Conditional Decoding for X Channels

by

Mostafa El Gamal

A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Master of Applied Science
in
Electrical and Computer Engineering

Waterloo, Ontario, Canada, 2012

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

I understand that my thesis may be made electronically available to the public.

Abstract

The conditional decoder is a low complexity and optimal decoder for multiplexed orthogonal designs in point-to-point channels. We extend this notion to X channels and differentiate between two interference scenarios. First, the interference is perfectly aligned in a different sub-space than the intended information symbols. For this scenario, the conditional decoder is applied as a one stage and two stages decoder to cancel the interference and decode the desired signal. Second, the interference is misaligned. In this case, the conditional decoder attempts to jointly cancel the interference and decode the intended signal while achieving a performance gain over other interference cancellation schemes. We consider the two user scenario where Alamouti codes are used at the transmitters and then extend our investigation to three user channels with arbitrary signal structure. Our numerical results establish the superiority of our decoder to previously proposed zero-forcing and decoupling techniques, in terms of performance. It is further shown that the proposed decoder achieves the same performance as the sphere decoder; while enjoying a much lower implementation complexity.

Acknowledgements

I would like to thank my advisor, Dr. Damen, who made this possible. I would also like to thank my brother, Hesham, who was always there for me whenever I needed his advice or guidance.

Dedication

This is dedicated to my brothers and best friends, Hesham and Ali.

Table of Contents

List of Figures

Chapter 1

Introduction

The state-of-the-art interference alignment technique allows each receiver in a multi-input multi-output (MIMO) system to resolve the intended information symbols while ignoring the unintended ones. This is done by projecting the unintended symbols in a different subspace than the intended ones then a specific technique is used at the receiver for interference cancellation [1],[2] and decoding the information symbols. Zero-forcing technique [3] can be used to cancel the interference with optimal performance and a very simple linear processing if the interference is perfectly aligned. In other cases where interference misalignment occurs, the zero-forcing technique is not optimal. That motivated us to find another algorithm that can achieve performance improvement over the zero-forcing technique in canceling the interference for the misaligned interference case while achieving an optimal performance for the case of perfectly aligned interference.

X networks [4] consist of multi transmitter and multi receiver, each transmitter has a message to each receiver. The channel coefficients from a specific transmitter to the receivers are only known to that transmitter which means that transmitter A only knows the channel coefficients from itself to all the receivers but does not know any information about the channel coefficients from transmitter B to the receivers. Each receiver works on decoding the messages sent to it from the transmitters while canceling the interference coming from the messages sent to other receivers. In this thesis, we consider two system models and referring to them as the two user case and the three user case. The two user system has two transmitters and two receivers, each equipped with double antennas. The three user system has three transmitters and two receivers, each equipped with double antennas.

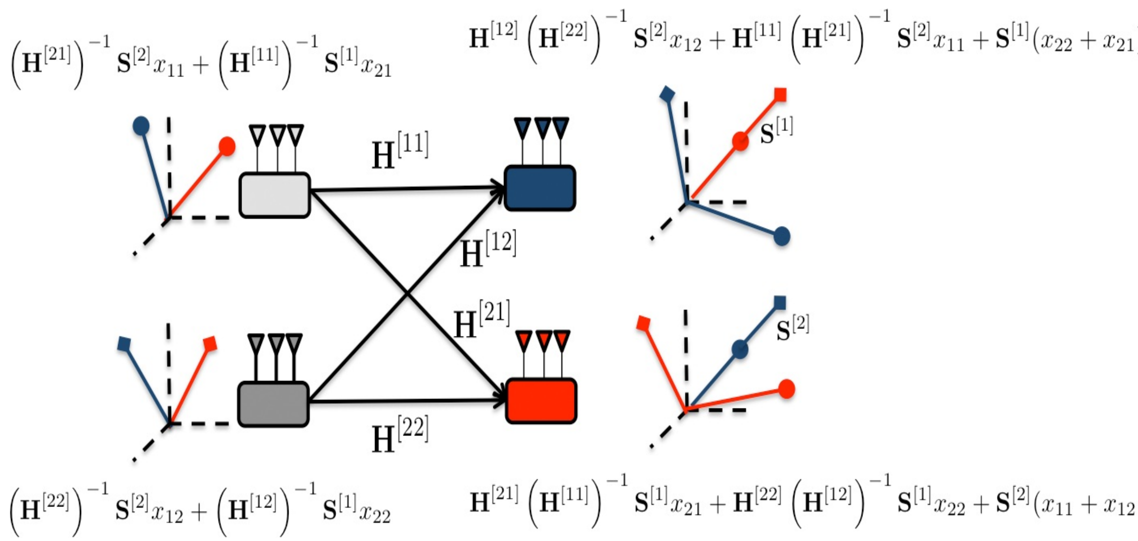

Figure 1.1: Interference Alignment for X channels

For the two user X channel case, it was shown that Alamouti scheme can be used along with beamforming matrices [4]-[6] at the transmitter to perfectly align the interference in a different sub-space than the intended signals so it's easy to cancel it at the receiver using a simple technique like the zero-forcing. For some other settings of the communication system, it's not an easy job to find the suitable coding scheme and beamforming matrices that can be used to align the interference perfectly. That would lead to a misalignment of the interference which means that a portion of the interference signal will be in the same sub-space as the intended signal and interfere with it. A simple interference cancellation technique like the zero-forcing technique is not optimal in this case which opens the door for trying other techniques trying to achieve a performance improvement while maintaining low complexity.

Alamouti scheme [7] along with beamforming matrices are used in the two user case to perfectly align the interference [8]. In this scheme, two symbols are sent from a double antenna transmitter in two time slots. The 2×2 transmitted matrix has an orthogonal structure which is an advantage in case of deploying the conditional decoder at the receiver.

The conditional decoding algorithm is optimal for multiplexed orthogonal designs and it has a reduced complexity [9] so it can be used in canceling the interference and decoding the information symbols especially if the transmitted coding scheme has an orthogonal structure. The reduced complexity comes from conditioning on some of the information symbols and calculating the remaining ones so the exhaustive search works on a specific set of the information symbols instead of searching all of them to reach the ML solution. The conditional decoder is essentially optimal for other codes such as Golden code [10] while it's optimal for orthogonal transmission schemes such as Alamouti code.

Throughout our research, we focused on applying the conditional decoder to the interference cancellation problem for X channels using different scenarios to examine the performance of the conditional decoder in the two cases of perfectly aligned and misaligned interference. We used two system models for the two user and three user cases while deploying the Alamouti scheme for the two user case and introducing an arbitrary transmission scheme for the three user case. We applied the conditional decoder as a two stage and one stage decoder. In the two stage decoder, it works only on decoding the information symbols as it's preceded by the zero-forcing stage for the interference cancelation. In the one stage decoder, it jointly cancels the interference and decodes the information symbols in a single step. The simulation results helped to conclude the benefit of applying the conditional decoding algorithm to solve the interference cancellation problem for X channels.

In this thesis, we start with the introduction in **Chapter 1** then the rest of the thesis material is organized as following:

- **Chapter 2**

Interference alignment concept is introduced through presenting the theory and the benefit of applying it on achieving the degrees of freedom for the channel unlike other techniques such as the cake cutting algorithm. Then two examples on applying the interference alignment technique are illustrated. The examples are applying the interference alignment for X networks and cellular networks.

- **Chapter 3**

The concept behind the conditional decoding algorithm is described and the steps for applying it are clearly stated. The optimality theorem of the conditional decoder is given and explained to show how it can be applied for interference cancellation. Then two applications are presented to show the performance of the conditional decoder through an example when the conditional decoder is essential optimal and another one when it is optimal.

- **Chapter 4**

In this chapter, our contribution of applying the conditional decoder to cancel the interference and decode the information symbols for X channels is demonstrated. We explain our simulations steps for the two scenarios of perfectly aligned and misaligned interference. The simulation results show the benefit of applying the conditional decoding algorithm to cancel the interference and decode the information symbols for X channels especially in the case of misaligned interference which is our main concern.

- **Chapter 5**

In this chapter, we conclude the results of our research on applying the conditional decoder algorithm for canceling the interference and decode the intended information symbols then we explain the reason behind the performance gain achieved by the conditional decoder over other interference cancellation methods such as the zero-forcing technique. Finally, we discuss the future work of our research and how the conditional decoder can be used in the more general case of interference alignment applications.

Chapter 2

Interference Alignment

Interference alignment technique works on achieving the degrees of freedom of the channel when the received signal at each receiver is a combination of the intended signals and the unintended ones. In this chapter, we present the theory behind it then we give examples on applying the interference alignment for X channels and cellular networks.

2.1 Theory

In multi user communication systems, interference can be an extra limit to the maximum data rate of sending the information symbols from each transmitter. One simple way to share the communication medium among K users is the cake cutting algorithm [11] where all transmitters have an equal share of the channel bandwidth and this share is $1/K$ of the total available bandwidth. This algorithm was shown to be suboptimal.

