Cross-layer Optimization in Wireless Multihop Networks

by

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I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

In order to meet the increasing demand for higher data rates, next generation wireless networks must incorporate additional functionalities to enhance network throughput. Multihop networks are considered as a promising alternative due to their ability to exploit spatial reuse and to extend coverage. Recently, industry has shown increased interest in multihop networks as they do not require additional infrastructure and have relatively low deployment costs.

Many advances in physical and network layer techniques have been proposed in the recent past and they have been studied mostly in single-hop networks. Very few studies, if any, have tried to quantify the gains that these techniques could provide in multihop networks. We investigate the impact of simple network coding, advanced physical layer and cooperative techniques on the maximum achievable throughput of wireless multihop networks of practical size. We consider the following advanced physical layer techniques: successive interference cancellation, superposition coding, dirty-paper coding, and some of their combinations. We achieve this by formulating several cross-layer frameworks when these techniques are jointly optimized with routing and scheduling. We also formulate power allocation subproblems for the cases of continuous power control and superposition coding. We also provide numerous engineering insights by solving these problems to optimality.

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To my parents,

Batma Sadyrova and Melis Shabdanov

Contents

List of Figures

List of Abbreviations

CPC   Continuous Power Control

CR   Cooperative Relaying

D-AC   Distributed Alamouti Coding

DPC   Dirty-paper Coding

D-STBC   Distributed Space-Time Block Coding

JRS   Joint Routing and Scheduling

JRS-NC   Joint Routing, Scheduling, and Network coding

ISet   Independent set

LP   Linear program

MAC   Medium Access Control

NC   Network Coding

NUM   Network Utility Maximization

SH   Single Hop

SIC   Successive Interference Cancellation

SINR   Signal to Interference plus Noise Ratio

SPC   Superposition Coding

S-TDMA   Spatial Time Division Multiple Access

List of Symbols

$a_{w,\ell}$ Binary variable for decoding order w and link ℓ

$\mathcal{D}(\ell)$ Collection of all decoding orders for link ℓ

d_0 Reference distance

$d_{i.j}$ Distance between nodes i and j

$d(f)$ Destination node of link f

$\mathcal{D}(\ell)$ Set of destination nodes of link ℓ

$d(\ell)$ Destination node of link ℓ

$\mathcal{DO}(\ell)$ SIC decoding order for link ℓ

$\mathcal{F}$ Set of flows

G Gateway

G_{max} Maximum relative gain

$G_{i,j}$ Channel power gain between nodes i and j

$g_{i,j}$ Fading power gains between nodes i and j

$\mathcal{L}$ Set of feasible links

N Number of nodes in the network

N_0 Receiver's background noise

$\mathcal{N}$ Set of nodes

$o(f)$	Origin node of flow f
$\mathcal{O}(\ell)$	Set of origin nodes of link ℓ
$o(\ell)$	Origin node of link ℓ
[**P1**]	JRS problem formulation
[**P2**]	JRS-NC problem formulation
[**P3**]	JRS with CR problem formulation
P	Maximum transmission power level
$PL(d)$	Path loss at distance d
$P(\ell)$	Transmission power level over link ℓ
P_{min}	Minimum network connectivity power
$\mathbf{P_s}$	Power allocation vector of links in s
$\mathbf{R}$	Vector of all flow rates
R	max-min throughput
R^*	Nodal max-min throughput
$\mathcal{R}$	Set of transmission rates
R_f	Rate of flow f
$r(\ell)$	Transmission rate of link ℓ
r_m	Maximum transmission rate
s	ISet
$\mathcal{V}$	Set of virtual nodes
w_f	Weighting factor of flow f
$\mathbf{x}$	Routing vector of all flows
$x_f(\ell)$	Amount of flow f over link ℓ
$\mathbf{y}$	Routing vector of all NC flows

$y_{f_1,f_2}(\ell_1, \ell_2)$	Amount of NC flows f_1 over link ℓ_1 and f_2 over link ℓ_2
$\boldsymbol{\alpha}$	Scheduling vector
α_s	Fraction of time ISet s is scheduled
$\beta(r(\ell))$	Minimum SINR threshold to support link rate $r(\ell)$
Δ	Global indicator variable
$\mathcal{I}$	Generic collection of ISets for problem [**P1**]
I_ℓ	Total received signal at the receiver of link ℓ
$\mathcal{I}_{\mathsf{int}}$	Collection of ISets in the network with single rate and power
$\mathcal{I}_{\mathsf{x}}$	Collection of ISets in the network where technique x is enabled
λ	Wavelength
μ	Path loss exponent
$\boldsymbol{\phi}$	Vector of all indicator variables ϕ_i
ϕ_i	Indicator variable for constraint i

Chapter 1

Introduction

1.1 Overview

Multihop networks: Wireless networks can be classified as traditional and multihop networks. In traditional networks, the data is delivered to the destination in a single hop, while in multihop networks, the data is delivered to the destination in multiple hops through several intermediate nodes or relays. Multihop networks can be further classified by their application as mobile ad-hoc networks and fixed multihop networks. A mobile ad-hoc network consists of self-managed mobile nodes that do not rely on a fixed infrastructure such as routers or access points to communicate with each other. Each node forwards its data to another node based on the current network connectivity. Examples of mobile ad-hoc networks are vehicular ad-hoc, military, mobile sensor or emergency networks, etc. In contrast, a fixed multihop network consists of nodes that have static locations. Typically, fixed multihop networks are centrally managed in terms of routing and medium access schemes to provide higher performance gains,
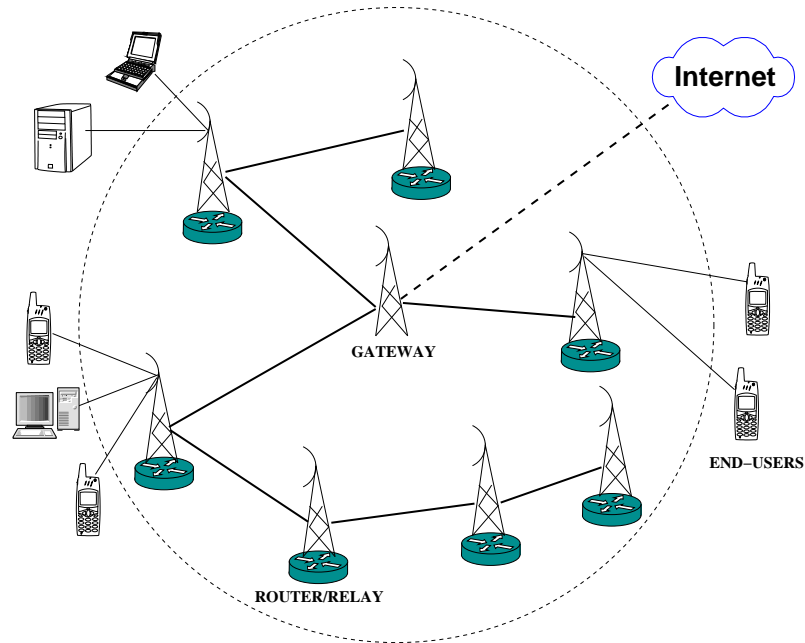
Figure 1.1: A wireless multihop backhaul network with a mesh-like topology

increased coverage, reduced power consumption and low deployment costs.

Centralized multihop networks are usually organized in a mesh-like topology. A mesh multihop network consists of a central gateway and fixed nodes that can act as sources of data as well as routers with relaying capabilities. Fig. 1.1 illustrates an example of wireless multihop network with mesh-like topology and a single gateway. Mesh-like networks often serve as backhaul networks to provide access services to the end clients. Besides this, the mesh network can be configured to provide communication directly between nodes. As for relays, there are two relaying strategies: amplify-and-forward and decode-and-forward. A relay with amplify-and-forward strategy first amplifies the received signal and then transmits it to the next node without any decoding. Typically, in multihop networks, relays employ the decode-and-forward strategy where relays must first decode the signal and then forward it.

2
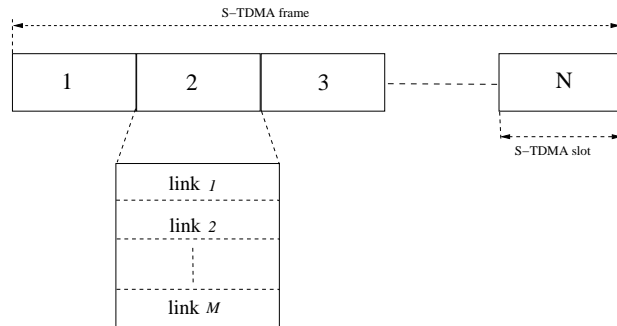
Figure 1.2: Link scheduling in S-TDMA

Nowadays, most of the backhaul networks are based on the leasing of bundled copper wires, e.g., in Europe, about 30% of the cellular backhaul networks are wireless, in the North America, it is less than 5%. Although, cooper wires are relative cheap, the deployment of wired backhaul can be either very expensive to deploy or even not possible due to the regulations or the locations of end-users. For this reason, wireless multihop backhaul can be an alternative to the wired-based backhaul as it allows network operators to reduce the costs of deployment and to provide better coverage.

In multihop networks, the only resource that nodes need to share is the common wireless channel. This common channel is shared by nodes using medium access schemes that can be broadly categorized as frequency, code, and time division multiple access schemes. All these schemes involve orthogonalization of the common channel into sub-channels. These channel access schemes are managed at nodes using a Medium Access Control (MAC) protocol. There are two broad classes of MAC protocol that are used in multihop networks: random and scheduling-based access protocols. While a scheduling-based MAC significantly outperforms a random access protocol in networks with high traffic load, the practical deployment of scheduling-based MAC is not always possible. It is because a scheduling-based MAC can only

be deployed in centrally managed and fixed networks where the channels remain relatively static or quasi-static. Examples of such networks are the wireless backhaul of WiMAX or LTE as well as sensor networks for activity monitoring. Moreover, scheduling-based protocols can serve as a performance benchmark for random access protocols, since the achievable throughput in networks with optimal scheduling is an upper bound on performance of random access schemes. In this work, we focus only on scheduling-based MAC.

For an efficient utilization of the bandwidth, the channel can be spatially reused by many nodes at the same time in a network while meeting the Quality-of-Service requirements. This spatial reuse can be effectively managed by a centralized scheduler using the Spatial Time Division Multiple Access (S-TDMA) protocol, first introduced by [1]. During the offline configuration phase, a centralized schedule allocates for each link a number of slots during which, links are allowed to transmit. Each time slot may consist of a number of links that can transmit concurrently on the same channel as shown in Fig. 1.2. Certainly, there is a trade-off between the number of links in a slot and the level of spatial reuse and the data rate at which nodes can transmit. In Fig. 1.2, a S-TDMA frame is a scheduling transmission cycle that is known by all nodes in a network. If the network is perfectly time synchronized then nodes can transmit interference free over the same channel based on a pre-configured S-TDMA frame. This is only possible when the set of links that are activated at the same time are chosen adequately. Clearly, for the efficient utilization of the channel, the S-TDMA frame must be optimally configured by a scheduler based on multihop routing, the channel conditions, the link rates and the fairness policy for end-users.

Cross-layer optimization: To optimally configure a network, it may seem sufficient to optimize separately routing and scheduling. However, studies [2,3] show that separate optimization of network and link layer, namely routing and scheduling, may not achieve the optimal network performance. Moreover, it was shown in [3] that simple routing such as the shortest path routing may be far from optimal in scheduling-based networks. In addition, in most wireless networks, the layering of network functions is not strict as there is an inherent coupling between different layers that allows an adjustment of scheduling and physical layer parameters such as the transmission power and the modulation rate. Under such coupled layers, optimizing only within layers may not achieve optimal network performance. Therefore, cross-layer optimization across the network, link and physical layers is necessary to achieve an optimal network performance by joint configuration of flow routes, scheduling and physical layer parameters.

According to [4], cross-layer optimization methods can be classified as loosely, tightly and extremely tightly coupled designs. In the loosely coupled design, optimization is performed on each layer by passing parameters from other layers. In the tightly coupled design, the parameters across different layers are optimized jointly as one optimization problem. In extremely tightly coupled design, different layers merge into one layer. In this work, we consider only the tightly coupled design.

While the end-to-end throughput is the main objective of cross-layer optimization, the fairness policy among end-users is another critical performance metric that must be taken into account. The Network Utility Maximization (NUM) problem, presented by [5], is a generalized model for cross-layer optimization problems that incorporates

into one formulation the fairness among users as well as inter-layer interactions.

$$max \quad \sum_i U_i(R_i)$$

$$\boldsymbol{xR} \leq \boldsymbol{c} \tag{1.1}$$

In the NUM model, the throughput rates of users R_i are maximized based on the utility function U_i and the routing matrix $\boldsymbol{x}$ such that the link capacity constraints $\boldsymbol{c}$ are not violated. The problem with the NUM formulation is that the link capacities are fixed as it was initially formulated for wired networks.

In wireless networks, the link capacities are not fixed and vary according to the level of interference, channel conditions and signal power. When the NUM problem was formulated for wireless networks like in [2], due to the increased complexity, it was common to use simplified protocol interference models. The cross-layer formulation based on a realistic physical interference model was formulated in [3] that was among the first to show how to optimally configure a network in terms of routing, scheduling, and physical layer parameters.

1.2 Motivation and Contributions

Wireless multihop networks have a potential advantage over traditional single-hop networks due to their ability to improve coverage while providing end-to-end throughputs similar to those of single-hop networks. However, this advantage can only be obtained if spatial reuse is efficiently managed in the multihop network. There are two main limitations on the increase of spatial reuse in a network: 1) the half-duplex

characteristics of wireless interfaces and 2) the interference that is produced by concurrently transmitting nodes. When a network is optimally configured, then an increase in spatial reuse results in a throughput improvement.

There are a number of recent techniques that allow an increase in spatial reuse, we consider the following conceptually different techniques: advanced physical layer techniques, network coding and cooperative techniques. While the achievable rates of these techniques are well addressed in point-to-point communications, there are only few studies that address the performance gains of these techniques in a realistic network scenario. The main objective of this work is to characterize the throughput gains that can be achieved in a multihop network when these techniques are jointly optimized with routing and scheduling. This is an offline network configuration study. Our main contributions on the modeling and engineering fronts can be summarized as follows:

- We formulate cross-layer frameworks for the optimal offline configuration of fixed wireless networks that use the following techniques:

  - Successive Interference Cancellation

  - Superposition Coding

  - Dirty-paper Coding

  - Network Coding without opportunistic listening

  - Cooperative Relaying based on the distributed Alamouti code.

- These formulations are based on the physical interference model and be easily adapted for any other similar techniques or energy consumption models.

7

- We also formulate power allocation subproblems for the cases of continuous power control and superposition coding.

- By solving these problems to optimality, we able to obtain the max-min achievable throughputs as well as the jointly-optimal offline configuration of flow rates, scheduling, power allocation and in the case of cooperative relaying, the optimal selection of cooperative node pairs.

- These problems are large scale linear programs with an exponentially growing number of variables as a function of the network size. We develop efficient tools based column generation method to solve these problems and provide only exact solutions.

- We provide numerous engineering insights for the network operators by quantifying the throughput gains of these techniques in isolation or in combinations in small to medium size wireless mesh networks.

1.3   Outline

The rest of the thesis is organized as follows. Chapter 2 presents the literature review and the related work. In Chapter 3, we describe definitions and a baseline system model for the joint routing and scheduling problem with continuous power control. In Chapters 4 and 5, we formulate the joint routing and scheduling problem for advanced physical layer techniques, network coding and cooperative relaying technique. We also provide numerical results along with engineering insights. Chapter 6 concludes this thesis by discussing the practical applicability of this work and the future work.

Chapter 2

Literature Review and

Related Work

Optimal configuration of wireless networks: The first framework to optimize the performance of communication systems was presented in [5] for wired networks. It was an innovative idea that originated much research in the area of resource allocation for wireless multihop networks. In [2], a joint routing and scheduling problem was formulated to compute an optimal throughput in any given network. The results in [2] were limited to only lower and upper bounds on the throughput, and were only computed for small networks. In addition, because of the complexity of the problem, the interference model used for computing all results presented therein was the simple protocol interference model. The importance of using the right interference model was studied in [6], where it was concluded that the physical or SINR-based interference model, as opposed to the protocol interference model, should be used to provide meaningful results. Prior to [2], studies such as [7] focused mostly on asymptotic

throughput bounds under assumptions of homogeneity of node locations. However, this kind of study does not answer the question on on how to optimally configure networks or what performance to expect for a medium size network. The max-min capacity of wireless mesh networks was addressed in [8, 9] but their models do not take into account the interference and the results are obtained for a given routing.

In [3], the authors extended the work of [2] to provide a cross-layer framework for the throughput-optimal configuration of wireless networks. The work of [3] provided exact numerical results for the max-min throughput by solving a joint routing and scheduling problem in small to medium-size networks. In [3], it was also shown that cross-layer optimization of routing and scheduling jointly with rate and power control parameters provides a significant improvement in throughput over the case that does not include rate and power control. In [10], a joint routing and medium control problem is formulated for a random access wireless networks.

Much research has been dedicated to developing efficient computational techniques for the cross-layer resource allocation problem. The work of [11] developed a computational technique based on a column generation method to solve a problem similar to the one in [2]. Later, the authors of [12] extended the work [11] by providing an efficient enumeration algorithm and a column generation technique to solve the joint routing and scheduling problem for a network with discrete power control and discrete rate adaptation. By using this technique, the authors were able to provide exact solutions for large wireless networks for both max-min and proportional fair throughputs. Other approaches, such as in [13], study the joint routing and scheduling problem as an integer LP to minimize a number of time slots that is required to achieve the throughput-optimal network configuration. In [14], it was shown that the

path-based formulation of the joint routing and scheduling problem is favorable than the link-based formulation as it allows the column generation method to converge faster to an optimal solution. In [15], a non-linear column generation method was proposed to solve the throughput-optimal configuration of wireless networks. It was also shown that this method converges to a globally optimal solution.

Interference cancellation and coding techniques: Successive interference cancellation was proposed and studied for its capacity regions in broadcast channels by [16]. The work of [17] was among the first to study the impact of successive interference cancellation on capacity regions in scheduled wireless multihop networks. The authors presented a mathematical model for finding maximum sum rates between source and destination nodes in a network. The study was based on the SINR interference and continuous link rate models. A limitation of this study is that the numerical results are limited to small-sized (six nodes) networks. In contrast, in [18], successive interference cancellation was studied with jointly optimal routing, scheduling, rate and power control in multihop networks with up to 25 nodes. Also, in [18], it was shown that successive interference cancellation can achieve significant gains at low transmission power and overcomes the fundamental throughput limit of $\frac{r_m}{N-1}$ in wireless mesh networks. The work of [18] was later extended by [19] to provide a problem formulation for a generic number interference cancellation stages in mesh networks with jointly optimal routing and scheduling[1].

Superposition coding, first proposed in [16] allows a wireless transmitter to send several signals, possibly intended for different users, as a composite signal. In [20],

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[1] [18, 19] are our prior works

a joint routing and power allocation problem with superposition coding was formulated as a non-linear problem for a wireless broadcast network, where all nodes are transmitting simultaneously using a continuous link rate model. The work of [20] was among the first that studied superposition coding for the throughput-optimal network configuration. In this work, the power partition of the superimposed signals is jointly optimized with routing for single and multiple power levels at each transmitter. However, the work of [20] does not consider scheduling. Another limitation of this work is that the interference cancellation technique is restricted to decode only superimposed signals originating from a common node. Thus, direct signals and superimposed signals originating from other nodes are treated as noise and cannot be canceled out. Later, the authors of the work [19] formulated a joint routing and scheduling flow-based problem of finding a jointly optimal parameters for superposition coding, interference cancellation, routing and scheduling for the max-min throughput. This work also quantified gains and determined that the combination of superposition coding with interference cancellation allows a network to double the maximum throughput in wireless mesh networks. There is also a number of work that addressed the practical implementation of superposition coding. The authors of [21] proposed the first design and implementation of an access protocol with superposition coding for wireless mesh networks. The authors reported the average gains in the range of 10% to 20% when superposition coding was implemented according with the 802.11 protocol standard.

Dirty-paper coding was first presented in [22,23] a special case of Gelfand-Pinkser coding for channels with side-information. This technique allows a transmitter to transmit interference-free to the particular receiver, but only when the interference

at this particular receiver is perfectly known at the transmitter. The work of [24] proposed a joint Wyner-Ziv and dirty-paper coding by using a modulo-lattice modulation to achieve the interference-free transmission. To the best of our knowledge, there was no prior work that quantified the throughput gains of dirty-paper coding in wireless mesh networks. The work of [19] quantified the gains of dirty-paper coding in a mesh network that is optimally configured for dirty-paper coding, routing and scheduling. The authors used dirty-paper coding only on links that originate from a central node or gateway since in a mesh network, only the gateway is capable to estimate the interference at any node in the network. The authors of [19] quantified the gains of dirty-paper coding in mesh networks and concluded that dirty-paper coding does not provide any significant throughput improvement[2].

Network coding: A theoretical framework for network coding was first introduced by [25]. In that work it was shown that for a class of block codes, i.e., the so-called α-codes, it is theoretically possible to achieve multicast capacity by encoding messages with network coding at intermediate nodes or relay nodes. This work was followed by [26], where network coding based on linear codes was proved to be sufficient to achieve maximum capacity for multicast traffic. In [27, 28], the first system architecture using linear network coding was introduced. It was shown that employing network coding allows a significant increase of throughput for a wireless network with unicast traffic. Prior to the work of [27], network coding was considered primarily for wired networks. As an extension of [7], an asymptotic study of achievable throughput bounds in a wireless network using XOR-based network coding was presented by [29].

