Optimal Routing in Battery-Powered Vehicles

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

The increased interest in reducing greenhouse gas emissions has motivated renewed interest in electric vehicles technology as an alternative to current fossil-fuel based transportation equipment. Electric vehicles (EVs) are envisioned as a promising viable technology because of their friendly impact on the environment and higher efficiency over conventional vehicles that rely on fossil fuel. However, the EVs' limited battery capacity, resulting in limited cruising range and long recharging time, hinders the widespread adoption of EVs. An essential requirement of EV motors is the ability to operate with minimum energy consumption in order to provide at least the same driving range as their Internal Combustion Engine (ICE) counterparts. Energy-optimal routing, which aims to find the least energy consuming routes, under battery constraints has been recognized as a viable approach to prolonging the cruising range of the EV battery.

This thesis addresses the problem of optimal routing for EVs and proposes a solution to overcome the difficulties of optimal energy/time routing under battery constraints. A multi-criteria path-finding technique A^* is proposed. The proposed technique functions in two modes and solves the problem of optimal energy/time routing in EVs with worst time complexity of $O(n^2)$. First, an energy mode to solve the problem of energy-optimal routing under battery constraints is introduced. This mode computes the most energy-efficient route from a source to a destination, thus extending the limited cruising range of a battery. Second, a time mode to solve the problem of optimal travel time routing under battery constraints, by computing the most efficient travel-time route from a source to a destination, is proposed. An EV can operate under these two modes to strike a balance between power consumption and travel time so as to satisfy user constraints and needs.

In addition, a technique to reduce the effects of range anxiety on the vehicle operator is proposed. This technique computes a robust estimate of driving range. Furthermore, the technique analyzes an EV's battery capacity required by the vehicle in order to reach a charging station. The thesis reports experimental work conducted to test and validate the proposed techniques under various driving conditions.

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Dedication

This thesis is dedicated to the memory of my father. He was a very dear friend, and he will always be missed. I dedicate this work to his memory. I also dedicate this thesis to my mother. She is more than a mother. Her strength, endurance, character, friendliness, and love mean a lot to me. Thank you Mom for your sacrifices.

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Nomenclature

$G(V,E)$	Directed graph with V vertices and E edges
(a,b)	Road segment between nodes (a) and (b)
$u(a)$	Elevation of node (a)
$l(a,b)$	Length of the road segment (a,b)
P^k	A path with k vertices
$C_{\max}$	Battery maximum capacity
J	Battery charge level
U	Remaining free capacity
s	Source
t	Destination
$g(n)$	Real cost to reach node n
$h(n)$	Heuristic from node n to reach the destination
m	Mass of the electric vehicle
g	Gravitational acceleration factor
Δ	Amount of energy consumed or gained along a path
f_r	Friction coefficient
ρ	Air density coefficient
A	Cross sectional area of the vehicle
c_w	Air drag coefficient
$S(a,b)$	Average speed on the road segment (a,b)
r	Diameter of the vehicle's tire
g_r	Gear ratio of the vehicle
w_r	Angular velocity of the vehicle's tire
w_m	Angular velocity of the vehicle's motor

T	Torque of the motor
P	Power of the motor
t_A	Acceleration time
t_D	Deceleration time
C_{AE}	Acceleration energy cost function
C_{DE}	Deceleration energy cost function
C_{LE}	Loss of energy cost function
C_{PE}	Potential consumed energy cost function
C_{PG}	Potential gained energy cost function
C_E	Energy cost function
η	Efficiency factor
DS_{coeff}	Driving style coefficient
q	Traffic flow
k	Traffic concentration
t_s	Waiting time at traffic signals
C_t	Travel time cost function
C_{ED}	On-board electric devices energy cost function
$P_{ED(i)}$	Power drawn by the on-board electric device i
t_i	Time that the on-board electric device i takes in the status on
EM	Electric Motor
EVs	Electric Vehicles
ICs	Internal Combustion Engines
ICEVs	Internal Combustion Engine Vehicles
PHEVs	Plug-in Hybrid Electric Vehicles
PSS	Power Storage System
ZEP	Zero Energy Point

Chapter 1: Introduction

The negative effects of climate change are becoming increasingly obvious. The Intergovernmental Panel on Climate Change (IPCC) [50] has reported that in some 100 physical and 450 biological processes, scientists and researchers have addressed climate-induced changes. Over the years, global climate changes have become more extreme and severe, with increases in floods, storms, and heat waves [28]. A major contributor to global warming is fossil-fuel-powered vehicles. EVs could reduce the impacts of global warming and thereby provide a transportation system that is friendlier to the environment.

The increasing public desire for an alternative to fossil-fuel transportation systems is motivating renewed interest in EVs as means for reducing greenhouse gas emissions. To address this increased interest, most automobile manufacturers are planning to produce EVs or Plug-in Hybrid Electric Vehicles (PHEVs). EVs emit no tailpipe pollutants and hence can significantly reduce greenhouse gas emissions. The following section provides a brief introduction to EVs, illustrating the benefits and the challenges associated with them.

1.1 Electric Vehicles

Technically, an EV is defined as one that utilizes electricity as its power source and that can be charged through an electrical outlet at one’s place of residence or business. The main reasons for considering EVs as alternatives to fossil-fuel based vehicles constitute both economical and environmental factors. EVs, often referred to as Battery-Operated Electric Vehicles (BOEVs), emit no pollutants into the air since they are powered by electricity that is stored in a battery—charged with electricity generated by the same power plants that supply homes and businesses. Consequently, EVs are considered to be a perfect example of what is known as Zero-Emission Vehicles (ZEVs), since their motors create no tailpipe exhaust, fuel evaporation, fuel refining or any other greenhouse gas producing activity harmful to the environment. Moreover, EVs help create a much cleaner environment by consuming electricity that is generated partly or entirely by renewable energy sources such as solar, wind, tidal or nuclear. EVs operate with much fewer units of energy than conventional Internal Combustion Engine Vehicles (ICEVs) for the same mileage; therefore, EVs are generally associated with less cost to own and operate [48]. Figure 1.1 shows a modern sample of the Tesla Roadster electric car.

Figure 1.1: Tesla Roadster Electric Car.

EVs have three main components: an electric motor, a controller, and a battery. Figure 1.2 illustrates the main components of EVs. The first is an electric motor (EM). The most popular EMs are the 3-Phase AC and Series Wound DC motors, which are usually inexpensive and easily found. EMs have high torque at a wide range of speeds, with incredibly high efficiencies of up to 90 percent compared to conventional ICEs, which have efficiencies of only around 30 percent [48]. Hence, EVs are considered extremely suitable for stop-start urban driving. A portion of the EMs' efficiency comes from their ability to provide quieter and smoother operations as well as shorter acceleration/deceleration phases with less maintenance than their ICE counterparts. In addition to running extremely smoothly and quietly, EMs are more reliable than conventional ICEs that rely on fluids such as engine oil and transmission oil that are prone to leak, causing engine failure. EMs do not require the use of these fluids [48]. Another important feature of the EM is that there is only one moving part, unlike the ICE, which typically has several moving parts.

The second main component of the EVs is the controller. The controller's job is to deliver electric current from the battery to the motor, which is controlled by the accelerator pedal of the vehicle. Therefore, the further a driver presses the accelerator pedal down, the greater the power delivered to the motor and the greater the kinetic energy the vehicle gains. During idling phases, no electrical current is being processed, which means energy is not being used during idling phases.

The third main and final component of EVs is the battery. EVs use rechargeable batteries, occasionally referred to as Power Storage Systems (PSSs), which are different from the ignition or lighting batteries. An EV's battery is designed to maintain power for long times. There are mainly three types of rechargeable batteries currently in use in EVs: "lithium-ion batteries, lead-acid batteries, and nickel-metal-hydride batteries" [35]. Batteries for EVs are usually the most expensive part of the vehicle and are classified based on their power-to-weight ratio and energy density. For these reasons, smaller, lighter, and higher-efficiency batteries are desired, given that these battery types reduce the weight of the EV, improving its performance. An EV's battery must be periodically recharged from the power grid, which itself is powered by a variety of resources, such as coal, steam, solar, wind, or others, at home or using a street or business recharging point.

Figure 1.2 (a): Tesla Roadster lithium-ion Battery.

Figure 1.2 (b): Series Wound Electric Car DC Motor.

Figure 1.2 (c): Soliton Jr Controller 340V/500A.

1.2 Challenges and Solutions

EVs have limited battery capacity, resulting in limited cruising range; long charging time; high battery cost, resulting in a high total purchase price. These unique constraints lead to a number of challenges that must be addressed and problems that must be resolved before EVs can become a practical reality. EVs require special mechanisms to efficiently utilize their own limited energy supply (i.e., limited battery capacity). The following section summarizes some of these challenges and required solutions.

- The limited cruising range of their battery is a critically significant issue of EVs. Cruising range of the battery is defined as the distance that a vehicle can travel over time until the battery runs out of energy. Due to the limited energy supply and the difficulty of improving rechargeable batteries' lifetime, EVs have stringent power consumption requirements. The electric energy consumed by EVs depends on driving conditions, environment conditions, and the use of energy-consuming technology, and it is often higher than the amount expected by manufacturers. EVs can recuperate some of their consumed energy, i.e. they can regenerate energy during downhill and deceleration phases, thus extending their cruising range by approximately 20 percent in urban areas; however, the widespread use of EVs is still limited by a cruising range of only 150 to 200 kilometres for a single battery charge [3]. Therefore, energy-efficient mechanisms for energy-optimized driving and accurate driving-range estimation technique are significantly important requirements, strongly needed to, first, save energy thereby prolonging the limited cruising range of the batteries, and second, help estimate the actual remaining driving range and so prevent drivers from being stranded.

- Long charging time has been another significant barrier to popularizing the use of EVs. The long charging time of batteries creates a real problem: travelers have to plan ahead before proceeding en route, to ensure and accommodate enough time to recharge the EVs' batteries. The anticipated mass production of EVs in the near future and demand for long-distance travel indicate the strong need for an electric service infrastructure capable of providing the considerable amount of power in a time at least similar to that of conventional service infrastructure. Charging stations ought to be designed as such, with the capability to deal with a huge number of vehicles simultaneously and provide all required electrical charging in a short period of time. Other related ideas regarding rapid charging time and charging-station positioning have been addressed, including the establishment of battery-charging stations at home, at work, and in and around cities [56].

- Expensive and toxic batteries were one of the main drawbacks resulting in previous EV models failure to achieve any significant market share. The increasing production of rechargeable batteries could be an indication that the price of the key metals used in manufacturing is likely to remain stable or perhaps even increase. The improvements in battery technology have indicated the possibility of manufacturing EVs with enhanced performance and improved efficiency in comparison to the ICEVs; however, the high cost of giant battery packs makes the initial purchase price of EVs much higher. Furthermore, the electronic parts used in battery packs may increase in cost; therefore, the trade-off of the batteries' size, efficiency, lifetime, and price should be more carefully considered by manufacturers to find a more economical approach that aids in reducing the purchase price of EVs.

1.3 Motivation and Objective

One of the major and essential requirements for EVs is the ability to operate using less energy in order to provide at least the same range of driving as ICEVs. The unique characteristic of limited battery capacity in EVs demonstrates the necessity for achieving energy consumption reductions. Regenerating energy during downhill or deceleration phases can somewhat help prolong the cruising range. Energy-optimal routing has been proven to effectively prolong the driving-range by reducing energy consumption [3, 36, and 29]. Essentially, an optimal routing technique computes the most energy-efficient route among all possible routes from a start point to end point, in any road network. The battery of the EV has two main constraints: (1) a route cannot be used by the vehicle if it has an energy cost that is greater than the battery charge level and, (2) when the battery is already fully charged, then a route cannot be used if it has negative energy cost—gained energy from downhill or deceleration phases. Therefore, the energy-optimal routing technique for EVs should not be limited only to finding the cheapest route in terms of energy cost but should also take the battery constraints into account. The first main objective of this thesis is to propose a routing technique that can be employed in EVs to resolve the battery constraints problem and compute the most energy-efficient route in any road network. The limited driving-range of EVs can then be prolonged.

Travel time for a road trip is an extremely important and integral part of traffic information for drivers; hence, it has become important for traffic designers to accurately estimate travel time. Drivers need accurate travel time estimation in order to make better choices in their traveling and avoid unnecessary delays. When considering delays caused by congested traffic conditions, the optimal path in travel time may not be the shortest distance. Going further, the optimal path in travel time may perhaps be estimated when the travel time cost includes the accident risk instead of just congestion conditions. Obviously, this approach means that a shorter route may be more expensive than a longer one under congested, highly risky traffic conditions. For electrically-powered vehicles with battery constraints, it is not efficient for drivers to use a route that is less expensive in travel time cost but violates the constraints of the battery, because the battery-constraint issue is the crucial factor in reaching the final destination. Thus, for all trips, EV battery constraints must be taken into account and not violated, even when the driver's journey is travel-time-cost based and not energy-cost based. In fact, this consideration imposes an algorithmic challenge for travel-time-optimal routing in EVs. Therefore, the second main objective of this thesis is to propose a routing technique that can address the battery constraints issue while finding the most efficient path in terms of travel-time cost.

Range anxiety is a major barrier to the widespread use of EVs. This condition is defined as the concern of running out of energy and being stranded on the way, caused by the limited cruising range of the battery. Drivers need to be aware of how far they can travel, paying particular attention to the constraints associated with EV batteries. Because of these constraints, an accurate driving-range estimation technique is required in order to help alert drivers of their maximum driving range. A variety of additional techniques to reduce the effects of range anxiety on drivers to the lowest level are greatly needed. Therefore, the third main objective of this thesis is to propose a range-anxiety reduction technique that includes an accurate and reliable driving-range estimation to reduce driver concern.

1.4 Thesis Organization

This thesis is organized into seven chapters: Chapter 2 presents a literature review to provide the broad background necessary for a general understanding of the development and challenges related to EV technology. The particular background and previous work on optimal energy/time routing and driving-range estimation are also addressed. Chapter 3 considers the complexity and computational considerations in resolving the problem of optimal energy/time routing for EVs. Chapter 4 describes theoretical information and introduces the proposed solution to the problem of optimal energy/time routing for EVs. Chapter 5 reports the experimental work performed to validate the proposed algorithmic technique, including the simulation environment created and results. Chapter 6 introduces a range-anxiety reduction technique, including a robust driving-range estimation approach, as a solution to reduce the effects of range anxiety; experimental work conducted to validate the technique is also reported. Finally, concluding remarks and future work are presented in Chapter 7.

Chapter 2:

Background and Literature Review

This chapter provides background and a literature review on a broad range of EVs' development, challenges and problems. Firstly, it addresses a number of research studies conducted with regards to economical and environmental impacts of adopting EVs as an alternative to conventional vehicles that depend on fossil fuels. After which, issues and challenges associated with the development of EVs are discussed, with special attention given to recent research work completed on optimal routing and driving-range estimation, both of which improve efficient utilization of EVs on roads as well as support the technology behind EV success.