The interference alignment technique offer a method that enables each transmitter to send the data at half the rate it can achieve if the channel was interference free which means that each transmitter can have a share equals to half the bandwidth. This can be explained through considering a simple example of solving a set of linear equations.

$$
\begin{aligned}
y_1 &= h_{11}x_1 + h_{12}x_2 + \ldots + h_{1K}x_K \\
y_2 &= h_{21}x_1 + h_{22}x_2 + \ldots + h_{2K}x_K \\
&\vdots \\
y_B &= h_{B1}x_1 + h_{B2}x_2 + \ldots + h_{BK}x_K
\end{aligned} \tag{2.1}
$$

Assume $y_1, y_2, \ldots, y_B$ represent different realizations of the transmitted information symbols at one of the receivers. The number of realizations depends on the channel bandwidth B. $x_1, x_2, \ldots, x_K$ are the information symbols from K transmitters given that each transmitter only sends 1 symbol. h_{ij} are the channel coefficients.

To solve that set of linear equations at one of the receivers, the number of equations B has to be larger than the number of unknowns K. This condition means that we should have a large bandwidth so the receiver can resolve all information symbols. This is true if the receiver is interested in resolving all the information symbols. To explain that, we need to re-write the set of linear equations as following

$$
Y = H_1x_1 + H_2x_2 + \ldots + H_Kx_K \tag{2.2}
$$

Where Y is the received vector at one of the receivers and H_i can be refered to as the received beam direction for symbol x_i

$$
Y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_B \end{bmatrix} \qquad , \quad H_i = \begin{bmatrix} h_{1i} \\ h_{2i} \\ \vdots \\ h_{Bi} \end{bmatrix}
$$

Now imagine that the receiver is only trying to resolve one of the information symbols, e.g., x_1 and the rest of the symbols $x_2, x_3, \ldots, x_k$ are considered to be interference symbols. To do that the receiver still have to resolve all interference symbols to decode the desired one unless the information symbol is projected in a different sub-space than the unintended ones which is the main idea of interference alignment.

If H_1 lies in a different vector space than the other received beam directions $H_2, H_3, \ldots, H_k$ then the receiver can resolve the intended symbol x_1. This means that B can be less than K and the receiver is still able to resolve the desired symbol. The same condition can be met at other receivers since each receiver sees a different set of linear equations.

There are many applications for the interference alignment technique such as index coding problem [12],[13], X channels [14], cellular networks, and MISO broadcast channel [15]. In this chapter, we will shed light on the X channels and cellular networks applications.

2.2 Examples

2.2.1 X Channels Application

The degrees of freedom for an $M \times N$ X network that consists of M single antenna transmitters and N single antenna receivers is $\frac{MN}{M+N-1}$ [16]. The interference alignment scheme that is used to achieve the degrees of freedom of the channel can be described through an example of a 3×3 X network as shown in **Fig 2.1**.

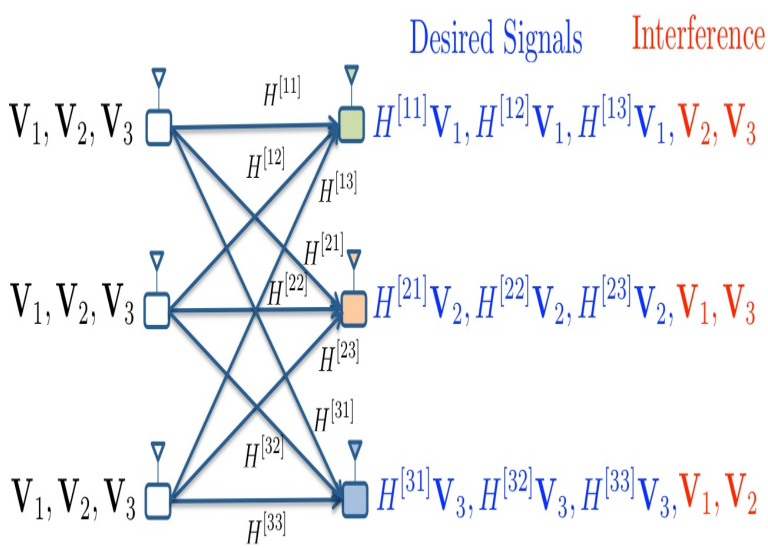

Figure 2.1: Interference Alignment for 3×3 X channel

The total number of transmitted messages is 9 with 3 desired messages at each receiver and 6 interference messages. The job of the interference alignment scheme is to project the desired messages in a different sub-space than the interference ones. This is done through sending the intended messages to receiver 1 on the signal space V_1, the intended messages to receiver 2 on the signal space V_2, and the intended messages to receiver 3 on the signal space V_3.

All transmitters send their messages to receiver 1 on the signal space V_1 so they are considered as interference messages to receivers 2 and 3. The same procedure is repeated for the sent messages to receivers 2 and 3. To decode the received messages, receiver 1 cancels the messages on V_2 and V_3 while decoding the messages on V_1, receiver 2 cancels the messages on V_1 and V_3 while decoding the messages on V_2, and receiver 3 cancels the messages on V_1 and V_2 while decoding the messages on V_3.

The challenge is to design the transmitted matrices in a way that can achieve the required alignment of the interference messages in a different sub-space than the intended information messages. One of the very effective designs for the double antenna 2×2 X channel deploys the Alamouti coding scheme along with beamforming matrices to perfectly align the interference [8]. The transmitted matrices are designed as

$$x_1 = \sqrt{\frac{3P}{2}} \begin{bmatrix} s_{11}^1 & s_{11}^2 \\ -s_{11}^{2*} & s_{11}^{1*} \\ 0 & 0 \end{bmatrix} v_{11} + \sqrt{\frac{3P}{2}} \begin{bmatrix} 0 & 0 \\ -s_{12}^{2*} & s_{12}^{1*} \\ s_{12}^1 & s_{12}^2 \end{bmatrix} v_{12} \tag{2.3}$$

$$x_2 = \sqrt{\frac{3P}{2}} \begin{bmatrix} s_{21}^1 & s_{21}^2 \\ -s_{21}^{2*} & s_{21}^{1*} \\ 0 & 0 \end{bmatrix} v_{21} + \sqrt{\frac{3P}{2}} \begin{bmatrix} 0 & 0 \\ -s_{22}^{2*} & s_{22}^{1*} \\ s_{22}^1 & s_{22}^2 \end{bmatrix} v_{22} \tag{2.4}$$

The transmitted matrices x_1 and x_2 have Alamouti structure for the information symbols. Beamforming matrices v_{ij} are used to project the interference on a different sub-space than the intended signals and each entry in beamforming matrices is kept smaller than 1 to avoid high power peaks.

$$v_{11} = \sqrt{\frac{1}{tr(H_{12}^{-1} H_{12}^{-1*})}} H_{12}^{-1}, \ v_{12} = \sqrt{\frac{1}{tr(H_{11}^{-1} H_{11}^{-1*})}} H_{11}^{-1}$$

$$v_{21} = \sqrt{\frac{1}{tr(H_{22}^{-1}H_{22}^{-1*})}}H_{22}^{-1}, \; v_{22} = \sqrt{\frac{1}{tr(H_{21}^{-1}H_{21}^{-1*})}}H_{21}^{-1}$$

In three time slots, each transmitter sends two symbols to each receiver. s_{ij}^k is the transmitted symbol from transmitter i to receiver j and the symbol number is k. The achieved symbol rate in this case is $\frac{8}{3}$ which is the maximum achievable rate [4]. The received signals at both receivers are written as

$$\begin{aligned} y_1 &= \sqrt{\frac{3P}{2}} \begin{bmatrix} s_{11}^1 & s_{11}^2 \\ -s_{11}^{2*} & s_{11}^{1*} \\ 0 & 0 \end{bmatrix} v_{11}H_{11} + \sqrt{\frac{3P}{2}} \begin{bmatrix} s_{21}^1 & s_{21}^2 \\ -s_{21}^{2*} & s_{21}^{1*} \\ 0 & 0 \end{bmatrix} v_{21}H_{21} \\ &+ \sqrt{\frac{3P}{2}} \begin{bmatrix} 0 & 0 \\ -as_{12}^{2*} - bs_{22}^{2*} & as_{12}^{1*} + bs_{22}^{1*} \\ as_{12}^1 + bs_{22}^1 & as_{12}^2 + bs_{22}^2 \end{bmatrix} + w_1 \end{aligned} \tag{2.5}$$

$$\begin{aligned} y_2 &= \sqrt{\frac{3P}{2}} \begin{bmatrix} s_{12}^1 & s_{12}^2 \\ -s_{12}^{2*} & s_{12}^{1*} \\ 0 & 0 \end{bmatrix} v_{12}H_{12} + \sqrt{\frac{3P}{2}} \begin{bmatrix} s_{22}^1 & s_{22}^2 \\ -s_{22}^{2*} & s_{22}^{1*} \\ 0 & 0 \end{bmatrix} v_{22}H_{22} \\ &+ \sqrt{\frac{3P}{2}} \begin{bmatrix} 0 & 0 \\ -cs_{11}^{2*} - ds_{21}^{2*} & cs_{11}^{1*} + ds_{21}^{1*} \\ cs_{11}^1 + ds_{21}^1 & cs_{11}^2 + ds_{21}^2 \end{bmatrix} + w_2 \end{aligned} \tag{2.6}$$

Where y_1 and y_2 are the received signals. w_1 and w_2 are the noise matrices. $a = \sqrt{2tr(H_{11}^{-1}H_{11}^{-1*})}$, $b = \sqrt{2tr(H_{21}^{-1}H_{21}^{-1*})}$, $c = \sqrt{2tr(H_{12}^{-1}H_{12}^{-1*})}$, $d = \sqrt{2tr(H_{22}^{-1}H_{22}^{-1*})}$.