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[2] [19] is our prior work

13

The bounds were derived for both protocol and physical interference models. In a network with network coding, the broadcast rate of linearly combined packets at a relay node is bounded by the lowest rate of the incoming links to the relay node. This limitation of network coding was addressed in [30] where the authors also considered cooperative communication. A theoretical study of jointly optimized network coding and scheduling was presented in [31]. In [32], a cross-layer design of wireless multi-channel mesh networks is studied for a network code construction in a joint routing and MAC scheduling problem. The problem of rate control with pairwise inter-session network coding for distributed networks was formulated in [33] and in directed networks in [34].

The first study that addressed network coding for the throughput-optimal configuration of wireless networks with network coding was the work of [35]. The authors provided a framework for the joint routing and network coding problem for networks with unicast flows. The proposed approach was an extension of the theoretical framework of [2] to allow broadcast transmission and to optimize routing with XOR-based network coding with and without opportunistic listening, initially presented in [28]. The study in [35] has a number of limitations: it is based on the protocol interference model, it uses an approximate model for computing a broadcast rate, it only computes bounds and the problem was formulated for a given routing. These limitations were addressed in the work of [36], where a joint routing, scheduling, and network coding was formulated as a LP based on the SINR interference model. With the use of this framework, the authors were able to quantify gains of network coding in a medium size networks and provided engineering insights[3].

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[3] [36] is our prior work

The idea of network coding was later extended by [37] to employ it on the physical layer. With physical layer network coding, signals are mixed not after decoding but coherently combined at antennas for decoding as symbols in a network coded constellation. The receiver with physical layer network coding is enabled to receive from two and more transmitters at a time while signals are not treated as interference but summed for decoding. Recently, network coding was also combined with various techniques such as with the amplify-and-forward relay [38], successive interference cancellation [39] or cooperative techniques [40].

Cooperative communication: The concept of cooperative relaying was first established in [41], where a three-terminal communication channel was introduced and its capacity bounds were derived. Later in [42], achievable lower bounds on capacity were obtained for a general relay channel. The use of cooperative relays via virtual distributed antennas was proposed in [43] and [44] for amplify-and-forward and decode-and-forward strategies. Outage analysis was studied in the work of [45] for these two uncoded cooperative strategies and also in [46] to compute optimal outage probability jointly with routing in networks with a string-based topology. A problem of joint resource allocation with optimal relay node selection between any source and destination pairs was studied in [47] but only for networks that are restricted to the two hop routing.

The work of [48] considered a problem to find the jointly optimal configuration of cooperative technique with routing and scheduling. The authors quantified the end-to-end performance gains that can be achieved in a given network when cooperative relaying is optimized jointly with routing and scheduling. In [48], it was also shown

that in a single rate and single power network, the cooperative technique between two nodes provides only marginal gains. There exist several studies that address cross-layer design with cooperative relaying in a random access network [49] or in scheduling-based networks, however, these works are based on simplified models or restricted cases. The works of [50] and [51] considered only a fixed selection of relay pairs in a network with no spatial reuse during scheduling and with a string-based topology. Similar assumptions were used in [52] to optimize the outage probability in a string network with a fixed number of hops and mean channel gains over each path. The authors of [53] proposed distributed algorithms to solve the problem of joint of routing and cooperative relaying with power control, but these algorithms were derived for a simple protocol interference model. A joint routing and relay node assignment problem was formulated as a mixed-integer LP in [54], however, this model is based on the use of multiple orthogonal channels for interference-free transmission. An interesting approach of cluster-based cooperation was presented in [55], where it was shown that such a hierarchical architecture allows a network to achieve linear scaling in capacity.

Chapter 3

Joint Routing and Scheduling

In this chapter, we define a system model and formulate a joint routing and scheduling problem with rate adaptation and continuous power control. This baseline flow-based formulation is used in the following chapters and is extended in Section 4.2. It is based on the joint routing and scheduling model provided and studied in [3,12] but has been enhanced to include continuous power control.

3.1 Assumptions and Definitions

We consider a wireless multihop network where 1) the traffic is heavy and static enough to allow for a static and central configuration of scheduling and routing parameters and to enable a flow-based model 2) all nodes are fixed and located about 20m above the ground so that it is reasonable to assume that the channel gains are known and quasi time-invariant. The first assumption is reasonable in the case of a mesh network since mesh nodes carry an aggregated traffic from end-users [3]. Previ-

ous studies like [2] considered a simplistic protocol interference model. However, this type of interference models does not provide meaningful results [6]. In the following, we formulate our flow-based framework based on the realistic physical interference model or SINR-based interference model.

We consider a single-channel network where nodes are equipped with half-duplex wireless interfaces and omni-directional antennas. We model a wireless multihop network as a set of nodes $\mathcal{N}$ and a set of feasible links $\mathcal{L}$. We assume that nodes in a network can support a finite set of available rates $\mathcal{R}$ and can transmit at power levels in the interval $(0, P]$, where P is the maximum transmission power.

Let us define link ℓ by a triple $(o(\ell), d(\ell), r(\ell))$, where $o(\ell) \in \mathcal{N}$ and $d(\ell) \in \mathcal{N}$ are the origin and the destination nodes of link ℓ and $r(\ell) \in \mathcal{R}$ is the link rate. A link ℓ is said to be feasible if, in the absence of interference, it meets the following condition[1]:

[**S3.1**]
$$\frac{P \, G_{o(\ell),d(\ell)}}{N_0} \geq \beta(r(\ell)).$$

Condition [**S3.1**] is the feasibility requirement for link ℓ, i.e., if link ℓ cannot support the link rate $r(\ell)$ at the maximum transmission power P then it is not possible to have a feasible link between nodes $o(\ell)$ and $d(\ell)$. If condition [**S3.1**] is satisfied then link ℓ is feasible for any transmission power level $P(\ell)$ in the interval $\left[\frac{\beta(r(\ell)) \, N_0}{G_{o(\ell),d(\ell)}}, P\right]$. In [**S3.1**], N_0 is the receiver's background noise, $\beta(r(\ell))$ is the Signal to Noise Ratio (SNR) threshold to support link rate $r(\ell)$ and $G_{o(\ell),d(\ell)}$ is the power gain of the channel between nodes $o(\ell)$ and $d(\ell)$. The channel power gain $G_{o(\ell),d(\ell)}$ is a combination of

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[1]Conditions for nodes are denoted as [**Cx.x**] and for SINR are denoted as [**Sx.x**]. Conditions are labeled in the increasing order within the chapter for the second digit while the first digit corresponds to the chapter number.

fading $g_{o(\ell),d(\ell)}$ and the path loss $PL(d_{o(\ell),d(\ell)})$ at distance $d_{o(\ell),d(\ell)}$:

$$G_{o(\ell),d(\ell)} = PL(d_{o(\ell),d(\ell)}) \cdot g_{o(\ell),d(\ell)}. \qquad (3.1)$$

Let $\mathcal{F}$ denote the set of flows, where each flow $f \in \mathcal{F}$ is specified by an ordered pair of nodes $f = (o(f), d(f))$ with $o(f) \neq d(f)$, where $o(d))$ and $d(f)$ are the origin and the destination nodes of flow f, respectively. Also, R_f denotes the rate of flow f or the throughput of flow f.

Usually, centralized multihop networks are based on a mesh-like topology. For this reason, in the following chapters, we provide numerical results for wireless mesh networks with a single gateway. The flow pattern in mesh networks is typically from each node to the gateway (uplink flow) and from the gateway to each node (downlink flow).

3.2 Conflict-free Scheduling and ISets

We focus on multihop networks that are managed by a scheduling-based MAC. We assume that all nodes in a network are perfectly synchronized to allow for conflict-free scheduling. A conflict-free schedule is an assignment of S-TDMA slots to a set of links that can transmit concurrently without causing harmful interference to each other, i.e., all corresponding receivers can decode the signals that are intended for them. When a scheduling-based network is throughput-optimally offline configured then its performance can serve as an upper bound for any random access MAC. Also, such a conflict-free schedule allows the centralized node to be fully aware of any

transmission that occurs in a network at all times, i.e, the centralized node is fully aware of which nodes are transmitting at any given time and which corresponding nodes are successfully receiving data.

We define an ISet s as a set links that can transmit conflict-free on the same channel and in the same S-TDMA slot. By activating only ISets, the scheduling is guaranteed to be conflict-free. In a scheduling-based network, a schedule cycle or S-TDMA frame consists of a sequence of slots during which ISets are assigned for transmission. Usually, such a time-slotted network is optimally configured by solving a binary LP similar to [56] for the minimum length S-TDMA frame. To overcome the complexity of an integer LP, we assume the S-TDMA frame to be infinite long to allow a fractional and flow-based model similar to [2, 12]. Denote by α_s the fraction of time ISet s is activated for transmission, i.e. an ISet s is scheduled if $\alpha_s > 0$.

ISets when continuous power control is enabled: With each ISet s, we associate a power allocation vector $\mathbf{P_s} = [P(\ell)]_{\ell \in s}$ that consists of the transmission powers of all links in s. A set of links $s \subseteq \mathcal{L}$ is an ISet, i.e., all links in s can transmit concurrently when interference is treated as noise, if the following conditions are satisfied [2]:

[**C3.1**] for each $n \in \mathcal{N}$: $\qquad \sum_{\ell \in s} \mathbf{1}_{\{o(\ell)=n\}} \leq 1,$

[**C3.2**] for all $\ell_1, \ell_2 \in s$, $\ell_1 \neq \ell_2$: $\quad o(\ell_1) \neq d(\ell_2),$

[**C3.3**] for each $n \in \mathcal{N}$: $\qquad \sum_{\ell \in s} \mathbf{1}_{\{d(\ell)=n\}} \leq 1,$

[**S3.2**] there exists $\mathbf{P_s}$ such that $\Delta = 0$ in the following subproblem:

---

[2]$\mathbf{1}_{\{A\}}$ is the indicator function: $\mathbf{1}_{\{A\}} = 1$ if condition A is true or otherwise, $\mathbf{1}_{\{A\}} = 0$.

$$\Delta = \min_{\mathbf{P_s}} \sum_{\ell \in s} \phi_\ell$$

$$P(\ell)\, G_{o(\ell),d(\ell)} - \beta(r(\ell))\, N_0$$

$$-\beta(r(\ell)) \sum_{\substack{\ell' \in s \\ \ell' \neq \ell}} P(\ell') G_{o(\ell'),d(\ell)} + \phi_\ell \geq 0 \qquad \forall \ell \in s$$

$$\boldsymbol{\phi}, \mathbf{P_s} \geq 0$$

$$\mathbf{P_s} \leq P$$

Conditions [**C3**.**1**] and [**C3**.**3**] specify that no two distinct links in s can share a source or destination, while [**C3**.**2**] is the half-duplex constraints.

Condition [**S3**.**2**] is the power allocation subproblem for $\mathbf{P_s}$. Each link ℓ in s is required to meet the minimum SINR threshold $\beta(r(\ell))$ for the conflict-free transmission at rate $r(\ell)$. It is only possible when $\Delta = 0$, i.e., there exists such an allocation of transmission powers $P(\ell)$ that allows all links in s to meet the SINR requirements. The variables $\boldsymbol{\phi} = [\phi]_{\ell \in s}$ are indicator variables in the sense that if all $\phi_\ell = 0$, then there exists a power allocation vector $\mathbf{P_s}$ for all transmitting nodes in s such that the SINR conditions for all links in s are satisfied. In the case if $\Delta > 0$ then a set of links s is not an ISet since there does not exist a transmission power vector $\mathbf{P_s}$ that allows the corresponding nodes to transmit concurrently. In practice, due to numerical scaling issues, Δ should be compared against some small number (we took 10^{-12}).

Denote $\mathcal{I}_{CPC}$ as a collection of all ISets in a network where nodes are enabled with Continuous Power Control (CPC) in the interval $(0, P]$ and rate adaptation from the set of available rates $\mathcal{R}$, and when interference is treated as noise.

ISets in a single-power network: For comparison purposes with results in the following chapters, we also define ISets in a network where nodes can support a set of rates $\mathcal{R}$ and transmit at a fixed power P. Denote by $\mathcal{I}_{\mathsf{int}}$ a collection of all ISets in a single-rate and single-power network, then each ISet s in $\mathcal{I}_{\mathsf{int}}$ must meet conditions [**C3**.1], [**C3**.2], [**C3**.3] and

[**S3.3**] for each $\ell \in s$:
$$\frac{P\,G_{o(\ell),d(\ell)}}{N_0 + P\sum_{\substack{\ell' \in s \\ \ell' \neq \ell}} G_{o(\ell'),d(\ell)}} \geq \beta(r(\ell)).$$

3.3 Problem Formulation

Let us denote by $x_f(\ell)$ the amount of flow f transmitted over link ℓ, by $\boldsymbol{x} = [x_f(\ell)]_{\ell \in \mathcal{L}, f \in \mathcal{F}}$ an aggregated routing vector of all flows allocated over all links, and by $\boldsymbol{R} = [R_f]_{f \in \mathcal{F}}$ a vector of all flow rates. Also, denote by $\boldsymbol{\alpha} = [\alpha_s]_{s \in \mathcal{I}}$ a scheduling vector that characterizes the conflict-free scheduling in a network with a collection of all ISets $\mathcal{I}$. Since, α_s is a fraction of time an ISet is scheduled in one schedule cycle, then $\sum_{s \in \mathcal{I}} \alpha_s = 1$.

The Joint Routing and Scheduling (JRS)[3] problem for the max-min throughput can be formulated for a given $\mathcal{I}$ as follows [3]:

[**P1**] :
$$\max_{\boldsymbol{\alpha}, \boldsymbol{x}, \boldsymbol{R}} R \tag{3.2}$$

$$\sum_{\substack{\ell \in \mathcal{L} \\ o(\ell)=n}} x_f(\ell) - \sum_{\substack{\ell \in \mathcal{L} \\ d(\ell)=n}} x_f(\ell) = \begin{cases} R_f, & n=o(f) \\ \text{-}R_f, & n=d(f) \\ 0, & \text{else} \end{cases} \quad \begin{aligned} &\forall n \in \mathcal{N} \\ &\forall f \in \mathcal{F} \end{aligned} \tag{3.3}$$

---

[3]The results for JRS with continuous power control in small to medium networks are presented in Chapter 5.

$$r(\ell) \sum_{s \in \mathcal{I}} \alpha_s \mathbf{1}_{\{\ell \in s\}} \geq \sum_{f \in \mathcal{F}} x_f(\ell) \qquad\qquad \forall \ell \in \mathcal{L} \qquad (3.4)$$

$$\sum_{s \in \mathcal{I}} \alpha_s = 1 \qquad\qquad\qquad (3.5)$$

$$R_f \geq R \geq 0 \qquad\qquad\qquad \forall f \in \mathcal{F} \qquad (3.6)$$

$$\boldsymbol{\alpha}, \boldsymbol{x}, \boldsymbol{R} \geq 0.$$

Condition (3.3) specifies the flow conservation constraints for each node and flow in the network. Link scheduling constraints are given in (3.4) and (3.5). Specifically, constraints in (3.4) restrict the total amount of flow that can be scheduled over a link ℓ to its link rate capacity $r(\ell) \sum_{s \in \mathcal{I}} \alpha_s \mathbf{1}_{\{\ell \in s\}}$. Constraint (3.5) states that ISets must be scheduled over a unit period of time. This framework is general enough to accommodate any quasi-static channel model.

In fact, [**P1**] is a variant of the NUM problem that can be used for the throughput-optimal configuration of fixed wireless mesh or ad-hoc networks. In the case of proportional fair, the objective function (3.2) can be replaced by $max \sum_{f \in \mathcal{F}} log(R_f)$ and the constraints (3.6) can be removed from the problem formulation [**P1**]. We formulate [**P1**] for the max-min throughput for the reason that we provide our numerical results in mesh networks. It is a reasonable objective function in mesh networks [12] as it prevents the relative starvation of nodal flow rate while trying to maximize the system throughput. It was shown in [3,8] that the gateway is the bottleneck in wireless mesh networks. The per node throughput under max-min policy is upper bounded by $\frac{r_m}{N-1}$ in a network with a single gateway and $N-1$ nodes, where r_m is the maximum possible rate in $\mathcal{R}$ for the given set of modulation and coding schemes.

Clearly, if each node is able to communicate with the gateway in single hop at rate r_m then this upper bound is feasible. It was shown that this bound can typically be reached at much lower transmit power by using multihop communications [12]. It is possible to increase the per-node throughput beyond this upper bound for the max-min policy by employing advanced physical layer techniques that are discussed in Chapter 4.

Typically in mesh networks, a downlink flow has a larger rate than an uplink flow. To take this remark into consideration, problem [**P1**] can be formulated as a weighted optimization problem for the max-min throughput by replacing constraint (3.6) with

$$\frac{R_f}{w_f} \geq R \geq 0 \qquad \qquad \forall f \in \mathcal{F}, \tag{3.7}$$

where w_f is a per flow weighting factor.

The solution to problem [**P1**] for $\mathcal{I} = \mathcal{I}_{CPC}$ provides jointly optimal routing and scheduling offline configuration in a network with continuous power control and rate adaption when interference is treated as noise. There are generally more than one solution to problem [**P1**].

Problem complexity: Although the problem [**P1**] is formulated as a Linear Program(LP), it is NP-hard. The maximum number of links $|\mathcal{L}|$ in a network with a set of rates $\mathcal{R}$ and with N nodes grows in $O(N^2|\mathcal{R}|)$. The number of variables is an $O(|\mathcal{L}||\mathcal{F}| + |\mathcal{L}|^M)$, where M is the maximum ISet size. In order to enumerate all possible ISets for $\mathcal{I}_{\mathsf{int}}$ and $\mathcal{I}_{CPC}$, all elements in the power set of $\mathcal{L}$ must be checked as possible candidates for being an ISet. In addition, in the case of CPC, the subproblem

in condition [**S3.2**] must be solved for each ISet. This approach is intractable even for reasonable size networks due to the fact that the power set of $\mathcal{L}$ grows exponentially in $O(2^{|\mathcal{L}|})$. However, an optimal solution use only a very small subset of ISets, i.e, at most $|\mathcal{L}|+1$ ISets are needed to obtain an optimal solution as the number of non-zero (basis variables) α_s is at most $|L| + 1$. In [12], the use of column generation was proposed to solve these type of problems by avoiding enumeration of all ISets. The only challenge is that even with the use of column generation, the convergence to an optimal solution might degenerate due to the instability in dual variables [57].

In the column generation method, the solution is determined to be optimal if no ISets can be found with strictly positive reduced costs. The reduced costs are computed for new off-basis columns or ISets s as follows:

$$-(\zeta + \sum_{\ell \in s} r(\ell) v_\ell), \tag{3.8}$$

where v_ℓ and ζ are the dual variables for (3.4) and (3.5), respectively[4].

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[4]We obtain our results with the use of Cplex 12.3 solver on the machine with the following HW specifications: 6-core CPU with 2.93 GHz and RAM of 24GB. The computation times vary from several hours up to 2 days depending on the problem and the size of the network.

Chapter 4

Cross-layer Optimization in Conventional Multihop Networks

The advanced techniques on network and physical layer involve fundamentally differ-
ent principles that allow an increase of the spatial reuse in a network and as a result,
may lead to a throughput improvement. In Section 4.1, we consider several physical
layer techniques and their combinations that are based on modulation pre-coding or
interference cancellation techniques. In Section 4.2, we consider a network coding
technique that improves the spatial reuse in a network by exploiting the broadcasting
properties of the wireless channel. We also provide engineering insights and quantify
throughout gains that can be achieved in medium size networks that employ these
techniques when jointly optimizing routing and scheduling. The baseline and starting
point for our framework is the JRS problem in Chapter 3.

4.1   Advanced Physical Layer Techniques [1]

First we formulate a joint routing and scheduling problem with advanced physical layer techniques for the optimal offline configuration of fixed wireless networks. In fact, we formulate a flow-based model by just revisiting the notion of ISets. In particular, we consider the following techniques: Successive Interference Cancellation (SIC), Superposition Coding (SPC) and Dirty-paper Coding (DPC) and some of their combinations. While these advanced techniques are well known, they are mainly studied in an information theory framework. Very few studies, if any, have tried to quantify the gains in maximum achievable throughput that can be obtained by using these techniques in a practical size network with the SINR-based interference model.

In Section 4.1.1, we consider SIC, first proposed in [16], which is a technique that enables a wireless receiver to decode multiple signals at a time to either partially cancel the interference or receive more than one packet [58]. We study the cases where SIC is enabled at all nodes or only at the gateway. As SIC requires proper modeling of the interference by all nodes, only interference models that can capture the cumulative effect of interference (such as the SINR-based model) can be used [18]. The work of [17] was among the first to study the impact of SIC on capacity regions in scheduled wireless multihop networks. However, the results were limited to very small networks.

Also initially proposed in [16], SPC is a technique that enables a wireless transmitter to send several signals at the same time, possibly intended for different users, as a composite signal. This technique is studied in Section 4.1.2. In order for a user

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[1]The main results of this study were presented in our works [18, 19].

to decode its own signal from such a composite signal, a SIC receiver is necessary. We study the cases where 1) SPC and SIC are both enabled at all nodes, 2) the case when SIC is enabled only at the gateway and 3) SIC is enabled at all nodes while SPC is enabled only at the gateway. To the best of our knowledge, [20] was the only prior study that considered an optimal power partition of the superimposed signals with routing in a wireless broadcast network with power control. However, scheduling was not considered in their model as all nodes were allowed for simultaneous transmission, i.e., full-duplex operation was assumed. Another limitation of the work of [20] is that the SIC technique was restricted to decode only superimposed signals originating from a common node. Thus, non-SPC and SPC signals originating from other nodes were treated as noise and could not be canceled out. In our work of [19], SPC was considered along with full capability SIC receivers in a single-rate and single-power network that was jointly optimized with routing, scheduling, SIC, and SPC power allocation.