The economical and environmental impact of EV technology has been well researched. Recent studies demonstrate that the widespread adoption of EVs has significant economical advantages over the use of ICEVs, even though the initial purchase price of an EV is currently higher than that of a conventional vehicle. The Sustainable Energy Authority of Ireland has revealed in [48] that EVs have greater energy cost savings. EVs require far fewer units of energy than their ICEV counterparts; thus, reducing the cost to own and operate one through up to 70 percent lower fuel costs. In addition, the EM requires less maintenance because it has fewer moving parts, and is less likely to leak, because it has no fluids, such as engine oil. A broader study about the impact of EVs on the domestic economy of the US in [37], demonstrates that the total cost of EV ownership is $7,203 less than that of ICEV. Moreover, at 39 percent EV adoption, the net oil imports of the US will decline by about $20 Billion. The numerical results in this study prove that by 2013, EV operators will in total gain benefits of $80 Billion from savings due to less maintenance and reduced energy costs.

The technology of EVs promises to reduce the negative environmental effects of the current transportation system. Reportedly, the overall fuel emission reductions from the use of EVs in Ireland would be about 30 percent; this estimation could reach 100 percent reduction if the electricity consumed by EVs was supplied through renewable sources, such as wind, solar, tidal, or nuclear [48]. In [53], the WWF Climate Change and Energy Program of Canada developed a simulation model that considers different parameters, such as the average kilometers travelled and vehicle retirement rate with three scenarios for EV sale growth, in order to estimate the greenhouse gas emission reductions. This study proves that the short-term greenhouse gas reduction benefits from the use of EVs are low, but once EVs become widely used, the reduction becomes highly significant. Furthermore, under a scenario in which 12,000 EVs are on the roads by the end of 2012, carbon dioxide emissions would be reduced by around 1.3 and 6.7 mega tones per year by 2020 and 2025, respectively. Moreover, the electricity generation mixes of different provinces have a great effect on greenhouse gas reduction levels. For example, emission reduction would be greater in British Colombia, where the electricity is mostly generated from renewable resources, than in

Alberta, where coal is dominant. The results of this study prove that EVs can be a powerful contributor to greenhouse gas reductions over the long term.

In addition to the positive economical and environmental impacts that they offer, EVs operate with higher efficiency than ICEVs. Recent investigations [35, 48, and 49] confirm that the EM is more efficient than the best ICE. EMs may have efficiency of up to 90 percent, while the best conventional ICEs can have efficiency of only 30 percent. In addition, while the ICE has a small amount of torque at low *rpm*, providing a reasonably small amount of horsepower, the Tesla Roadster Incorporation [35] demonstrates that the Tesla Roadster EM delivers a huge amount of torque at zero *rpm*, providing almost the same torque up to 6,000 *rpm*; this EM is able to deliver a huge amount of power up to 13,500 *rpm*. Furthermore, the ICE requires a huge amount of horsepower in order to speed up hastily, resulting in poor gas mileage. Conversely, the high horsepower of an EM results in an efficient, quiet and smooth operation with the ability to accelerate and decelerate quickly. EMs also provide high reliability since they have lower waste heat output and less vibration.

2.1 Challenges and related issues

The introduction of EVs presents new challenges for road drivers. This section addresses problems that must be resolved before EVs can be widely used on roads. In particular, four major challenges hamper the growth of widespread use of EVs: limited driving range, long charging time, charging infrastructure [41], and battery durability concerns [40].

Limited battery capacity has restricted EVs' driving range to that possible on a single battery charge. Range anxiety, which is considered a major barrier to the successful use of EVs, is a term that captures drivers' concern about not reaching their destination and being stranded on the way. Range anxiety has emerged due to the technical constraints of rechargeable batteries used in EVs [10]. Driving style is only one of the factors that may affect the range of a battery charge. For instance, aggressive or high speed driving, road conditions, environment conditions, and the lifetime of batteries are all important factors that influence cruising range [34]. According to several studies [54, 52, and 10], range anxiety is the most important factor that hampers the penetration of EVs into the market. A survey conducted recently in the U.S. by Deloitte Global Services [15] reported that 90 percent of the people surveyed tend to travel around 75 miles a day. The same study reported that 63 percent of respondents expect the range to be 300 miles for a battery charge, which is not supported by current EV models. Eventually, public and workplace charging infrastructure installation may help reduce concerns about range anxiety. However, the public charging infrastructure required does not exist yet. The Tokyo Electric Power Company (TEPCO) predicts that EV operators will feel relaxed during their traveling once minimal but fast charging infrastructure is in place [2]. Some private and public stakeholders are convinced that a complete public and workplace charging infrastructure is necessary to reduce the impacts of range anxiety [41]. The energy density of their batteries may be the factor that determines the range of EVs. Their range might be limited to 160 to 190 miles for a single charge if there are no new advances and developments in battery technology [6]. In the future, lithium-ion batteries will probably use advanced technology that increases energy density [17].

In addition to limited driving range, EVs take a long time to recharge their batteries. Although a standard Level-1 charger (i.e., a 120-volt electrical outlet) is able to charge the battery of an EV, the charging time of around 17 hours is incredibly long for operators [41]. The battery may be fully charged overnight when most charging is expected, but many EV owners need a shorter charging time; therefore, some owners may need to install a Level-2 charger (i.e., a 240-volt electric outlet) at their homes. For instance, while the Nissan LEAF can use a portable 120-volt charger, most EV owners will likely prefer a 240-volt charger that can fully recharge the battery in less than 8 hours [18]. With a 240-charger, the Ford Focus Electric, powered by a lithium-ion battery, can fully recharge its battery in as little time as 3 to 4 hours [18]. However, 240-volt outlets may not be common in most houses and businesses [42]. Moreover, even when utilizing high voltage chargers, the time that EVs take to recharge is still longer than that taken by ICEVs to refuel, and therefore, charging can still be inconvenient for owners [27]. Swapping batteries at battery switching stations could eventually provide a better solution and thus overcome the problem of long recharging time [7].

The integration of EVs with the electrical grid faces the challenge of implementing a charging infrastructure, which is proposed to be concentrated in residential areas [12].

Several benefits can be obtained with residential charging infrastructure, such as charging during off-peak hours, when power is inexpensive, a timing that could sustain power network reliability [41]. Cost, time, and access are other challenges facing home charging stations. Consumers might need to use Level-2 charging stations, which may be expensive. Those living in multi-unit buildings may suffer more inconvenience, and the process could become complicated for them if they do not have reserved parking spots or are not authorized to access charging infrastructure [11]. The installation of residential charging infrastructure requires collaboration between governments and stakeholders to facilitate the process, reduce the cost, and develop solutions for multi-unit buildings. On the other hand, non-residential charging infrastructure may be necessary for the popularization of EVs. In addition to being beneficial for EV owners who do not utilize residential charging infrastructure, this infrastructure can help extend the daily range of driving [11]. EV operators might depend primarily on residential charging services, but augment these with a non-residential charging service, a joint approach that can be another key factor for further improvement [12]. Establishing non-residential charging infrastructure may be associated with several challenges, such as the effects on the power network when use grows [11]. To overcome the obstacles associated with non-residential charging service, research determining the highest charging demand and time of EV use is strongly needed. For example, public and private stakeholders could collaborate to integrate charging infrastructure networks and try to maximize the coverage they provide and access to them [41].

There are two ways of defining the lifetime of a battery: age of the battery in years or the point when the battery is no longer able to power the vehicle because of the charge-and discharge cycle number [40]. Three factors can be used to estimate the lifetime of a battery: temperature, charge rates, and depth of discharge swings [33]. Most manufacturers are now designing batteries with larger capacity to meet energy storage needs over the lifetime of an EV [41]. The size, weight, and cost of a battery are increased by larger capacity, but on the other hand, the efficiency of the battery is reduced [6]. Alternatively, manufacturers could install short-lifetime batteries but replace them every 5 to 7 years [41]. A battery leasing model, for instance, separates the vehicle lifetime from the battery lifetime and reduces the high initial price of purchasing an EV [6]. However, the U.S. Department of Energy estimates that manufacturers will have been capable of manufacturing batteries with a lifetime of approximately 14 years by the year 2015 [17].

2.2 Plug-in Hybrid Electric Vehicles (PHEV)

Plug-in Hybrid Electric Vehicles (PHEVs) differ from EVs in various ways, including overall cost, driving range, complexity, and battery pack size. PHEVs are introduced as a practical solution to the problem of the constrained PSS of EVs. PHEVs use both an ICE and an EM as an energy transformation medium and a battery with sufficient capacity to store the extra energy from the engine or regenerative breaking [26]. The battery powers the EM when needed, either to allow the engine to be turned off during some phases, such as at low speeds, or to provide auxiliary motive power to the engine. PHEVs offer drivers the chance to rely on the electricity sector while maintaining the driving range of ICEVs. Hence, they combine the efficiency advantages of hybridization by traveling part-time on electricity provided by the power grid. In addition, PHEVs are significantly important technology for reducing greenhouse gas emissions since they can operate on electricity for a limited distance, depending on their battery capacity. PHEVs have been marketed over the past decade in developed countries such as the U.S., and they have market penetration of around three percent worldwide, with more than 1.5 million PHEVs in use over the past decade [26]. The cost challenge of PHEVs is more complicated than that of EVs because they require an ICE with other associated components as well as a battery pack. The battery capacity required by PHEVs is less, and therefore, these vehicles have a lower cost battery pack than EVs [26]. PHEVs are able to overcome the phenomenon of range anxiety since they can run completely on gasoline if the battery runs out of energy [9].

2.3 Routing

Routing in general is defined as the process of computing routes in networks. Routing can be performed for multiple kinds of networks, including transportation networks, telephone networks, and electronic networks, such as the Internet. The optimization routing problem of vehicles can be defined as a combinatorial optimization process for finding the route, from a source to a destination, with minimal cost in a network. Traditionally, the focus was on finding the shortest paths in networks, with positive edge costs that represent distances between the start node and end node. In what follows, we address a brief overview of the most common traditional routing techniques, and then we proceed to recent studies that have been conducted on optimal routing under battery constraints in EVs.

The traditional shortest path problem has been broadly researched and studied. The best known static shortest path algorithm is Dijkstra's [16]. This algorithm, introduced by the Dutch computer scientist, Edsger Dijkastra, is a graph search technique for efficiently computing the shortest path between any two nodes in networks, with non-negative edge costs. Basically, when a source node is determined, the algorithm computes the most efficient route (e.g., the route with the shortest distance) from the source node to each other node in the graph. This algorithm may also be applied to compute the most efficient route from a known source to a known destination by terminating it when the optimal route to the destination is found. The performance of Dijkastra's algorithm using the array data structure achieves a running time of $O(n^2)$, and the binary heap achieves a running time of $O(m \log n)$, where m and n are the numbers of links and nodes respectively. Label-correcting algorithms with optimality condition are required if the network has some negative edge costs. These algorithms are able to change the labels of edge costs until all edge costs satisfy the optimality condition. The performance of such algorithms with a first-in-first-out (FIFO) queue achieves a running time of $O(mn)$.

2.3.1 Optimized Routing in EVs

EVs, powered by constrained rechargeable batteries, are expected to shape future traffic. Designing optimal energy/time routing algorithms has become an essential requirement for broader use of EVs. The EV battery constraints of limited battery capacity and regenerating energy during downhill and deceleration phases require novel routing techniques. The optimal energy/time routing therefore creates novel algorithmic challenges for navigation system designers and route planners since their aim is to compute the most energy/time-efficient routes rather than the fastest or shortest ones. The following paragraphs address recent research studies and existing work conducted on optimal EV routing.

Artmeier et al. [3], proposed certain shortest path techniques that tackle energy-optimal routing. They formalized efficient energy routing using constrained batteries as an example of the constrained shortest path problem (CSP) and also classified the battery constraints into hard and soft ones. They presented a shortest path algorithm that takes into account the battery constraints and solves the problem in a running time of $O(n^3)$ .They also showed that by an unfolding of a weighted routing graph, acceleration and deceleration cost values for road edges can be considered.

Jochen et al. [29] showed that the battery constraints of EVs are formed as cost functions on road segments satisfying the FIFO property, and thus, a Bellman-Ford algorithm can be used to solve the problem. They employed a result by Johnson [30] and some significant observations about Dijkstra [16] under non-constant edge costs to obtain an $O(n \log n + m)$ query time after an $O(nm)$ pre-processing phase for any road network weighted with energy edge costs. They also demonstrated that if the energy recuperation was induced in a very natural way, the pre-processing stage could be omitted.

Martin et al. [36] have proposed a solution to the problem of energy-optimal routing taking into account the battery constraints using a framework of the $A*$ search algorithm. They modified the $A*$ algorithm and showed that specific domain knowledge could be exploited to give rise to a heuristic to solve the problem in a running time of $O(n^2)$. To model the energy cost function of each road segment, they established two different types of energy: potential energy, which can be either consumed or recuperated, and loss of energy. The battery constraints were incorporated into the modified algorithm by adjusting them dynamically in order to compute the most efficient energy-based path during the search. Thus, they proved that the battery constraints could be tackled in the same way as other parameters given at query time.

Noticeably, optimal routing of EVs is different from the traditional routing that has no constraints associated with it. For EV energy/time-optimal routing, apparently the edge costs cannot be assumed to be distance values, so understanding edge costs as energy/time values and applying traditional techniques is not feasible. The battery constraints as well as the dependence of the graph weights on energy or travel time values have made routing of EVs more complex. Dijkstra's algorithm can be integrated with other pre-processing techniques such as contraction hierarchies [21], highway hierarchies [44], and transit vertex routing [5] to form state-of-the-art route planning. In addition to the exclusion of traditional techniques and algorithms from EV energy/time-optimal routing, some other techniques based on global graph analysis to eliminate negative weights [30] are not applicable because some parameters involved in the computations of energy/time cost functions are known only at query time. Bellman [8] has introduced a solution for finding the shortest path of weighted edges working with graphs with arbitrary weights. This solution, however, does not consider the EVs' battery constraints. While there are extensions of the shortest path problem to consider these constraints [31], these extensions are generally known to be NP-complete [20]. The recent studies that have been conducted to tackle optimal routing of EVs [3, 36, and 29] are still incomplete since all of them have focused on optimal routing that is based only on the energy cost aspect and neglected the problem of travel time-optimal routing under battery constraints. Thus, optimal routing based on travel time costs of road segments under battery constraints has not been addressed yet. In addition, time complexity matters in any solution proposed to the problem of EV routing. For example, the solution proposed by Artmeier et al. in [3] has a worst case time complexity of $O(n^3)$, which makes this solution not a preferred choice of navigation system designers and route planers. In fact, it is shown in [3] that a modified version of Bellman-Ford is able to solve the problem; however, Bellman-Ford can only ensure a running time of $O(mn)$. While the solution introduced by Martin et al. in [36] has a time complexity of $O(n^2)$, it is still incomplete. First, the energy cost function used to model road segment costs is also not complete. The energy cost function proposed by Martin et al. breaks down into potential energy, which can be consumed or recuperated along the path, and loss of energy due to aerodynamic and friction resistance. This energy cost function does not consider other energy forms that may occur along a certain path, such as the energies dissipated and recuperated during acceleration and deceleration phases or the dissipation of energy by on-board electric devices, such as air conditioners, radios, etc. Moreover, other factors that affect the energy consumption of EVs are not considered, such as the driving-style coefficient, which represents different styles of driving for different drivers. Second, the solution computed by the A^* algorithm here is not verified in its optimality. In other words, it is not proven that the solution obtained is optimal by satisfying the optimality conditions of the A^* algorithm. Finally, the solution introduced by Jochen et al. in [29] has a time complexity of $O(mn)$ and is dependent on a pre-processing phase for any road network. Techniques that are based on pre-processing and global graph analysis to eliminate negative weights cannot be applied since some parameters involved in the computations of energy/time cost functions are only known as real-time information at query time.