It can be shown from the structure of the received signals that the intended information symbols are in a different sub-space than the interference. A specific algorithm has to be used at the receiver to cancel the interference prior to decoding the intended information symbols.

Recent research [17] shows that the degrees of freedom for a multiple-antenna MIMO X channel is $\frac{AMN}{M+N-1}$ where M and N are the number of transmitters and receivers, each node is equipped with A antennas. To explain this result, we consider an example of 3×3

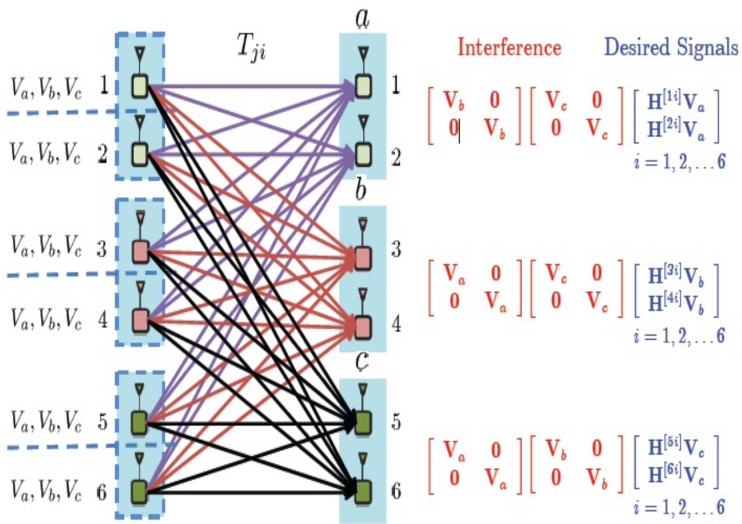

Figure 2.2: Double Antenna 3×3 MIMO X Channel

X network with double antenna at each node. We can imagine that the system has six virtual transmitters, each antenna is considered as a separate virtual transmitter as shown in **Fig 2.2**.

All transmitters send the information symbols over signal space V_a, V_b, and V_c to receivers a, b, and c respectively. Transmitter 1 and 2 send their messages to receiver a over a signal dimension $2|V_a|$ which is considered as interference at receivers b and c. The other messages sent from transmitters 3, 4, 5, and 6 to receiver a span the same signal space $2|V_a|$ at receivers b and c. That means that the messages intended for receiver a constitute interference at receivers b and c and span a signal dimension of $V_a \times V_a$.

The same argument applies to the interference caused by the messages sent to receivers b and c as they occupy a signal dimension of $V_b \times V_b$ and $V_c \times V_c$ respictively. Assuming all signal spaces have the same size, i.e., $|V_a| = |V_b| = |V_c| = |V|$, the desired signals at each receiver occupy a signal dimension of $6|V|$ while the interference signals occupy a signal dimension of $4|V|$.

For a symbol duration of n time slots, the signal dimension seen by each receiver is $2n$. To make sure that the desired and interference signals will not overlap, the signal space seen by each receiver has to be larger than $10|V|$. The available degrees of freedom in this system is $\frac{3\times6|V|}{n}$ with $2n = 10|V|$. That gives a $DOF = \frac{18}{5} = A(\frac{MN}{M+N-1})$.

2.2.2 Cellular Networks Application

For K cells and each cell consists of one base station and M users. The degrees of freedom that can be achieved for each cell using the interference alignment technique is shown to be $\frac{M}{M+1}$ [18]. This result means that for a large number of users, the degrees of freedom per cell approaches the one for interference-free network.

To explain the application of interference alignment technique for cellular networks, we take an example of three cells where each cell consists of one base station and three users. The uplink signals are shown in **Fig 2.3**.

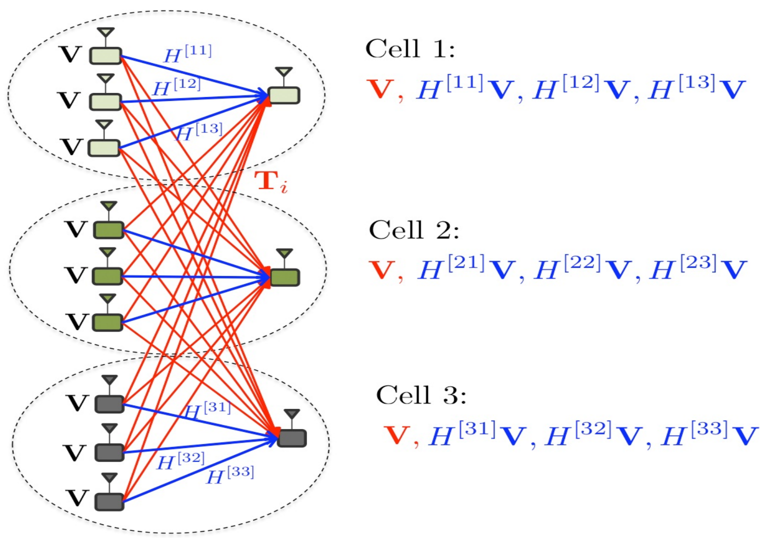

Figure 2.3: Interference Alignment for 3 Cells, 9 Users

Each transmitter sends its information with the same signal space V. At cell 1, the overall coming interference from the users connected to the other two cells is combined within $|V|$ signal dimensions while the useful signals from cell 1 users occupy $3\times|V|$ signal dimension. The same applies to cell 2 and cell 3. In general, the interference occupies $|V|$

signal dimensions and the desired signals occupy $M \times |V|$ signal dimensions. This means that the degrees of freedom per cell equals to $\frac{M\times|V|}{(M+1)\times|V|}$ which is $\frac{M}{M+1}$

Fig 2.4 shows four cellular networks having the same degrees of freedom when applying the interference alignment technique. $DOF = K(\frac{M}{M+1})$ with $K > M$. (a) is a SIMO network with K users and M antennas at each receiver, (b) is a MISO network with K users and M antennas at each transmitter, (c) is an interfering MAC channel with K cells and M users in each cell, and (d) is an interfering BC channel with K cells and M users in each cell.

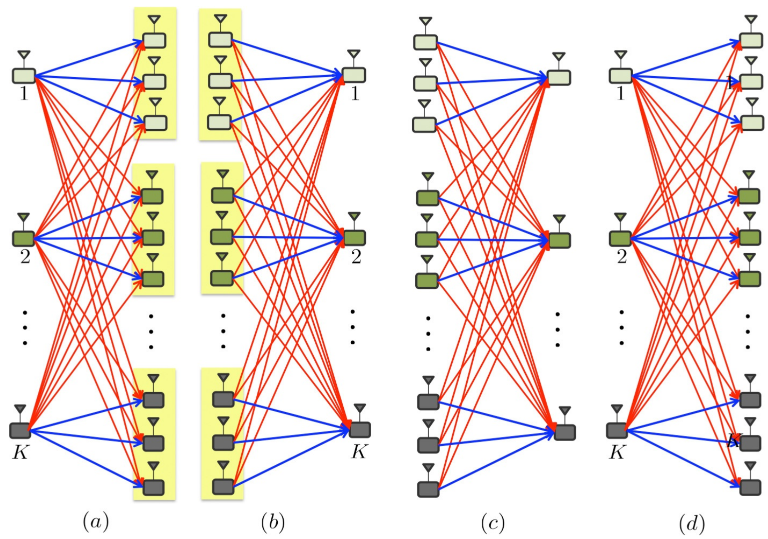

Figure 2.4: Four Cellular Networks With The Same DOF

Interference cancellation can be an easy job if the interference is perfectly aligned in a different sub-space than the intended signals. But, if interference misalignment occurs due to the difficulty of finding suitable coding scheme and beamforming matrices then a simple interference cancellation scheme such as the zero-forcing technique is not optimal. That motivated us to find another scheme that works in a different way than the zero-forcing technique. In the next chapter, we study the conditional decoder algorithm to show the benefit of using it for the application of interference cancellation especially in the scenario of misaligned interference.