DPC, first presented in [22], is a technique used at a transmitter to encode a signal with prior knowledge of the interference at a particular receiver so that, at this receiver, the harmful interference is perfectly mitigated. As a result, it allows a receiver to effectively benefit from an interference free transmission at no extra power cost to the transmitter. We only study the case when DPC is enabled at the gateway and not in other nodes due to practical implementation challenges that are discussed in Section 4.1.3. Hence, in the following DPC is to be understood as DPC at the gateway only. To the best of our knowledge, there were no prior work that quantified gains of DPC when jointly optimized with routing and scheduling except our work in [19], where we quantified gains of using DPC in wireless mesh networks.

System model:. First, for simplicity of description, we define our system model for a single-power and multi-rate network. At the end of this section, we will show how to adapt this model to networks that allow continuous power control. Consider a multihop network where nodes can support a set of rates $\mathcal{R}$ and can transmit at fixed power P. In the case of SIC and DPC techniques, we define link ℓ by a triple $(o(\ell), d(\ell), r(\ell))$ that needs to meet condition [**S3.1**] to be feasible. We define links for SPC later in this section. All other model descriptions and definitions are described in Chapter 3.

4.1.1   Successive Interference Cancellation

Background: Interference cancellation schemes can be categorized in three main groups: parallel [59], successive [60] and the combination of parallel and successive schemes [61]. There are different trade-offs between these schemes in terms of decoding latency, performance and complexity. A parallel scheme is preferable when received powers are fairly equal and a successive scheme operates best with an unequal received power distribution [58]. Due to the practical complexity of parallel schemes, we consider only the interference cancellation based on a successive scheme. The SIC technique allows a receiver to decode multiple signals to either partially cancel the interference or receive more than one packet at a time.

We denote by SIC(k) a generic SIC receiver that can perform up to $k-1$ rounds of successive interference cancellation, i.e., up to k signals can be decoded at any node in a network. Fig. 4.1 shows the structure of a SIC filter bank that allows a node to obtain a stream of data at one of the outputs. In Fig. 4.1, Y denotes the total received signal, $\hat{x}_1, .., \hat{x}_k$ denotes the streams of decoded symbols and $\hat{h}_1, .., \hat{h}_{k-1}$ are
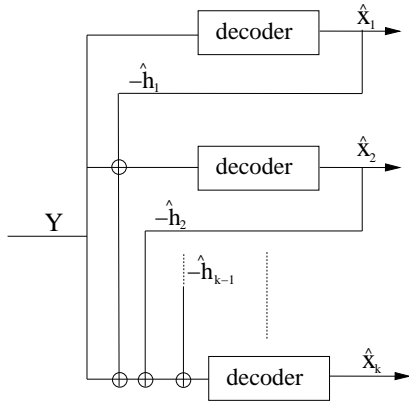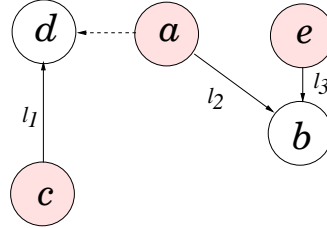
Figure 4.1: SIC filter bank

Figure 4.2: Illustration of SIC(2)

the channel estimates for subtracting the decoded signals from Y. A receiver with SIC(k) can be built either by placing in parallel k such SIC filter banks or by using $k-1$ delay lines with one SIC filter bank.

Let us consider the example[2] in Fig. 4.2. Node b without a SIC receiver cannot receive from both nodes a and e at the same due to the half-duplex radio, and node d cannot receive a signal from c due to the strong interference from node a. Now when SIC is enabled at the receivers, it is possible for node d to first decode the strong interfering signal from a. Node d can then subtract it from its compound received signal so that it can now decode the signal from c. In this case, d has partially canceled the interference to decode the signal from c successfully. With SIC, node b can also decode both signals from a and e. Node b first decodes the strong signal from e, cancels its interference out to decode successfully the signal from a. This example illustrates how SIC can help to improve the network performance, i.e., the max-min flow rate.

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[2]Dashed line denotes the interference.

ISets when SIC is enabled: In order to incorporate $\text{SIC}(k)$ into the JRS problem described in Section 3, we need to revisit under which conditions an ISet s exists. We denote by $\mathcal{DO}(\ell)$ a decoding order for link ℓ at the $\text{SIC}(k)$ receiver of node $d(\ell)$. Each link ℓ in s must have at least one decoding order $\mathcal{DO}(\ell)$. We define $\mathcal{DO}(\ell)$ as an ordered set of links $\mathcal{DO}(\ell) = (\ell_1, \ldots, \ell_{k_\ell})$, where all ℓ_j must be in s, the last decoding link ℓ_{k_ℓ} must be equal to ℓ, all ℓ_j must be unique so that $\ell_j \neq \ell_i$ for $j \neq i$. Also, the length of decoding order $\mathcal{DO}(\ell)$ cannot exceed k so that $k_\ell \leq k$. $\text{SIC}(k)$ allows the receiver to decode up to k signals sent to it or to allow partial decoding of the interference from up to $k - 1$ other sources. For partial decoding of the interference, the ordered set $\mathcal{DO}(\ell)$ may include links that do not have $d(\ell)$ as a destination node, i.e. to allow partial decoding of the interference.

If all nodes in a network are enabled with $\text{SIC}(k)$ capabilities, then a set of links s is an ISet if it satisfies the node conditions [**C3.1**], [**C3.2**], as well as:

[**C4.1**] for each $n \in \mathcal{N}$: $\qquad \sum_{\ell \in s} 1_{\{d(\ell)=n\}} \leq k,$

[**S4.1**] for all $\ell \in s$: there exists $\mathcal{DO}(\ell)$ such that for each $\ell_j \in \mathcal{DO}(\ell)$

$$\frac{P(\ell_j)\, G_{o(\ell_j),d(\ell)}}{N_0 + I_\ell - \sum_{i=1}^{j} P(\ell_i)\, G_{o(\ell_i),d(\ell)}} \geq \beta(r(\ell)).$$

In condition [**S4.1**], I_ℓ denotes the total received signal power $I_\ell = \sum_{\ell' \in s} P(\ell') G_{o(\ell'),d(\ell)}$ at link ℓ. For a set of links s to be an ISet, each link ℓ in s must have at least one decoding order $\mathcal{DO}(\ell)$ to satisfy condition [**S4.1**]. With the use of this decoding order, the receiver at node $d(\ell)$ is capable to decode signals from $o(\ell)$ and to read the correct output stream as shown in Fig. 4.1.

In a single-power network, for small k, the decoding order $\mathcal{DO}(\ell)$ can be found by

iteratively checking all possible decoding order sets of length up to k for transmission power levels $P(\ell) = P$ for all $\ell \in s$, i.e., all possible combinations of up to k must be checked for [**S4.1**]. If no decoding order can be found for at least one link in s, then the set of links s is not an ISet. In the case when CPC is enabled with SIC(k), it is not possible to check condition [**S4.1**] without an additional power allocation subproblem that will be discussed later in this section.

Denote by $\mathcal{I}_{\mathsf{SIC}(k)}$ the collection of all ISets in the network with SIC(k).

Since the maximum number of links to any node is now k, the max-min throughput is now upper bounded by $\frac{k \times r_m}{N-1}$ for uplink flows and still by $\frac{r_m}{N-1}$ for downlink flows.

We also study the case where SIC is only enabled at the gateway G. We consider this variant of SIC because we provide numerical results for mesh networks. In this case, a set of links s is an ISet if it meets: i) [**C3.1**] and [**C3.2**] for all links, ii) [**C3.3**] and [**S3.3**] for all links ℓ such that $d(\ell) \neq G$, and iii) [**C4.1**] and [**S4.1**] for all links ℓ such that $d(\ell) = G$.

4.1.2 Superposition Coding

Background: SPC is well-known as an efficient technique to increase the throughput of multiuser systems. The idea of SPC is to allow a node to transmit several signals at a time intended for different nodes in a network as a composite signal. When transmitters are enabled with SPC, the use of SIC at the receivers is required for optimal decoding. This is a challenging task to find such a jointly optimal configuration for SPC with SIC since 1) we allow SPC at each node 2) we allow SIC at each node.

We denote by SPC(m)-SIC(k) a generic SPC scheme with full capability SIC receivers. In a network where SPC(m)-SIC(k) is enabled at each node, a transmitter
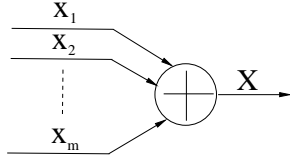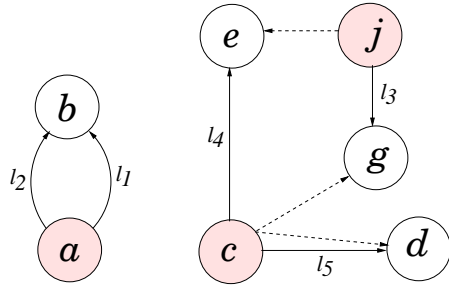
Figure 4.3: SPC encoder

Figure 4.4: Illustration of SPC(2)-SIC(2)

can superimpose up to m signals simultaneously and the receiver can decode a maximum of k signals. Fig. 4.3 shows a SPC encoder, where the composite signal X is the sum of up to m modulated signals $x_1, .., x_m$ that are intended for different nodes. We define a full capability SIC receiver as a receiver that is able to decode any signal either direct or superimposed from any node in a network as well as partially cancel the interference. We denote by SPC(m) a restricted variant of superposition coding where a SIC receiver is used for decoding only superimposed signals from a common source. This variant of SPC with restricted SIC is sometimes studied in the literature as in the work of [20].

Let us consider the example in Fig. 4.4 where all nodes transmit with single power P and are enabled with SPC(2)-SIC(2). Nodes a and c transmit composite signals and node j transmits a direct signal to g. All nodes transmit with the same power P. The composite signal at node c is the sum of two signals with powers $P(\ell_4)$ and $P(\ell_5)$ destined to node e and node d, respectively. The composite signal from node a is the sum of two signals with powers $P(\ell_1)$ and $P(\ell_2)$ both destined to node b. It is certainly possible to transmit from node a to b at higher rate but this requires node a to increase its transmission power, while with SPC the same rate is achieved

without an increase in transmission power. To allow concurrent transmissions of links $\{\ell_1, .., \ell_5\}$ without harmful interference to each other, the powers at nodes a and c must be partitioned jointly with respect to all link powers. If at node a the power is split such that $P(\ell_1) > P(\ell_2)$ and $P(\ell_1) + P(\ell_2) = P$, then the receiver first decodes a signal over link ℓ_1 and after canceling it out, the receiver at node b decodes a signal over link ℓ_2. At node c the power is split such that $P(\ell_4) > P(\ell_5)$ and $P(\ell_4) + P(\ell_5) = P$, then at nodes d and g the main interferer is the superimposed signal over link ℓ_4 and at node e the main interferer is the direct signal from node j. SIC receivers first cancel out their strong interference signals from the compound received signals, and then decode their own signals. This example in Fig. 4.4 is only possible because we allow SIC receivers to decode the main interference from any node in a network. If only SPC(2) is enabled, then in this example, node e cannot decode signal from node c since it cannot cancel the strong interference from node j.

ISets when SPC and SIC are enabled: In a network with SPC(m)-SIC(k), any node can transmit a composite signal destined to multiple destinations. If a composite signal is the superposition of m signals then it results in up to m links $\ell_1, \ldots, \ell_m$ leaving a common source node, and necessarily $\sum_{i=1}^{m} P(\ell_i) = P$, i.e., for these links with a common source node $P(\ell_i) \neq P$. Since, the total transmission power of a composite signal is still P, SPC does not introduce additional interference in a network. Also, the choice of power partition of SPC signals does not impact the level of interference at other nodes. However, to fully utilize SPC capabilities, the transmission powers over superimposed links must be allocated optimally and jointly among all SPC destination nodes to maximize the spatial reuse in a network. At first

glance, it is not trivial to find such a jointly optimal power allocation since we also allow SIC(k) at each node, except SPC(1)-SIC(k) which is equivalent to the SIC(k) case.

In the case of SPC(m), link ℓ is defined by a quadruple $\ell = (o(\ell), d(\ell), r(\ell), i)$, where i is a unique sequence number to distinguish it from parallel links with the same origin and destination nodes, and to be feasible it needs to meet [**S3.1**]. Denote by $\mathbf{P_s} = [P(\ell)]_{\ell \in \mathbf{s}}$ a power allocation vector for all the links in a given set of links s. If in a network all nodes are enabled with SPC(m)-SIC(k) capabilities, then a set of links s is an ISet if it satisfies conditions [**C3.2**], [**C4.1**], [**S4.1**] and

[**C4.2**] for each $n \in \mathcal{N}$: $\qquad \sum_{\ell \in s} 1_{\{o(\ell)=n\}} \leq m,$

[**C4.3**] for each $n \in \mathcal{N}$: $\qquad \sum_{\substack{\ell \in s \\ o(\ell)=n}} P(\ell) = P.$

Each link ℓ in s is either a direct link with $P(\ell) = P$ or a superimposed link with $P(\ell) < P$. It is easy to check if a set of links s is an ISet for all conditions except [**S4.1**]. To check the feasibility of links for [**S4.1**], it is necessary to enumerate all inequalities [**S4.1**] for each possible decoding order $\mathcal{DO}(\ell)$ of length up to k for each link in s. The set s is an ISet if we can find at least one decoding order $\mathcal{DO}(\ell)$ for each link that satisfies [**S4.1**] for a common power vector $\mathbf{P_s}$.

We address this SINR feasibility [**S4.1**] of an ISet by formulating a resource subproblem for $\mathbf{P_s}$. The purpose of this subproblem is to find a vector power $\mathbf{P_s}$ and a decoding order $\mathcal{DO}(\ell)$ that satisfy the condition [**S4.1**] for each link ℓ in s.

For a fixed set of links s, denote by $\mathcal{D}(\ell)$ the collection of all possible decoding

order sets of length up to k for link ℓ. Let us define a binary variable $a_{w,\ell}$ as follows:

$$a_{w,\ell} = \begin{cases} 1, & \text{if } w \text{ is a feasible decoding order set for link } \ell \\ 0, & \text{otherwise}, \end{cases}$$

where $w \in \mathcal{D}(\ell)$ and $\ell \in \mathcal{L}$.

If s satisfies the node constraints given in [**C3.2**], [**C4.1**] and [**C4.2**], then the optimal power allocation subproblem for $\mathbf{P_s}$ can be formulated as follows:

$$\Delta = \min_{\mathbf{P_s}, \boldsymbol{a}, \boldsymbol{\phi}} \sum_{\substack{w \in \mathcal{D}(\ell) \\ \ell_j \in w \\ \ell \in s}} \phi_{w,\ell_j,\ell} \tag{4.1}$$

$$P(\ell_j)G_{o(\ell_j),d(\ell)} - \beta(r(\ell))(N_0 + I_\ell)a_{w,\ell} +$$

$$\beta(r(\ell)) \sum_{i=1}^{j} P(\ell_i)G_{o(\ell_i),d(\ell)} + \phi_{w,\ell_j,\ell} \geq 0 \qquad \begin{aligned} &\forall \ell \in s, \forall \ell_j \in w \\ &\forall w \in \mathcal{D}(\ell) \end{aligned} \tag{4.2}$$

$$\sum_{\substack{\ell' \in s \\ o(\ell')=o(\ell)}} P(\ell') = P \qquad \forall \ell \in s \tag{4.3}$$

$$\sum_{w \in \mathcal{D}(\ell)} a_{w,\ell} \geq 1 \qquad \forall \ell \in s \tag{4.4}$$

$$a_{w,\ell} \in \{0,1\} \qquad \begin{aligned} &\forall \ell \in s \\ &\forall w \in \mathcal{D}(\ell) \end{aligned} \tag{4.5}$$

$$\mathbf{P_s}, \boldsymbol{\phi} \geq \mathbf{0}. \tag{4.6}$$

In the formulation above, we denote $\boldsymbol{\phi}$ as the vector of all indicator variables $\phi_{w,\ell_j,\ell}$ and $\boldsymbol{a}$ as the vector of all binary variables $a_{w,\ell}$. Constraints (4.2) are the SINR

conditions [**S4.1**] for each link in s. Constraints (4.3) are the SPC power conditions [**S4.1**], and constraints (4.4) specifies that each link ℓ in s should have at least one decoding order $\mathcal{DO}(\ell)$ that is feasible. The variables $\phi_{w,\ell_j,\ell}$ are indicator variables in the sense that if all $\phi_{w,\ell_j,\ell} = 0$, then there is a feasible solution to [**S4.1**]. Specifically, if the solution to the subproblem results in $\Delta = 0$ (Δ is the value of the objective function), then there must be a common power vector $\mathbf{P_s}$, and for each link ℓ in s, there is a decoding order $\mathcal{DO}(\ell)$ such that $a_{\mathcal{DO}(\ell),\ell} = 1$. Hence, for these decoding order sets and since all $\phi_{m,\ell_j,\ell} = 0$, constraints (4.2) imply that the condition [**S4.1**]is satisfied, and therefore, a set of links s is an ISet. Conversely, if $\Delta > 0$, then there does not exist a power vector $\mathbf{P_s}$, and decoding orders $\mathcal{DO}(\ell)$ that satisfy [**S4.1**] for s. In practice, due to numerical scaling issues, to check if an optimal power allocation is feasible for [**S4.1**], we need to check for $\Delta \in [-\delta, \delta]$, where δ is a small value (of the order of 10^{-12}).

Denote by $\mathcal{I}_{\mathsf{SPC}(m)-\mathsf{SIC}(k)}$ the collection of all ISets in a network where nodes are enabled with SPC(m)-SIC(k) capabilities.

Since the maximum number of incoming links and outgoing links that can be achieved simultaneously from any node is k and m respectively, the max-min throughput for uplink flows is upper bounded by $\frac{k \times r_m}{N-1}$, whereas for downlink flows it is upper bounded by $\frac{\min(m,k) \times r_m}{N-1}$.

For comparison purposes with [20], we also consider the case of SPC(m) where SIC is only employed to cancel interference from a single composite source at a time, i.e., [**C3.2**], [**C4.1**], [**C4.2**], [**C4.3**] and [**S4.1**] hold, where $\mathcal{DO}(\ell)$ in [**S4.1**] is limited to links from the same source as link ℓ. $\mathcal{I}_{\mathsf{SPC(m)}}$ denotes the collection of all ISets in a network using SPC(m) with restricted variant of SIC.

ISets when SPC, SIC and CPC are enabled: Now, we formulate a problem for the joint optimal configuration of SIC-SPC with continuous power control. Consider a network where each node can support a set of rates $\mathcal{R}$, transmit at power levels in the interval $(0, P]$ and can support SIC and SPC. Then a set of links s is an ISet if it meets conditions [**C3.2**], [**C4.1**], [**C4.2**] and $\Delta = 0$ for the subproblem (4.1)-(4.6) with constraints (4.3) replaced by

$$\sum_{\substack{\ell' \in s \\ o(\ell')=o(\ell)}} P(\ell') \leq P \qquad \forall \ell \in s$$

Solution to this subproblem is an optimal power allocation $\mathbf{P_s}$ for ISet s and decoding orders $\mathcal{DO}(\ell)$ for each link in s. It is an optimal power allocation in the sense that in this subproblem the feasible region is an optimal region. In the case when only SIC is enabled with CPC, then a set of links s is an ISet when it meets node conditions [**C3.1**], [**C3.2**] and $\Delta = 0$. Hence, we are enabled to formulate the SIC-SPC-CPC problem by only revisiting the notion of ISets.

4.1.3 Dirty-paper Coding

Background: DPC is a pre-coding technique that is used at a transmitter to encode a signal with prior knowledge of interference at a particular receiver such that at this receiver the harmful interference is "canceled out" without requiring any additional transmission power. While details on DPC are beyond the scope of this thesis, a general idea of DPC is shown in Fig. 4.5. A transmitter with DPC needs to encode a signal x and the interference I with a modulation function $f(x, I)$, e.g., using modulo-
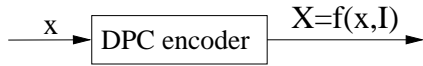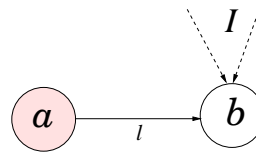
Figure 4.5: DPC encoder

Figure 4.6: Illustration of DPC

lattice modulation [24]. Referring to Fig. 4.6, signal x is modulated with DPC then it allows a receiver at node b to effectively benefit from an interference free transmission. However, DPC is only possible if the transmitter with DPC knows the data being sent by other transmitters along with the corresponding channel gains to estimate the interference I at a particular receiver.

It is hard to benefit from DPC in a general network. However, in the context of a scheduled and centralized wireless mesh network, a gateway on the downlink has perfect knowledge of the data being sent in a network since all packets originate from the gateway itself. With appropriate feedback, it can be made aware of any links along with their channel gains. Thus, in a scheduled network, the gateway is able to estimate the aggregated interfering signal I at any node from other active links for downlink flows only. For uplink flows, in general, it is not possible for any node to be aware of the interfering signals as uplink flows do not have a common node of origin. Thus, for DPC, we separate the scheduling of uplink and downlink flows. For downlink flow scheduling, links that originate from the gateway G are interference free using DPC, while all other links will be subject to interference. For uplink flows, all links are subject to interference.