2.4 Driving Range Estimation in EVs

Driving-range estimation is becoming important to enhancing the popularization of EVs. To reduce drivers' concerns about range of driving, techniques to accurately estimate how far drivers can travel are needed. Only one research study, presented recently by Yuhe et al.

[55], tackles the problem of remaining driving range estimation as a solution to reduce the effects of range anxiety on drivers. This study proposes a telemetric basic service for EVs that is designed to provide an estimate of the remaining driving distance, classifying the process into rough range and precise range estimation. The rough range estimation is based on the maximum driving distance determined by the EV maker as well as the battery charge level and its maximum capacity. The approach starts by performing rough range estimation until the battery charge level reaches a preset threshold value that is determined in advance. Then, precise range estimation, which is based on computing the energy cost values of road segments, is performed and displayed to the user.

However, this approach is still incomplete for many reasons. First, the most important problem for EVs, battery constraints, is not addressed. Even when the EV is moving in a random manner without having a specific destination, the battery constraints must be taken into account. Second, the energy cost function used in the precise range estimation approach is not complete. For instance, the energy cost function does not integrate the acceleration and deceleration energies at traffic lights. Also, the loss of energy due to rolling resistance is not integrated into the energy cost function. Third, the study shows that precise range estimation is very expensive in terms of time and computing resources because the estimation process is performed through six stages, including a map-matching approach and shortest distance path finding approach, which also address inaccuracy in the process. Rough range estimation is generally less expensive; however, it is also considered to be inaccurate because it is based on the maximum driving distance specified by the EV maker. Therefore, this approach is generally not sufficient to reduce drivers' anxiety about being stranded. Reducing range anxiety may be achieved by some other strategy by which drivers are guaranteed to reach at least one charging station during their traveling, especially when they have no specific destination.

2.5 Summary

This chapter presented a broad background on EVs, including their environmental and economical impacts as well as the efficiency provided by the use of EVs on roads. It also briefly discussed challenges and related issues associated with the growth of EV technology and electrifying the transportation infrastructure, such as the limited cruising range of the battery, battery durability concerns, long charging time, and charging infrastructure. A brief background on PHEVs was presented, showing the difference between PHEVs and EVs. A brief background on traditional routing techniques and algorithms such as Dijkastra's algorithm was provided. Then, research work that aimed at optimized routing under battery constraints in EVs was reviewed. Finally, a concise background on driving-range estimation as a solution to reduce the effects of range anxiety was also presented.

It has been shown that despite the diversity of research in the area of optimal routing in EVs, existing work does not fully solve the problem of optimal EV routing; specifically, optimal routing of EVs has been limited to energy-optimal routing without representation of a complete energy cost function, while optimal travel time routing under battery constraints has not been addressed at all yet. Moreover, the work conducted to date on driving-range estimation has not resulted in an approach to fully reduce drivers' range anxiety.

Chapter 3:

Optimal Energy/Time Routing in EV

3.1 Introduction

Among the main barriers to widespread adoption of EVs is their driving range limitation caused by batteries. The limited cruising range between battery charges has become a fundamental obstacle for manufacturers wishing to broaden the adoption of EVs. The sensitivity of power consumption in EVs is critically important; hence, early studies in the area of optimal routing for EVs [3, 36, and 29] have focused on optimal routing that is based only on energy costs. Due to their limited capacity, batteries have two main constraints that cannot be violated while finding the optimal energy route. The first constraint is that a path cannot be used if it has an energy cost that exceeds the battery charge level. For the EV driver, taking such a path results in not reaching the destination and being stranded en route. The second constraint is that if the battery is fully charged, then any path with a negative energy cost cannot be used. Since the battery has only a limited capacity, storing energy from downhill or deceleration phases in the battery is only possible if there is sufficient free capacity, which means recuperation is no longer possible when the battery is fully charged. These two constraints demonstrate that the optimal energy routing problem of EVs is complex and requires an efficient algorithmic solution.

On the other hand, optimal travel-time routing under battery constraints in EVs has not been studied. Optimal travel-time routing in EVs is also important and is a major concern for drivers wishing to manage their trips conveniently. The estimation of travel-time should take into account congested conditions and accident risk factors so that drivers can avoid unexpected delays and take the most efficient travel-time based routes to their destinations. The battery constraints must not be violated in the energy and travel-time aspects. Obviously, there is a necessity to resolve the battery constraints problem for optimal energy routing as doing so will allow EVs to extend the cruising range of the battery. However, the battery constraints problem must also be resolved when the optimal routing is travel-time based in order that an EV driver is certain to reach his/her destination safely. In other words, to prevent the driver from being stranded during or at any point of travel, the battery constraints problem must be resolved while finding the travel-time route.

3.2 Energy-Optimal Path Problem under Battery Constraints

Assume a directed graph is given, $G = (V, E)$ having $|V| = n$ and $|E| = m$, to represent a road network, where vertices $v \in V$ represent points, and edges $e \in E$ represent connections between these points corresponding to road segments. Assume for each vertex, an elevation $u : v \rightarrow R_0^+$ is given, and for each edge, a length $l : E \rightarrow R^+$ and a speed limit $S{:}E \rightarrow N$ are given. A path P can be defined as a sequence of k vertices $(v_1, v_2, ..., v_k)$, and the edge is two vertices $(v_i, v_{i+1}) \in E$ with $i = 1, 2, ..., k-1$. Assume that the vehicle is traveling on a route at the average speed of each road segment, and that when it transits from one segment (v_i, v_{i+1}) to the following one (v_{i+1}, v_{i+2}), it adapts to the new higher or lower speed. Making this setting, consider the graph depicted in Figure 3.1 as a simple theoretical example to explain the complexity of resolving the energy-optimal routing problem under battery constraints. To do so, we define the following parameters: the battery maximum capacity, $C_{\max} \in \mathbb{R}^+$, the battery charge level, $J \in \mathbb{R}^+$, where $J \le C_{\max}$, and the remaining free capacity of the battery, $U \in \mathbb{R}_0^+$, where $U = C_{\max} - J$.

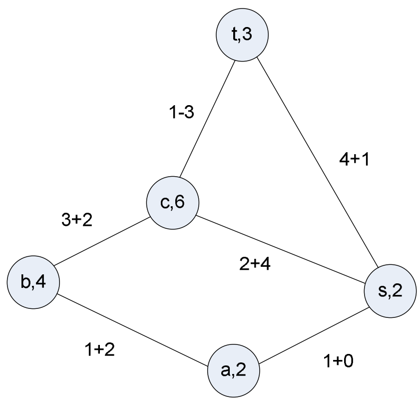

Figure 3.1: Simple Example of Energy-optimal Path.

Given the graph depicted in Figure 3.1 above as an energy weighted graph $G\ (V, E, c_E)$, two vertices $s, t \in V$, an initial charge level J, and a maximum battery capacity $C_{\max}$, the energy-optimal routing problem is determining a path, P in G, from source s to destination t with minimal energy cost. The energy-optimal routes here correspond to paths that are feasible—ones that satisfy the battery constraints—and where the remaining battery charge at the end of the path is maximal, or equivalently, where the remaining free capacity of the battery is minimal. Each vertex u in the graph has an elevation $z(u)$ resulting in a potential energy $E_P(z(u)....)$, which is shown next to the label of each vertex. For instance, the potential energy of vertex b is 4 and vertex a is 2, meaning that the total energy cost $c_E(a,b)$ of the edge (a,b) can be computed as $1+(4-2)=3$.

Considering the two battery constraints of limited energy supply and the ability to recuperate energy into the available free capacity, assume a fully charged battery at s with $J=C_{\max}=5$. Now if the battery constraints are not considered, then the energy-optimal path is (s,c,t), with a total energy cost of 4 energy units. However, it is not possible for the EV to travel over the road segment (s,c) under the effect of battery constraints because it would require 6 energy units. Therefore, the energy-optimal path under battery constraints is (s,t), with energy costs of 5 units. Analogously, assume the EV starts at vertex c with $C_{\max}=5$ and $J=4$. The road segment (c,t) offers a negative energy cost $c_E(c,t)=-2$ units, but the battery in this case can store only $C_{\max}-J=5-4=1$ units. Resolving the battery constraints is extremely important, and necessitates flexibility in the search technique required to solve the problem. Hence, the algorithmic solution posed to the energy-optimal routing problem must carefully take into account the battery constraints while computing the energy-optimal path in any road network.

3.3 Travel Time-Optimal Path Problem under Battery Constraints

The optimal travel-time path problem corresponds to computing a path, P, in a graph, G, from source s to destination t with minimal travel-time cost. Although the optimal routing problem here is travel-time based and not energy based, the battery constraints must not be violated; otherwise, reaching the driver's destination is not possible. The optimal travel-time routes here correspond to paths that are feasible—ones that satisfy the battery constraints—and where the time spent travelling from source s to destination t is minimal. In this case, the energy cost along the path is not necessarily minimal since the major concern for drivers is the optimality of travel-time cost. Rather, the energy cost of a travel-time-optimal path is the accumulated energy along the path.

Let us consider the weighted graph depicted in Figure 3.2 as a simple theoretical example for studying the travel-time-optimal path problem under battery constraints. Given this graph as a travel-time and energy weighted graph $G\ (V, E, c_E, c_t)$, two vertices $s, t \in V$, an initial charge level J and a maximum battery capacity $C_{\max}$, the optimal travel-time routing problem corresponds to finding a path, P in G, from source s to destination t with minimal travel-time cost. Let us assume that weights in blue represent energy costs, while weights in black represent travel-time costs, including congestion conditions and accident risk.

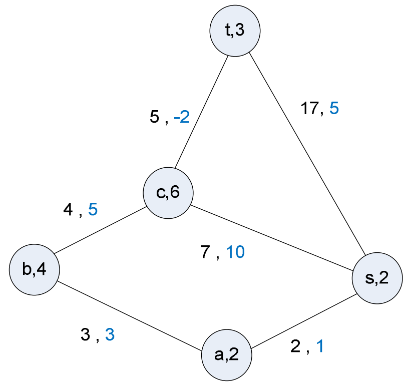

Figure 3.2: Simple Example of Travel Time-optimal Path.

Taking into account the two battery constraints of limited energy supply and the ability to recuperate energy into the available free capacity, assume a fully charged battery at s with $J = C_{\max} = 9$ energy units. Even though the path (s,c,t) is the most efficient travel-time one from source s to destination t, it is not possible for the EV to travel over this path because the road segment (s,c) requires 10 energy units, which exceeds the battery charge level, and such a path must not be selected by the path-finding technique during the search process. Therefore, the travel-time-optimal path under battery constraints is (s,a,b,c,t), with total energy costs of 7 units and total travel-time costs of 14 units. The path (s,t) is not considered optimal here despite the fact that it has less energy costs, because it has higher travel-time costs. Hence, the algorithmic solution required to resolve the travel-time-optimal path problem must carefully take into account the battery constraints and not violate them while computing the optimal path.

3.4 Computational Considerations

The complexity of solving problems by searching and, in particular, finding optimal solutions in graph networks is measured using what is known as asymptotic complexity, that is, $O(\ )$ notation and NP-completeness. A number of criteria can be used to measure the performance of any algorithmic solution to the problem of finding optimal routes for EVs; The performance of the search technique can be assessed based on four criteria: completeness—defined as the assurance of obtaining a solution if there is one; optimality—describing the verification of the search technique in finding the optimal solution (one may verify whether the optimality conditions of the search technique are satisfied so that the solution found is guaranteed to be optimal); time complexity—defined as the time that the search technique takes in finding a solution; and space complexity—defined as the memory space required to find a solution and finish the search. Because the problem of optimal routing under battery constraints in EVs is generally classified to be NP-complete [20], the following chapter introduces the solution proposed in this thesis to solve the problem. The posed solution relies on a framework for a multi-criteria routing technique that uses a heuristic function during its search to solve the problem in polynomial time.

3.5 Summary

This chapter has addressed the problem of optimal energy/time routing under battery constraints in EVs. The effect of battery constraints on the optimal routing process with respect to energy and travel-time aspects was discussed. Thus, the chapter stressed that the algorithmic solution posed to solve the problem of optimal energy/time routing in EVs must carefully take into account the battery constraints and not violate them while computing the most efficient routes. To this end, the complexity and computational considerations of the problem were briefly discussed.

Chapter 4:

Optimal Path Finding: A Multi-criteria Model

4.1 Introduction

This chapter presents the posed solution to the problem of optimal energy/time routing under battery constraints in EVs. The recommended solution considers not only energy, but rather it provides drivers with more freedom and convenience by including travel time in finding optimal routes. The proposed technique is a multi-criteria model within a framework of the A^* search technique that relies on a heuristic function during its search, and thus the problem is solved with a worst case time complexity of $O(n^2)$. This model functions in two modes: the A^* search algorithm has been modified such that it can be run for both energy and travel-time modes based on drivers' needs. To give drivers more freedom to plan their trips, two separate algorithms have been created by developing one modified A^* algorithm for each mode. The problem of battery constraints is solved by dynamically adjusting the energy cost function in the algorithm during the search process.

For the energy mode, the modified algorithm is called *Energy Mode* A^* *Algorithm;* it takes the battery constraints into account and excludes the optimality of travel time, thus computing the most energy-efficient path among all possible paths. For the travel-time mode, the modified algorithm is called *Time Mode* A^* *Algorithm;* it takes the battery constraints into account and excludes the optimality of energy, computing the most travel-time-efficient path among all possible paths. The battery constraints are taken into account by the both modified A^* algorithms and not violated in either; consequently, drivers have assurance of reaching their destinations if they use the optimal solution.

