Relay-aided Interference Alignment in Wireless Networks

by

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A thesis
presented to the University of Waterloo
in fulfillment of the
thesis requirement for the degree of
Doctor of Philosophy
in
Electrical and Computer Engineering

Waterloo, Ontario, Canada, 2011

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

Resource management in wireless networks is one of the key factors in maximizing the overall throughput. Contrary to popular belief, dividing the resources in a dense network does not yield the best results. A method that has been developed recently shares the spectrum amongst all the users in such a way that each node can potentially utilize about half of all the available resources. This new technique is often referred to as Interference Alignment and excels based on the fact that the amount of the network resources assigned to a user does not go to zero as the number of users in the network increases. Unfortunately it is still very difficult to implement the interference alignment concepts in practice. This thesis investigates some of the low-complexity solutions to integrate interference alignment ideas into the existing wireless networks.

In the third and fourth chapters of this thesis, it is shown that introducing relays to a quasi-static wireless network can be very beneficial in terms of achieving higher degrees of freedom. The relays store the signals being communicated in the network and then send a linear combination of those signals. Using the proposed scheme, it is shown that although the relays cannot decode the original information, they can transform the equivalent channel in such a way that performing interference alignment becomes much easier. Investigating the required output power of the relays shows that it can scale either slower or faster than the output power of the main transmitters. This opens new doors for the applications that have constraints on the accessible output powers in the network nodes. The results are valid for both X Channel and Interference Channel network topologies.

In Chapter Five, the similarities between full-duplex transmitters and relays are examined. The results suggest that the transmitters can play the relay roles for offering easier interference alignment. Similar to the relay-based alignment, in the presented scheme full-duplex transmitters listen to the signals from other transmitters and use this information during the subsequent transmission periods. Studying the functionality of the full-duplex transmitters from the receivers' side shows the benefits of having a minimal cooperation between transmitters without even being able to decode the signals. It is also proved that the degrees of freedom for the N-user Interference Channel with full-duplex transmitters can be $\sqrt{\frac{N}{2}}$. The results offer an easy way to recover a portion of degrees of freedom with manageable complexity suited for practical systems.

Acknowledgements

I would like to express my deep and sincere gratitude to my supervisor Professor Amir Khandani for his endless support, understanding, kindness, and great supervision. During my PhD studies in Waterloo, I learned a lot from him not only about research but also about life.

I would like to thank the members of my dissertation committee, Professors Tim Davidson, Paul Fieguth, Oussama Damen and Murat Uysal for taking the time out of their busy schedules to carefully review my thesis and providing me with their insightful comments and suggestions. I am also thankful to Dr. Ladan Tahvildari who accepted to be a delegate member on such a short notice.

I have been very fortunate to work among members of the Coding and Signal Transmission (CST) laboratory. I specially acknowledge Dr. Abolfazl Motahari for his collaboration on this research and co-authorship of several papers. I would like to thank Vahid Pourahmadi who has been of tremendous support over more than three years that we shared our office. I am also grateful to my dear friend Arash Tabibiazar who has always been very supportive to me. My thanks go to all the other members including but not limited to Dr. Alireza Bayesteh, Dr. Mahmoud Taherzadeh, Dr. Shahab Oveis Gharan, Dr. Hamid Farmanbar, Dr. Saeed Changiz Rezaei, Dr. Hossein Bagheri and Ali Ahmadzadeh.

Finally, I am indefinitely indebted to my wife, Nasim, who has put up with me for reasons not always obvious, endured countless sacrifices so that I can follow my dreams and supported me in any way that she could. I owe all of my accomplishments to her.

Dedication

This thesis is dedicated to the memory of my parents and also to my dear wife, Nasim.

I would have never been here without their support.

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List of Abbreviations

AWGN	Additive White Gaussian Noise
CSI	Channel State Information
DOF	Degrees of Freedom
IA	Interference Alignment
IC	Interference Channel
MIMO	Multiple-Input Multiple-Output
MG	Multiplexing Gain
MMSE	Minimum Mean Square Error
SNR	Signal to Noise Ratio
TDMA	Time Division Multiple Access
FDMA	Frequency Division Multiple Access
CDMA	Code Division Multiple Access