Chapter 3

Conditional Decoding Algorithm

The conditional decoding algorithm is optimal and has low complexity in decoding the information symbols for multiplexed orthogonal designs [9]. It can be essentially optimal for other applications such as the golden code [10]. In this chapter, we are presenting the theory behind the conditional decoding algorithm and how to apply it through two examples with the golden code and Alamouti scheme.

3.1 Theory

The main idea of the decoder is to split the channel matrix into two matrices with one matrix has mutually orthogonal rows then it does exhaustive search over a reduced number of the transmitted symbols while calculating the remaining ones.

$$y = xH + n \tag{3.1}$$

Where y is the received signal, x is the transmitted vector, H is the channel matrix, and n is the noise vector.

To get the optimal solution, it is required to maximize the likelihood function $P(y|x)$, the transmitted vector x is uniformly distributed over a constellation C of size Q. The optimal solution can be written as

$$\hat{x} = \arg\max_{x \in C} P(y|x) \tag{3.2}$$

The optimal solution can be obtained by an exhaustive search over all symbols in x which gives a complexity of $O(Q^K)$, where K is the dimension of x. If the channel matrix H has N mutually orthogonal rows then the optimal solution can be obtained with a complexity of $O(Q^{K-N})$.

The channel matrix H can be divided into two matrices H_1, H_2 where H_1 has the N mutually orthogonal rows and H_2 has the remaining M rows. The transmitted victor x can also be divided into two victors x_1, x_2.

$$y = x_1 H_1 + x_2 H_2 + n \tag{3.3}$$

The conditional decoder does exhaustive search over the M symbols in x_2 and calculates the N symbols of x_1 [10]

$$\tilde{x_1} = (y - x_2 H_2) H_1^T (H_1 H_1^T)^{-1} \tag{3.4}$$

$\hat{x}_1$ is the quantized vector of $\tilde{x_1}$ according to the transmitted constellation. Maximizing the likelihood functions leads to

$$\hat{x}_2 = \arg\min_{x_2} ||y - x_2 H_2 - x_1 H_1||^2 \tag{3.5}$$

The final decoded symbols are $\hat{x}_1$ and $\hat{x}_2$ and the decoder has a reduced complexity of $O(Q^M)$ so the complexity of the conditional decoder depends on the number of mutually independent rows in the channel matrix.

The conditional decoder can be essentially optimal or optimal depending on the structure of the channel matrix so for a quasi-static fading channel, the optimality of the conditional decoder depends on the structure of the coding scheme.

Theorem 1 *The conditional decoding algorithm is optimal for multiplexed orthogonal designs.*

For multiplexed orthogonal designs, the transmitted codes have orthogonal structures that makes the channel matrix has mutually orthogonal rows in the case of quasi-static fading channel. The received signal can be written as

$$r = sH + cG + n \tag{3.6}$$

Where s and c are the transmitted information symbols, H and G are the channel matrices, and n is the received noise. The conditional decoder job is to estimate the information symbols, first it conditions on one of the transmitted codewords then it calculates the other one. If the channel matrix G has better conditions than H then the decoder will first condition on s and calculates c as following

$$c = (r - sH)G^{-1} \tag{3.7}$$

Since the rows of the channel matrix G are mutually orthogonal so the zero-forcing technique can solve the above equation optimally to obtain the codeword c. That makes the conditional decoder optimal in this case of using orthogonal multiplexed codes. The decoded symbols $\hat{s}$ and $\hat{c}$ are the ones that maximize the likelihood function

$$p(r|s, c) \propto exp(-\frac{1}{2\sigma^2}\|r - sH - cG\|^2) \tag{3.8}$$

Maximizing the likelihood function will lead to the optimal solution for the information symbols as following

$$\hat{c} \;\; = \;\; Q((r - sH)G^{\dagger}(GG^{\dagger})^{-1}) \tag{3.9}$$

Where Q is a quantization operator according to the transmitted constellation. The codeword s is selected to minimize the following argument

$$\hat{s} = \arg\min_{s} \|r - sH - \hat{c}G\|^2 \tag{3.10}$$

3.2 Examples

3.2.1 Golden Code Application

The conditional decoding algorithm can be deployed for a point to point communication system, each node is equipped with double antennas. For this setup, the transmitter sends 4 information symbols in 2 time slots using the golden code [19]-[21] as following

$$x = \frac{1}{\sqrt{5}}\begin{pmatrix} \alpha & 0 \\ 0 & \bar{\alpha} \end{pmatrix}\begin{pmatrix} s_1 + s_2\tau & s_3 + s_4\tau \\ i(s_3 + s_4\mu) & s_1 + s_2\mu \end{pmatrix} \tag{3.11}$$

Where s_j are the four information symbols. $\tau = \frac{1+\sqrt{5}}{2}$ is the golden number and $\mu = \frac{-1}{\tau} = \frac{1-\sqrt{5}}{2}$ is the algebraic conjugate of τ. $\alpha = 1 + i\mu$ and its algebraic conjugate $\bar{\alpha} = 1 + i\tau$.

The transmitted matrix can be written as

$$x = \frac{1}{\sqrt{5}}\begin{pmatrix} \alpha & 0 \\ 0 & \bar{\alpha} \end{pmatrix}\left[\begin{pmatrix} s_1 & s_3 \\ is_3 & s_1 \end{pmatrix} + \begin{pmatrix} \tau & 0 \\ 0 & \mu \end{pmatrix}\begin{pmatrix} s_2 & s_4 \\ is_4 & s_2 \end{pmatrix}\right] \tag{3.12}$$

The received signal after passing through an AWGN channel [3] with quasi-static fading coefficients can be written as

$$\begin{aligned}(r_{11}, r_{12}) &= \frac{1}{\sqrt{5}}(\alpha h_{11}, \bar{\alpha} h_{21})\begin{pmatrix} s_1 & s_3 \\ is_3 & s_1 \end{pmatrix} \\ &+ \frac{1}{\sqrt{5}}(\tau h_{11}, \mu h_{21})\begin{pmatrix} s_2 & s_4 \\ is_4 & s_2 \end{pmatrix} + (n_{11}, n_{12})\end{aligned} \tag{3.13}$$

$$\begin{aligned}(r_{21}, r_{22}) &= \frac{1}{\sqrt{5}}(\alpha h_{12}, \bar{\alpha} h_{22})\begin{pmatrix} s_1 & s_3 \\ is_3 & s_1 \end{pmatrix} \\ &+ \frac{1}{\sqrt{5}}(\tau h_{12}, \mu h_{22})\begin{pmatrix} s_2 & s_4 \\ is_4 & s_2 \end{pmatrix} + (n_{21}, n_{22})\end{aligned} \tag{3.14}$$

Where r_{ij} are the received signal at antenna i and time slot j, h_{ij} are the fading coefficients, and n_{ij} are the complex Gaussian noise values. A simple form of the received signal is given as

$$r = aH + bG + n \tag{3.15}$$

Where

$$r = (r_{11}, r_{12}, r_{21}, r_{22})$$

$$n = (n_{11}, n_{12}, n_{21}, n_{22})$$

$$a = (s_1, s_3)\ ,\ b = (s_2, s_4)$$

$$H = \frac{1}{\sqrt{5}} \begin{pmatrix} \alpha h_{11} & \bar{\alpha} h_{21} & \alpha h_{12} & \bar{\alpha} h_{22} \\ i\bar{\alpha} h_{21} & \alpha h_{11} & i\bar{\alpha} h_{22} & \alpha h_{12} \end{pmatrix}$$

$$G = \frac{1}{\sqrt{5}} \begin{pmatrix} \alpha\tau h_{11} & \bar{\alpha}\mu h_{21} & \alpha\tau h_{12} & \bar{\alpha}\mu h_{22} \\ i\bar{\alpha}\mu h_{21} & \alpha\tau h_{11} & i\bar{\alpha}\mu h_{22} & \alpha\tau h_{12} \end{pmatrix}$$

The decoding problem is to maximize the likelihood function which means finding the codewords a and b that achieve that.

$$p(r|a, b) \propto exp(-\frac{1}{2\sigma^2}\|r - aH - bG\|^2) \tag{3.16}$$

This can be done through an optimal exhaustive search over all information symbols which means a complexity of $O(N^4)$ where N is the size of the transmitted constellation. The conditional decoder can decode the information symbols with an essential optimal performance and a complexity of $O(N^2)$.

Maximizing the likelihood function conditioning on a value for a will lead to

$$\hat{b} = Q((r - aH)G^{\dagger}(GG^{\dagger})^{-1}) \tag{3.17}$$

Where $\hat{b}$ is the decoded result for symbols s_2 and s_4 given the codeword a, Q operator quantizes the input according to the transmitted constellation. Choosing the codeword a is done through an exhaustive search to minimize the following argument

$$\hat{a} = \arg\min_{a} \|r - aH - \hat{b}G\|^2 \tag{3.18}$$

The complexity of applying the conditional decoder to this decoding problem is of $O(N^2)$. Of course, the algorithm can reverse the process and starts with selecting a value for b through the exhaustive search then calculates a. This decision is made based on the channel matrix that has better conditions so if $\det(GG^{\dagger}) \geq \det(HH^{\dagger})$ then the decoder selects a through the exhaustive search and calculates b and vice versa.