4.2 A-Star Search Technique

A^* Search, pronounced "A-star search", is one of the most commonly known forms of best-first search. The A^* algorithm has two main significant properties. First, if a route exists, it returns the most efficient route from a given source to a given destination. Second, A^* uses a heuristic function (i.e., an estimate) to search nodes that are considered more likely to have the cheapest cost, which allows one to obtain the optimal route without searching the whole network. A^* Algorithm generates two node lists: a closed list, which contains all the nodes that the algorithm has explored so far, and an open list, which has all the nodes that the algorithm is currently working on. In order to evaluate nodes, A^* Algorithm combines $g(n)$, the real cost value to arrive at node n, and $h(n)$, the heuristic (i.e., the estimated cost value) from node n to the destination.

$$f(n) = g(n) + h(n) \qquad (1)$$

In Equation (1), $f(n)$ is the most efficient solution-estimated cost through node n. A^* Algorithm states that if one wants to compute the optimal path cost, then the node having the smallest value of $f(n)$ is the best choice. This strategy is more than appropriate: it is reported in [47] that if the optimality conditions of A^* are satisfied by $h(n)$, then A^* technique is both complete and optimal.

4.2.1 Heuristics

The performance of A^* search is critically dependent upon selecting an appropriate heuristic. A^* Search is ideal in its performance when the heuristic $h(n)$ equals the actual path cost. If $h(n)$ is chosen to be equal to the actual cost of reaching the destination through node n, then A^* follows only the most efficient path and never investigates nodes that are not in the solution. The actual cost of the path is generally not known, and obtaining it is the reason for running a path-finding technique. If $h(n)$ is chosen to be greater than the real cost of the path, then A^* can be faster but is less accurate in finding the solution; as a result, it is no longer guaranteed that the solution found is optimal. Therefore, $h(n)$ must never be greater than the actual path cost.

4.2.2 Conditions for Optimality

A^* Search requires two conditions for optimality. The first is to have $h(n)$ be an admissible heuristic, one that does not overvalue the true path cost. As defined previously, $g(n)$ is the real cost to arrive at node n, and we have $f(n)$ as stated in Equation (1); therefore, for $h(n)$ to be an admissible heuristic, $f(n)$ must not overestimate the true cost of a path to arrive at node n. ”Admissible heuristics are by nature optimistic because they think the cost of solving the problem is less than it actually is" [47]. Since a straight line is always the shortest distance between any two nodes, and it can never be overestimated, it is considered a simple example of an admissible heuristic. The second condition, which is much stronger and a more important condition for optimality, is consistency. If a heuristic $h(n)$ satisfies the condition stated in Equation (2) below, then it is considered a consistent heuristic. In Equation (2), $h(n)$ is the heuristic from node n to arrive at the destination, $c(n,n')$ is the true cost between node n and the following node n', and $h(n')$ is the heuristic of travelling from n' to arrive at the destination.

$$h(n) \le c(n,n') + h(n') \qquad (2)$$

4.2.3 Optimality of A^*

For the solution provided by A^* to be optimal, the following properties have to be satisfied: "the tree-search version of A^* is optimal if $h(n)$ is admissible, while the graph-search version is optimal if $h(n)$ is consistent" [47]. This research pays more attention to the second of these two properties because, as stated in [47], the consistency of a heuristic $h(n)$ implies that $h(n)$ is admissible. Therefore, the following claim is established: "*if $h(n)$ is consistent, then the values of $f(n')$ along any path are non-decreasing*" [47]. This claim has been proven in [47]. For the convenience of the reader, the proof is presented as follows: let us assume that n' is a following node of n, then

$$g(n') = g(n) + c(n,n') \qquad (3)$$

$$f(n') = g(n') + h(n') = g(n) + c(n, n') + h(n') \geq g(n) + h(n) = f(n) \tag{4}$$

4.3 Energy Mode

The A^* search algorithm for this mode will compute the most efficient path in terms of energy cost among all possible paths. Working on stochastic networks, the algorithm is modified to satisfy the battery constraints. The first constraint of the battery is that a path can no longer be used if it has an energy cost that is greater than the battery charge level. This constraint problem seems to be more significant since ensuring that the driver is not stranded en route is extremely important. We propose that this constraint problem be solved by turning the path energy cost value into infinity, thus excluding the path from the search process.

The second battery constraint, which matters only with edges having negative energy costs, is that recuperation, that is, gaining energy from downhill edges and during deceleration phases, is not possible if the battery is already fully charged. For this constraint problem, we propose dynamically adjusting the energy cost function in the A^* algorithm such that the energy gained from downhill edges and during deceleration phases is stored in the available free capacity of the battery until the battery is full. The rest of any energy gained is lost. If this process is implemented along the path whenever there is recuperated energy, this energy can be made use of, extending the cruising range of the battery.

Assume a directed graph is given, $G = (V, E)$ having $|V| = n$ and $|E| = m$, to represent a road network, where vertices $v \in V$ represent points, and edges $e \in E$ represent connections between these points corresponding to road segments. Assume that for each vertex an elevation $u : v \to R_0^+$ is given, and for each edge a length $l : E \to R^+$ and a speed limit $S{:}E \to N$ are given. A path P is defined as a sequence of k vertices $(v_1, v_2, ..., v_k)$, and the edge is two vertices $(v_i, v_{i+1}) \in E$ with $i = 1, 2, ..., k-1$. When making this setting, we consider the amount of energy needed to travel through a path in the network as well as give an idea about how the battery constraints can be modeled; therefore, the following parameters are defined: $C_{\max}$ is the maximum capacity of the battery, J is the charge level of the battery where $J \leq C_{\max}$, U is the remaining free capacity of the battery, where $U = C_{\max} - J$, and Δ^k is the amount of energy consumed or gained along a path. We consider different forms of energy costs that can occur from taking a path $P^k = (v_1, v_2, ..., v_k)$ as follows:

Potential Consumed Energy

We define a function $E_p(u(a))$ that represents an elevation of a vertex (a). When the EV travels over an edge (a,b), the potential energy $E_p(a,b) = u(b) - u(a)$ is consumed or drawn from the battery only if the EV is going uphill $(i.e.,\ u(b) > u(a))$. Then, the energy cost of an edge (a,b) induced by the potential consumed energy function is defined to be

$$C_{PC}(a,b) = \frac{1}{\eta_c}[mg(u(b) - u(a))] \tag{5}$$

where, m is the mass of the vehicle, including payload, g is the gravitational acceleration factor, and η_c is the efficiency factor. This potential consumed energy on the road segment (a,b) takes the following values:

$$C_{PC}(a,b)=\begin{cases}0 & if \quad k=1\\ 0 & if \quad k>1, \qquad \Delta^k<0\\ \Delta^k & if \quad k>1, \quad 0\le\Delta^k\le J\\ \infty & if \quad k>1, \qquad \Delta^k>J\end{cases}$$

Potential Gained Energy

We define a function $E_G(u(a))$ that represents an elevation of a vertex (a). When the EV travels over an edge (a,b), the gained energy $E_G(a,b)=u(a)-u(b)$ is regenerated and stored in the battery only if the EV is going downhill $(i.e,\ u(a)>u(b))$. During downhill phases, the motor can be turned by the wheels acting as a generator to recharge the battery. Then, the energy cost of an edge (a,b) induced by the potential gained energy function is defined to be

$$C_{PG}(a,b)=\eta_r[mg(u(a)-u(b))] \tag{6}$$

This potential gained energy is stored in the battery and is lost only if the battery is fully charged. It takes the following values:

$$C_{PG}(a,b)=\begin{cases}0 & if \quad k=1\\ 0 & if \quad k>1, \quad 0\le\Delta^k\le J\\ -\Delta^k & if \quad k>1, \quad 0>\Delta^k\le U\\ \Delta^k & if \quad k>1, \quad 0>\Delta^k>U\end{cases}$$

Note: When the road segment is a flat surface, meaning that the start node and the end node of the edge *(a, b)* have equal elevations, then the potential energy is negligible.

Loss of Energy

Due to the aerodynamic and rolling resistances, we define a function $E_L(l(e), s(e))$ that models loss of energy to the environment. When the EV travels over an edge (a,b), the loss of energy occurs even if the vehicle is going downhill, which means this energy cost value cannot be recuperated. However, the aerodynamic resistance component, which is the right hand side of Equation (7), is not dissipated and has a value of zero if and only if two conditions are satisfied: first, the wind is in the same direction as the vehicle, and second, the wind has a speed that is greater than or equal to the speed of the vehicle.

$$C_{LE}(a,b) = \frac{1}{\eta_c}[f_r mgl(a,b) + \frac{1}{2}\rho A c_w S(a,b)^2 l(a,b)] \qquad (7)$$

where f_r is the friction coefficient, ρ is the air density coefficient, A is the vehicle's cross sectional area, c_w is the air drag coefficient, S is the average speed on the edge *(a, b)*, and l is the length of the edge *(a, b)*. This loss of energy on the road segment (a,b) takes the following values:

$$C_{LE}(a,b) = \begin{cases} 0 & if \quad k = 1 \\ \Delta^k & if \quad k > 1, \qquad \Delta^k > 0 \\ \Delta^k & if \quad k > 1, \qquad \Delta^k < 0 \\ \infty & if \quad k > 1, \qquad \Delta^k > J \end{cases}$$

Acceleration and Deceleration Energy

We define a function $E_{a,d}(P, t_{a,d}(e))$ that models energy consumption due to mechanical loss, which consists of acceleration energy required to bring the EV back up to the average speed, and then deceleration energy used to bring the vehicle to a stop. The energy dissipated by the EV during idling is zero. When the accelerator pedal is pressed down, electric current flows from the battery to the motor to turn the vehicle's wheels. This energy cannot be recuperated, and it is spent only during acceleration phases. However, when the driver's foot comes off the accelerator pedal, the motor can still be turned by the wheels acting as a generator to recharge the battery [55]. If the EV has a tire with diameter r, then the angular velocity of each tire is

$$w_r = \frac{S}{r} \tag{8}$$

where S is the linear speed (i.e., average speed on the edge in m/s), r is the diameter of the tire in m, and w_r is the angular velocity of the tire in rad/s. If the EV has a gear ratio of g_r, then the angular velocity and power of the motor are as follows:

$$w_m = w_r \; g_r \tag{9}$$

$$P = T \; w_m \tag{10}$$

where T is the torque in $N.m$, w_m is the angular velocity of the motor in rad/s, and P is the power of the motor in $watts$. Thus, the energies dissipated and recuperated during acceleration and deceleration phases on the road segment (a,b) are

$$C_{AE}(a,b) = \frac{1}{\eta_c} P\ t_A \qquad (11)$$

$$C_{DE}(a,b) = \eta_r\ P\ t_D \qquad (12)$$

where t_A is the time that the vehicle takes to accelerate back up to the average speed, and t_D is the time that the vehicle takes to come to a complete stop.

Driving Style

In addition to the four forms of energy represented above, we also consider a driving style coefficient to represent different styles of driving. After calculating the four energy forms of road segment (a,b), the total energy cost function on a road segment (a,b) is multiplied by the driving style coefficient. For normal driving, the driving style coefficient may take a value of 1, and for aggressive driving, the coefficient may take the value 1.2. The driving style type may be set by drivers according to their driving behaviors.

On-Board Electric Devices

We define an energy form for the energy consumed by EVs' on-board electric devices, such as air-conditioners, windshield wipers, etc. This type of consumed energy is determined by the power drawn by the electric device, which is a static value provided by the vehicle maker and the time that the electric device is in use. In addition, this type of consumed energy is considered to be spent directly from the battery and not part of the energy cost function occurring from taking a path. Therefore, the battery charge level must be periodically updated. For example, if one of the on-board electric devices is turned on/off, the battery charge level is updated. Updating the battery charge level can also be timer specified; for instance, users may want the update to be made every two minutes. This type of energy is defined by the following form:

$$C_{ED} = \sum_{i=1}^{n} (P_{ED(i)}\ t_i) * Status_{(i)} \tag{13}$$

where $Status_{(i)}$ has a value of 0 if the electric device i is *off;* otherwise, it has a value of 1; $t_{(i)}$ is the time that the electric device i takes in the status *on;* $P_{ED(i)}$ is the power drawn by the electric device i, and n is the EV's number of on-board electric devices. The battery charge level is updated using the following form:

$$J_{updated} = J - C_{ED} \tag{14}$$

Total Energy Cost

The complete form of the total energy cost function on the edge (a,b) is as follows:

$$C_E(a,b) = [C_{AE}(a,b) + C_{LE}(a,b) + C_{PC}(a,b) + C_{PG}(a,b) + C_{DE}(a,b)] * DS_{coeff} \tag{15}$$

where the total cost of a path $P^k = (v_1, v_2, ..., v_k)$ is

$$C_E(P^k) = \sum_{j=1}^{j=k-1} C_E(v_j, v_{j+1}) \tag{16}$$

One interesting strategy to solve the problem of energy-optimal routing in EVs is to first transform the weight function C_E into a positive reduced weight function C_Π, in which we use a potential function Π assigning to each vertex a potential as described by Mehlhorn and Sanders, 2008, in [39]. Assuming no negative cycles exist, it is proven that whenever a function Π satisfies the fact that $\Pi(b) - \Pi(a) \leq C_E(a,b)$ and also C_Π is determined as

$C_\Pi(a,b) = C_E(a,b) + \Pi(a) - \Pi(b)$, then the optimal routes in the weighted graph (V, E, c_Π) are also the optimal in the weighted graph (V, E, c_E). This idea is used in Johnson's algorithm [30] by applying the Bellman-Ford algorithm as a pre-processing stage. Then the problem of the shortest path in the weighted graph (V, E, c_Π) is solved by Dijkastra's algorithm. The important observation here, and also stated in [36], is that according to the energy cost function defined in Equation (15), the potential function Π can be inherently obtained without performing a pre-processing stage. In other words, the potential energy function π—resulting from an elevation of a vertex in the weighted graph (V, E, c_E)—implies a potential function Π resulting in a positive reduced weight function C_Π.

Lemma 1: π implies a positive reduced weight function C_Π.

Proof:

$$\begin{aligned} C_\Pi(a,b) &= C_E(a,b) + \Pi(a) - \Pi(b) \\ &= C_{AE}(a,b) + C_{LE}(a,b) + C_{DE}(a,b) + \Pi(b) - \Pi(a) + \Pi(a) - \Pi(b) \\ &= C_{AE}(a,b) + C_{LE}(a,b) + C_{DE}(a,b) \geq 0 \end{aligned}$$

where energies during acceleration and deceleration phases on a road segment (a,b) take the following values:

$$C_{AE}(a,b) = \begin{cases} 0 & if \quad k = 1 \\ \Delta^k & if \quad k > 1, \quad \Delta^k > 0 \\ \Delta^k & if \quad k > 1, \quad \Delta^k < 0 \\ \infty & if \quad k > 1, \quad \Delta^k > J \end{cases} \qquad C_{DE}(a,b) = \begin{cases} 0 & if \quad k = 1 \\ -\Delta^k & if \quad k > 1, \quad 0 > \Delta^k \leq U \\ \Delta^k & if \quad k > 1, \quad 0 > \Delta^k > U \\ -\Delta & if \quad k > 1, \quad \Delta_\Pi \geq 0 \\ \Delta & if \quad k > 1, \quad \Delta_\Pi < 0 \end{cases}$$

where Δ_Π is the amount of energy consumed or gained on the road segment (a,b) in the weighted graph (V,E,c_Π). Therefore, there is no need to perform the pre-processing stage that is performed in Johnson's algorithm, and the problem can be solved in $O(n^2)$ by means of the A^* algorithm. The A^* algorithm for the energy mode is modified such that the energy-optimal path is determined in the weighted graph (V,E,c_Π). The battery constraints for the optimal path, however, are dynamically adjusted and resolved based on the energy costs in the weighted graph (V,E,c_E). Figure 4.1 in Section 4.2.2 depicts a slightly modified version of the A^* algorithm to resolve the problem of energy-optimal routing.