ISets when DPC is enabled: Thus, if $\mathcal{I}_{\mathsf{DPC}}$ is the collection of all ISets with DPC, then each ISet $s \in \mathcal{I}_{\mathsf{DPC}}$ must satisfy the requirements of [**C3.1**], [**C3.2**], [**C3.3**] and

in addition the SINR condition [**S3.3**] if $o(\ell) \neq G$ or [**S1.1**] for $o(\ell) = G$. $\mathcal{I}_{\mathsf{DPC}}$ is only used to schedule downlink flows. Uplink flows are scheduled using $\mathcal{I}_{\mathsf{int}}$ described in Section 3, and the two schedules are combined by time division duplexing so as to maximize the minimum flow rate over all uplink and downlink flows. In the case of DPC, the max-min throughput in a mesh network is bounded by $\frac{r_m}{N-1}$ since all downlink flows are originated from a gateway which can operate only over a single link at a time.

4.1.4 Problem Formulations

Denote by $\mathcal{I}$ a collection of ISets that can be constructed based on the conditions described above for each physical layer technique, i.e., $\mathcal{I}$ is either $\mathcal{I}_{\mathsf{SIC}(k)}$, $\mathcal{I}_{\mathsf{SPC}(r)-\mathsf{SIC}(k)}$, $\mathcal{I}_{\mathsf{SPC}(r)}$, or $\mathcal{I}_{\mathsf{int}}$ described in Section 3.2.

The JRS problem with physical layer technique for a given $\mathcal{I}$ is then formulated identically to the problem [**P1**] in Section 3.3. It is a flow-based formulation that allows us to compute solutions not only in terms of R but also in terms of the optimal network configuration for routing, scheduling, and physical layer parameters, i.e. an optimal decoding orders $\mathcal{DO}(\ell)$ for each ISet and an optimal power allocation $\mathbf{P_s}$ for each ISet.

For DPC, we solve the JRS problem twice: first for downlink flows only using $\mathcal{I}_{\mathsf{DPC}}$, and then for uplink flows only using $\mathcal{I}_{\mathsf{int}}$. These two schedules are then combined using time division duplexing so as to maximize the minimum flow rate over all uplink and downlink flows per node.

Problem complexity: These problems are of larger scale than the original JRS

problem [**P1**] in Section 3.3. The maximum number of links $|\mathcal{L}|$ is in the order of $O(N^2)$ and the number of ISets is in the order of $O(|\mathcal{L}|^{M'})$, where $M' > M$ is the maximum ISet size. It is not tractable to directly enumerate all ISets as this requires testing all $2^{|\mathcal{L}|} - 1$ elements of the power set of $\mathcal{L}$ to check if they are ISets. In addition, in the case of SPC, for each element in the power set, a subproblem must be solved to check if there exists a power allocation for SPC signals. This subproblem is a binary LP problem for which it is necessity to construct all possible decoding orders for each link. Thus, an enumeration search is not a viable technique.

We use the same approach as in the work of [12] to develop a column generation method with additional enhancements: e.g. we find ISets that allow an increase in capacity of bottleneck cliques, for addition of those ISets that allow for faster convergence, as well as elimination of redundant variables and some other techniques to stabilize duals. With the use of the column generation method, we determine a solution to be an optimal solution if no ISets with strictly positive reduced costs can be found in the next iteration. The reduced costs for new ISets are computed as in (3.8). Using this approach we are able to avoid the enumeration of all ISets and to solve the problems within reasonable time. We provide only optimal solutions to the problems.

4.1.5 Numerical Results and Insights

We provide exact numerical solutions for medium size wireless mesh networks with a total of $N = 20$ nodes, where $N - 1$ nodes are placed uniformly at random in a 2km by 2km square, and a gateway node G is placed in the center of the square. We show our results in terms of nodal max-min throughput denoted as R^* where $R^* = R$ in

a network with only uplink (resp. downlink) flows, and $R^* = 2R$ when there are an uplink flow and a downlink flow per node. Solutions R are obtained by solving to optimality problem [**P1**] with the corresponding physical layer technique and hence, the right definition of ISets. We label by P_{SH} the minimum transmission power at which all nodes are able communicate with the gateway in single hop.

We assume that each node uses a fixed transmit power budget P and the same single modulation scheme yielding a normalized rate $r = 1$ with a corresponding SINR threshold of $\beta = 6.4$dB. We use the channel model in (3.1) but for simplicity and without loss of generality, we model channels without fading using the following path loss model:

$$PL(d_{o(\ell),d(\ell)}) = \left(\frac{d_{o(\ell),d(\ell)}}{d_0} \right)^{-\mu}, \tag{4.7}$$

where $d_0 = 10$m is the near-field crossover distance, $\mu = 3$ is the path loss exponent and $d_{o(\ell),d(\ell)}$ is the distance between a transmitting node $o(\ell)$ and a receiving node $d(\ell)$. The receiver's background noise is $N_0 = -100$dBm.

We label by *JRS* the solutions to [**P1**] with $\mathcal{I} = \mathcal{I}_{\text{int}}$ case where interference is treated as noise. Techniques SIC(2), SIC(3), SPC(2)-SIC(2) and DPC are labeled as such in the figures.

We have studied multiple realizations of 20-nodes networks. In Fig. 4.7, we show the relative gains of each technique with respect to the baseline results for the JRS problem with $\mathcal{I}_{\text{int}}$ as a function of the transmission power P in networks with uplink
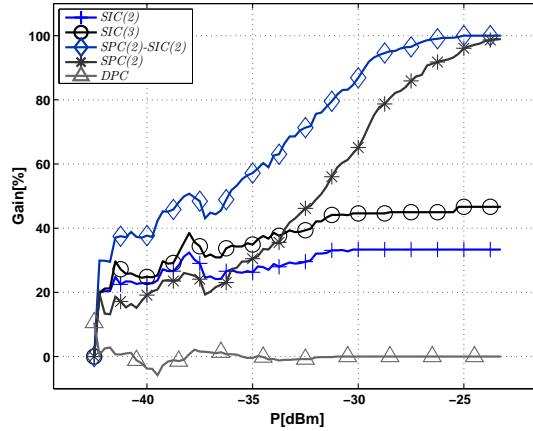
Figure 4.7: Relative gain vs transmission power P, uplink+downlink flows

and downlink flows. The relative gain of each technique is computed as follows:

$$\mathsf{gain}(P) = \frac{\bar{R}(P) - \bar{R}(P)_J}{\bar{R}(P)_J} \, 100\%, \tag{4.8}$$

where $\bar{R}(P)$ is the averaged throughput at power P over 10 random network realizations and $\bar{R}_J(P)$ is the averaged throughput at power P for the JRS problem. Each network realization has the same number of nodes with the gateway located in the center. We provide these results to show general "average" trends for throughput in random topology networks. The results for DPC show that it provides no gains and even underperforms compared to the JRS case where uplink and downlink flows are jointly optimized. For SIC(2) and SIC(3), high throughput gains are obtained across the entire range power. However, even SIC(3) lags the throughput gains that can be offered by SPC(2)-SIC(2). The case of SPC(2)-SIC(2) provides the maximum possible throughput increase from 100% at high powers and $60 - 80\%$ at intermediate power levels. SPC(2)-SIC(2) even outperforms SIC(3) at low power regime with gains

(a) Net-1 (b) Net-2

Figure 4.8: Placement of nodes

(a) Net-1 (b) Net-2
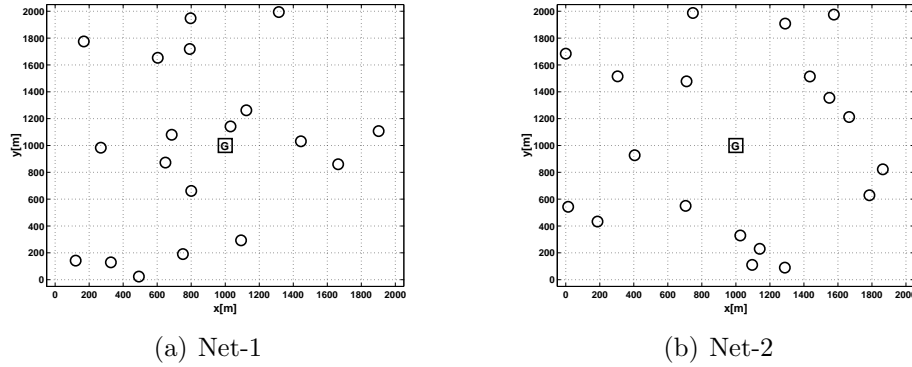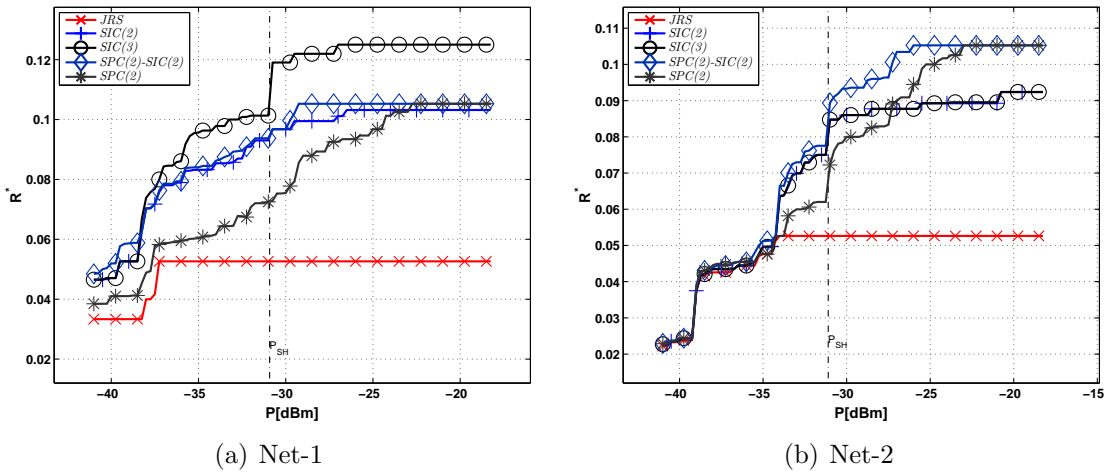
Figure 4.9: max-min throughput vs transmission power P, uplink flows

of up to 40%.

For comparison, we choose two networks Net-1 and Net-2, shown in Fig. 4.8(a) and in Fig. 4.8(b). Net-1 and Net-1 were selected because among the many realizations that we have performed, Net-1 in general had the largest performance gains, while Net-2 had the lowest.

In Fig. 4.9, we show R^* as a function of the transmission power P when the flow pattern is uplink-only for Net-1 and Net-2. In the JRS case, the maximum achievable
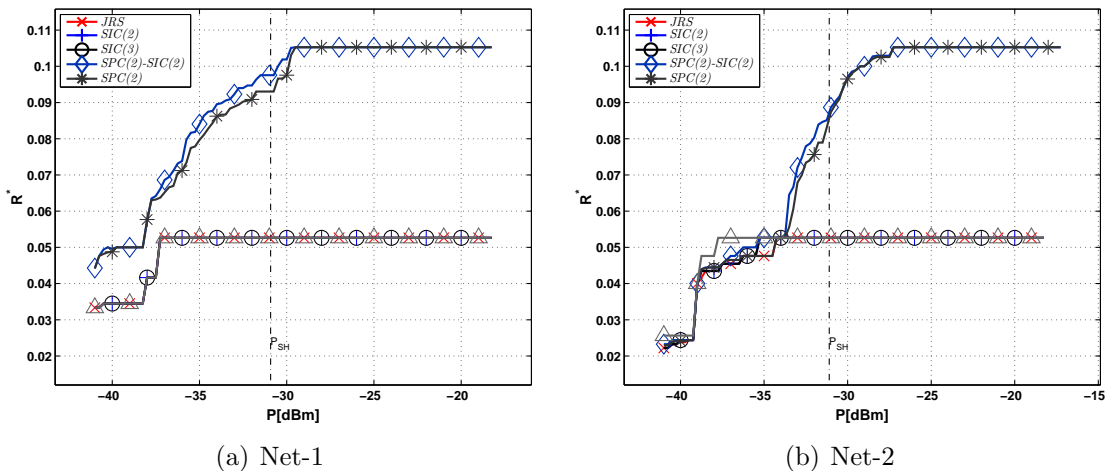
(a) Net-1   (b) Net-2

Figure 4.10: max-min throughput vs transmission power P, downlink flows.

throughput is bounded to $1/(N-1) = 0.0526$, which is achieved in both networks at much lower powers than P_{SH} due to the multihop advantage reported in [12]. The theoretical maximum throughput for SIC(2) is $2/(N-1) = 0.1053$ and for SIC(3) it is $3/(N-1) = 0.1579$, and yet at no transmission power these bounds are achieved using SIC(2) or SIC(3). It is because with Net-1 and Net-2 network topologies, it is not possible to have three simultaneous transmissions to the gateway at all times.

In Net-2, neither SIC(2) and SIC(3) can provide any gains at all in the low power regime due to the fact that with this network topology with path-loss, the channel gains do not provide a sufficient unequal received power distribution. As for Net-1, SIC(2) and SIC(3) provide large gains across all transmission power range by allowing the gateway (and other nodes) to receive multiple transmissions simultaneously, and thus, increasing spatial reuse in the bottleneck clique. Also, SIC(3) does not provide significant gains compared to SIC(2) at low transmission powers in Net-2.

In the case of SPC(2)-SIC(2), we observe that, in both networks, the maximum

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(a) Net-1 (b) Net-2
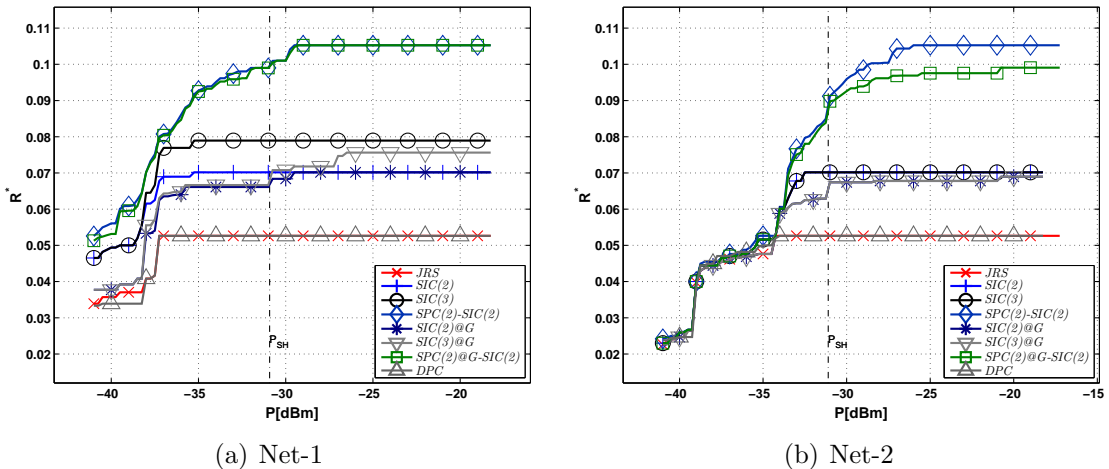
Figure 4.11: max-min throughput vs transmission power P, uplink+downlink flows

throughput of $2/(N - 1)$ can be obtained at powers $P \geq P_{SH}$. In fact, this is to be expected, as at sufficiently high power, each node can then transmit in single hop fashion to the gateway two parallel links with superposition coding, thus doubling the rate. If we now focus on low transmission powers in both networks, we observe that SPC(2)-SIC(2) does not outperform SIC(2) significantly as in this regime the achievable throughput is bounded by the decoding capabilities of the gateway node. We also show results for SPC(2), i.e., with restricted SIC(2) decoders, that is used in the work of [20]. As shown in Fig. 4.9, this approach has a significant performance penalty as SPC(2) underperforms significantly at low and medium transmission powers. We do not show the results for DPC in Fig. 4.9 since we consider only uplink flows.

In Fig. 4.10, we show R^* in Net-1 and Net-2 as a function of the transmission power P when the flow pattern is downlink-only. In this case, the optimal max-min rate is limited by the transmission capabilities of the gateway G, and is $1/(N - 1)$
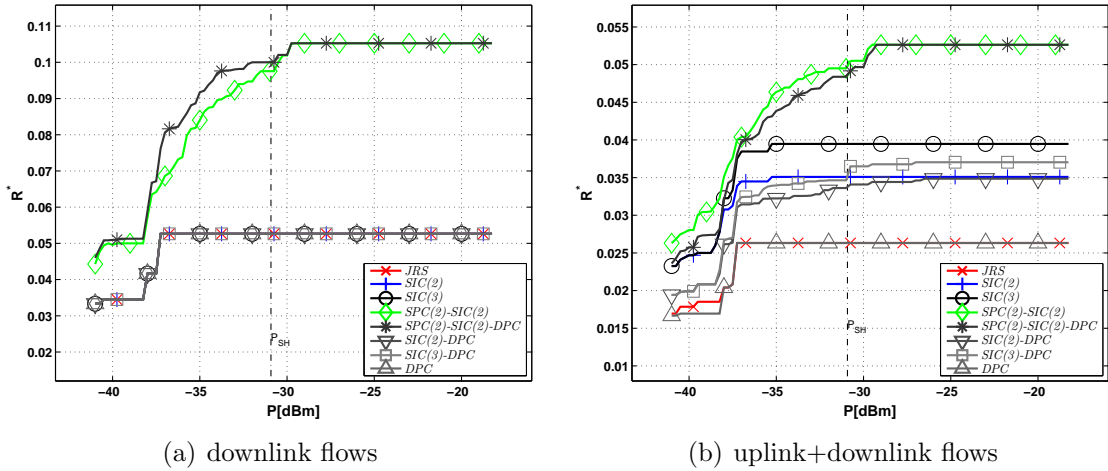
(a) downlink flows

(b) uplink+downlink flows

Figure 4.12: max-min throughput vs transmission power P, Net-1

per node except when SPC is enabled. Although, the use of SIC can potentially improve throughput at low transmission powers, it cannot overcome the bound of $1/(N-1)$ on the downlink regardless of the number of decodings k as the gateway is the bottleneck. However, with the use of SPC(2)-SIC(2), it achieves the theoretical maximum throughput of $2/(N-1)$ for both networks. This combination of SPC(2)-SIC(2) allows two (or more) outgoing transmissions from any node simultaneously, and at high powers allows for parallel links between two nodes. Thus, SPC with full capability SIC may double the throughput in both uplink and downlink. Comparing with SPC(2), these results show that this variant of SPC operates near the performance of SPC(2)-SIC(2) for downlink flows. As shown in Fig. 4.10, DPC does not provide any gain for Net-1, and only marginal gains for Net-2 at low powers.

In Fig. 5.6, we consider the case where there are uplink and downlink flows. We also show results for restricted variants of SIC(2), SIC(3) and SPC(2) at the gateway only that are labeled as *SIC(2)@G*, *SIC(3)@G* and *SPC(2)@G-SIC(2)*, respectively.

We jointly optimize uplink and downlink flows for networks Net-1 and Net-2 for all cases except DPC for which uplink and downlink flows are optimized separately. With the use of SPC(2)-SIC(2), the theoretical maximum throughput of $2/(N-1)$ is achieved for both Net-1 and Net-2. In the variant $SPC(2)@G\text{-}SIC(2)$, SPC is enabled only at the gateway and SIC is enabled at each node. By enabling these techniques at the gateway only, we can achieve significant throughput improvements in a network with uplink and downlink flow pattern. For both networks in Fig. 5.6, these restricted variants show significant portion of gains at medium to high powers when compared to the case when no physical layer techniques are enabled in any node. This indicates that at medium to high power the max-min throughput is mainly limited by the transmission capabilities of the gateway. Interestingly, the SPC(2)@G+SIC(2) variant shows almost no decrease in terms of throughput compared with SPC(2)-SIC(2). It is because on the uplink all the gains are attributed to SIC(2) and on the downlink all gains are attributed to SPC(2) by allowing two transmissions from the gateway.

In the case of DPC, the throughput per node is obtained by time time division duplexing of uplink and downlink flows that are optimized such that overall flow rates are equal. Thus, the overall flow rate is $R^* = 2\frac{R_{UL}R_{DL}}{R_{UL}+R_{DL}}$, where R_{UL} and R_{DL} are the uplink and downlink flow rates that are obtained by solving the problem [**P1**] with $\mathcal{I}_{\mathsf{int}}$ for R_{UL} and with $\mathcal{I}_{DPC}$ for R_{DL}. Fig. 5.6 shows that DPC is not justified in a network with mixed uplink and downlink flows and separate optimization of flows can even result in throughput losses as seen in Net-1. To support this, we show additional results for Net-1 when DPC is combined with SIC and SPC in Fig. 4.12.

DPC when combined with SPC(2)-SIC(2) does provide some small gains over SPC(2)-SIC(2) on downlink flows in Fig.4.12(a) and yet, it cannot improve SPC(2)-

SIC(2) for the case with uplink and downlink flows in Fig. 4.12(b). It is for the reason that time-sharing separately optimized uplink and downlink flows is not an optimal in terms of network configuration. Therefore, we conclude that the use of DPC is not worth the implementation complexity when DPC is enabled only at the gateway in networks with mixed uplink and downlink flows.

4.1.6 Conclusions

We have studied the throughput-optimal joint configuration of routing, scheduling, and advanced physical layer parameters in fixed multihop networks. We have formulated the optimization framework that is generic and can be relatively easily adopted for other physical layer techniques by redefining the conditions for ISets for the problem [**P1**], e.g., for such techniques as Directional Antennas[3], Full-Duplex[4] [64] or Interference Alignment [65]. In the case when each node is enabled with SPC, we also formulated a power allocation subproblem. By solving this subproblem, we are able to obtain the optimal power partition for each set of concurrently transmitting links in the optimal schedule. We also provide practical engineering insights for network operators on how much performance gain can be obtained by using these techniques in isolation or in combinations, and at all nodes or by restricting the use of these techniques only at the gateway. With the use of this framework, we have obtained the maximum achievable throughputs and the gains that can be achieved in networks that employ SIC, SPC or DPC. Specifically, we show that:

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[3]The use of Directional Antennas was studied in [62] and in our work [63].