Fig 3.1 shows a comparison between the ML decoder and the conditional decoder for the golden code application. The constellation size is 4 QAM and 16 QAM. The performance of the conditional decoder for this application is essential ML with reduced complexity of $O(N^2)$.

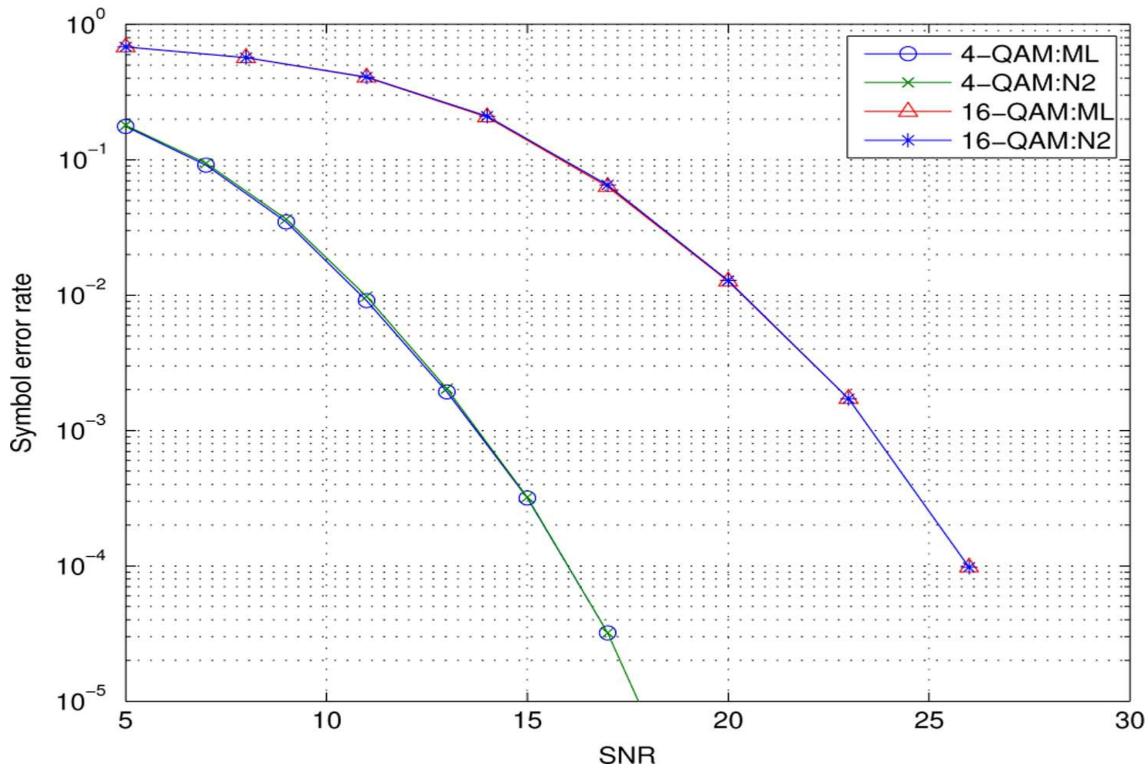

Figure 3.1: Comparison Between ML decoder and Conditional Decoder for Golden Code Application

3.2.2 Alamouti Scheme Application

Consider a communication system which consists of two transmitters and one receiver. Each node is equipped with double antennas and each transmitter sends two symbols in an Alamouti block. The received signal at the two antennas of the receiver is given by

$$r_1 = sH_1 + cG_1 + n_1 \tag{3.19}$$

$$r_2 = sH_2 + cG_2 + n_2 \tag{3.20}$$

Where r_1 and r_2 are the received signals at the receiver's first and second antenna respectively. $s = (x_1, x_2)$ are the symbols sent from transmitter 1, $c = (x_3, x_4)$ are the symbols sent from transmitter 2. n_1 and n_2 are the complex Gaussian noise observed at the two antennas of the receiver. H_1 and H_2 are the channel matrices from transmitter 1 to the receiver, G_1 and G_2 are the channel matrices from transmitter 2 to the receiver. The channel matrices can be written as

$$H_1 = \begin{pmatrix} h_{11} & h_{21} \\ h_{21}^* & -h_{11}^* \end{pmatrix} \quad , \quad H_2 = \begin{pmatrix} h_{12} & h_{22} \\ h_{22}^* & -h_{12}^* \end{pmatrix}$$

$$G_1 = \begin{pmatrix} g_{11} & g_{21} \\ g_{21}^* & -g_{11}^* \end{pmatrix} \quad , \quad G_2 = \begin{pmatrix} g_{12} & g_{22} \\ g_{22}^* & -g_{12}^* \end{pmatrix}$$

h_{ij} are the channel coefficients from antenna i of transmitter 1 to antenna j of the receiver, g_{ij} are the channel coefficients from antenna i of transmitter 2 to antenna j of the receiver. It's obvious that the channel matrices have Alamouti structure which means that its rows are mutually orthogonal. The received signal can be re-written as

$$r = sH + cG + n \tag{3.21}$$

Where $r = (r_1, r_2)$, $n = (n_1, n_2)$, $H = (H_1, H_2)$, and $G = (G_1, G_2)$. The conditional decoder in this case can select either s or c to perform the exhaustive search over it. This is because both H and G have mutual orthogonal rows so they have the exact same conditions.

The complexity of the conditional decoder is of $O(N^2)$ where N is the constellation size and the decoded symbols are given by

$$\hat{c} \;=\; Q((r - sH)G^{\dagger}(GG^{\dagger})^{-1}) \tag{3.22}$$

$$\hat{s} = \arg\min_{s} \|r - sH - \hat{c}G\|^2 \tag{3.23}$$

Where $\hat{c}$ and $\hat{s}$ are the decoded symbols. Q is the quantization operator according to the transmitted constellation. The conditional decoder is optimal for this application because of the orthogonal structure of Alamouti scheme that reflects on the rows of the channel matrices, making them mutually orthogonal.

In conclusion, the conditional decoder can be optimal for interference cancellation if the induced channel matrices at the receiver have mutually orthogonal rows. That motivated us to study applying the conditional decoder in canceling the interference for the two user x channel where the channel matrices have an orthogonal structure and the three user X channel where the interference matrix has an orthogonal structure.

Chapter 4

Conditional Decoding for X Channels

4.1 Introduction

For X channels, the system consists of multi transmitters and multi receivers. Each transmitter has a message to each receiver and the challenge is to cancel the interference [1],[2] caused by the unintended signals at the receiver end. One way to do that is by projecting the unintended signals in a different sub-space than the intended ones. This is done at the transmitter end using beamforming matrices [5]-[6] and a specific technique to cancel the interference effect at the receiver end. The use of Alamouti scheme [7] along with beamforming matrices to send the information messages makes it possible to exploit the simplicity of the zero-forcing technique at the receiver end to cancel the interference [8].

In the two user case, the interference is perfectly aligned in a different sub-space using previously proposed beamforming matrices along with the orthogonal Alamouti coding scheme [8]. That makes the zero-forcing technique optimal in cancelling the interference. In this case, we apply the conditional decoder as a two stage decoder preceded by the zero-forcing technique and we compare it to the decoupling technique and the sphere decoder. Then we apply it as a one stage decoder and we compare it to the zero-forcing technique followed by an ML decoder.

In the three user case, the interference is misaligned since we used the same beamforming matrices as in the two user case. We also proposed an arbitrary coding scheme. That makes the zero-forcing technique suboptimal so we applied the conditional decoder as a one stage decoder to achieve a performance improvement for the misaligned interference case.

In this chapter, we focus on applying the conditional decoding algorithm in two different scenarios. First, we use it in for the perfectly aligned interference case and we deploy it as a two stage and one stage decoder Section 4.3. In this scenario, we compare the two stage conditional decoder to the decoupling technique and the ML sphere decoder [22] then we compare the one stage decoder to the zero-forcing technique. Second, we use the conditional decoder for the misaligned interference scenario as we deploy it as a one stage decoder to cancel the interference and we compare it to the zero-forcing technique Section 4.4.

In Section 4.2, we introduce the system model and in Section 4.5, we demonstrate the simulation results.

4.2 System Model

The system model we adopt has M transmitters and N receivers as shown in **Fig 4.1**. Each transmitter has a message to each receiver so the received signal at each receiver is a combination of the intended messages and the unintended ones, the transmitted symbols are independent. H_{ij} is the channel matrix between transmitter i and receiver j, the channel coefficients are i.i.d. with Gaussian distribution $CN(0, 1)$, the channel is constant during transmission. Each transmitter knows only the channel information between itself and each receiver, each transmitter and receiver has double antenna. The noise is additive white Gaussian noise with unit variance.