4.3.1 Energy Mode Heuristic Function

For the heuristic function of the energy cost in the weighted graph (V,E,c_Π), the air-line distance and the minimum speed over all speed limits are used. Let us define two vertices, u and v, and a destination, t. Obviously, the air-line distance l' is a consistent heuristic for a road length.

$$l'(u,t) \le l(u,v) + l'(v,t) \qquad (17)$$

Therefore, the following heuristics are defined:

$$h_L(u,t) = f_r m g l'(u,t) + \frac{1}{2}\rho A c_w S_{\min}^2 l'(u,t) \qquad (18)$$

$$h_A(u,t) = T \ \frac{S_{\min}}{r} \ \mathrm{g}_r \ t_A \qquad (19)$$

$$h_D(u,t) = T \ \frac{S_{\min}}{r} \ \mathrm{g}_r \ t_D \qquad (20)$$

where

$$h(u,t) = h_L(u,t) + h_A(u,t) + h_D(u,t) \tag{21}$$

Thus, we write

$$h(u,t) = f_r mg l'(u,t) + \frac{1}{2}\rho A c_w S_{\min}^2 l'(u,t) + T \;\frac{S_{\min}}{r}\; \mathrm{g}_r \; t_A + T \;\frac{S_{\min}}{r}\; \mathrm{g}_r \; t_D \le C_\Pi(u,t)$$

Lemma 2: The heuristic $h(u,t)$ is consistent in the weighted graph (V, E, c_Π).

Proof: Since $h(u,t)$ is linearly increasing in l, $h(u,t) \le h(u,v) + h(v,t)$. As $h(u,t)$ is monotonic in S and l, $h(u,v) \le C(u,v)$ for all u and v for which $C(u,v)$ is defined, and therefore,

$$h(u,t) \le C(u,v) + h(v,t).$$

In addition, the following is defined:

$$V(u,v) = f_r mg + \frac{1}{2}\rho A c_w S^2(u,v) + T \;\frac{S(u,v)}{r}\; \mathrm{g}_r \; t_A + T \;\frac{S(u,v)}{r}\; \mathrm{g}_r \; t_D$$

$$V' = f_r mg + \frac{1}{2}\rho A c_w S_{\min}^2 + T \;\frac{S_{\min}}{r}\; \mathrm{g}_r \; t_A + T \;\frac{S_{\min}}{r}\; \mathrm{g}_r \; t_D$$

Since $S_{\min}$ is a lower bound of $S_{(u,v)}$, V' is also a lower bound of $V(u,v)$ and therefore consistency follows down from

$$h(u,t) \le V(u,v) l(u,v) + V' l'(v,t)$$

$$\therefore\; h(u,t) \le C(u,v) + h(v,t)$$

which proves that the heuristic $h(u,t)$ is consistent in the weighted graph (V, E, c_Π).

4.3.2 Energy Mode A-Star Algorithm

The A^* algorithm, depicted in Figure 4.1, is used for the energy mode in order to compute the optimal path in terms of energy cost. Working on stochastic networks with weights representing energy costs, the algorithm is modified to handle the battery constraints. The battery constraints are resolved and verified based on the energy costs in the weighted graph (V, E, c_E), while the energy-optimal path is determined in the weighted graph (V, E, c_Π). The algorithm is modified to tackle the first constraint problem by turning the cost value of the path into infinity and thus excluding the path from the search process. The solution to this constraint problem is stated in line 7 in the algorithm, which says that any possible road segment to use in (V, E, c_E) must have a cost value, $g(u)$, that is less than the battery charge level J. In addition to this modification in line 7, in every iteration, the algorithm will always choose the vertex u in Q with minimal $g(u)+h(u)$, (i.e., for which the current costs plus the estimated cost in (V, E, c_Π) are minimal, is removed from Q and expanded). During the expansion, a successor v of u is added to Q if its new path costs given by $g(u)+C(u,v)$ are smaller than the so-far known cost value. In order to be able to return the cheapest path (i.e., the path having the least energy costs), the choices for building up a shortest path are recorded via function p in lines 3 and 16. Then, the optimal path is returned via function p in line 11. The energy cost of the source, $g(s)$, is initialized with zero in line 4. The second constraint problem is solved by dynamically adjusting the energy cost functions that are included in the total energy cost function C_E as stated in Section 4.2.

Algorithm 1: Energy mode - A^* algorithm

Inputs: directed weighted graph with C_E and C_Π, the energy cost functions; C_{max}, the maximum Capacity of the battery; J, the battery charge level; s, source; and t, destination.

Output: energy-optimal path from source to destination.

Begin

  Foreach vertex v **in** V **do**

    $g(v) \leftarrow \infty$; $p \leftarrow null$;

  $g(s) \leftarrow 0$;

  $Q \leftarrow \{s\}$;

  While $Q \neq \varnothing$ **do**

    Choose u from Q with minimal $g(u) + h(u, t)$ in (V, E, c_Π) and with $g(u) < J$ in (V, E, c_E)

    if all paths in Q have $g(u) > J$ **then**

      break;

    if $u = t$ **then**

      return p;

    $Q \leftarrow Q \setminus \{u\}$

    Foreach successor v of u **do**

      $g' \leftarrow g(u) + C_\Pi(u, v)$

      if $g' < g(u)$ **then**

        $g(v) \leftarrow g'$; $p(v) \leftarrow u$;

        $Q \leftarrow Q \cup \{v\}$

Figure 4.1: Energy Mode A-Star Algorithm.

4.4 Time Mode

As mentioned earlier, in this mode, the A^* algorithm is used to compute the most efficient path in terms of travel-time cost among all possible paths. The travel-time cost is based on real time information of traffic density, so congestion and accident risk are included in the estimation of travel-time cost. During the search for the most efficient travel-time path, the battery constraints are taken into account. A path is not feasible if it does not satisfy the battery constraints. Even when drivers care only about time in their traveling, any path with an energy cost that is greater than the battery charge level is excluded from the search process by turning its travel-time cost value into infinity. Each road segment has an average speed that is assumed to be a function of the traffic density depending on traffic flow and concentration.

$$S(tr) = \frac{q}{k} \tag{22}$$

where q is the traffic flow in *vehicle/hour* and k is the traffic concentration in *vehicle/km*. Jan Rouwendal states in [43] the following observations about Equation (22):

1) The speed decreases if the traffic concentration increases $S(tr) \to 0 \quad as \quad k \to \infty$, which means it becomes more costly in terms of accident risk if the speed increases and more costly in terms of time if the speed decreases when the EV takes a path that has high traffic concentration.

2) The free flow speed S^* is finite and is defined as the speed chosen when the traffic density approaches zero (e.g., $S^* = 50 \;\; km/hour$ inside cities).

$$S^* = \lim_{tr \to 0} S(tr) \tag{23}$$

Then the travel-time cost function can be written as

Travel cost = time cost + safety cost

$$C_T = \frac{t_{travel}}{S(tr)} + b(tr)\, S^2 \tag{24}$$

If the cost of waiting time at traffic lights is included in the time cost component in Equation (24), then the travel-time cost function can be written as

Travel cost = time cost+ waiting time at signals + safety cost

$$C_T = \frac{t}{S(tr)} + b(tr)\, S^2 \tag{25}$$

where $t = t_{travel} + t_{stops}$. According to Jan Rouwendal [43], $b(tr)$ must be increased in traffic density, and it can be determined as

$$b(tr) = \frac{t}{2S^3(tr)} \tag{26}$$

By substituting (26) into (25), the travel-time cost function of a road segment (a,b) takes the following form:

$$C_T(a,b) = \frac{3\ t}{2\ S(tr)} \tag{27}$$

Equation (27) states that when accident risk is involved in the travelling, the true value of travel-time cost is about 50 percent higher than that of time cost [43]. Thus, the travel-time cost function of a path $P^k = (v_1, v_2, ..., v_k)$ is

$$C_T(P^k) = \sum_{j=1}^{j=k-1} C_T(v_j, v_{j+1}) \tag{28}$$

4.4.1 Time Mode Heuristic Function

For the heuristic function of the travel time cost, the air-line distance and maximum speed over all speed limits are used. Let us define two vertices u, v and a destination t. As mentioned earlier, in Section 4.2.1, the air-line distance l' is a consistent heuristic for a road length $l'(u,t) \le l(u,v) + l'(v,t)$. Therefore, the travel-time heuristic function is written as follows:

$$h_t(u,t) = \frac{3}{2}\frac{l'(u,t)}{S_{\max}^2} \tag{29}$$

Lemma 3: The heuristic $h_t(u,t)$ is consistent in the weighted graph (V, E, c_t).

Proof: Since $h_t(u,t)$ is linearly increasing in l, $h_t(u,t) \le h_t(u,v) + h_t(v,t)$. As $h_t(u,t)$ is monotonic in l and S, then

$\frac{3}{2}\frac{l'(u,v)}{S_{\max}^2} = h_t(u,v) \le C_t(u,v) = \frac{3}{2}\frac{l(u,v)}{S^2(u,v)}$ for all u and v for which $C_t(u,v)$ is defined, and

therefore, $h_t(u,t) \le C_t(u,v) + h_t(v,t)$

In addition, the following is defined:

$$V(u,v) = \frac{3}{2S^2(u,v)} \quad \text{and} \quad V' = \frac{3}{2S_{\max}^2}$$

Since $S_{\max}$ is a higher bound of $S(u,v)$, V' is a lower bound of $V(u,v)$ and therefore,

$$h_t(u,t) \le V(u,v)l(u,v) + V'l'(v,t)$$

$$\therefore\ h_t(u,t) \le C_t(u,v) + h_t(v,t)$$

which proves that the heuristic $h_t(u,t)$ is consistent in the weighted graph (V,E,c_t).

4.4.2 Time Mode A-Star Algorithm

The A^* algorithm, depicted in Figure 4.2, is used for the time mode in order to compute the optimal path in terms of travel-time cost. Working on stochastic networks with weights representing travel-time costs, the algorithm is modified to tackle the battery constraints. The focus in this algorithm is to compute the optimal path in terms of travel-time cost; however, the battery constraints must be satisfied along the path so that the driver is ensured not to be stranded. Therefore, a path with less travel-time cost cannot be used if it does not satisfy the battery constraints. Any path with an energy cost that is greater than the battery charge level is excluded from the search process by turning its travel-time cost into infinity. The solution to this constraint problem is stated in line 7 in the algorithm: any possible road segment must have an energy cost value, $g_e(u)$, that is less than the battery charge level J.

In every iteration, the algorithm will choose the vertex u in Q with minimal $g(u)+h(u)$ (i.e., for which the current travel time costs plus the estimated cost are minimal, is removed from Q and expanded). During the expansion, a successor v of u is added to Q if its new path cost given by $g(u)+C_t(u,v)$, is smaller than the so-far known cost value. The choices for building up a shortest path are recorded via function p in lines 3 and 17. The optimal path (i.e., the path having the cheapest travel time cost), is returned via function p after reaching the destination in line 11.The travel time cost of the source, $g(s)$, is initialized with zero in line 4. The second battery constraint here is solved in the same way as in the energy mode section. The energy cost of the optimal travel time path is accumulated along the path from the source to the destination in line 15.

Algorithm 2: Time mode - A^* algorithm

Inputs: directed weighted graph with C_t , time cost function; C_E, energy cost function; C_{max}, maximum capacity of the battery, J, battery charge level; s, source; and t, destination.

Output: travel time-optimal path from source to destination.

Begin

  Foreach vertex v **in** V **do**

    $g(v) \leftarrow \infty$; $p \leftarrow null$;

  $g(s) \leftarrow 0$;

  $Q \leftarrow \{s\}$;

  While $Q \neq \varnothing$ **do**

    Choose u from Q with $g_e(u) < J$ and minimal $g(u) + h(u, t)$

    if all paths in Q have $g_e(u) > J$ **then**

      break;

    if $u = t$ **then**

      return p;

    $Q \leftarrow Q \setminus \{u\}$

    Foreach successor v of u **do**

      $g' \leftarrow g(u) + C_t(u,v)$

      $g'_e \leftarrow g_e(u) + C_E(u,v)$

      if $g' < g(u)$ **then**

        $g(v) \leftarrow g'$; $g_e(v) \leftarrow g'_e$; $p(v) \leftarrow u$;

        $Q \leftarrow Q \cup \{v\}$

Figure 4.2: Time Mode A-Star Algorithm.

4.5 Summary

This chapter has presented a solution to the problem of optimal energy/time routing under battery constraints in EVs. The proposed solution is a multi-criteria model that functions in two modes: energy mode and travel-time mode. We first introduced the search technique posed to solve the problem and its optimality conditions. Then, we presented in detail the energy mode, including the energy cost function that combines different energy forms used to represent energy weights on road segments in any road network. We explained how the battery constraints can be dynamically resolved and incorporated into the search technique during the search process. Finally, a time mode to provide the travel-time-optimal path, including the travel-time cost function used to represent travel-time weights on road segments in any road network, was presented.

Chapter 5: Experimental Work

5.1 Simulation Environment

This chapter reports experimental work performed to test and validate the proposed multi-criteria routing technique. Matlab was used to construct the test and validation environment. A 40-intersection road network is used to construct working scenarios to analyze the performance of the proposed technique (Figure 5.1). The road network is constructed such that roads intersect at various elevations. Road segments vary with respect to length, speed limits (min/max), and average speed. The road network spans a 30kmx30km area. The performance of the technique is tested on various source/destination scenarios. Sources and destinations are selected randomly. In Figure 5.1, the source node is marked in red and the destination node is marked in blue. The technique can operate in either of two modes, namely, time mode, to compute the travel-time-optimal path from source to destination, or energy mode, to compute the energy-optimal path from source to destination.

The parameters of the EV and the environment are as follows: mass of the vehicle including payload, $m = 1200\ kg$, the efficiency of the EM, $\eta = 0.8$, the air drag coefficient, $c_w = 0.24$, the cross sectional area of the vehicle, $A = 1.85\ m^2$, the friction coefficient, $f_r = 0.9$,the gravitational factor, $g = 9.81\ m/s^2$, the air density coefficient, $\rho = 1.2\ kg/m^3$, the radius of the vehicle's tires, $r = 0.25\ m$, the maximum torque of the vehicle, $T = 250\ N.m$, the gear ratio of the vehicle, $g_r = 1:6.572$, and the maximum capacity of the battery, $C_{\max} = 25\ kwh$.