[4]We studied Full-Duplex for the max-min and proportional fair throughputs. In the case of proportional fair, we solve the problem using nonlinear column generation.

- The use of SIC allows a network to improve significantly the per node max-min throughput across the entire transmission power range. i.e. the gains of up 30% for SIC(2) and of up 40% for SIC(3) can be expected in a wireless mesh networks at high transmission power range and up to 25% at the moderate transmit power levels.

- DPC enabled only at a gateway is not justified for use in a network with uplink and downlink flows since it is necessary to separately optimize uplink and downlink flows which is shown to be not optimal for the max-min throughput.

- On the other hand, SPC with full SIC capabilities outperforms significantly any other technique across all transmission power range for both uplink and downlink flows and achieves the maximum theoretical throughput at high power, i.e., $\frac{k \times r_m}{N-1}$ for uplink and $\frac{\min(m,k) \times r_m}{N-1}$ for downlink flows, and at low to medium transmission powers the gains of up to 70% can be obtained.

- Implementing SPC-SIC only at the gateway is sufficient to provide significant portion of performance gains.

4.2 Network Layer Technique [5]

The concept of Network Coding (NC) was first introduced in [25]. Since then, NC has attracted a lot of attention, primarily within the information theory community [26, 30]. We study a simple linear NC, which we define below. While many experiments and testbeds have demonstrated the potential benefits of NC, there are

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[5]The main results of this study were presented in our work [36].

no studies that have quantified the gains in a wireless network which is jointly optimized for scheduling and routing with NC. The work of [35] was the first contribution in this area. However, the work in [35] has several limitations. It is based on the simple protocol interference model and uses an approximate model for computing the broadcast rate for NC. We consider the SINR-based interference model, and enable NC at each node and between any pairs of flows. We restrict the use of NC to a maximum of two flows at a time and to the case without opportunistic listening. We do this restriction to obtain an exact formulation that remains tractable while not compromising on the interference model.

System model: We use the same system model as described in Chapter 3, except that we consider only networks where all nodes transmit with the same fixed power P and support a single rate r, i.e., $\mathcal{R} = \{r\}$.

4.2.1   Network Coding

Background: NC is a network layer technique that is employed at intermediate routing nodes for mixing the data packets from different sources. An increase in throughput is achieved by taking advantage of wireless broadcast medium to reduce the number of transmissions. We consider an XOR-like Network Coding identical to the scheme described in COPE [27]. This scheme is a simple linear NC without opportunistic listening. Binary packets are combined at intermediate nodes using XOR-operation before the broadcast and at destination nodes, the desired packets are extracted with the use of original packets.

Let us consider the example in Fig. 4.13(a), where a small multihop network is
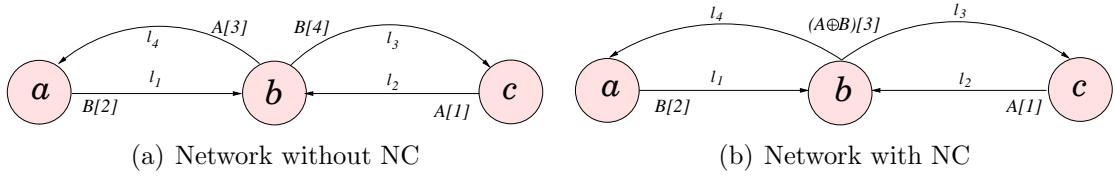
(a) Network without NC   (b) Network with NC

Figure 4.13: Illustration of NC

depicted with three nodes $\{a, b, c\}$ and the set of directed links $\{\ell_1, .., \ell_4\}$. In this network, node a needs to send a packet B to c and node c needs to send a packet A for node a. Node b acts as an intermediate node between a and c. If NC is not allowed, four time units will be necessary for node a to receive A and for node c to receive B. Therefore, a per flow throughput of only 1/4 can be achieved if all links have an unit rate $r = 1$. Now, if NC without opportunistic listening is enabled in this network, then node b after receiving packets A and B (takes two time units) can broadcast a composite packet $A \oplus B$ that is obtained by performing XOR operation on packets A and B as shown in Fig. 4.13(b). At nodes a and c, the destination packets A and B are extracted from $A \oplus B$ by using the XOR operation with packets B and A, respectively. Therefore, by enabling NC, only three time units are necessary as opposed to four time units in the network without NC. In this example, the use of NC yields a throughput increase of 33% by only taking advantage of the flow patterns and wireless broadcast.

ISets when NC is enabled: Contrary to the previous section, it is not enough to revisit only the notion of ISets to formulate a flow-based problem for NC. An ISet may now contain at least two links that originate from a common source node and may contain mutiple pairs of NC links. We define a set of links s is an ISet if it meets [**C3.2**], [**C3.3**], [**S3.3**] and

[**C4.4**] for each $n \in \mathcal{N}$: $\qquad \sum_{\ell \in s} \mathbf{1}_{\{o(\ell)=n\}} \leq 2$.

Although, we allow the use of NC between any flows, i.e. intra- and inter-session NC, we restrict it for use between two flows only, as specified in the condition [**C4.4**].

Denote by $\mathcal{I}_{NC}$ the collection of all ISets in a network where nodes are enabled with NC capability. We also note that the max-min throughput in a mesh network with NC is upper bounded by $\frac{r_m}{N-1}$ as NC cannot offer to a gateway the capabilities to support multiple incoming or outgoing links at a time, since the gateway is the origin or the destination of all flows.

4.2.2 Problem Formulation

In this section, we aim to formulate a joint routing and scheduling problem with NC that is without opportunistic listening and that only allows coding between the data of two flows transiting through an intermediate relay node. In our system model, we allow all paths a priori and we allow each intermediate node to send part of each flow it receives with NC and its remaining part without NC. Consequently, we are able to obtain an optimal network configuration, where routing and scheduling parameters determine an optimal amount of each flow to be carried on each feasible link and determine where and if NC is required.

Denote by $y_{f_1,f_2}(\ell_1, \ell_2)$ the amount of flow $f_1 \in \mathcal{F}$ transmitted over link $\ell_1 \in \mathcal{L}$ and the same amount of flow $f_2 \in \mathcal{F}$ transmitted over link $\ell_2 \in \mathcal{L}$ using NC. This new variable can only be defined if links ℓ_1 and ℓ_2 are such that $o(\ell_1) = o(\ell_2)$ and $\ell_1 \neq \ell_2$. The variable $y_{f_1,f_2}(\ell_1, \ell_2)$ represents the amount of flow f_1 (resp. f_2) that is network coded and transmitted over link ℓ_1 (resp. link ℓ_2). To better illustrate it,

consider Fig. 4.13(b). In this example, only the following pair of NC variables can be defined: $y_{f_2,f_1}(\ell_2, \ell_3) = y_{f_1,f_2}(\ell_3, \ell_2)$. Let us define by $\bar{\ell}$ the inverse link of link ℓ, i.e., $o(\bar{\ell}) = d(\ell)$ and $d(\bar{\ell}) = o(\ell)$, and denote by $\boldsymbol{y} = [y_{f_1,f_2}(\ell_1, \ell_2)]_{\substack{f_1,f_2 \in \mathcal{F} \\ \ell_1, \ell_2 \in \mathcal{L}}}$ the network coded routing vector. Also, let $x_f(\ell)$ denote the amount of flow $f \in \mathcal{F}$ transmitted over link $\ell \in \mathcal{L}$ that is not encoded with NC.

The Joint Routing, Scheduling, and Network coding (JRS-NC) problem for the max-min throughput in a network where each node is enabled with NC allowed between any two flows can be formulated as follows:

$$[\mathbf{P2}]: \qquad \max_{\boldsymbol{\alpha,x,y,R}} R \tag{4.9}$$

$$\sum_{\substack{\ell_1 \in \mathcal{L} \\ o(\ell_1)=n}} x_{f_1}(\ell_1) - \sum_{\substack{f_2 \in \mathcal{F};\, \ell_1,\ell_2 \in \mathcal{L} \\ d(\ell_1)=n;\, o(\ell_2)=o(\ell_1)}} y_{f_1,f_2}(\ell_1, \ell_2) - \sum_{\substack{\ell_1 \in \mathcal{L} \\ d(\ell_1)=n}} x_{f_1}(\ell_1) \tag{4.10}$$

$$+ \sum_{\substack{f_2 \in \mathcal{F};\, \ell_1,\ell_2 \in \mathcal{L} \\ o(\ell_1)=n;\, o(\ell_2)=n}} y_{f_1,f_2}(\ell_1, \ell_2) = \begin{cases} R_{f_1}, & n = o(f_1) \\ -R_{f_1}, & n = d(f_1) \\ 0, & \text{else} \end{cases} \quad \begin{array}{l} \forall n \in \mathcal{N}, \\ f_1 \in \mathcal{F} \end{array} \tag{4.11}$$

$$\sum_{s \in \mathcal{I}_{\mathcal{NC}}} \alpha_s \mathbf{1}_{\{\ell \in s\}} \geq \sum_{f \in \mathcal{F}} x_f(\ell) \qquad \forall \ell \in \mathcal{L} \tag{4.12}$$

$$\sum_{s \in \mathcal{I}_{\mathcal{NC}}} \alpha_s \mathbf{1}_{\left\{\{\ell_1,\ell_2\} \subseteq s\right\}} \geq \sum_{f_1,f_2 \in \mathcal{F}} y_{f_1,f_2}(\ell_1, \ell_2) \qquad \forall \ell_1, \ell_2 \in \mathcal{L} \tag{4.13}$$

$$\sum_{f \in \mathcal{F}} x_f(\bar{\ell}_1) + \sum_{\substack{f_1,f_2 \in \mathcal{F};\, \ell_2 \in \mathcal{L} \\ o(\ell_2)=o(\bar{\ell}_1)}} y_{f_1,f_2}(\bar{\ell}_1, \ell_2) \geq \sum_{\substack{f_1,f_2 \in \mathcal{F} \\ \ell_2 \in \mathcal{L} \\ o(\ell_2)=o(\ell_1)}} y_{f_1,f_2}(\ell_1, \ell_2) \qquad \ell_1 \in \mathcal{L} \tag{4.14}$$

$$y_{f_1,f_2}(\ell_1, \ell_2) = \begin{cases} 0, & o(\ell_1) = o(f_1) \\ 0, & o(\ell_1) = d(f_1) \\ y_{f_2,f_1}(\ell_2, \ell_1), & \text{else} \end{cases} \quad \begin{array}{l} \forall \ell_1, \ell_2 \in \mathcal{L}, \\ f_1, f_2 \in \mathcal{F} \end{array} \tag{4.15}$$

54

$$R_f \geq R \geq 0 \qquad\qquad \forall f \in \mathcal{F} \qquad (4.16)$$

$$\sum_{s \in \mathcal{I}_{\mathcal{NC}}} \alpha_s = 1 \qquad\qquad (4.17)$$

$$\boldsymbol{\alpha}, \boldsymbol{x}, \boldsymbol{y} \geq 0 \qquad\qquad (4.18)$$

Conditions in (4.11) state the flow conservation constraints that require the amount of each flow with and without NC to be conserved at each node $n \in \mathcal{N}$. Conditions in (4.12) and (4.13) are the link rate constraints for flows without and with NC, respectively. For the description of link scheduling and other non-NC variables and constraints, we refer to the JRS framework in Section 3.3.

The NC constraints (4.14) require that the total amount of network coded flows transmitted over the link ℓ_1 by a node $o(\ell_1)$ cannot be larger than the total amount of non-network coded and network coded flows a node $o(\ell_1)$ receives over the inverse link $\bar{\ell}_1$. To describe these NC constraints, we refer to the pair of links (ℓ_3, ℓ_2) and NC variable $y_{f_1,f_2}(\ell_3, \ell_2)$ in Fig. 4.13(b). In this network, the amount of network coded flow $y_{f_1,f_2}(\ell_3, \ell_2)$ cannot be larger than the amount of non-network coded flow the node b receives over the inverse link ℓ_4 for flow f_2.

In general, the total amount of network coded flow transmitted over link ℓ_3 by node b cannot be larger than the total amount of direct and network coded flow the node b receives over inverse link $\bar{\ell}_3$. The same must hold for link ℓ_2 in Fig. 4.13(b). The RHS of (4.14) shows the total amount of all flows that is received by node $o(\ell_1)$ and restricts the total amount of encoded flows that can be transmitted over link ℓ_1.

The additional network coding constraints (4.15) on variables $y_{f_1,f_2}(\ell_1, \ell_2)$ state that NC between flows f_1 and f_2 for node $n = o(\ell_1) = o(\ell_2)$ is not allowed if node n

is the source or destination node for any flows $f_1, f_2 \in \mathcal{F}$.

Problem complexity: The JRS-NC problem is formulated as an LP, but it is an NP-hard problem and is of larger scale than the original JRS problem in Section 3. The maximum number of links $|\mathcal{L}|$ in a network with N nodes grows in $O(N^2)$. In the JRS-NC problem, the number of variables is in $O(|\mathcal{L}||\mathcal{F}| + |\mathcal{L}|^2|\mathcal{F}|^2 + |\mathcal{L}|^{M''})$, where $M'' > M$ is the maximum ISets size. In order to enumerate all possible ISets for $\mathcal{I}_{NC}$, all elements of the power set of $\mathcal{L}$ must be checked for being an ISet. This approach is intractable even for reasonable size networks due to the power set of $\mathcal{L}$ grows exponentially in $O(2^{|\mathcal{L}|})$. We solve the JRS-NC problem with the column generation method that allows us to use only a subset of $\mathcal{I}_{NC}$ to obtain optimal solutions. We also use some elaborate techniques for faster convergence of the column generation method to optimal solutions.

The reduced costs are computed for new off-basis columns or ISets as follows:

$$r_s = -(\zeta + \sum_{\ell_1 \in \mathcal{K}_1(s)} \upsilon'_{\ell_1} + \sum_{(\ell_1, \ell_2) \in \mathcal{K}_2(s)} \upsilon''_{\ell_1, \ell_2})$$

$$\mathcal{K}_1(s) = \{\ell_1 \in s / \nexists \ell_2 \in s, o(\ell_1) = o(\ell_2), \ell_1 \neq \ell_2\}$$

$$\mathcal{K}_2(s) = \{(\ell_1, \ell_2) \in s^2 / o(\ell_1) = o(\ell_2), \ell_1 \neq \ell_2\}, \tag{4.19}$$

where ζ, υ' and υ'' are the dual variables for (4.17), (4.12) and (4.13), respectively. The solutions are determined to be optimal only when no other distinct ISets can be found with strictly positive reduced costs.

4.2.3   Numerical Results and Insights

We provide solutions for wireless mesh networks with $N = 20$ nodes, where $N - 1$ nodes are randomly and uniformly placed in a rectangular grid, and a gateway is deterministically placed in the corner for type-A networks and in the middle of a grid for type-B networks. Type-A networks with a gateway in the corner are typical of sectored systems with directional antennas, while a gateway in the middle is typical to non-sectored systems. Typical node placements of these two types of networks are shown in Fig. 4.16.

We use the channel model in (3.1) but for simplicity and without loss of generality, we model channels without fading using the path loss model in (4.7), where $d_0 = 0.1$m is the reference distance and $\mu = 3$ is the path loss exponent. In all network realizations, the receiver's background noise is $N_0 = 100$dBm and the SINR threshold is $\beta = 6.4$dB to support the normalized rate $r = 1$.

In order to provide additional insights on NC when flow rates are not equal, we also study a weighted optimization problem for the max-min throughput. Specifically, in the JRS-NC problem (resp. JRS problem), we replace (4.16) (resp. (3.6)) by (3.7). We consider two types of weighting factors: equal weighting $1 : 1$ and unequal weighting $1 : 2$, with $w_f = 2$ for all downlink flows to ensure that downlink flows obtain twice the throughput of uplink flows. By solving the JRS-NC problem, we obtain an optimal network configuration for routing and scheduling such that R is maximized. The total nodal throughput $R^* = 2R$ for $1 : 1$ and $R^* = 3R$ for $1 : 2$. All our results are in terms of R^*.

We obtained results for 10 network realizations for each type-A and type-B net-

works. We denote by P_{SH} the minimum power required for all nodes to communicate with the gateway G node in single hop and $\bar{P}_{SH}$ denotes the minimum single hop power averaged over 10 realizations.

The optimal solutions for problem [**P2**] when NC is allowed between any two flows and at any node are denoted by *Full NC* and the results for the baseline problem [**P1**] for $\mathcal{I} = \mathcal{I}_{\text{int}}$ without any NC are denoted by *JRS*.

We also study three variants of the JRS-NC problem which are denoted as: *NC@G*, *Bidirectional NC* and *Bidirectional NC@G*, respectively. In the first variant (*NC@G*), only the nodes that are adjacent to the gateway can perform network coding, i.e., the variables $y_{f_1,f_2}(\ell_1, \ell_2)$ are only defined if $d(\ell_1) = G$ or $d(\ell_2) = G$ (the number of nodes that are adjacent to the gateway is a function of the transmit power P). In the second variant (*Bidirectional NC*), network coding is permitted only between bidirectional flows, where two flows f_1 and f_2 are defined as bidirectional if $o(f_1) = d(f_2)$ and $o(f_2) = d(f_1)$. In the third variant *Bidirectional NC@G*, only the nodes adjacent to the gateway can perform NC, and even then, only between bidirectional flows. These variants are of interest for two reasons. First, they help to understand the interplay between the network topology and NC that provides most of the gain. Second, they result in formulations that involve considerably fewer variables, and thus significantly reduce the complexity of the optimization and computation time.

Fig. 4.14 and Fig.4.15 show the relative gains in throughput averaged over 10 type-A and type-B network realizations for $1:1$ and $1:2$ cases. The relative gains in averaged R^* are computed as given in (4.8). These results are shown for the *Full NC* and the three NC variants with respect to the JRS solutions (without NC). These relative gains are used to obtained the general trend in throughput and cannot be

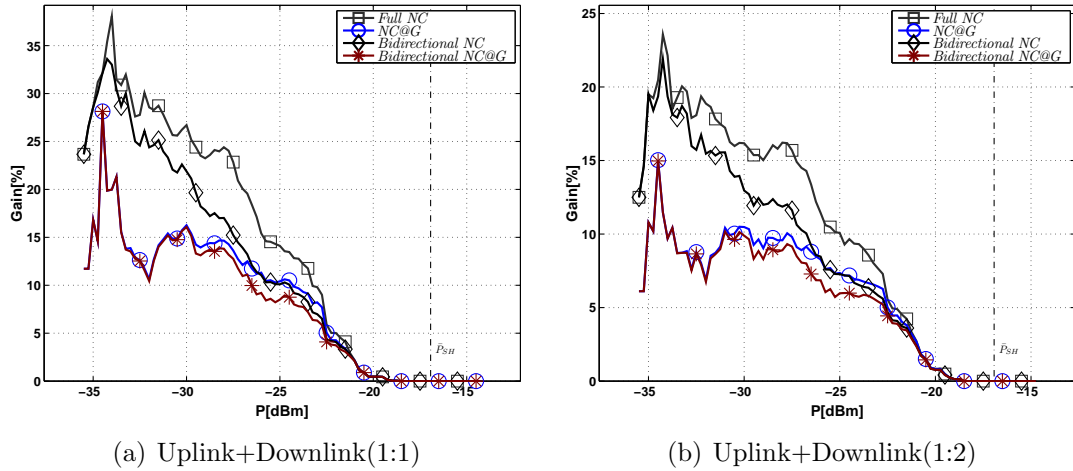(a) Uplink+Downlink(1:1)

(b) Uplink+Downlink(1:2)

Figure 4.14: Relative gain vs transmit power P (type-A networks)

used to obtain the expected gain in a random network with NC due to the small number of network realizations.

These results show that the simple XOR-based network coding (*Full NC*) can yield significant gains at low to medium transmission power: as high as 35% when the gateway is in the corner and as high as 15% when the gateway is in the center of a grid. The difference in gains is attributed to the fact that NC is topology dependent and is less effective when the nodes are placed close to the sources of flows.

Restricting NC only at nodes adjacent to the gateway (*NC@G*) yields a significant portion of the gain. This is because the nodes that are adjacent to the gateway need to transmit over bottleneck links. Thus, the use of NC at nodes that are distant from the gateway provides little benefit, while doing so at nodes near the gateway results in a significant improvement in throughput. Limiting NC to bidirectional flows (*Bidirectional NC*) does not perform as well as the *Full NC* case due to the fact that some bidirectional flows may traverse non-identical optimal routes in terms of

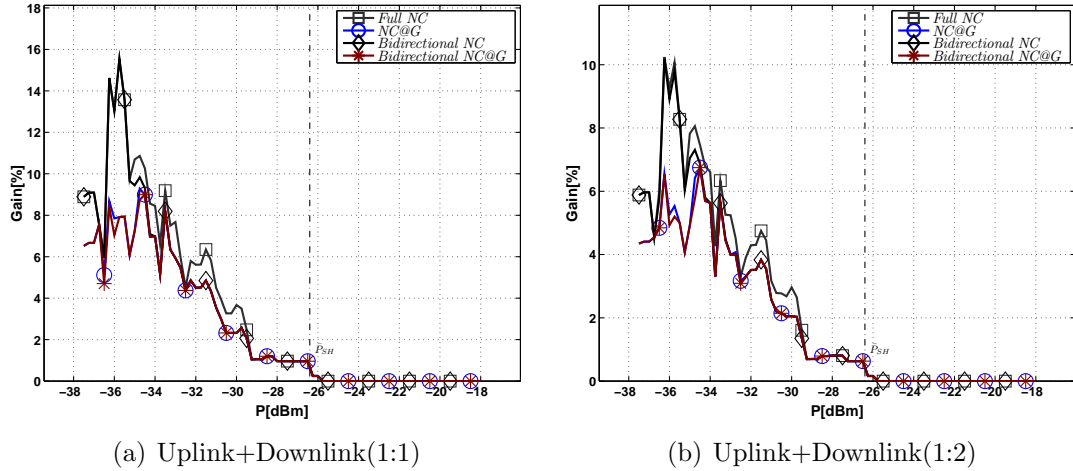(a) Uplink+Downlink(1:1) (b) Uplink+Downlink(1:2)

Figure 4.15: Relative gain vs transmit power P (type-B networks)

intermediate nodes and hence, it prohibits the use of NC on such flows.