4.3 Perfectly Aligned Interference

In this scenario, the interference is perfectly aligned in a different sub-space than the intended signal using beamforming matrices along with Alamouti coding scheme. The conditional decoder is deployed as a two stage decoder and compared to the decoupling technique and the ML sphere decoder then it is deployed as a one stage decoder and compared to the zero-forcing technique followed by any ML decoder. In this scenario, the system is 2×2.

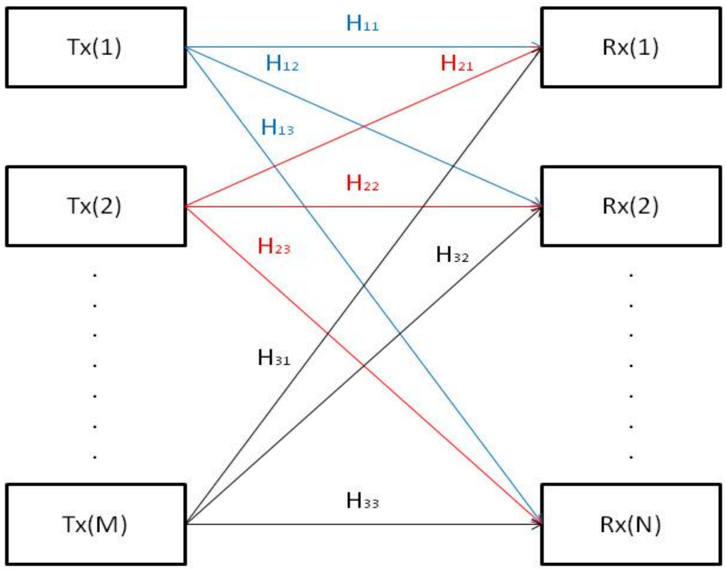

Figure 4.1: System Model

4.3.1 Two Stage Decoder

For a two stage decoder, the zero-forcing technique is used at the first stage to cancel the interference then the conditional decoding algorithm is applied to decode the transmitted symbols. A maximum likelihood algorithm can be applied to decode the symbols optimally through an exhaustive search over all possible solutions. The conditional decoder does the same job with a reduced complexity. The transmitted matrices are designed as

$$x_1 = \sqrt{\frac{3P}{2}} \begin{bmatrix} s_{11}^1 & s_{11}^2 \\ -s_{11}^{2*} & s_{11}^{1*} \\ 0 & 0 \end{bmatrix} v_{11} + \sqrt{\frac{3P}{2}} \begin{bmatrix} 0 & 0 \\ -s_{12}^{2*} & s_{12}^{1*} \\ s_{12}^1 & s_{12}^2 \end{bmatrix} v_{12} \tag{4.1}$$

$$x_2 = \sqrt{\frac{3P}{2}} \begin{bmatrix} s_{21}^1 & s_{21}^2 \\ -s_{21}^{2*} & s_{21}^{1*} \\ 0 & 0 \end{bmatrix} v_{21} + \sqrt{\frac{3P}{2}} \begin{bmatrix} 0 & 0 \\ -s_{22}^{2*} & s_{22}^{1*} \\ s_{22}^1 & s_{22}^2 \end{bmatrix} v_{22} \tag{4.2}$$

The transmitted matrices x_1 and x_2 have Alamouti structure for the information symbols. Beamforming matrices v_{ij} are used to project the interference on a different sub-space than the intended signals and each entry in beamforming matrix is kept smaller than 1 to avoid high power peaks.

$$v_{11} = \sqrt{\frac{1}{tr(H_{12}^{-1}H_{12}^{-1*})}}H_{12}^{-1},\ v_{12} = \sqrt{\frac{1}{tr(H_{11}^{-1}H_{11}^{-1*})}}H_{11}^{-1}$$

$$v_{21} = \sqrt{\frac{1}{tr(H_{22}^{-1}H_{22}^{-1*})}}H_{22}^{-1},\ v_{22} = \sqrt{\frac{1}{tr(H_{21}^{-1}H_{21}^{-1*})}}H_{21}^{-1}$$

In three time slots, each transmitter sends two symbols to each receiver. s_{ij}^k is the transmitted symbol from transmitter i to receiver j and the symbol number is k. The achieved symbol rate in this case is $\frac{8}{3}$ which is the maximum achievable rate [4]. The received signals at both receivers are written as

$$\begin{aligned} y_1 &= \sqrt{\frac{3P}{2}}\begin{bmatrix} s_{11}^1 & s_{11}^2 \\ -s_{11}^{2*} & s_{11}^{1*} \\ 0 & 0 \end{bmatrix} v_{11}H_{11} + \sqrt{\frac{3P}{2}}\begin{bmatrix} s_{21}^1 & s_{21}^2 \\ -s_{21}^{2*} & s_{21}^{1*} \\ 0 & 0 \end{bmatrix} v_{21}H_{21} \\ &+ \sqrt{\frac{3P}{2}}\begin{bmatrix} 0 & 0 \\ -as_{12}^{2*} - bs_{22}^{2*} & as_{12}^{1*} + bs_{22}^{1*} \\ as_{12}^1 + bs_{22}^1 & as_{12}^2 + bs_{22}^2 \end{bmatrix} + w_1 \end{aligned} \tag{4.3}$$

$$\begin{aligned} y_2 &= \sqrt{\frac{3P}{2}}\begin{bmatrix} s_{12}^1 & s_{12}^2 \\ -s_{12}^{2*} & s_{12}^{1*} \\ 0 & 0 \end{bmatrix} v_{12}H_{12} + \sqrt{\frac{3P}{2}}\begin{bmatrix} s_{22}^1 & s_{22}^2 \\ -s_{22}^{2*} & s_{22}^{1*} \\ 0 & 0 \end{bmatrix} v_{22}H_{22} \\ &+ \sqrt{\frac{3P}{2}}\begin{bmatrix} 0 & 0 \\ -cs_{11}^{2*} - ds_{21}^{2*} & cs_{11}^{1*} + ds_{21}^{1*} \\ cs_{11}^1 + ds_{21}^1 & cs_{11}^2 + ds_{21}^2 \end{bmatrix} + w_2 \end{aligned} \tag{4.4}$$

Where y_1 and y_2 are the received signals. w_1 and w_2 are the noise matrices. $a = \sqrt{2tr(H_{11}^{-1}H_{11}^{-1*})}$, $b = \sqrt{2tr(H_{21}^{-1}H_{21}^{-1*})}$, $c = \sqrt{2tr(H_{12}^{-1}H_{12}^{-1*})}$, $d = \sqrt{2tr(H_{22}^{-1}H_{22}^{-1*})}$. The received signal at receiver 1 can be written as

$$\tilde{y}_1 = \sqrt{\frac{3P}{2}} \begin{bmatrix} \tilde{h}_{11} & \tilde{h}_{21} & \tilde{g}_{11} & \tilde{g}_{21} & 0 & 0 \\ \tilde{h}^*_{21} & -\tilde{h}^*_{11} & \tilde{g}^*_{21} & -\tilde{g}^*_{11} & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ \tilde{h}_{12} & \tilde{h}_{22} & \tilde{g}_{12} & \tilde{g}_{22} & 0 & 0 \\ \tilde{h}^*_{22} & -\tilde{h}^*_{12} & \tilde{g}^*_{22} & -\tilde{g}^*_{12} & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s^1_{11} \\ s^2_{11} \\ s^1_{21} \\ s^2_{21} \\ i_1 \\ i_2 \end{bmatrix} + \tilde{w}_1 \tag{4.5}$$

Where $\tilde{h}_{ij}$ are the entries for $\tilde{H} = v_{11}H_{11}$ and $\tilde{g}_{ij}$ are the entries for $\tilde{G} = v_{21}H_{21}$. $i_1 = as^1_{12} + bs^1_{22}$, $i_1 = as^2_{12} + bs^2_{22}$. $\hat{w}$ is a 6x1 additive white Gaussian noise (AWGN) vector with entries that have zero mean and unit variance.

The zero-forcing technique cancels the interference with some simple linear processing forming a new version of the received vector. $\hat{y} = [\tilde{y}_1,\ \tilde{y}_2 + \tilde{y}_6,\ \tilde{y}_4,\ \tilde{y}_5 - \tilde{y}_3]^T$ where $\tilde{y}_i$ are the entries of the vector $\tilde{y}$

$$\hat{y} = \hat{H} \begin{bmatrix} s^1_{11} \\ s^2_{11} \end{bmatrix} + \hat{G} \begin{bmatrix} s^1_{21} \\ s^2_{21} \end{bmatrix} + \hat{w} \tag{4.6}$$

$$\hat{H} = \begin{bmatrix} \tilde{h}_{11} & \tilde{h}_{21} \\ \tilde{h}^*_{21} & -\tilde{h}^*_{11} \\ \tilde{h}_{12} & \tilde{h}_{22} \\ \tilde{h}^*_{22} & -\tilde{h}^*_{12} \end{bmatrix}, \hat{G} = \begin{bmatrix} \tilde{g}_{11} & \tilde{g}_{21} \\ \tilde{g}^*_{21} & -\tilde{g}^*_{11} \\ \tilde{g}_{12} & \tilde{g}_{22} \\ \tilde{g}^*_{22} & -\tilde{g}^*_{12} \end{bmatrix}$$

And $\hat{w} = [\tilde{w}_1,\ \tilde{w}_2 + \tilde{w}_6,\ \tilde{w}_4,\ \tilde{w}_5 - \tilde{w}_3]^T$ where $\tilde{w}_i$ are the entries of the vector $\tilde{w}$

To apply the conditional decoder, a comparison between the two channel matrices is done to determine which one of them has better conditions. If $\det(GG^T) > \det(HH^T)$ then the conditional decoder does the exhaustive search over s^1_{11} and s^2_{11}. Otherwise, it does the exhaustive search over s^1_{21} and s^2_{21}. Since both of the two resulted channel matrices $\hat{H}, \hat{G}$ have the orthogonal Alamouti structure so the conditional decoder is optimal for this application and the decoder has a complexity of $O(Q^2)$.