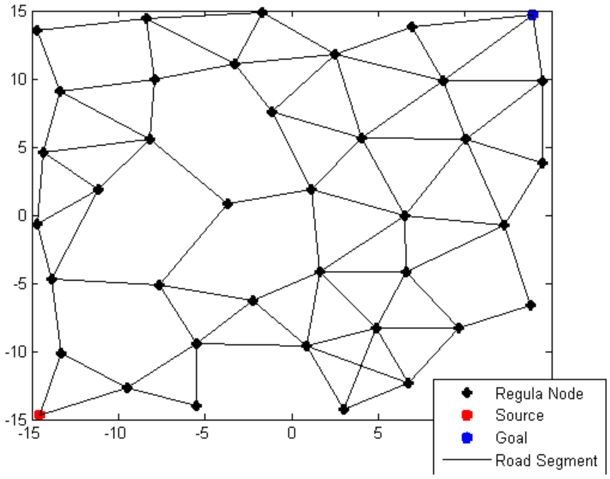

Figure 5.1: Road Network with 40 Nodes.

The vehicle is assumed to be able to accelerate up to the speed of 100 $km/hour$ in 4.5 *seconds* and decelerate from the same speed to come to a complete stop in 4.2 *seconds.* The total energy cost function, C_E, of each road segment in the network is in kwh. The driving style is assumed to be normal, with a driving style coefficient $DS_{coeff} = 1$. The waiting time at a traffic light is assumed to be 3 *minutes.* All the on-board electric devices are assumed to be *off,* and therefore the energy cost function C_{ED}, the energy consumed by the on-board electric devices, is negligible. The following sections present the results of the experiments conducted under various operation modes, various road conditions, and various source/destination scenarios.

5.2 Energy Mode Results

In this mode, the technique excludes the optimality of travel time and thus computes the travel path with minimum energy cost. Therefore, the computed optimal path here may be longer in travel distance and in travel time, but optimal in energy. In order to make certain that the computed paths are optimal, we verify the optimality conditions of the A^* search algorithm stated in Sections 4.1.3 and 4.1.4. The results are provided in two tables (Tables 5.1 and 5.2) in which the admissibility and consistency conditions along the computed optimal path are satisfied.

The results also include a number of figures illustrating the constructed road network marking the optimal path in green from the source node to the destination node, the battery charge level along the optimal path, the energy cost of each road segment, and the total energy cost of the optimal path. Figure 5.2 depicts the constructed road network with the source node marked in red, the destination node marked in blue, and the energy-optimal path, computed by the technique, marked in green. For the path marked in green to be optimal, heuristic $h(n)$ must be admissible and consistent along the path. For the admissibility of $h(n)$, the function $f(n)$, stated in Equation (1), must never overestimate the true energy cost of the path.

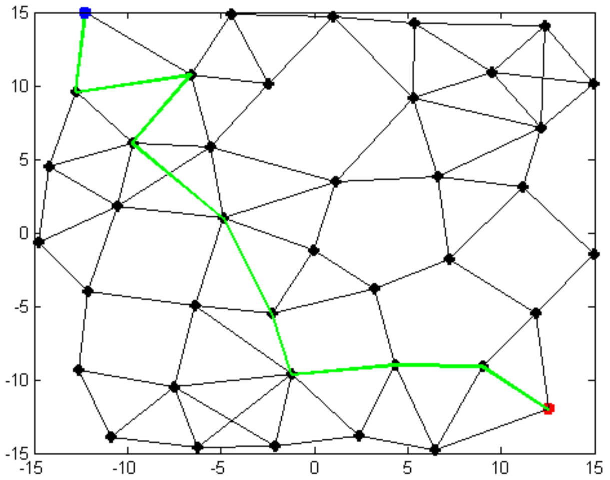

Figure 5.2: Optimal Path Energy-based Network.

Table 5.1, depicted below, illustrates the admissibility condition satisfaction in the weighted graph (V, E, c_{Π}). The first column of Table 5.1 is evidence for the node indices of the optimal path computed by the technique. The path starts with the source node having the index 3 and ends with the destination node having the index 23. In this operation scenario, the EV has gone through ten nodes, including the source and destination nodes. Additionally, $g(n)$, the real cost to arrive at node n along the optimal path, is shown in the second column. The value of $g(n)$ is accumulated along the path until it becomes the total path cost when the vehicle reaches the destination. The third column demonstrates that the values of $h(n)$ along the path decrease as the vehicle moves closer to the destination. The fourth column shows the values of $f(n)$, which is equal to $g(n)+h(n)$, along the path. The fifth column demonstrates the total path cost, which is greater than the values of $f(n)$, proving that heuristic $h(n)$ is admissible along the path. Thus, Table 5.1 demonstrates that the admissibility condition in the weighted graph (V, E, c_{Π}) is satisfied along the computed path.

Table 5.1: Admissibility Information of Energy-Optimal Path**.**

Path Nodes	*g(n)*	*h(n)*	*f(n)*	Path Cost
3	0	0.13619	0.13619	0.27303
36	0.026846	0.11949	0.14634	0.27303
13	0.063123	0.10864	0.17176	0.27303
32	0.090594	0.10063	0.19122	0.27303
22	0.11373	0.085166	0.1989	0.27303
2	0.14115	0.059536	0.20069	0.27303
37	0.16991	0.035197	0.20511	0.27303
21	0.1987	0.027231	0.22593	0.27303
14	0.22886	0.021142	0.25	0.27303
23	0.27303	-0.0016633	0.27137	0.27303

Table 5.2: Consistency Information of Energy-Optimal Path.

h(n, t)	*c(n, v)*	*h(v, t)*
0.13619	0.026846	0.11949
0.11949	0.036277	0.10864
0.10864	0.027471	0.10063
0.10063	0.023141	0.085166
0.085166	0.027416	0.059536
0.059536	0.028759	0.035197
0.035197	0.028788	0.027231
0.027231	0.030161	0.021142
0.021142	0.044175	-0.0016633
-0.0016633	0	0

Table 5.2 proves that heuristic $h(n)$ satisfies the consistency condition stated in Equation (2). In Table 5.2, $h(n,t)$ is the estimated energy cost to arrive at the destination from node n, $c(n,v)$ is the real energy cost from node n to node v, the successor of n, and $h(v,t)$ is the estimated energy cost to arrive at the destination from the successor node v. Another way of proving consistency as stated in Equation (4) is by having the values of $f(n)$ be non-decreasing along the computed path, starting from the source node up to the destination node, which is demonstrated in Table 5.1. Thus, heuristic $h(n)$ is consistent in the weighted graph (V,E,c_{Π}) along the computed path.

Figure 5.3, depicted below, illustrates the battery charge level and the effects of negative energy costs on the battery charge along the optimal path in the weighted graph (V,E,c_E). In this operation scenario, the EV started along the path with a fully charged battery (that is, $25\ kwh$), and it ended reaching the destination with a charge level of $24.618\ kwh$. It can be seen from the graph that during this run, the first road segment of the travel path has a negative energy cost value, and therefore, there is no energy consumed from the battery over this road segment. Due to the dynamic adjustment of the travel energy cost, the negative energy cost incurred by taking the first road segment was lost since the battery was fully charged when the EV first started traveling. All the other negative energy costs incurred by taking this path were stored in the battery until the destination was reached. Therefore, prolonging of the limited cruising range of the battery is noticeable along the optimal path.

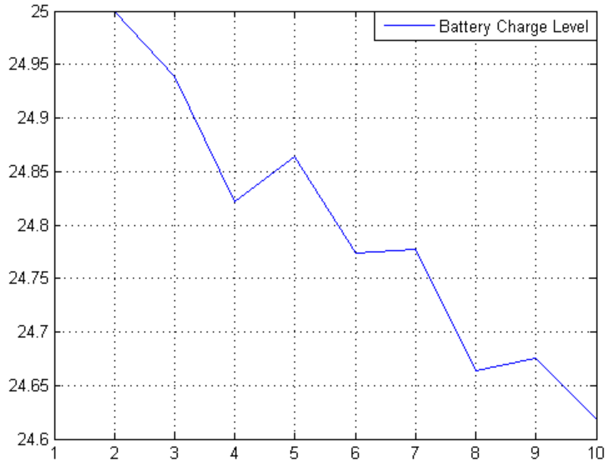

Figure 5.3: Battery Charge Level along the Energy-optimal Path.

We also illustrate the energy cost of each road segment the EV has gone through along the optimal path. Figures 5.4 and 5.5 depict the road segment costs in the weighted graphs (V,E,c_{Π}) and (V,E,c_E), respectively, recorded at the end of each road segment. As can be seen from the graphs, the EV has gone over nine road segments from the source node to the destination node, with each road segment having a different energy cost value.

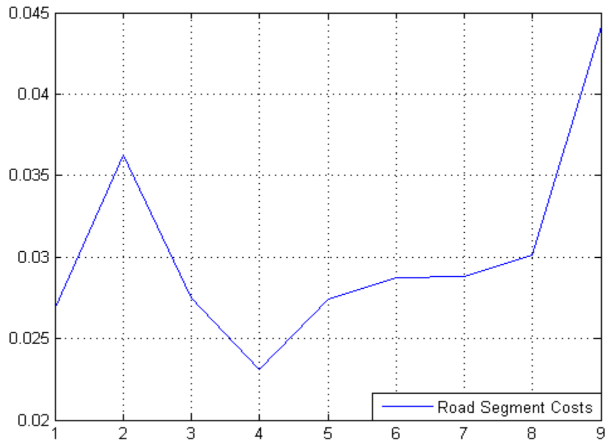

Figure 5.4: Road Segment Costs of the Energy-optimal Path in (V,E,c_{Π}).

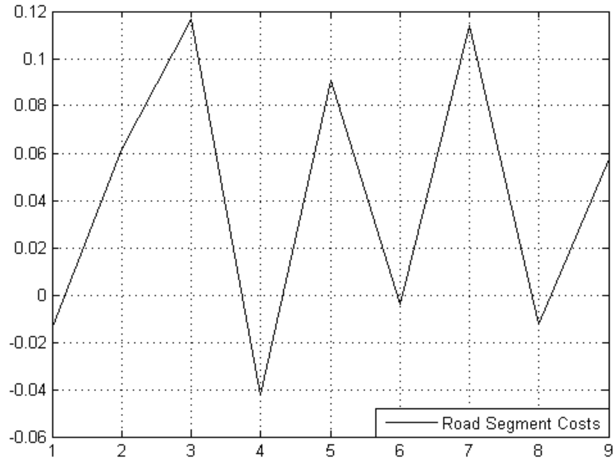

Figure 5.5: Road Segment Costs of the Energy-optimal Path in (V, E, c_E).

The total path energy cost in the weighted graph (V, E, c_E) is depicted in Figure 5.6 below. This graph illustrates the effects of potential energies on the path energy cost. The total path energy cost here refers to the total energy that resulted from taking the optimal path and not that consumed from the battery.

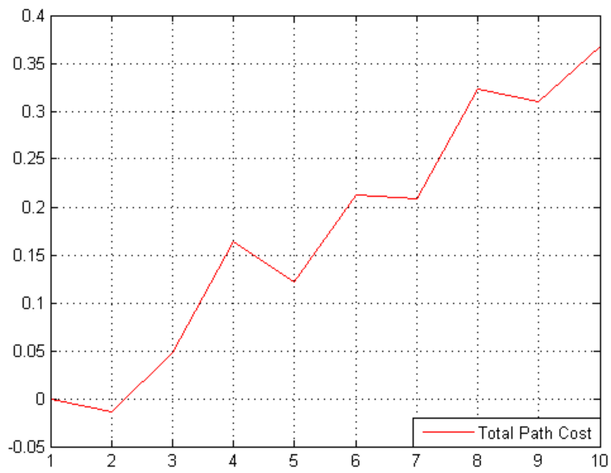

Figure 5.6: Total Path Energy Cost in (V, E, c_E).

During this run, the negative energy cost value incurred by taking the first road segment was lost from the battery due to the dynamic adjustment of battery constraints, and all of the other negative energy costs of taking the path were stored in the battery.

5.3 Time Mode Results

In this mode, the technique excludes the optimality of energy cost and thus computes the path with minimum travel-time cost. Therefore, the computed path here may be more energy consuming and longer in travel distance but optimal in travel time. Although this mode is used by drivers when the concern is travel time and not energy, the battery constraints are considered and not violated. We verify the computed path by checking the optimality conditions of the A^* search technique stated in Sections 4.1.3 and 4.1.4. The results of this operation mode are provided in two tables (Tables 5.3 and 5.4) in which the satisfaction of the admissibility and consistency conditions along the computed path is proven.

The results also include a number of graphs illustrating the constructed road network marking the computed path from the source node to the destination node in green, the battery charge level along the computed path, the travel-time cost of each road segment, the energy cost of each road segment, and the total energy cost of the computed path. Figure 5.7, depicted below, illustrates the constructed road network with the source node marked in red, the destination node marked in blue, and the travel-time-optimal path, computed by the technique, marked in green. For the path marked in green in Figure 5.7 to be optimal, heuristic $h(n)$ must be admissible and consistent along the path. For the admissibility of $h(n)$, the function $f(n)$, stated in Equation (1), must never overestimate the true travel-time cost of the path.

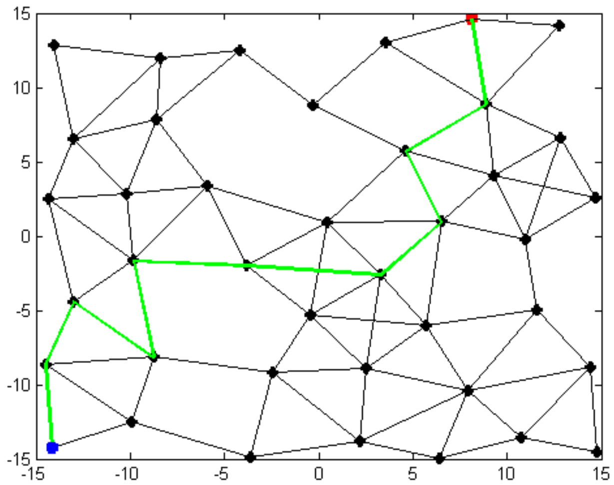

Figure 5.7: Optimal Path Travel Time-based Network.

Table 5.3 proves the satisfaction of the admissibility condition along the computed path. The first column of Table 5.3 demonstrates the computed path node indices chosen by the technique. The path starts with the source node having the index 1 and ends with the destination node having the index 36. In this operation scenario, the EV has gone through eleven nodes, including the source and destination nodes. Additionally, $g(n)$, which is the real cost to arrive at node n along the computed path, is shown in the second column. The value of $g(n)$ is accumulated along the path until it becomes the total path cost when the vehicle reaches its destination. The third column demonstrates that $h(n)$ decreases along the computed path as the vehicle moves toward the destination. The fourth column illustrates the values of $f(n)$ along the path, which are equal to $g(n)+h(n)$. The fifth column illustrates the true cost value of the entire path, which must be greater than the values of $f(n)$ in order to prove that heuristic $h(n)$ is admissible along the path. As can be seen from Table 5.3, the admissibility condition is satisfied along the computed path, marked in green in Figure 5.7.