When there is an imbalance in uplink and downlink flow rates as in the 1:2 case, NC does not perform as well since there are less opportunities for XORing opposing flows due to the constraint imposed by (4.14) and the unequal amount of incoming traffic at intermediate nodes. Fig. 4.14(b) shows a significant reduction in gains in the $1 : 2$ case as compared to the $1 : 1$ case in Fig. 4.14(a) for type-A networks. In type-B networks, the reduction in gains is less significant (see Fig. 4.15(b)).

We compare the solutions in terms of R^* in two selected networks, Net-1 and Net-2, that are depicted in Fig.4.16(a) and Fig.4.16(b), respectively. NC in Net-1 results in fairly typical performance gains among type-A network realizations. In Net-2 of type-B, NC results in gains that are insignificant.

Fig. 4.17 shows the max-min throughput R^* as a function of transmission power P in Net-1 for both $1 : 1$ and $1 : 2$ cases. A network with *Full NC* can provide the maximum per node throughput of $1/(N-1)$ at significantly lower transmission
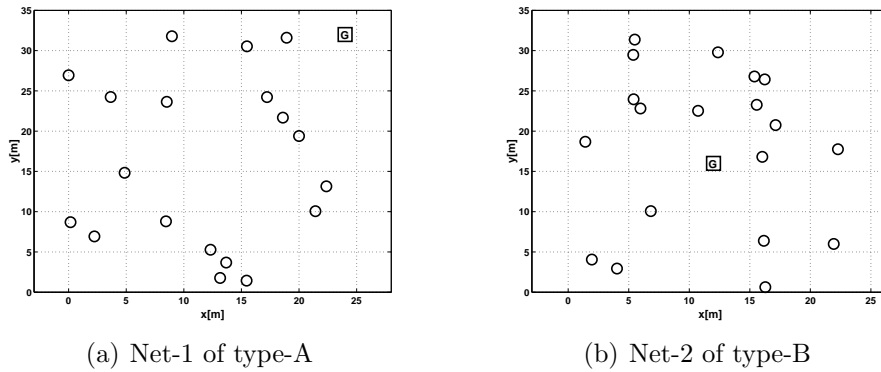
(a) Net-1 of type-A

(b) Net-2 of type-B

Figure 4.16: Placement of nodes

power (on average about 3dB lower) than *JRS*. As discussed in [3,12], using multihop communications enables a mesh network operator to offer this maximum achievable throughput at a transmit power which is much lower than P_{SH}. With NC, a network operator can offer the maximum throughput at even lower transmission power as shown in Fig. 4.17. It is the case of Net-1, where the gateway is placed in the corner of a grid. However, NC does not guarantee this improvement in all networks. The change of a network topology may result in no gain as shown in Fig. 4.18(a) or in Fig. 4.18(b), where the gateway is placed in the center.

NC is also less effective in terms of throughput in type-B networks with the gateway placed in the center. It is because in such networks, there is less number of potential nodes that may act as intermediate relay nodes for NC. The number of potential intermediate nodes between the gateway and the nodes in a random network is topology dependent. Therefore, it is possible to have almost no NC gains in such network realizations, e.g. in Fig.4.18(a) for the Net-2. This has to do with the relatively small size of the networks. However, in some networks as in the case for Net-1, NC allows a network to achieve significant gains as shown in Fig. 4.17. Even
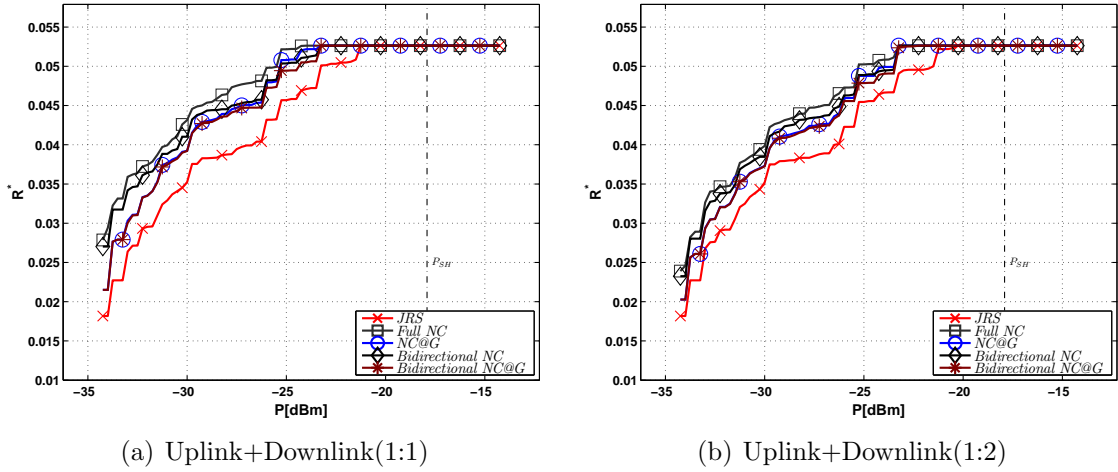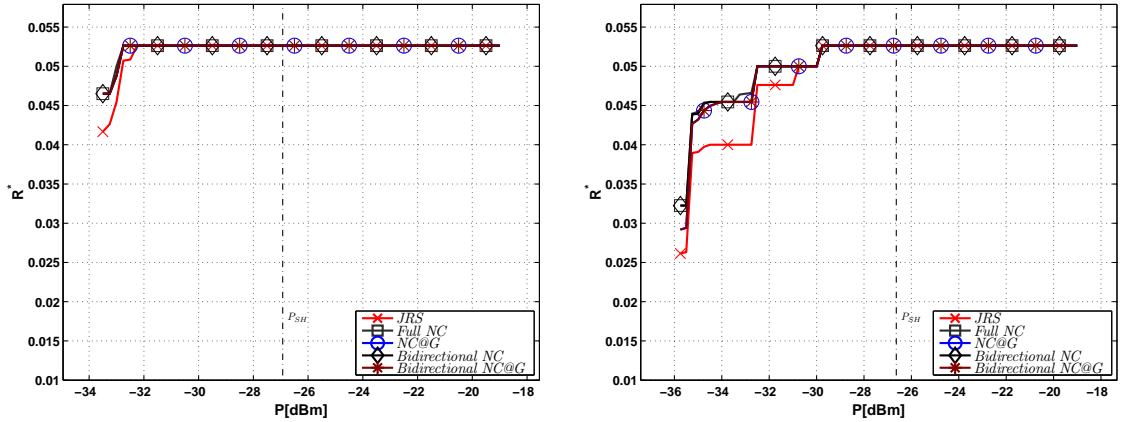
(a) Uplink+Downlink(1:1)

(b) Uplink+Downlink(1:2)

Figure 4.17: max-min throughput vs transmission power P, Net-1 in Fig. 4.16(a)

the variants of NC yield a significant throughput improvement in this network. NC is less effective at high power since the number of hops required for optimum routing decreases and hence there are less opportunities for NC.

4.2.4   Conclusions

We have formulated a comprehensive optimization framework for the throughput-optimal configuration of joint routing, scheduling, and NC parameters under the SINR-based interference model. This formulation is generic and can be used to obtain the maximum achievable throughput in a given network. By redefining the conditions for ISets, this framework can be used for combinations of NC with SIC or distributed space-time coding, or for variants of NC such as physical NC [37]. We provide exact solutions for medium size networks and quantify the throughput gains that can be achieved by using the XOR-based NC without opportunistic listening. The results that we obtain may be interesting for network operators to justify the use of NC during

(a) Uplink+Downlink(1:1), Net-2 in Fig. 4.16(b) (b) Uplink+Downlink(1:1), a type-B network

Figure 4.18: max-min throughput vs transmission power P

the offline network configuration phase. In particular, we conclude the following:

- The efficiency of NC in a wireless mesh network is topology dependent, i.e., it is affected by the placement of the gateway and the nodes in a network. In some networks NC, can provide significant gains as high as 35% at low to medium transmission power, while in some others, NC is ineffective. However, even for small gains, NC is justified due to the fact that it is an extremely cost effective technique. NC provides no advantage at high transmission powers, as single hop transmission to the gateway is optimal and therefore, no relaying is needed. NC is a technique for the multihop regime only and it cannot improve the throughput bound of $\frac{r_m}{N-1}$ in mesh networks.

- With the use of our formulation [**P2**], network operators can quantify the gains in any medium size networks as well as obtain the optimal network configuration in terms of routing and scheduling.

- To provide maximum gains, NC must be jointly optimized with routing and

scheduling, as the efficiency of NC depends on routing.

- The restricted variants of NC provide significant throughput gains. Thus, when the complexity is high, the restricted variants of NC can be considered to reduce the complexity of a problem.

- Performance of NC is more effective in a mesh network where uplink and down-link flow rates are fairly symmetric.

Chapter 5

Cross-layer Optimization in Cooperative Networks

In conventional multihop networks, the data flows are routed from source to destination over point-to-point links with intermediate nodes acting as relays. The use of only point-to-point links with single antennas at the transmitters and receivers cannot offer the performance gains that can be achieved in multiple antenna systems. In cooperative networks, a group of nodes can forward data flows over multipoint-to-point links by forming a virtual antenna array. This group of nodes act as relays that share their single antennas to transmit a common message to the destination. This use of multiple antennas among multiple nodes is only possible when these nodes are perfectly synchronized, and the signals from the distributed antennas do not interfere at the receiver. As a result, it allows the receiver to benefit from the increased signal power while decoding a combined signal from multiple nodes. To avoid interference, signals from cooperating nodes can be transmitted either separately on different chan-

nels or simultaneously on the same channel, with the use of cooperative techniques such as distributed beamforming or distributed space-time coding.

While it is well known that cooperative communication can yield performance gains in wireless network [42, 44, 45], there are only few studies that quantify these gains. Typically, these works address a cross-layer approach by considering simplified models or restricted cases. The works of [50, 51] consider only a fixed selection of relay pairs in small size networks with no spatial reuse during scheduling. In [53], optimization framework is proposed that is based on a simple protocol interference model, and in [54], interference-free transmission is achieved by using multiple orthogonal channels.

We neither restrict the selection of cooperating nodes with which to form virtual antenna arrays nor do we consider a simplified interference model. We aim to quantify the throughput gains and other performance metrics of cooperative communication in a single channel network as well as to provide engineering insights. We achieve this by formulating a cross-layer framework that allows a network to be optimally configured for routing, scheduling, and the parameters for cooperative communication. We mainly focus on centrally managed and fixed broadband networks with a scheduling-based access scheme that can be configured offline beforehand for optimal operation.

5.1 Background

We consider a network where nodes are equipped with a single antenna and transmit on the same channel. Each node can act as a flow source as well as a relay. We refer
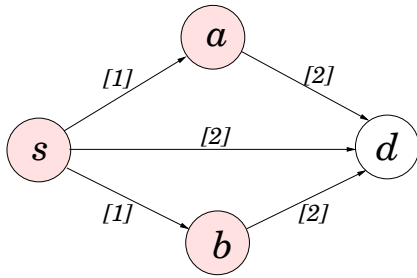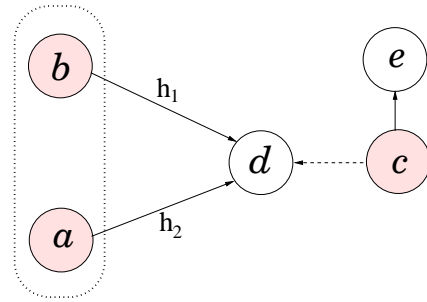
Figure 5.1: Cooperative network

Figure 5.2: With spatial reuse

to the use of distributed multiple antennas via forming a virtual antenna array as Cooperative Relaying (CR) or cooperative communication.

Fig. 5.1 shows a cooperative multihop network where node s needs to send a message to node d. Let us assume that neither a, b nor s can communicate directly with node d, i.e., node d cannot decode signals transmitted from any of these nodes alone since all SNRs are below a certain minimum threshold. Thus, node s cannot send a message to node d using only point-to-point transmission. However, it is still possible for node s to deliver a message to d with the use of cooperative techniques. In the first hop, node s transmits the same message to both a and b, and in the second hop, nodes a,b and s cooperatively forward this message to node d by forming a distributed antenna array. Node d can only receive a message from s if all multipoint-to-point links are feasible in each hop, i.e. the signal is successfully decoded at the receivers in each hop. This example shows that in a cooperative network, it is possible to have two types of improvement: one in terms of in connectivity and one in terms of throughput.

Signals from a and b can be spatially combined at node d with the use of Distributed Space-Time Block Coding (D-STBC) [43] or distributed beamforming [44].

Both these techniques require nodes a and b to transmit a common message simultaneously with the same rate. While beamforming can be used between more than two nodes, this technique requires perfect channel knowledge at the transmitters to synchronize the phases such that signals from a and b can add coherently at d. Space-time codes do not require perfect channel knowledge at the transmitters, but among all classes of codes, only the Alamouti code can provide full-rate and it can only be used for cooperation between two nodes. The loss of rate with the use of other codes such as quasi-orthogonal space-time codes is unavoidable for cooperation between more than two nodes.

The example in Fig. 5.1 shows that using CR, nodes a and b are able to convey a message over links which are infeasible otherwise. However, it comes at the expense of increased interference to other nodes in the network since nodes a and b must transmit at the same time. On the other hand, node d is able to mitigate the interference from node c. In Fig. 5.2, node c is the main interferer to node d but its interference can be mitigated due to the increased power of the received signal at node d, i.e, with the use of cooperative techniques, nodes a and b are able to send successfully to d together. Therefore, on the one hand, cooperation may improve network performance by providing better connectivity, but on the other hand, it creates additional interference that may decrease spatial reuse in a network. This example shows that CPC is needed to take full advantage of CR, hence, we study CR with CPC by optimizing jointly routing, scheduling, transmission powers, and cooperative relaying parameters.

Distributed Alamouti coding: We consider the D-STBC technique for coopera-

tion between two nodes only. Although, with the use of non-orthogonal space-time block codes, cooperation is possible between any number of nodes, it is practically hard to achieve this due to the increasing complexity of decoding at the receivers [66]. D-STBC is a technique that requires the channel knowledge at the receivers. In the case of two antenna array, D-STBC is the Distributed Alamouti Coding (D-AC). It is a relatively simple technique to implement as it does not require challenging phase synchronization, and nodes can be perfectly synchronized in the context of a scheduling-based network.

As an example, let us consider the two cooperating nodes a and b in Fig. 5.2 that need to send a message X to node d with the use of the distributed Alamouti scheme. The channels h_1 and h_2 are assumed to be zero mean Gaussian with unit variance that experience flat fading and remain constant during the transmission of message X. Let us assume that message $X = [X_1 \; X_2]$ consists of N symbols that can be split in X_1 and X_2 row vectors with $N/2$ symbols each, so that in the first half of a slot of duration $N/2$ symbols, nodes a and b transmit X_1 and X_2, respectively, and in the second half of the slot, node a sends $-X_2^*$ and node b sends X_1^*. The baseband signal Y at the receiver of node d is then

$$Y = \begin{bmatrix} Y_1[1] \\ Y_2^*[2] \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ h_2^* & -h_1^* \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} w[1] \\ w^*[2] \end{bmatrix}, \qquad (5.1)$$

where $w[1]$ and $w[2]$ are $N/2$ row vectors of additive white Gaussian noise with variance N_0 in the first and the second half of a slot, respectively. After a slot of N symbols, node d can decode X from $H^H * Y$, where H^H is a Hermitian of the channel coefficient matrix in (5.1). There is no rate loss as it takes one time slot to transmit a

69

message of N symbols. The average SNR at the receiver is now $P_x(|h_1|^2 + |h_2|^2)/N_0$, where P_x is the signal power of X. Hence, D-AC allows for an increase in the signal power received at node d by combining non-interfering signals from two distributed antennas.

5.2 System Model

We model a wireless network as a set of virtual nodes $\mathcal{V}$, a set of flows $\mathcal{F}$ and a set of feasible links $\mathcal{L}$. We assume that each node in the network can support a set of available rates $\mathcal{R}$ and can transmit at power levels in the interval $(0, P]$, where P is the maximum transmission power.

We define two types of nodes: physical and virtual. A physical node represents the actual physical node in the network and the set of all physical nodes is denoted as $\mathcal{N}$. A virtual node n in $\mathcal{V}$ is either a physical node, defined by a singleton $n = \{a\}, a \in \mathcal{N}$, or a cooperative node, defined by an unordered pair of distinct physical nodes $n = \{a, b\}$ such that $a \neq b \in \mathcal{N}$. The pair $\{a, b\}$ indicates that two nodes a and b participate jointly in cooperative transmission or form a virtual two-antenna array to convey a common message. We allow cooperative transmission between any two physical nodes. Therefore, the set $\mathcal{V}$ must include all unordered pairs over $\mathcal{N}$, i.e., $\mathcal{V} = \{\{a, b\} : a, b \in \mathcal{N}; \quad a \neq b\} \cup \{\{a\} : a \in \mathcal{N}\}$.

Each flow f in $\mathcal{F}$ can only originate at a physical node, i.e., $o(f) \in \mathcal{N}$, and can only be destined to a physical node, i.e., $d(f) \in \mathcal{N}$. A pair of nodes $\{a, b\}$ can cooperatively transmit a message only when they both have received this message from a source of the flow or from another virtual node.

Let us define a by a triple $\ell = (n_s, n_d, r(\ell))$, where $n_s \in \mathcal{V}$ and $n_d \in \mathcal{V}$ are the origin and destination nodes of link ℓ and $r(\ell)$ is the link rate. Link ℓ can be defined between any pair of virtual nodes. We denote by $\mathcal{O}(\ell)$ and $\mathcal{D}(\ell)$ the mapping operators from link ℓ to the corresponding origin and destination nodes, respectively. A link ℓ is feasible to support the link rate $r(\ell)$ at the maximum transmission power P if, in the absence of interference, it meets the following conditions:

[**C5.1**] $$\mathcal{D}(\ell) \backslash \mathcal{O}(\ell) \neq \emptyset,$$

[**S5.1**] for each $j \in \mathcal{D}(\ell) \backslash \mathcal{O}(\ell)$: $$\frac{P \sum_{i \in \mathcal{O}(\ell)} G_{i,j}}{N_0} \geq \beta(r(\ell)).$$

Condition [**C5.1**] states that any feasible link ℓ must have at least one physical node in $\mathcal{D}(\ell)$ that is not in $\mathcal{O}(\ell)$. If all nodes in $\mathcal{D}(\ell)$ are in $\mathcal{O}(\ell)$, then link ℓ is meaningless in the sense that all physical nodes in $\mathcal{D}(\ell)$ already have the message intended for transmission from $\mathcal{O}(\ell)$. Condition [**S5.1**] requires each physical node in $\mathcal{D}(\ell)$, that is not in $\mathcal{O}(\ell)$, to meet the minimum threshold SINR $\beta(r)$ to successfully decode a message at rate $r(\ell)$ from virtual node $\mathcal{O}(\ell)$. The nodes in $\mathcal{D}(\ell)$ that are shared with $\mathcal{O}(\ell)$ do not need to meet [**S5.1**] as these nodes already have the message sent by nodes in $\mathcal{O}(\ell)$. In [**S5.1**], $G_{i,j}$ is the power gain of the channel between physical nodes i and j, and is modeled as in (3.1) by an aggregation of the fading gain $g_{i,j}$ and the path loss $PL(d_{i,j})$ at distance $d_{i,j}$. In condition [**S5.1**] , the signal power received at each physical node in $\mathcal{D}(\ell)$ is now cooperatively combined from all nodes in $\mathcal{O}(\ell)$ by enabling the D-AC. The set of all feasible links $\mathcal{L}$ is then defined as $\mathcal{L} = \{(n_s, n_d, r(\ell))$: $\ell = (n_s, n_d, r(\ell))$ must meet [**C5.1**] and [**S5.1**], $n_s, n_d \in \mathcal{V}, r(\ell) \in \mathcal{R}$ $\}$.