4.3.2 One Stage Decoder

For a one stage decoder, the conditional decoder cancel the interference and decode the intended symbols in one step and we compare it to the zero-forcing technique followed by any ML decoder. The received signal at receiver 1 is written as

$$
\begin{aligned}
\tilde{y}_1 \quad &= \quad \sqrt{\frac{3P}{2}} \begin{bmatrix} \tilde{h}_{11} & \tilde{h}_{21} & \tilde{g}_{11} & \tilde{g}_{21} \\ \tilde{h}^*_{21} & -\tilde{h}^*_{11} & \tilde{g}^*_{21} & -\tilde{g}^*_{11} \\ 0 & 0 & 0 & 0 \\ \tilde{h}_{12} & \tilde{h}_{22} & \tilde{g}_{12} & \tilde{g}_{22} \\ \tilde{h}^*_{22} & -\tilde{h}^*_{12} & \tilde{g}^*_{22} & -\tilde{g}^*_{12} \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} s^1_{11} \\ s^2_{11} \\ s^1_{21} \\ s^2_{21} \end{bmatrix} \\
&+ \quad \sqrt{\frac{3P}{2}} \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} + \tilde{w}_1
\end{aligned}
\tag{4.7}
$$

The matrix multiplied by the interference elements has mutually orthogonal rows so the conditional decoder in this case will do exhaustive search over the intended symbols to receiver 1. The decoder has a complexity of $O(Q^4)$.

4.4 Misaligned Interference

In this scenario, the system is 3×2 but we still use the same beamforming matrices as in the 2×2 system which makes the interference misaligned. We apply the conditional decoder as a one stage decoder and we compare its performance to the zero-forcing technique followed by any ML decoder. In 3×2 system, the maximum achievable symbol rate is 3 [16] which means transmitting two symbols from each transmitter to each receiver in a four time slot duration. We proposed the following arbitrary scheme for the transmitted matrices

$$x_1 = \sqrt{2P}\left(\begin{bmatrix} s_{11}^1 & s_{11}^2 \\ -s_{11}^{2*} & s_{11}^{1*} \\ -s_{11}^2 & -s_{11}^1 \\ 0 & 0 \end{bmatrix} v_{11} + \begin{bmatrix} 0 & 0 \\ -s_{12}^{2*} & s_{12}^{1*} \\ s_{12}^1 & s_{12}^2 \\ -s_{12}^2 & -s_{12}^1 \end{bmatrix} v_{12}\right) \quad (4.8)$$

$$x_2 = \sqrt{2P}\left(\begin{bmatrix} s_{21}^1 & s_{21}^2 \\ -s_{21}^{2*} & s_{21}^{1*} \\ -s_{21}^2 & -s_{21}^1 \\ 0 & 0 \end{bmatrix} v_{21} + \begin{bmatrix} 0 & 0 \\ -s_{22}^{2*} & s_{22}^{1*} \\ s_{22}^1 & s_{22}^2 \\ -s_{22}^2 & -s_{22}^1 \end{bmatrix} v_{22}\right) \quad (4.9)$$

$$x_3 = \sqrt{2P}\left(\begin{bmatrix} s_{31}^1 & s_{31}^2 \\ -s_{31}^{2*} & s_{31}^{1*} \\ -s_{31}^2 & -s_{31}^1 \\ 0 & 0 \end{bmatrix} v_{31} + \begin{bmatrix} 0 & 0 \\ -s_{32}^{2*} & s_{32}^{1*} \\ s_{32}^1 & s_{32}^2 \\ -s_{32}^2 & -s_{32}^1 \end{bmatrix} v_{32}\right) \quad (4.10)$$

s_{ij}^k is the transmitted symbol from transmitter i to receiver j and the symbol number is k. The received signal at receiver 1 can be written as

$$\begin{aligned} y_1 &= \sqrt{2P}\begin{bmatrix} s_{11}^1 & s_{11}^2 \\ -s_{11}^{2*} & s_{11}^{1*} \\ -s_{11}^2 & -s_{11}^1 \\ 0 & 0 \end{bmatrix} v_{11}H_{11} \\ &+ \sqrt{2P}\begin{bmatrix} s_{21}^1 & s_{21}^2 \\ -s_{21}^{2*} & s_{21}^{1*} \\ -s_{21}^2 & -s_{21}^1 \\ 0 & 0 \end{bmatrix} v_{21}H_{21} \\ &+ \sqrt{2P}\begin{bmatrix} s_{31}^1 & s_{31}^2 \\ -s_{31}^{2*} & s_{31}^{1*} \\ -s_{31}^2 & -s_{31}^1 \\ 0 & 0 \end{bmatrix} v_{31}H_{31} \\ &+ \sqrt{2P}\begin{bmatrix} 0 & 0 \\ as_{12}^1 + bs_{22}^1 + cs_{32}^1 & as_{12}^2 + bs_{22}^2 + cs_{32}^2 \\ -as_{12}^{2*} - bs_{22}^{2*} - cs_{32}^{2*} & as_{12}^{1*} + bs_{22}^{1*} + cs_{32}^{1*} \\ -as_{12}^2 - bs_{22}^2 - cs_{32}^2 & -as_{12}^1 - bs_{22}^1 - cs_{32}^1 \end{bmatrix} + w_1 \end{aligned} \quad (4.11)$$

Where $c = \sqrt{2tr(H_{31}^{-1} H_{31}^{-1*})}$. The conditional decoder in this case does the exhaustive search over the six intended symbols to receiver 1. The decoder has a complexity of $O(Q^6)$.

4.5 Simulation Results

In this section, we demonstrate the simulation results for the conditional decoder in the two different scenarios, perfectly aligned and misaligned interference.

4.5.1 Perfectly Aligned Interference

We compare the two stage conditional decoder performance to the decoupling technique and the ML sphere decoder. The conditional decoder has the same optimal performance of the sphere decoder and it has a diversity gain over the decoupling technique. This gain is about 7 dB at 10^{-3} as shown in **Fig 4.2**. The one stage conditional decoder gives the same optimal performance as the zero-forcing technique followed by any ML decoder. The transmitted constellation size for this simulation is 4 QAM.

4.5.2 Misaligned Interference

We compare the one stage conditional decoder to the zero-forcing technique followed by an ML decoder in the misaligned interference case. The conditional decoder achieves a performance improvement compared to the zero-forcing technique. This improvement is about 3 dB as shown in **Fig 4.3**. The transmitted constellation size for this simulation is BPSK.