Table 5.3: Admissibility Information of Time-Optimal Path.

Path Nodes	*g(n)*	*h(n)*	*f(n)*	Total Path Cost
1	0	0.015192	0.015192	0.053064
18	0.0046393	0.013606	0.018245	0.053064
23	0.010432	0.011436	0.021868	0.053064
5	0.014457	0.010697	0.025154	0.053064
40	0.018835	0.0087469	0.027582	0.053064
9	0.02482	0.006699	0.031519	0.053064
22	0.028922	0.00556	0.034482	0.053064
4	0.034193	0.0034037	0.037596	0.053064
11	0.042056	0.0041382	0.046195	0.053064
26	0.049379	0.0023342	0.051713	0.053064
36	0.053064	0	0.053064	0.053064

Table 5.4 demonstrates that heuristic $h(n)$ satisfies the consistency condition stated in Equation (2), and thus heuristic $h(n)$ is consistent along the computed path. In Table 5.4, $h(n,t)$ is the estimated travel-time cost to arrive at the destination from node n, $c(n,v)$ is the real travel-time cost between node n and node v, the successor of n, and $h(v,t)$ is the

estimated travel-time cost to arrive at the destination from the successor node v. Another way of proving consistency, as stated in Equation (4), is by having the values of $f(n)$ be non-decreasing along the path, starting from the source node up to the destination node, as demonstrated in Table 5.3.

Table 5.4: Consistency Information of Time-Optimal Path.

h(n, t)	*c(n, v)*	*h(v, t)*
0.015192	0.0046393	0.013606
0.013606	0.005793	0.011436
0.011436	0.0040242	0.010697
0.010697	0.0043786	0.0087469
0.0087469	0.0059845	0.006699
0.006699	0.0041021	0.00556
0.00556	0.005271	0.0034037
0.0034037	0.0078637	0.0041382
0.0041382	0.0073229	0.0023342
0.0023342	0.0036852	0
0	0	0

Figure 5.8 depicts the battery charge level and the effects of negative energy costs on it along the computed path. When the EV started traveling over the path, the battery was fully charged—equal to its maximum capacity $25\,kwh$. The EV reached the destination with the battery having a charge level of $24.411\,kwh$. As can be seen from the graph, the negative energy costs incurred by taking this travel-time-optimal path were stored in the battery due to the adjustment of battery constraints. The travel-time cost and energy cost values of each road segment the EV has gone over along the computed path are also depicted in Figures 5.9

and 5.10, respectively. Figure 5.9 illustrates the travel-time costs in h^2 / km recorded at the end of each road segment along the computed path, while Figure 5.10 illustrates the energy costs in kwh recorded at the end of each road segment along the computed path. As can be seen from the graphs, the EV has gone over ten road segments, starting from the source node and ending with the destination node.

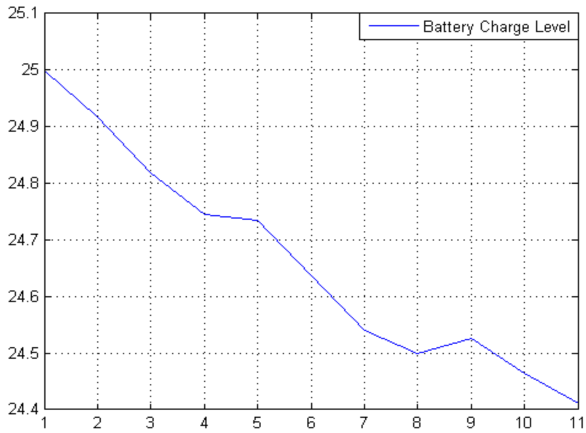

Figure 5.8: Battery Charge Level along the Travel-Time-optimal Path.

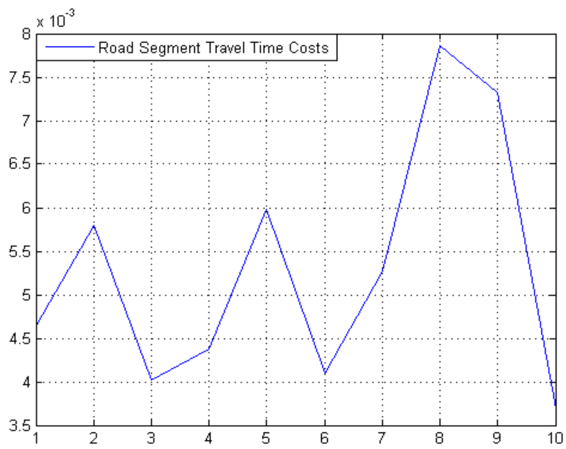

Figure 5.9: Road Segment Time Costs of the Time-optimal Path.

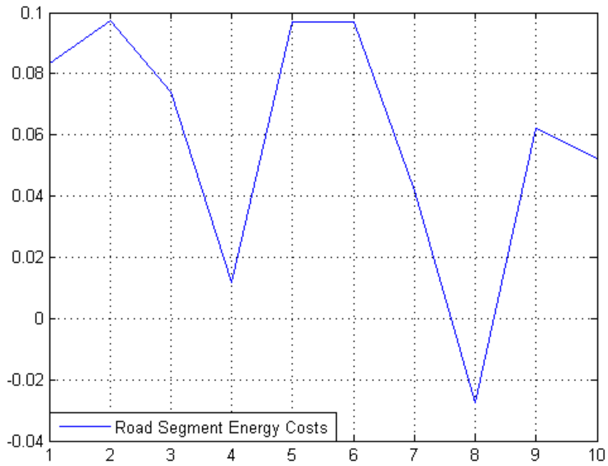

Figure 5.10: Road Segment Energy Costs of the Time-optimal Path.

Figure 5.11 illustrates the total energy cost of the travel-time-optimal path. Although this path is optimal in travel-time cost, it is not necessarily an optimal path in energy cost. The total energy cost here is the total energy incurred by taking the travel-time-optimal path and not that consumed from the battery. This path was verified by the technique to be feasible in the sense that its total energy cost is less than the battery charge level, so the path can be used by the EV.

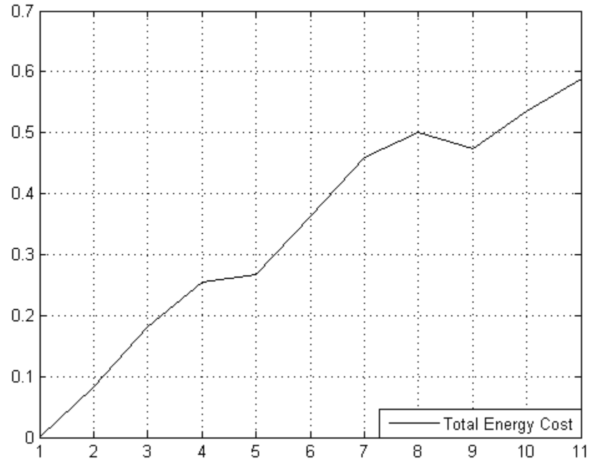

Figure 5.11: Total Energy Cost of the Travel-Time-optimal Path.

5.4 Summary

This chapter has reported the experimental work performed to validate the solution proposed for the EV energy/time routing problem. It addressed the simulation environment created to construct two complete weighted road networks for the energy mode and travel-time mode. Then, it reported the results of running and implementing the suggested multi-criteria technique. The results reported in this chapter, which include verification of the proposed search technique optimality conditions, prove that the solution posed is feasible.

Chapter 6: Range Anxiety

This chapter explores drivers' fear of and concerns about their EV running out of energy, leaving them stranded. A new technique is proposed as a solution to reduce the effects of range anxiety on drivers and help drivers travel confidently without much fear of being stranded. In addition, the technique includes a robust estimation of the remaining driving range on a specific path.

6.1 Introduction

The unique characteristic of the PSS of EVs, namely, the limited battery capacity resulting in limited driving range, has led to what is called range anxiety. Range anxiety is classified as a major barrier to the widespread adoption of EVs. While the single battery charge of EVs may, depending on conditions, support a driving range that is roughly just less than 200 km, the full tank of conventional vehicles can support a driving range of around 600 km or even further [55]. For operators, a shorter driving range is translated to a higher range anxiety [57]. The need to reduce range anxiety has led researchers to pay special attention to driving-range estimation. It is becoming an extremely important strategy because some believe that the only means to reduce driver concerns about being stranded is to make them aware of the remaining distance that their EVs can be driven. It is inaccurate to estimate remaining drivable distance based only on the maximum driving distance provided by the EV maker.

Many factors that affect the battery charge must be taken into account in order to make accurate estimates, including vehicle features, such as mass, power, torque, etc.; road and environment conditions; on-board electric device use; and driving style. All of these factors and more must be involved in the process of driving range estimation; otherwise, the estimation cannot be accurate. However, this thesis reports that driving-range estimation is not the only means to reduce drivers' range anxiety, proposing a technique that analyzes an EV's battery charge required by the vehicle in order to reach a charging station. A guarantee that the EV operators can always reach at least one charging station and recharge their drained batteries would be more useful for reducing range anxiety. Therefore, this thesis introduces the following contributions to the problem of range anxiety:

- Taking the battery constraints of EVs into account even when drivers have no specific destination,
- Presenting a new model that reduces range anxiety to its lowest level by analyzing an EV's battery charge required to reach at least one charging station, including an accurate estimation for the remaining driving range when a path is specified by the user.

6.2 Range Anxiety Reduction Model

This section presents a model posed for reducing range anxiety. The model is designed to analyze an EV's battery charge required by the vehicle to reach at least one charging station before the battery is completely drained, thus providing a guarantee to the drivers that wherever they travel, they will not be stranded en route. To do so, the locations of charging stations within a pre-determined area around an EV's current location are required. The first important point about the model is that it can be used mainly when drivers have no specific destination in their traveling (e.g., when drivers move around looking for a restaurant). In such cases, one of the important factors to reducing concerns about being stranded is that the battery constraints must be taken into account. When drivers have no specific destination, it is extremely important that they avoid any road segment with an energy cost that exceeds the battery charge level. Therefore, in the posed model, the two battery constraints are resolved dynamically in the same way as explained in Chapter 4, Section 4.2. The model performs three range-anxiety-reducing steps.

1) In addition to solving the battery constraints, as explained in Chapter 4, Section 4.2, for non-specific-destination traveling, locations of charging stations within a circular area around a vehicle's current location are determined. Forming a circle around the vehicle's current location is performed to establish a boundary that helps determine charging stations that may be reachable with the current battery charge. The formula in Equation (30) is used to determine the radius of the boundary circle, where $d_{\max}$, the maximum driving distance determined by the EV maker (e.g., 200 km for a single battery charge), is the radius of the circle, and the division of battery charge level by maximum capacity of the battery represents the remaining battery charge.

$$BR = d_{\max} \frac{J}{C_{\max}} \tag{30}$$

The formula stated in Equation (30) above is used in rough range estimation and in the first step of the precise range estimation approach presented in [55]. This formula is not accurate in providing a driving-range estimate because it is based only on maximum driving distance $d_{\max}$ and remaining battery charge. However, this formula is used in the beginning of the approach posed in this thesis as an approximation to help limit the number of charging stations that may be reachable with the remaining battery charge.

2) In the beginning, charging stations within the circular area are localized, and the path energy cost to each charging station is computed. A charging station is designated reachable or not based on the travel energy cost and not travel distance. The routing technique presented in Chapter 4, Section 4.2.2 is used to compute the most energy-efficient path among all possible paths to each charging station. The routing technique here uses the vehicle's current location as the source node and the charging stations as destinations in its computations. Therefore, a heuristic function, exactly as defined in Chapter 4, Section 4.2.1, is used to provide some knowledge about each charging station. The path energy cost used in this model is exactly the same as the path energy cost presented in Chapter 4, Section 4.2:

Potential Consumed Energy

$$C_{PC}(a,b) = \frac{1}{\eta_c}[mg(u(b)-u(a))] \tag{31}$$

Potential Gained Energy

$$C_{PG}(a,b) = \eta_r[mg(u(a)-u(b))] \tag{32}$$

Loss of Energy

$$C_{LE}(a,b)=\frac{1}{\eta_c}[f_r mgl(a,b)+\frac{1}{2}\rho A c_w S(a,b)^2 l(a,b)] \qquad (33)$$

Acceleration and Deceleration Energy

$$C_{AE}(a,b)=\frac{1}{\eta_c}P\ t_A \qquad (34)$$

$$C_{DE}(a,b)=\eta_r\ P\ t_D \qquad (35)$$

On-Board Electric Devices Energy

$$C_{ED}=\sum_{i=1}^{n}(P_{ED(i)}\ t_i)*Status_{(i)} \qquad (36)$$

Therefore, the complete form of the total energy cost on the road segment (a,b) is represented as follows:

$$C_E(a,b)=[C_{AE}(a,b)+C_{LE}(a,b)+C_{PC}(a,b)+C_{PG}(a,b)+C_{DE}(a,b)]*DS_{coeff} \qquad (37)$$

where the total cost of a path $P^k=(v_1,v_2,.....,v_k)$ is

$$C_E(P^k)=\sum_{j=1}^{j=k-1}C_E(v_j,v_{j+1}) \qquad (38)$$

3) After computing the energy-optimal paths to all reachable charging stations, the charging station that has the minimum path energy cost is compared to the battery charge level. If the battery charge level is within a preset threshold value, which is about twice the cost to the cheapest charging station, then a warning is displayed alerting drivers that they have insufficient energy to travel anywhere and to follow the energy-optimal path to the cheapest

charging station. The battery charge level is updated and the process is repeated every time the vehicle enters a new road segment. During the process, if a charging station within the circular area, determined in the beginning, is not reachable, its path energy cost is turned into infinity and thus excluded from the search process.

6.2.1 Remaining Driving Range Estimation of a Specific Path

Driving range estimation of a specific path cannot be performed based on the maximum driving distance determined by the EV maker along with the remaining battery charge, for two main reasons: 1) the battery constraints of limited capacity and the ability to recuperate energy during downhill and deceleration phases, and 2) the differences in road conditions, environmental conditions, and EV features. These two factors must be considered in any remaining-driving-range estimation approach in order to accurately provide drivers with the exact remaining range on a specific path. Therefore, the estimation here is based on using the path energy cost with the remaining battery charge, which together can provide accurate driving-range estimation. The path energy cost used in this estimation is exactly the same as that represented in Chapter 4, Section 4.2 and in Equation (38).

The graph illustrated in Figure 6.1 provides a simple theoretical example to explain the estimation approach. If the battery charge is not sufficient to supply the EV along the whole distance of a specified path, then from the vehicle's current location to the specified destination there must be a point on the path at which the battery charge level, J, equals the path energy cost, $C_E(P^k)$. We term this point the Zero Energy Point (ZEP), meaning that traveling beyond this point is no longer possible. The objective of the approach is to use the battery charge level and path energy cost to compute the ZEP. The strategy is to use the next node (i.e., the node following the vehicle's starting point) where the edge energy cost for the vehicle to travel between its initial location and next node is compared to the battery charge level. If the battery charge level is greater than the first edge energy cost on the path, then the edge energy cost is subtracted from the battery charge, and the next node is used for the next step. This process is repeated along the specified path until a road segment with an energy cost exceeding the battery charge level is found. It is then known that the ZEP occurs on this road segment.