The notion of virtual nodes allows us to incorporate all types of links in a network without restrictions on the choice of pairs of nodes participating in cooperative com-

(a) $\ell = (\{a\}, \{b\})$

(b) $\ell = (\{a\}, \{c, d\})$

(c) $\ell = (\{c, d\}, \{a\})$
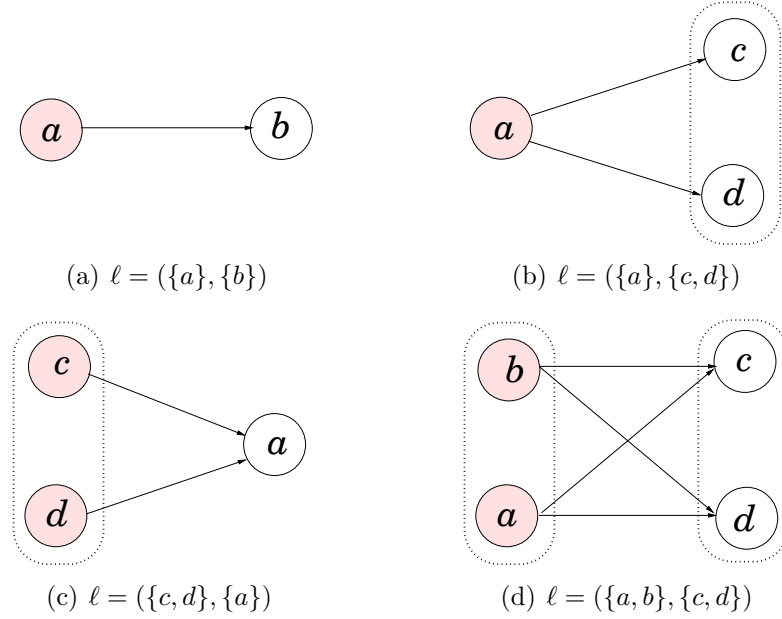
(d) $\ell = (\{a, b\}, \{c, d\})$

Figure 5.3: Illustration of types of links in a network

munication, unlike in [50, 51]. Fig. 5.3(a) shows a link between two physical nodes, also called a basic link. Fig. 5.3(b) and Fig. 5.3(c) show links between a cooperative node $\{c, d\} \in \mathcal{V}$ and a physical node $\{a\} \in \mathcal{N}$, and Fig. 5.3(d) shows a link between two cooperative nodes. The origin and destination of links in Fig. 5.3(b) and Fig. 5.3(d) may share a physical node, e.g., when $a = d$ then only node c needs to receive a message from $\{a, b\}$ so that in the next transmission, node d can form a cooperative pair with node c.

ISets when D-AC and CPC are enabled: Recall that in Section 3.2, we define an ISet as a set of links that can be spatially reused on the same channel. We address CPC by formulating a power allocation subproblem similar to the subproblem in [**S3.2**]. We denote $[\mathbf{P_s}] = [P_i(\ell)]_{\substack{\ell \in s \\ i \in \mathcal{O}(\ell)}}$ as the vector of transmission powers that is associated with all physical origins of each link ℓ in s. When each node in a network

72

is enabled with D-AC and CPC then, a set of links $s \subseteq \mathcal{L}$ is an ISet if it meets the following conditions:

[**C5.2**] for all $\ell_1, \ell_2 \in s, \ell_1 \neq \ell_2$: $\qquad \mathcal{O}(\ell_1) \cap \mathcal{O}(\ell_2) = \emptyset,$

[**C5.3**] for all $\ell_1, \ell_2 \in s, \ell_1 \neq \ell_2$: $\qquad \mathcal{D}(\ell_1) \cap \mathcal{D}(\ell_2) = \emptyset,$

[**C5.4**] for all $\ell_1, \ell_2 \in s, \ell_1 \neq \ell_2$: $\qquad \mathcal{O}(\ell_1) \cap \mathcal{D}(\ell_2) = \emptyset,$

[**S5.2**] there exists $\mathbf{P_s}$ such that $\Delta = 0$ in the following subproblem:

$$\Delta = \min_{\mathbf{P_s}} \sum_{\substack{\ell \in s \\ j \in \mathcal{D}(\ell) \setminus \mathcal{O}(\ell)}} \phi_{\ell,j}$$

$$\sum_{i \in \mathcal{O}(\ell)} P_i(\ell) \, G_{i,j} - \beta(r(\ell)) \, N_0$$

$$-\beta(r(\ell)) \sum_{\substack{\ell' \in s \\ \ell' \neq \ell}} \sum_{i \in \mathcal{O}(\ell')} P_i(\ell') \, G_{i,j} + \phi_{\ell,j} \geq 0 \qquad \begin{aligned} &\forall \ell \in s \\ &\forall j \in \mathcal{D}(\ell) \setminus \mathcal{O}(\ell) \end{aligned}$$

$$\boldsymbol{\phi}, \mathbf{P_s} \geq 0$$

$$\mathbf{P_s} \leq P$$

Conditions [**C5.2**] – [**5.4**] specify the half-duplex requirements that no two distinct links in an ISet s can share a physical node of origin or destination.

Condition [**S5.2**] requires each physical node in $\mathcal{D}(\ell)$ that is not in $\mathcal{O}(\ell)$ to meet the minimum SINR threshold $\beta(r(\ell))$ to support the link rate $r(\ell)$ for all links ℓ in an ISet s. We denote by $\boldsymbol{\phi} = [\phi_{\ell,j}]_{\substack{\ell \in s \\ j \in \mathcal{D}(\ell) \setminus \mathcal{O}(\ell)}}$ the vector of indicator variables for each SINR constraint in [**S5.2**]. If all $\phi_{\ell,j} = 0$, then there exists a power allocation vector $\mathbf{P_s}$ for all transmitting nodes in s such that the SINR conditions for all links in s

73

are satisfied. When $\Delta > 0$, then a set of links s is not an ISet since there does not exist a transmission power vector $\mathbf{P_s}$ that allows the corresponding nodes to transmit concurrently. In the subproblem of condition [**S5.2**], the interference at the receivers of link ℓ must be taken into account from all physical nodes that are not in $\mathcal{O}(\ell)$.

Denote by $\mathcal{I}_{CR-CPC}$ the collection of all ISets in a cooperative network where nodes are enabled with D-AC and CPC, and are capable to support a set of rates $\mathcal{R}$. In the case of a mesh network, the use of cooperative techniques cannot improve the max-min throughput bound $\frac{r_m}{N-1}$ since the gateway can only transmit and receive over a single link at a time.

ISets when D-AC is enabled a single-power network: Denote by $\mathcal{I}_{CR}$ a collection of ISets in a network where each node can transmit at fixed power P and can support a set of rates $\mathcal{R}$. Thus, each ISet s in $\mathcal{I}_{CR}$ must meet conditions [**C5.2**], [**C5.3**], [**C5.4**] and

[**S5.3**] for all $\ell \in s, \forall j \in \mathcal{D}(\ell) \backslash \mathcal{O}(\ell)$: $\dfrac{P \sum_{i \in \mathcal{O}(\ell)} G_{i,j}}{N_0 + P \sum_{\substack{\ell' \in s \\ \ell' \neq \ell}} \sum_{i \in \mathcal{O}(\ell')} G_{i,j}} \geq \beta(r(\ell)).$

5.3   Problem formulation

Problem formulation [**P3**] for finding jointly optimal routing, scheduling, rate adaptation, continuous power control, and cooperative relaying parameters, is identical to problem [**P1**], except that the set of physical nodes $\mathcal{N}$ in constrains (3.3) is now

replaced by the set of virtual nodes:

$$\sum_{\substack{\ell \in \mathcal{L} \\ o(\ell)=n}} x_f(\ell) - \sum_{\substack{\ell \in \mathcal{L} \\ d(\ell)=n}} x_f(\ell) = \begin{cases} R_f, & n=o(f) \\ \text{-}R_f, & n=d(f) \\ 0, & \text{else} \end{cases} \qquad \begin{aligned} &\forall n \in \mathcal{V} \\ &\forall f \in \mathcal{F} \end{aligned} .$$

Problem complexity: The maximum number of links in a network with N physical and $N(N+1)/2$ virtual nodes is of the order of $O(N^4)$ and the maximum number of ISets is of the order $O(2^{|\mathcal{L}|})$. Problem [**P3**] is of much larger scale than [**P1**] where the number of links is only of the order $O(N^2)$. Certainly, it is not tractable to solve [**P3**] by enumerating all elements of $\mathcal{I}_{CR}$ or $\mathcal{I}_{CR-CPC}$, i.e., by checking for each of the $2^{|\mathcal{L}|} - 1$ elements of the power set of $\mathcal{L}$ whether it is an ISet or not. In addition, for each element in the power set, the power allocation subproblem in condition [**S5.2**] must be solved to check for SINR requirements. We solve [**P3**] using the column generation method to avoid the extensive enumeration of all ISets. Problem [**P3**] is solved iteratively and at each iteration a new subset of ISets is added into the master program. If no ISets with strictly positive reduced costs can be found, then the current solution is optimal. The reduced cost for an ISet is computed as in (3.8)

5.4 Numerical Results and Insights

In this section, we provide exact numerical solutions for mesh networks with a single gateway. We denote by R^* the total max-min throughput per node, i.e., $R^* = 2R$ since problems [**P1**] and [**P3**] are solved for R in networks with equal uplink to downlink

flow rate ratios. We place nodes uniformly at random in a 2km by 2km square with the gateway G in the center of the square. For each network realization, we modeled the constant channel gains between any pairs of physical nodes as a combination of Rayleigh flat fading with path loss. As for the path loss in (3.1) we use the following model [67]:

$$PL(d_{i,j}) = \begin{cases} \left(\frac{\lambda}{4\pi d}\right)^2 & d_{i,j} < d_0 \\ \left(\frac{\lambda}{4\pi d_0}\right)^2 \left(\frac{d_{i,j}}{d_0}\right)^\mu & d_{i,j} \geq d_0, \end{cases} \tag{5.2}$$

where $\mu = -3.3$ is the path loss exponent, $\lambda = 0.3$m is the wavelength and $d_0 = 30$m is the reference distance of near field. The fading gains $g_{i,j}$ are modeled as exponentially distributed power gains with unit variance. The thermal noise is $N_0 = -100$dBm.

5.4.1 Single Rate and Single Power[1]

First we study the performance of CR in networks where nodes are capable to transmit only with single transmission power P and support single rate r. The results for R^* are obtained for two modulation/coding schemes: 1) $\beta = 3$dB corresponding to the rate $r = 1$ and 2) $\beta = 10$dB corresponding to the rate $r = 2$. Both problems [**P1**] and [**P3**] are solved for 200 random mesh networks with $N = 16$ nodes: $N - 1$ physical nodes and one gateway. We label by P_{SH} the minimum transmission power at which all nodes are able communicate with the gateway in single hop. The results for [**P1**] with $\mathcal{I} = \mathcal{I}_{\text{int}}$ are denoted as *JRS* and for [**P3**] with $\mathcal{I} = \mathcal{I}_{CR}$ are denoted as *CR*.

Fig. 5.8(a) shows the relative gains of *CR* with respect to *JRS* in max-min through-

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[1]The results for the case of single power and single rate were presented in our work [48].

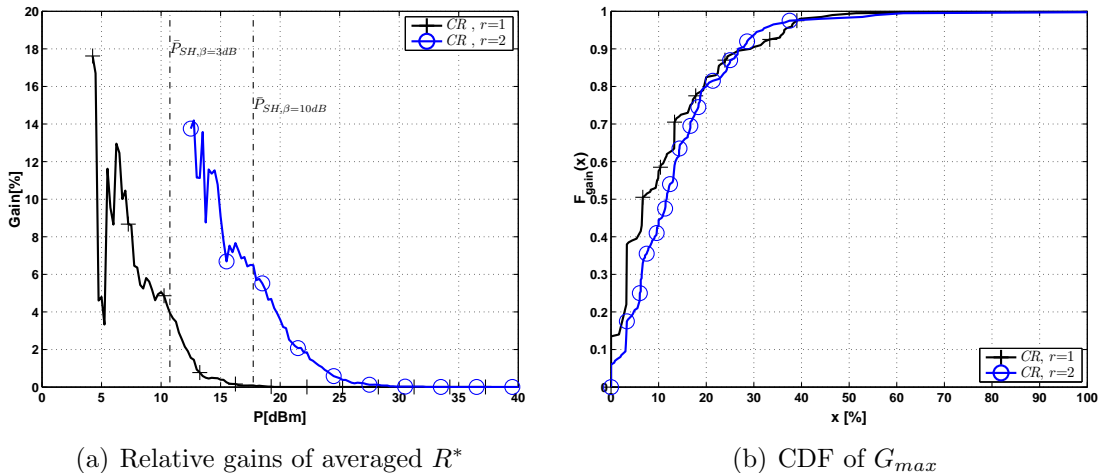(a) Relative gains of averaged R^* (b) CDF of G_{max}

Figure 5.4: Gains of CR in networks with single rate and single power

put averaged over 200 network realizations. The relative gains in averaged R^* are computed as in (4.8). These results show that CR can provide up to 15% gains at low power and about 8% gains at moderate transmit power level in a random topology network. Interestingly, the gains are smaller at high transmission power range. It is mainly due to the fact that at high transmission power, nodes need less number of hops to communicate directly with the gateway. Despite this, the results for R^* in 200 networks show that CR does not often improve the throughput in a network with single rate and power. Fig. 5.4(b) shows the empirical CDF for the maximum relative throughput gain G_{max} of *CR* over *JRS* for 200 networks, where G_{max} is obtained using (4.8) as a maximum across all transmission powers. The medians of G_{max} for $r = 1$ is 7% and for $r = 2$ is 12%. Clearly, the gains are marginal at best in a medium size network. Indeed, the CDF of G_{max} in Fig. 5.4(b) shows that in a random network, there is a 10% probability that CR does not provide any gain at all and "high gains" above 30% can be expected in only 10% of networks for both cases $r = 1$ and $r = 2$.
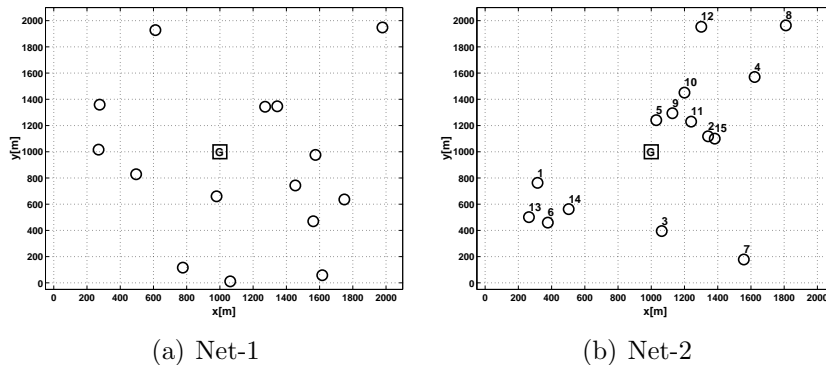
(a) Net-1 (b) Net-2

Figure 5.5: Placement of nodes

Hence, in some networks CR does provide gains in throughput as well as it improves other key network metrics, though the improvements then are only marginal. To illustrate, among the 200 networks we selected two networks, Net-1 and Net-2, shown in Fig. 5.5, with results that have significantly different outcomes. Fig. 5.6(a) shows R^* as a function of the transmit power for Net-1, a case where CR does not provide any gain at all with respect to JRS for both $r = 1$ and $r = 2$. In this network with the given fading realizations, the use of only the basic links is sufficient to provide the optimal max-min throughput. On the other hand, Fig. 5.6(b) shows the case for Net-2 where CR provides throughput gains across low to medium transmission power range for both $r = 1$ and $r = 2$.

We study also two other performance metrics called P_{min} and $P_{r/N}$, where P_{min} is the minimum transmit power at which a mesh network is fully connected and $P_{r/N}$ is the minimum transmit power at which a network can yield the maximum throughput $\frac{r}{N-1}$. ΔP_{min} is the difference between the minimum transmission power at which connectivity is possible when CR is enabled and when it is not. Similarly, $\Delta P_{r/N}$ is the difference between the minimum transmit power at which the network
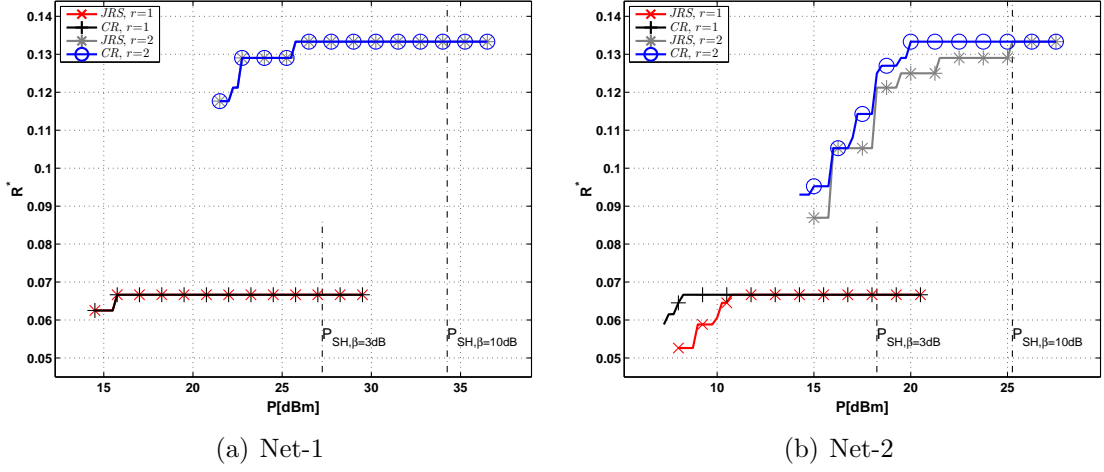
78

(a) Net-1   (b) Net-2

Figure 5.6: max-min throughput vs transmission power P, uplink+downlink flows

can provide the maximum achievable throughput of $\frac{r}{N-1}$ when CR is enabled and when it is not. ΔP_{min} is a metric allowing us to quantify the gain of connectivity at low transmit power while $\Delta P_{r/N}$ allows us to quantify the gain in transmit power to obtain the same throughput as a single hop network. In Fig. 5.6(b), a gain of up to $\Delta P_{min} = 0.75$dB for P_{min} and $\Delta P_{r/N} = 5$dB for $P_{r/N}$ for $r = 2$ can be achieved in the case of *CR* over *JRS* in Net-2.

    Fig. 5.7(a) shows the CDF of ΔP_{min} over the 200 network realizations, where medians for cases when $\Delta P_{min} > 0$ are 1.25dB for both $r = 1$ and $r = 2$. In 55% of cases, cooperative relaying does not improve the minimum connectivity power at all. Also, the choice of β does not have any impact on ΔP_{min}. Gains for P_{min} above 2dB are infrequent and occur in less than 1% of cases. Fig. 5.7(b) shows the CDF of $\Delta P_{r/N}$: no gains are observed in 30% of cases for $\beta = 3$dB and likewise in 55% of the cases for $r = 2$. Gains for $P_{r/N}$ above 3dB are seen in only 1% of cases for $r = 1$ and 10% of cases for $r = 2$. The medians of $\Delta P_{r/N}$ for cases when $P_{r/N} > 0$ are

(a) Gain ΔP_{min} in connectivity
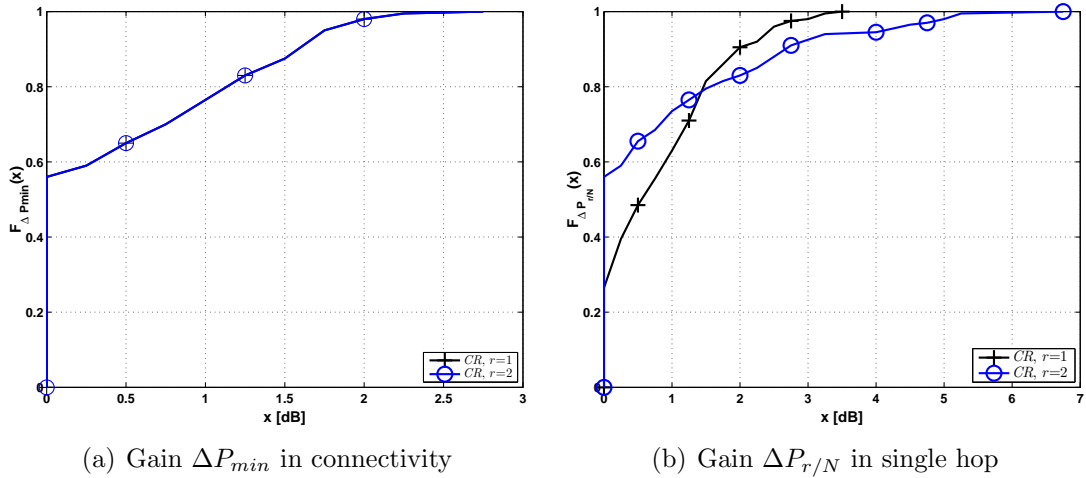
(b) Gain $\Delta P_{r/N}$ in single hop

Figure 5.7: CDFs of connectivity metrics

1.25dB for $\beta = 3$dB and 1.5dB for $r = 2$. Altogether, these results for P_{min} and $P_{r/N}$ show that CR does not provide significant gains in mid-size mesh networks in terms of connectivity gains. It is mainly due to the fact that the connectivity of a mesh network is determined by the node with the worst channel conditions. The maximum increase in the signal power in free-space is at best 3dB which might not be always sufficient to compensate the channel losses for the worst node in a network.

Solutions for the problems [**P1**] and [**P3**] also provide us with the optimal config-uration of a network in terms of routing and scheduling parameters, and in the case of CR with the optimal selection of relaying node pairs or also known as the optimal virtual node grouping. As an example, we show in Table 5.1 the optimal routing for downlink flow $(G, 13)$ in Net-2 using JRS and CR at the minimum power for the network to achieve the maximum throughput. In the case of CR, flow $(G, 13)$ must be routed along the two cooperative relay pairs $\{3, 5\}$ and $\{1, 3\}$.

downlink flow $(G, 13)$	
JRS	$\{G\} \to \{14\} \to \{13\}$
CR	$\{G\} \to \{3, 5\} \to \{1, 3\} \to \{13\}$

Table 5.1: e.g. Optimal relay selection in Net-2 for node 13, $\beta = 3$dB.