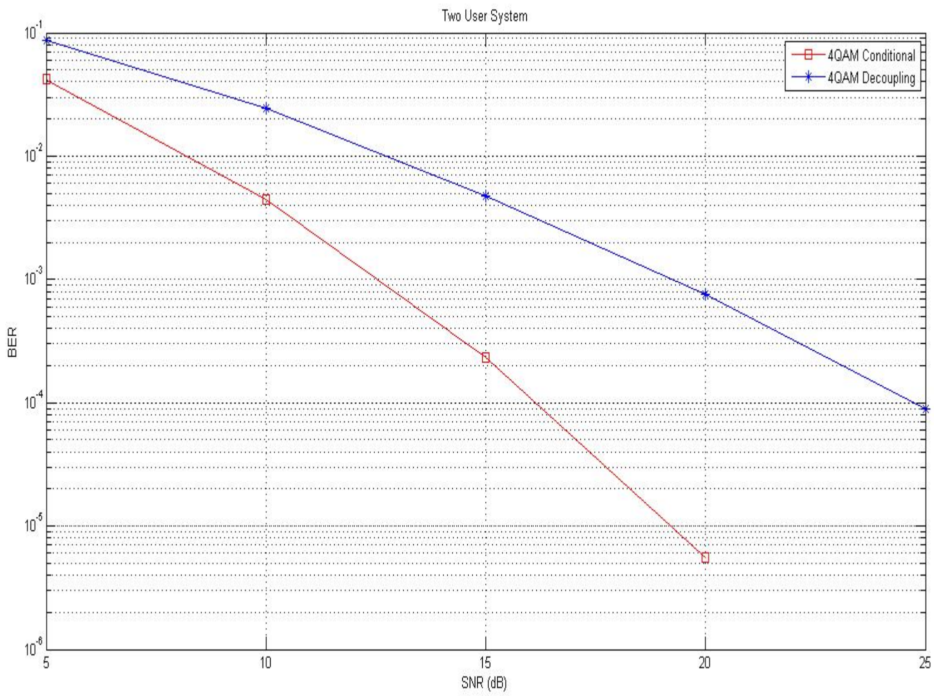

Figure 4.2: Conditional Decoder - Perfectly Aligned Interference

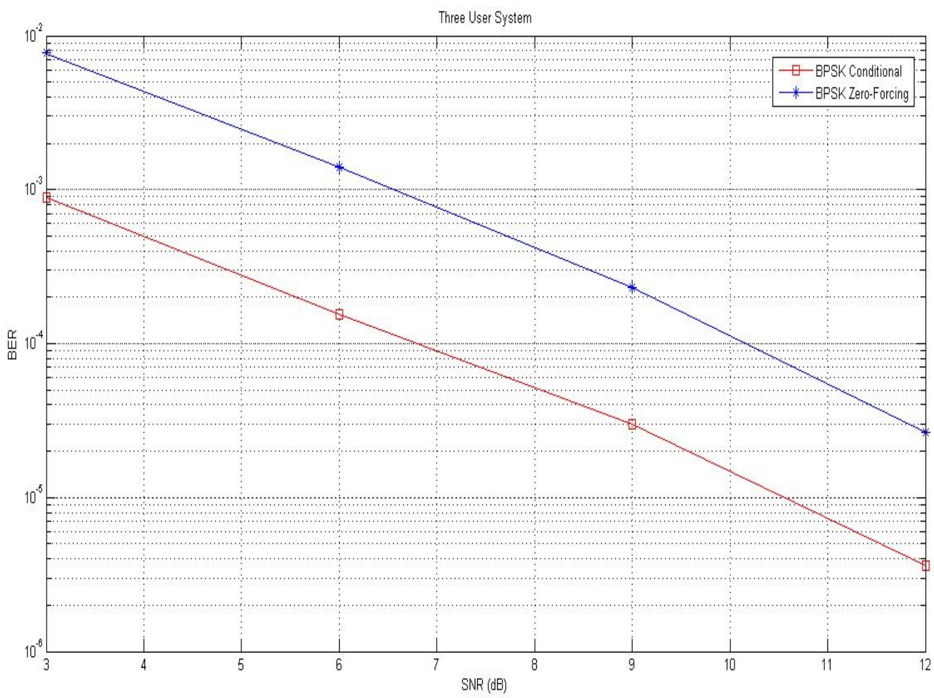

Figure 4.3: Conditional Decoder - Misaligned Interference

Chapter 5

Conclusion

We extended the notion of conditional decoding to X channels in two different scenarios, perfectly aligned interference and misaligned interference. For the perfectly aligned interference, it was shown that the conditional decoder achieves the optimal performance whether it is deployed as a one stage or two stage decoder. In this scenario, the conditional decoder was shown to outperform the previously proposed decoupling technique and achieve the same performance as the sphere decoder; but with a much lower implementation complexity. For the misaligned interference case, the one stage conditional decoder achieves a performance improvement in canceling the interference compared to the zero-forcing technique. This makes the conditional decoder a good candidate for canceling the interference when suitable beamforming matrices cannot be found.

The conditional decoder was shown to perform better than the zero-forcing technique for interference cancellation in the misaligned interference case. This can be explained since the way the conditional decoder works does not depend on having the interference in a different sub-space than the intended signals unlike the zero-forcing technique which can only give good results when the interference is perfectly aligned. The conditional decoder way of work depends on the number of mutually orthogonal rows in the channel matrices; this means that the conditional decoder can still be optimal even if the interference is misaligned.

To take advantage of the promising performance of the conditional decoder in interference cancellation for X channels, future work of this research should go to implementing the conditional decoding algorithm in more applications for the interference alignment technique especially when it's difficult to perfectly align the interference in a different sub-space than the intended signals. We applied the conditional decoder to cancel the interference and decode the intended signals for X channels. The conditional decoder can also be deployed for interference cancellation and information symbols decoding in many other communication systems especially if the interference is not perfectly aligned in a different sub-space than the desired information symbols.

References

[1] A. Naguib, N. Seshadri, and A. Calderbank, "Application of space-time block codes and interference suppression for high capacity and high data rate wireless systems," in Proc. of Asilomar Conf., Pacific Grove, CA, Oct. 1998.

[2] J. Kazemitabar and H. Jafarkhani, "Multiuser interference cancellation and detection for users with more than two transmit antennas," IEEE Trans on Comm., pp. 574-583, Apr. 2008.

[3] J. G. Proakis, "Digital Communications," 3rd. ed. New York: McGrawHill, 1995.

[4] S. Jafar and S. Shamai, "Degrees of freedom region for the mimo x channel," IEEE Transactions on Information Theory, vol. 54, no. 1, pp. 151-170, Jan. 2008.

[5] V. Cadambe and S. Jafar, "Interference alignment and the degrees of freedom for the k user interference channel," IEEE Transactions on Information Theory, vol. 54, pp. 3425-3441, Aug. 2008.

[6] M. Maddah-Ali and D. Tse, "On the degrees of freedom of miso broadcast channels with delayed feedback," EECS Department, University of California, Berkeley, Tech. Rep. UCB/EECS-2010-122, Sep. 2010.

[7] S. Alamouti, "A simple transmitter diversity scheme for wireless communications," IEEE J. Select. Areas Commun., vol. 16, pp. 1451-1458, 1998

[8] Liangbin Li,Hamid Jafarkhani, Syed Ali Jafar, "When Alamouti codes meet interference alignment: transmission shemes for two-user X channel," in *Proc. IEEE International Symposium of Information Theory (ISIT)*, 2011.

[9] S. Sirianunpiboon, Yiyue Wu, A. Robert Calderbank, Stephen D. Howard, "Fast Optimal Decoding of Multiplexed Orthogonal Designs by Conditional Optimization," in *Proc. IEEE International Symposium of Information Theory (ISIT)*, Mar. 2010.

[10] S. Sirinaunpiboon, A. Robert Calderbank, Fellow, Stephen D. Howard, "Fast essentially maximum likelihood decoding of the Golden code," in *Proc. IEEE International Symposium of Information Theory (ISIT)*, Jun. 2011.

[11] Syed A. Jafar, "Interference alignment A new look at signal dimensions in a communication network," Foundations and Trends in Communications and Information Theory, Vol. 7, No. 1 (2010)

[12] Y. Birk and T. Kol, "Informed-source coding-on-demand (ISCOD) over broadcast channels," in Proceedings of the Seventeenth Annual Joint Conference of the IEEE Computer and Communications Societies, 1998.

[13] Y. Birk and T. Kol, "Coding on demand by an informed source (ISCOD) for efficient broadcast of different supplemental data to caching clients," IEEE Transactions on Information Theory, vol. 52, no. 6, pp. 28252830, June 2006.

[14] M. Maddah-Ali, A. Motahari, and A. Khandani, "Communication over X channel: Signalling and multiplexing gain," in Technical Report. UW-ECE-2006-12, University of Waterloo, July 2006.

[15] H. Weingarten, S. Shamai, and G. Kramer, "On the compound MIMO broadcast channel," in Proceedings of Annual Information Theory and Applications Workshop UCSD, January 2007.

[16] V. Cadambe and S. Jafar, "Interference alignment and the degrees of freedom of wireless X networks," IEEE Transactions on Information Theory, no. 9, September 2009.

[17] Hua Sun, Chunhua Geng, Tiangao Gou and Syed A. Jafar, "Degrees of freedom of MIMO X Networks: spatial scale invariance, one-sided decomposability and linear feasibility"

[18] C. Suh and D. Tse, "Interference alignment for cellular networks," in Proceedings of 40th Annual Allerton Conference on Communication, Control and Computing, September 2008.

[19] J.-C. Belfiore, G. Rekaya, and E. Viterbo, "The golden code: A 2×2 full-rate space-time code with nonvanishing determinants," IEEE Trans. Inf. Theory, vol. 51, no. 4, Apr. 2005.

[20] H. Yao and G. W. Wornell, "Achieving the full MIMO diversity multiplexing frontier with rotation-based space-time codes," presented at the Allerton Conf. Commun., Control Comput., Monticello, IL, Oct. 2003.

[21] P. Dayal and M. K. Varanasi, "Optimal two transmit antenna spacetime code and its stacked extensions," IEEE Trans. Inf. Theory, vol. 51, no. 12, Dec. 2005.

[22] M. Damen, H. El Gamal, G. Cairei, "On maximum-likelihood detection and the search for the closest lattice point," in *Proc. IEEE International Symposium of Information Theory (ISIT)*, Oct. 2003.