For example, in Figure 6.1, at the node before (the node marked in yellow) the battery charge is greater than the path energy cost. However, at the node after (the node marked in orange) the battery charge is less than the path energy cost. Therefore, it is known that the ZEP occurs on the road segment connecting those two nodes. The difference between the battery charge level and path energy cost at the node before will be used in the path energy cost, represented in Equation (37), to compute the maximum distance the EV can reach on this road segment. Then, this distance is added to the lengths of previous road segments to compute the maximum distance of driving the entire path. Thus, an accurate estimation of the remaining driving-range on a specific path can be displayed to the driver as a distance in km.

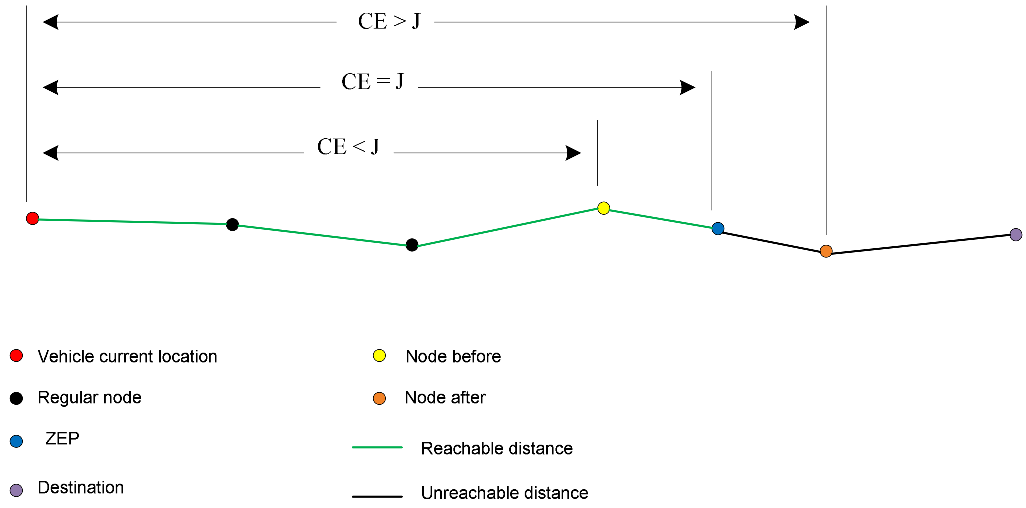

Figure 6.1: Zero Energy Point Determination.

6.3 Experimental Work

This section reports simulation experiments performed to test and validate the proposed techniques. Matlab was used to construct the test and validation environment. A 40-intersection road network is used to construct working scenarios to analyze the performance of the posed techniques. The road network is constructed such that roads intersect at various elevations. Road segments vary with respect to length, speed limits (min/max), and average speed. The road network spans a 30kmx30km area, and four charging stations are selected to be equal distances apart. The parameters of the EV and the environment are as follows: mass of the vehicle, including payload, $m = 1200\ kg$, the efficiency of the EM, $\eta = 0.8$, the air drag coefficient, $c_w = 0.24$, the cross sectional area of the vehicle, $A = 1.85\ m^2$, the friction coefficient, $f_r = 0.9$, the gravitational factor, $g = 9.81\ m/s^2$, the air density coefficient, $\rho = 1.2\ kg/m^3$, the radius of the vehicle's tires, $r = 0.25\ m$, the maximum torque of the vehicle, $T = 250\ N.m$, the gear ratio of the vehicle, $g_r = 1:6.572$, and the battery maximum capacity, $C_{\max} = 25\ kwh$. The vehicle is assumed to be able to accelerate up to the speed of 100 $km/hour$ in 4.5 *seconds* and decelerate from the same speed to come to a complete stop in 4.2 *seconds.* The total energy cost function, C_E, of each road segment in the network is in kwh. The driving style is assumed to be normal with a driving style coefficient of $DS_{coeff} = 1$. The waiting time at traffic lights is assumed to be 3 *minutes.* Only the air-conditioner is assumed to be *on*, with $P_{ac} = 650\ watts$, and all other on-board electric devices are assumed to be *off*. The travel-time path cost represented in Chapter 4, Equation (28), is used to represent the time that the air-conditioner is in the status *on.* The maximum driving distance determined by the EV maker, $d_{\max}$, is assumed to be 250km for a single battery charge. If a sequence of nodes representing a path is entered in the model, the model performs driving-range estimation and returns the result as distance in *km*. Otherwise, the model will perform range anxiety reduction until the battery charge is about twice the energy-optimal path cost to the cheapest charging station, and then returns the result to the user. The following section presents the results of the experiments conducted to validate the proposed model.

6.3.1 Results

This section reports the results gathered from testing the proposed model. Figure 6.2 depicts the constructed road network, including 4 charging stations marked in red and an EV marked in blue. The charging stations are assumed to be positioned at intersections. The EV is assumed to travel around the road network in a random manner but with the consideration of battery constraints. In this operation scenario, the battery charge level was assumed to be $5\,kwh$ when the EV started travelling. The energy-optimal path to the charging station having the cheapest energy cost is marked in green.

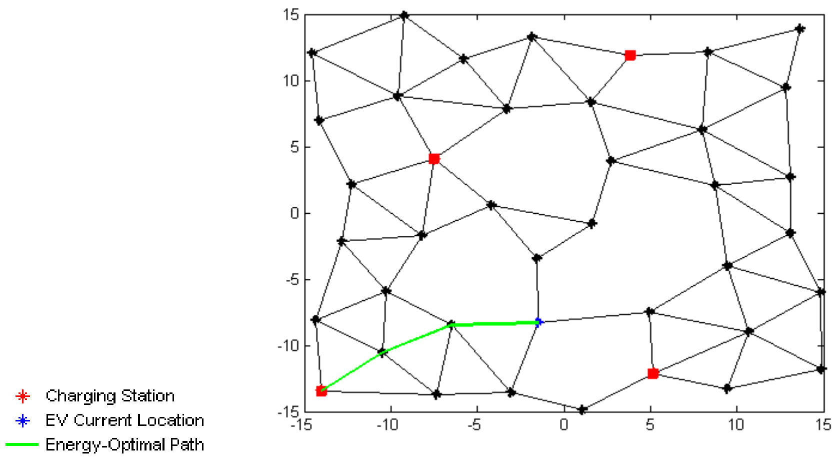

Figure 6.2: Range Anxiety Reduction Network with Four Charging Stations.

The first important observation in Figure 6.2 is that the nearest charging station to the EV's current location, which is about two road segments in length, was not selected by the routing technique. Apparently, this occurred because the optimal path to the charging station is energy cost-based and not distance-based. The second important observation proves the inaccuracy of estimating driving range based on the formula represented in Equation (30). The distance to the cheapest charging station is about 14 km, while the maximum driving distance according to the formula of Equation (30) is 3.226 km, computed as follows:

$$d = 250 \times \frac{0.3226}{25} = 3.226 \ km$$

This observation also demonstrates how driving range can be extended by recuperated energy from downhill and deceleration phases. The message displayed to the user is depicted in Figure 6.3 below; it shows the current battery charge, energy-optimal path cost to the cheapest charging station, and distance to the cheapest charging station.

```
You no longer have enough charge
Your battery charge is 0.3226  kwh
The energy cost to the cheapest energy cost charging station is 0.1641  kwh
Distance to the cheapest energy cost charging station is 14.0348  km
>>
```

Figure 6.3: Message Displayed to the Driver.

Table 6.1 provides the total path energy cost to each charging station in the weighted graph (V, E, c_E) along with the indices of charging stations. Noticeably, the charging station having index 29 has the minimum path energy cost over all charging stations, and hence, it was chosen by the search technique as the best charging station. Table 6.2 proves that the path marked in green, Figure 6.2, is the optimal path to the charging station in the weighted graph

(V, E, c_{Π}) by satisfying the admissibility and consistency conditions of the search technique stated in Chapter 4, Sections 4.1.3 and 4.1.4. The values of $f(n)$, which never overestimate the path cost along the optimal path, prove that heuristic $h(n)$ is admissible along the path. Heuristic $h(n)$ is proven to be consistent along the path by having the values of $f(n)$ be non-decreasing along the path. Thus, Table 6.2 proves that heuristic $h(n)$ is both admissible and consistent.

Table 6.1: Total Path Energy Costs to Charging Stations.

Indices	14	29	11	5
Path cost in kwh	0.16708	0.16407	0.17306	0.21157

Table 6.2: Satisfaction of Admissibility and Consistency Conditions.

Path Nodes	*g(n)*	*h(n)*	*f(n)*	*Path Cost*
24	0	0.051306	0.051306	0.095815
12	0.033869	0.035744	0.069613	0.095815
37	0.061435	0.02001	0.081445	0.095815
29	0.095815	-0.0061339	0.089681	0.095815

Finally, Figure 6.4, depicted below, demonstrates the result gathered from testing the model for driving-range estimation on a specific path. Simply, a sequence of nodes, chosen randomly to represent a path, is used to validate the model. The path starts with the source node (marked in red) and ends with the destination node (marked in blue). A low battery charge that is unlikely to supply the vehicle along the entire path is used to verify the success of the approach. The model marks the reachable distance on the path in yellow and the ZEP in purple as depicted in the graph. The maximum distance that the EV can reach in this operation scenario, 25.379 km, is returned by the model.

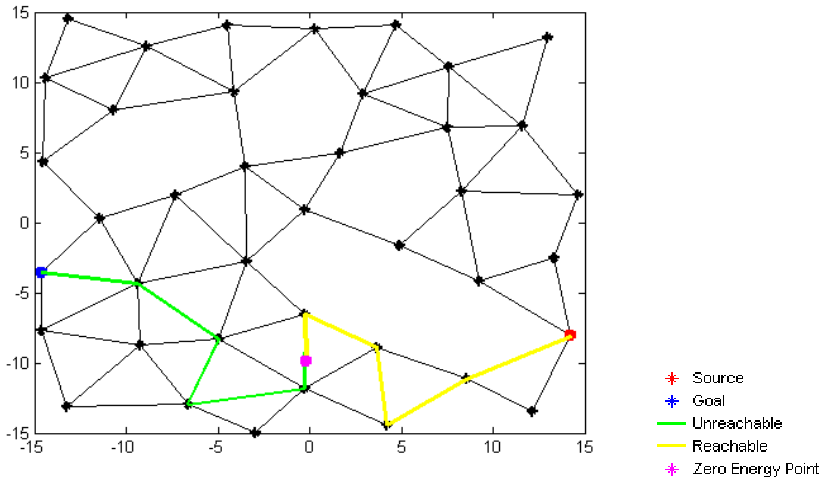

Figure 6.4: Driving Range Estimation on a Specific Path.

6.4 Summary

This chapter has introduced a new model to reduce range anxiety in EV drivers. The model analyzes an EV’s battery charge required by the vehicle to reach at least one charging station. It keeps computing the energy-optimal path cost to each charging station within a pre-determined circular area around an EV’s current location and compares that with the battery charge level. If the battery charge level is about twice the energy-optimal path cost needed to reach the cheapest charging station, then a warning and energy-optimal path to the cheapest charging station are displayed to the driver, prompting him/her to recharge the drained battery. Additionally, the model includes a robust driving-range estimation approach. The approach is used to accurately estimate the maximum driving distance on a specific path using the path energy cost and battery charge. The results reported in this chapter demonstrate that the proposed model is successful and can help make EVs more efficient on roads and reduce drivers’ range anxiety.

Chapter 7: Conclusions and Future Work

The environmental and economical advantages as well as the higher efficiency of EVs over their ICE counterparts are pushing industry and academia to pay more attention to this promising technology. Optimal energy/time-based routing for EVs under constrained rechargeable batteries as well as techniques to reduce range anxiety will become significantly important in the near future since the global trend now is to introduce the technology of EVs as a strategy to help reduce greenhouse gas emissions.

This thesis has formalized the problem of optimal energy/time routing in EVs within a framework of a multi-criteria routing technique in a graph context as a solution to finding optimal energy/time routes. The routing technique relies on using the A^* search algorithm, thus the problem is solved in $O(n^2)$. The A^* algorithm was modified such that it can be run on two modes based on driver needs: an energy mode with a modified algorithm called *Energy Mode* A^* *Algorithm* or a time mode with a modified algorithm called *Time Mode* A^* *Algorithm*.

For the energy mode, the algorithm computes the optimal path among all possible paths, taking into account the battery constraints. The first battery constraint is that a path is not useable if it has an energy cost that is greater than the battery charge level. This constraint problem was solved by turning the energy cost value of the path into infinity, so the algorithm excludes that path from the search process. The second battery constraint is that following a path that has negative energy cost is not feasible if the battery is fully charged. For this constraint problem, the energy cost value was dynamically modified such that the negative costs are stored in the battery based on the remaining free capacity. When the battery is fully charged, the additional negative costs are lost from the battery. For the time mode, the A^* algorithm computes the optimal path in travel time among all possible paths. The travel-time cost includes traffic congestion, accident risk, and waiting time at traffic lights. The A^* algorithm was modified to take into account the battery constraints and not violate them, so traveling over the optimal path becomes possible. However, it is not necessarily the case that the optimal travel-time path is also optimal in terms of energy cost, and the energy cost of the optimal travel-time path is accumulated along the path to ensure the path is feasible. The experimental results in this thesis obtained by testing and validating the multi-criteria routing technique demonstrate that the solutions provided by the technique are optimal. The results prove that the optimality conditions of the search technique are satisfied along the computed paths.

In addition, a new model to reduce drivers' range anxiety has been presented in this thesis. The recommended range-anxiety reduction model provides a guarantee to EV drivers that they will never be stranded en route. The underlying idea was to compare an EV's battery charge with the energy-optimal path costs that would be incurred driving to charging stations within reach. The routing technique proposed in this thesis computes the energy-optimal path to each charging station. If the battery charge level is about twice the path energy cost to the charging station having the cheapest energy cost over all charging stations, then a warning as well as directions to the cheapest charging station are displayed to the driver. Furthermore, the model includes a driving-range estimation approach to provide an accurate estimate of how far drivers can travel on a specific path. The driving-range estimation is based on travel energy cost and not distance. The experimental results reported in this thesis demonstrate that the posed model can help reduce range anxiety and so help remove one barrier to widespread.

Further research should be conducted in the area of optimal energy/time routing for designing a combined routing technique that considers the battery constraints and strikes a balance between the optimality of energy and travel time in EVs. The future work in this area should not compromise the optimality of energy or optimality of travel time but rather should concentrate on designing one routing technique for computing one path that is optimal in terms of both energy and travel-time costs. In addition, field experiments should be performed for further validation of the techniques introduced in this thesis.

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