5.4.2 Multiple Rates and Continuous Power Control

Now we characterize the performance of CR in networks where nodes can support multiple rates and continuous power control. We focus only on the throughput metric since connectivity metrics ΔP_{min} and $\Delta P_{r/N}$ do not depend on other available rates or the choice of transmit power level, i.e., ΔP_{min} and $\Delta P_{r/N}$ are determined by the minimum and the maximum rate in a network,respectively. We are mainly interested to understand two characteristics of CR: 1) if the multi-rate capability makes CR more attractive, i.e., provide better gains that the single rate case and 2) if continuous power control is an effective technique that can be used with CR to achieve better throughput gain.

We solve both problems [**P1**] and [**P3**] for 100 random small size[2] mesh networks. Each network consists of $N = 10$ nodes with $N - 1$ physical nodes and one gateway. We need to emphasize that the solutions are obtained for small size networks and these results cannot be generalized for larger networks. And yet, these results are still valuable for providing insights in small networks as well as for implication of CR in larger networks. We use the same 100 network and fading realizations to obtain solutions for both cases: multi-rate and continuous power control. In the following, $\bar{P}_{SH,\beta}$ denotes the minimum single hop power averaged over 100 networks with value

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[2]Due the complexity of problem [**P3**], we could obtain results for the case of CR with continuous power control only for 10-nodes networks.

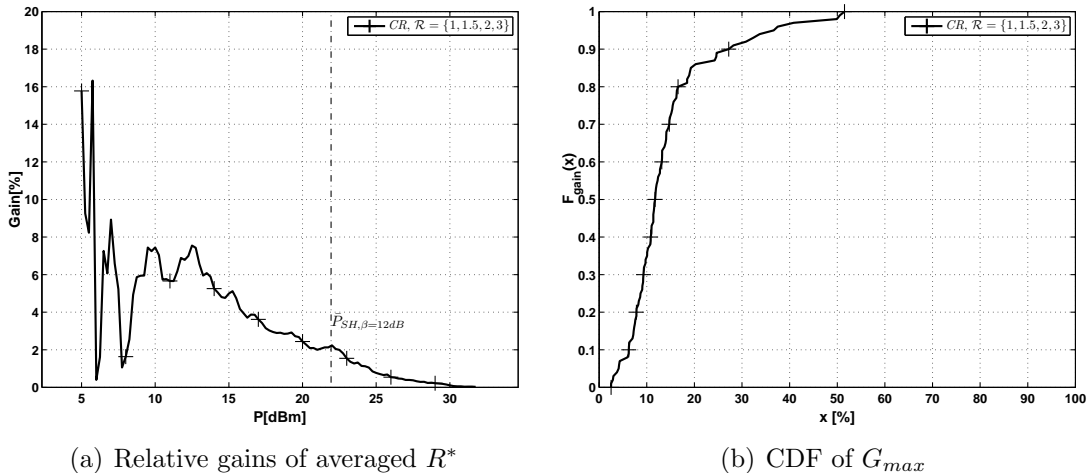(a) Relative gains of averaged R^*

(b) CDF of G_{max}

Figure 5.8: Gains of CR in small size networks with rate adaptation

for β that corresponds to the maximum rate in a network, i.e. $r_m = 3$.

Multiple rates and single power:. In the multi-rate case, we study CR in networks where nodes employ rate adaptation from the set of available rates $\mathcal{R} = \{1, 1.5, 2, 3\}$ but are allowed to transmit only at a fixed power level P. The corresponding SINR thresholds for the rates are chosen as follows: $\beta(1) = 3$dB, $\beta(1.5) = 6.4$dB, $\beta(2) = 10$dB and $\beta(3) = 12$dB. The results for [**P1**] with $\mathcal{I} = \mathcal{I}_{\text{int}}$ and $\mathcal{R} = \{1, 1.5, 2, 3\}$ are denoted as *JRS* and for [**P3**] with $\mathcal{I} = \mathcal{I}_{CR}$ and $\mathcal{R} = \{1, 1.5, 2, 3\}$ are denoted as *CR*.

Fig. 5.8(a) shows the relative gains of *CR* with respect to *JRS* in throughput averaged over 100 network realizations. The relative gains in averaged R^* are computed as in (4.8). These results show that CR can provide about 8% gains at intermediate power level and up to 15% gains at low power level. While these gains are marginal, they can only indicate the general trends for throughput in small networks. We also show in Fig. 5.8(b) the empirical CDF for the maximum relative gain G_{max}. Clearly, CR in multi-rate networks can provide better gains than in single-rate networks. In

(a) Net-3   (b) Net-4

Figure 5.9: Placement of nodes

(a) Net-3   (b) Net-4
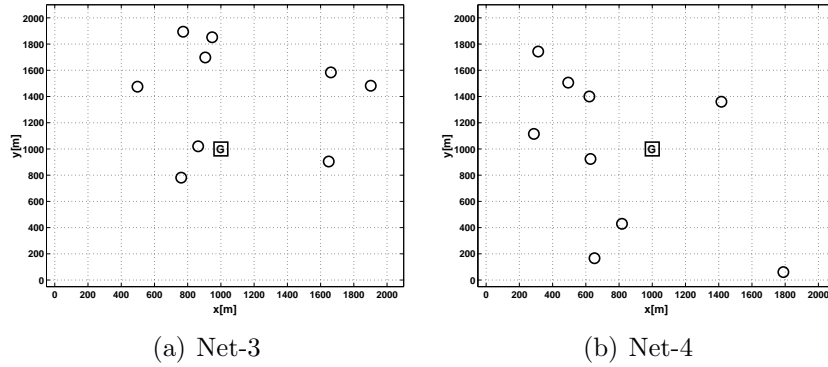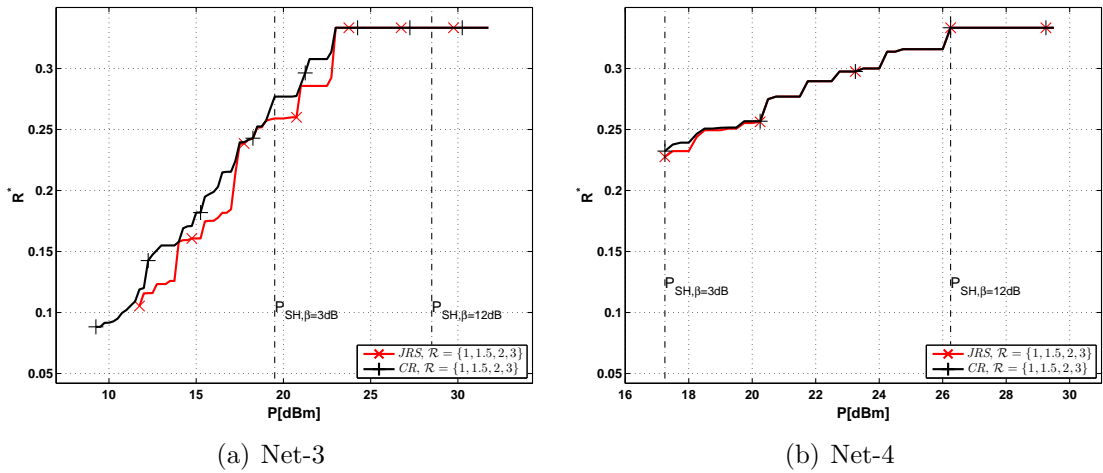
Figure 5.10: max-min throughput vs transmission power P, uplink+downlink flows

20% of the network realizations, there is a minimum gain of up to 10% and gains above 20% can be achieved in about 15% of network realizations. The median of G_{max} is 11.77% and the mean of $G_{max} = 14.35\%$. This improvement comes at no surprise as in multi-rate networks, nodes can effectively utilize an increase in the received signal power by transmitting at higher transmission rates.

In Fig. 5.10, we show the max-min throughput as a function of transmit power for the networks Net-3 and Net-4, illustrated in Fig. 5.9. While in Net-3, CR shows
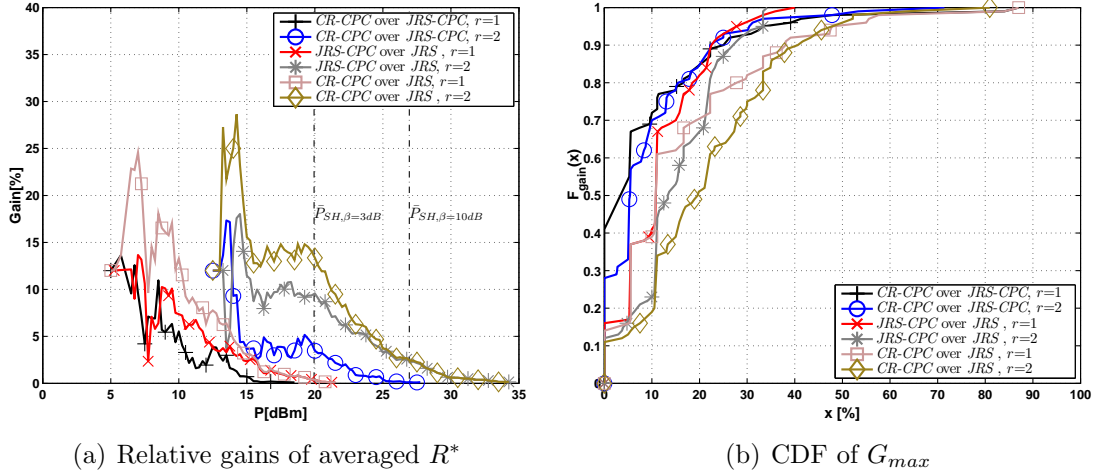
(a) Relative gains of averaged R^*

(b) CDF of G_{max}

Figure 5.11: Gains of CR and JRS in small size networks with CPC

gains across all transmission power range, in Net-4, the gains are insignificant. It is because Net-4 can only be connected when nodes transmit in single hop and can only provide the maximum throughput when nodes transmit with the single hop power of $\beta = 12$dB. For this reason, CR cannot offer any improvement in this network.

Continuous power control and single rate: Lastly, we study CR with CPC (JRS-CPC) and JRS with CPC (JRS-CPC) in networks where nodes are enabled with continuous power control and can support only single rate r. The results for R^* are obtained for the two cases of single rate: 1) $r = 1$ with $\beta = 3$dB and 2) $r = 2$ with $\beta = 10$dB. We are interested in continuous power control for the reason that in single-rate and single-power networks the use of CR induces an average 3dB increase in interference to other nodes. For this reason, without interference mitigation or power control techniques, the throughput improvement is severely limited by the total transmission power of cooperatively relaying nodes. The results for [**P1**] with $\mathcal{I} = \mathcal{I}_{CPC}$ are denoted as *JRS-CPC*, for [**P3**] with $\mathcal{I} = \mathcal{I}_{CR-CPC}$ are denoted as

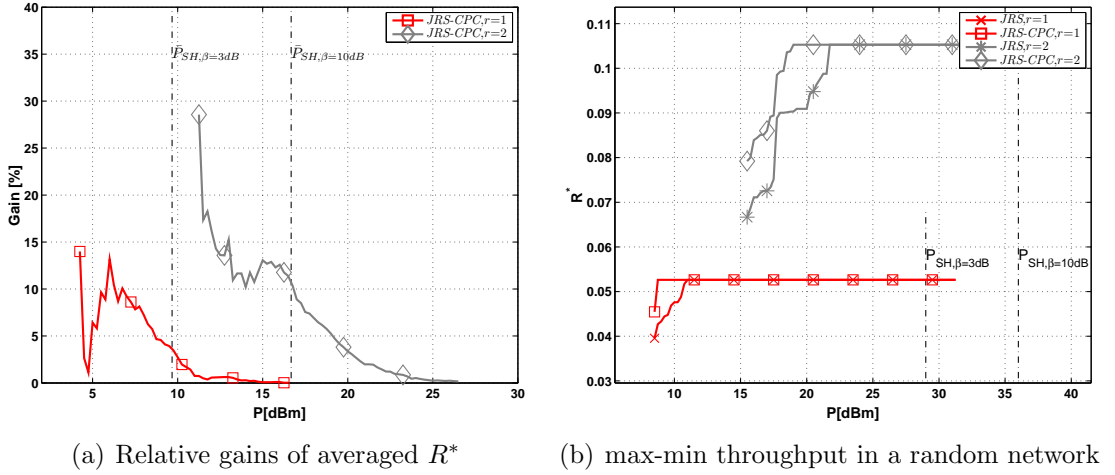(a) Relative gains of averaged R^* (b) max-min throughput in a random network

Figure 5.12: JRS with CPC in medium size networks

CR-CPC and for [**P1**] with $\mathcal{I} = \mathcal{I}_{\text{int}}$ are denoted as *JRS*.

Fig. 5.11(a) shows the relative gains in throughput averaged over 100 network realizations. The relative gains in averaged R^* are computed using (4.8) for CR-CPC and JRS-CPC with respect to the JRS case. Despite the fact that these results are obtained for R^* in small networks, they still clearly indicate that the use of CPC provides throughput improvement across low to medium transmission power range. Gains as high as 20% and 25% can be obtained at low transmission power level for $r = 1$ and $r = 10$, respectively, and up to 15% gains in the moderate transmission power range for both β. Interestingly, CPC outperforms significantly in terms of gains the use of multiple rates when compared with the results in Fig. 5.8(a). Fig. 5.11(a) also shows that most of the gains in the case of CR-CPC with respect to JRS can be attributed to the use of CPC with just JRS. In fact, the gains of CR-CPC with respect to JRS-CPC are relatively small. To characterize the impact of CPC on the throughput improvement in medium size networks, we also solved problem [**P1**]
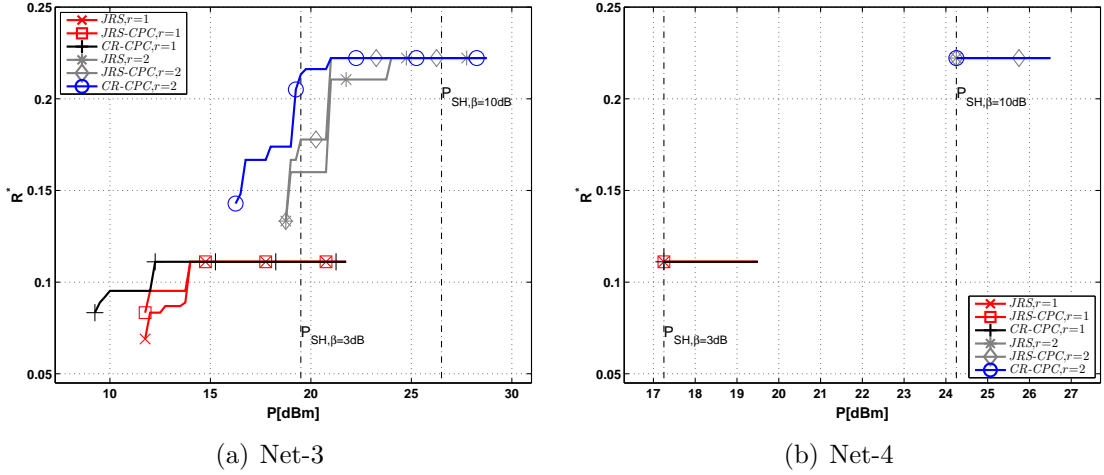
(a) Net-3

(b) Net-4

Figure 5.13: max-min throughput vs transmission power P, uplink+downlink flows

for 20-nodes mesh networks for both single-rate cases $r = 1$ and $r = 2$. Fig. 5.12(a) shows the relative gains of JRS-CPC with respect to JRS in terms of the throughput averaged over 100 network realizations. In medium size networks, CPC when jointly optimized for routing and scheduling can provide gains of up to 14% for $r = 1$ and up to 28% for the case of $r = 2$. Fig. 5.12(b) shows the max-min nodal throughput in a random network to illustrate how the use of only CPC can significantly improve the throughput at low to intermediate transmit power levels in this particular network.

The CDFs of the maximum relative gains G_{max}, shown Fig. 5.11(b), indicate that CPC does not always improve throughput in a random network. In about 15% of the network realizations, the use of CPC does not provide any gains with either JRS or CR in small size networks. We attribute it to the fact that the solutions are obtained for small networks. In spite of this, high gains above 30% can be obtained with CR in between 20% to 30% of the network realizations, while with JRS-CPC only happens in less than 10% of the networks. The medians of G_{max} for the case of JRS-CPC are

10% for $r = 1$ and 13.48% for $r = 2$. In the case of CR-CPC with respect to JRS, the medians of G_{max} are 11.11% for $r = 1$ and 19.72% for $r = 2$. These medians indicate that the use of CPC with CR is more effective for high rate links with respectively high values of β. These results on the relative and the maximum gains allow us to conclude that CPC can be effectively used in conjunction with CR to reduce the total transmission power from cooperatively relaying node pairs.

In Fig. 5.13, we show the max-min throughput as a function of the transmit power for networks Net-3 and Net-4, illustrated in Fig. 5.9, with results that have quite different outcomes. In Net-3, CR-CPC and JRS-CPC provide significant gains across low to moderate transmission power range for both cases $r = 1$ and $r = 2$. On the contrary, Net-4 is an example of networks where gains cannot be achieved in principal since this network can only be connected when nodes transmit at the minimum single hop power. This is a typical trend in small size networks when there is either no multihop regime or a small number of hops in the optimal routing. For this reason, in Fig. 5.11(b), CR-CPC shows no improvement for G_{max} in 15% of network realizations.

5.5 Conclusions

In this section, we have characterized the impact of cooperative relaying on the performance of fixed wireless networks. We achieve such a characterization by formulating a cross-layer optimization framework that allows us to obtain jointly-optimal configuration of flow rates, routing, scheduling, power allocation, and cooperatively relaying node pairs. This framework is based on the distributed Alamouti code for cooperation

between two nodes only. We provide the results for networks where rate adaptation and continuous power control are enabled at each node. When continuous power is enabled, we formulate a power allocation subproblem. By solving this subproblem, we can obtain a feasible power allocation vector corresponding to the nodes that are allowed to transmit conflict-free in the optimal schedule. In our formulation, we consider the SINR-based interference model, and allow cooperative transmission between any pair of nodes. This framework is generic in the sense that it can be easily adopted to schemes that allow cooperation among more than two nodes by employing techniques such as distributed beamforming. or higher order distributed space-time block codes. By using this framework, network operators can obtain the maximum achievable throughputs as well as the optimal selection of cooperative relaying node pairs. Due to the complexity of the problems, we could obtain results for the cases of continuous power control and multi-rate only for 10-nodes networks. By solving the formulated problem to optimality, we have been able to quantify the throughput and connectivity gains in a random network as well as to obtain the following engineering insights:

- In medium-size mesh networks where nodes can support only a single rate and a single power, cooperative relaying often does not improve throughput or connectivity in a random network. About 10% of network realizations showed no gains in throughput and about 50% of networks showed no gains in connectivity. Even when there is a gain in some realizations, this gain is marginal at best considering the medium size of networks.

- In multi-rate networks with a single power, the use of cooperative relaying can

provide better gains in throughput than in the single-rate networks. At worst, maximum gains of up to 10% can be obtained in 10% of network realizations and at best, the gains of up to 30% in 10% of networks realizations. Regardless of this, cooperative relaying does not provide significant improvement in networks of small size. We attribute this performance to the fact that solutions are obtained for small size networks and therefore, we do not make any conclusions on performance for networks of larger size.

• When CPC is enabled, cooperative relaying does not provide significant gains in small size networks since most of gains are attributed to the use of CPC with only routing and scheduling. Despite the fact that the solutions are obtained for small size networks, it is evident that CPC is an effective technique to improve the throughput performance in networks where CPC is jointly optimized with only routing and scheduling. We do not make any conclusion on the effectiveness of CPC when used jointly with cooperative relaying in large size networks.

Chapter 6

Summary

We conclude the thesis by discussing the practical applications of this work. Frankly speaking, there are only a few examples where our models and results can be useful for practical purposes. For the reason that nowadays, most of the wireless networks are either traditional single-hop networks, mobile, or are based on random access MAC. The wireless backhaul of cellular networks is one potential application since it is fixed, centrally managed using scheduling, and the channels are relatively static. Although the level of deployment of wireless backhaul is very low, network operators tend to move toward wireless and multihop cellular backhauls. Another application of this work is in sensor networks that are deployed in a relatively static surrounding environment.

The results of this work can be useful for network operators during the offline configuration of a wireless network. With the use of our problem formulations, the maximum throughput gains of advanced physical layer techniques, network coding, or cooperative technique can be obtained in any given network to allow network op-

Techniques	Complexity	Throughput Gains	Connectivity Gains	Deployment
CPC	low	medium	-	easy
NC	low	low/medium	-	easy
SIC	medium	high	-	medium
SPC+SIC	medium	very high	-	medium
DPC	very high	low	-	difficult
CR	medium	low	low	medium

Table 6.1: Comparison of different techniques

erators to justify the costs of deployment. These results for the maximum achievable throughputs can serve as an upper bound for the performance in a network with random access MAC.

In Table 6.1, we provide a general summary on the practical insights of the studied techniques in wireless mesh networks.

As for the future work, we can outline the following directions:

- We can consider a trivial extension of our formulations for combinations of these techniques, e.g. NC and SIC, CR and SIC, DPC and NC, and etc., as well as re-compute our results for the Rician fading model to address the line-of-sight communication networks. We can also compute results for successive interference cancellation and superposition coding when these techniques are jointly optimized with continuous power control.

- In the case of SIC, there is a potential improvement as the receiver does not utilize the decoded interfering signal while partially canceling the interference. This decoded interfering signal can be utilized if the problem formulation is adapted for the combination with cooperative relaying.

- Our problem formulations are deterministic in the sense that they do not capture the channel fluctuations, and the optimal network configuration is obtained for the given channel realizations. Therefore, our model does not characterize one of the benefits of cooperative relaying such as robustness against fading. We can extend our deterministic model to a stochastic model by averaging the link rates over the probability distribution of fading.

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