do not have any intersection. If we assume that op_j is the set of hyper-edges or bids of optimal solution that are associated with bid j, we can write

$$\sum_{i \in op_j} n_i^2 \le n_j^2 s_j \quad . \tag{D.5}$$

Therefore we can conclude $\sum_{i \in op} n_i^2 \le \sum_{i \in gr} n_i^2 s_i$ from (D.5) and the first part of theorem is proven.

For the second part, based on the above explanation, we conclude that $\sum_{i \in op_j} n_i \le n_j s_j$. From the definition of norm, we have $n_i = W_i \big/ \sqrt{s_i}$, therefore $\sum_{i \in op_j} n_i \le \sqrt{s_j} W_j$. The number of items in a bid is m at most, thus $m \le M$. By the same conclusion as in (D.5), we have $\sum_{i \in op} n_i \le \sqrt{m} \sum_{i \in gr} W_i$.

On the other hand, we have $SWF_{opt} = \sum_{i \in opt} W_i \le \sqrt{m} \sum_{i \in opt} n_i$ and $SWF_{opt} \big/ \sqrt{m} \le \sum_{i \in op} n_i$ therefore we conclude that $SWF_{opt} \le SWF_{gr} \min\left(m, \sqrt{M}\right)$. If $l \ne 1 \big/ \sqrt{2}$ it can be shown that the approximation algorithm will be $\max\left(1/m, 1/\sqrt{M}\right)$.

Appendix E
Solving Linear Equations in (5.9)

In this appendix, we want to solve set of linear equations in (5.9). (5.9) can be rewritten in matrix form as

$$\underbrace{\begin{bmatrix} X_1 - V_1 & Y_2 & \cdots & Y_N \\ Y_1 & X_2 - V_2 & \cdots & Y_N \\ \vdots & \vdots & \vdots & \vdots \\ Y_1 & Y_2 & \cdots & X_N - V_N \end{bmatrix}}_{\boldsymbol{A}} \underbrace{\begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_N \end{bmatrix}}_{\boldsymbol{p}} = \underbrace{\begin{bmatrix} l_1 - q_1 \\ l_2 - q_2 \\ \vdots \\ l_N - q_N \end{bmatrix}}_{\boldsymbol{Q}} . \tag{E.1}$$

Therefore to solve (E.1), first we should calculate $\boldsymbol{A}^{-1}$. There is a lemma in [103] for calculation of $\boldsymbol{A}^{-1}$.

Lemma: If we can write matrix $\boldsymbol{A} = \boldsymbol{G} + \boldsymbol{H}$ where $\boldsymbol{G}$ is non-singular and $\boldsymbol{H}$ has rank of one.

$$\boldsymbol{A}^{-1} = \left(\boldsymbol{G} + \boldsymbol{H}\right)^{-1} = \boldsymbol{G}^{-1} - \frac{1}{1+g}\boldsymbol{G}^{-1}\boldsymbol{H}\boldsymbol{G}^{-1} \tag{E.2}$$

where $g = tr(\boldsymbol{H}\boldsymbol{G}^{-1})$.

We can write $\boldsymbol{G}$ and $\boldsymbol{H}$ as

$$\boldsymbol{G} = \begin{bmatrix} X_1 - V_1 - Y_1 & 0 & \cdots & 0 \\ 0 & X_2 - V_2 - Y_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & X_N - V_N - Y_N \end{bmatrix}, \quad \boldsymbol{H} = \begin{bmatrix} Y_1 & Y_2 & \cdots & Y_N \\ Y_1 & Y_2 & \cdots & Y_N \\ \vdots & \vdots & \vdots & \vdots \\ Y_1 & Y_2 & \cdots & Y_N \end{bmatrix} . \tag{E.3}$$

Therefore $\boldsymbol{A}^{-1}$ can be calculated as $N \times N$ matrix which the elements on the diagonal of matrix are

$$a_{ii}^{-1} = \frac{1}{X_i - V_i - Y_i} - \frac{1}{1 + \sum_{i=1}^{N} \frac{Y_i}{X_i - V_i - Y_i}} \left(\frac{Y_i}{\left(X_i - V_i - Y_i\right)^2} \right) . \tag{E.4}$$

The other elements of $\boldsymbol{A}^{-1}$ are

$$a_{ij}^{-1} = -\frac{1}{1+\sum_{k=1}^{N}\frac{Y_k}{X_k - V_k - Y_k}}\left(\frac{Y_j}{\left(X_i - V_i - Y_i\right)\left(X_j - V_j - Y_j\right)}\right),\ i \neq j \tag{E.5}$$

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