Notation

Lower-case Letters	Scalars
Upper-case Letters	Matrices and Vectors
x^*	Conjugate of the complex scalar variable x
Y^H	Hermitian of the Matrix/Vector Y
Y^{-H}	$(Y^{-1})^H$
$\|z\|$	Modulus (absolute) value of the complex variable z
$\\|T\\|$	Norm of the column vector T, $\\|T\\|^2 = T^H T$
I	Identity Matrix
$\mathbf{0}$	All-zero Matrix
$\delta(\cdot)$	Discrete Dirac delta function
$\mathcal{E}\{\cdot\}$	Expectation Operator
$V_1 \parallel V_2$	The two vectors V_1 and V_2 are parallel, there is a complex scalar α such that $V_1 = \alpha V_2$
$U_1 \perp U_2$	The two vectors U_1 and U_2 are orthogonal, $U_1^H U_2 = 0$

Chapter 1

Introduction

Wireless networks have been the topic of extensive study in the last few years[8, 29, 48]. Various techniques are designed to achieve rates close to the fundamental limits of these networks. Equipping transmitters and/or receivers with multiple antennas is amongst the approaches by which the throughput of a network can be dramatically increased. The advantages of having more than one antenna are not limited to the rates only. In fact, back in earlier days of wireless communications, multiple antennas were used frequently to increase the reliability of the links [6]. Roughly speaking, multiple antennas create redundant paths between transmitter and receiver. Therefore, if the failure events of those paths are independent, the probability of all the paths failing simultaneously will be much smaller than the failure rate for a single link. This technique is often referred to as diversity. Mathematically, diversity gain is usually defined as the slope of the error probability with respect to Signal to Noise Ratio (SNR) as below

$$d \triangleq - \lim_{\mathrm{SNR} \to \infty} \frac{\log P_e\left(\mathrm{SNR}\right)}{\log \mathrm{SNR}}. \tag{1.1}$$

Receive diversity, one of the earliest diversity schemes, uses redundant receive paths to fight channel fading. Likewise, multiple antennas on the transmitter side can be used for the same effect which is usually referred to as transmit diversity or beamforming [2]. There are other types of diversity which are often seen in multi-user communications [63].

Back to the benefits of multiple antennas, the main drive behind their adaptation to almost all the recent wireless applications is to provide higher rates. Theoretically, a transmitter with M antennas sending data to a receiver with N antennas can transfer $\min(M, N)$ independent streams of information on each channel use. This number is an important property of every Multiple-Input Multiple-Output (MIMO) system which is often referred to as the Degrees of Freedom (DOF) or Multiplexing Gain (MG). Similar to diversity gain, DOF has a mathematical definition as the slope of the rate (R) with respect to SNR as the following

$$r \triangleq \lim_{\text{SNR}\to\infty} \frac{R(\text{SNR})}{\log \text{SNR}}. \tag{1.2}$$

For slowly varying channels, the channel information can be reported back to the transmitter. The channel structure is usually contained in a matrix whose elements correspond to the the channel coefficients from one of the antennas of the transmitter to one of the antennas of the receiver. Any knowledge about this matrix on the transmitter side has been proved to be very beneficial from the throughput point of view [61]. If the channels are not known on the transmitters but can be trained, estimated or tracked from the receiver side, there are a lot of techniques that can be applied for maximizing the rate [21, 60, 59]. If the channels are not known by the receiver, there are still a lot more complex schemes that can be applied for achieving the same amount of DOF [66]. Finally, for every point to point MIMO system the amount of diversity and multiplexing are related to each other and cannot vary independently [67].

Moving to more complex topologies, different multi-user scenarios have been investigated in the literature, such as Broadcast Channels (one transmitter and multiple receivers) [8, 17] and Multiple Access Channels (multiple transmitters and one receiver) [8]. The communication schemes for both channels are still rather simple, specially if the channel information is known on the transmitter side. As the wireless systems grow larger (notably, the cases where the number of transmitters and receivers are both more than one), the point to point assumptions are no longer valid. In these multi-user networks, the communication problem is more difficult to handle as the unwanted interference from different users is the major limiting factor in achieving the desired throughput. In fact, the traditional treatment of interference as Gaussian noise results in a decrease in the achievable throughput as the number of users increases. Some recent studies have shown that incorporating multiple antennas increases the dimensionality of the signal space [37, 38], which in turn allows reducing the effect of the multi-user interference by aligning the interfering signals at the receivers.

Interference management in wireless networks has always been a difficult problem with no simple answer that fits all the different conditions. During the transition from wired to wireless, management of the available resources becomes a lot more challenging. A large portion of this complexity arises from the fact that unlike wired networks, in wireless systems, the unwanted signals from the other users can be heard in all the nearby nodes. As a result, a minimal cooperation is inevitable when the number of transmitters and/or receivers is more than one. There are a lot of techniques that avoid the interference by splitting the resources between users (TDMA/FDMA), treating the interference as noise (CDMA is one example) or decoding a strong interference before decoding the intended signal. Unfortunately, none of them are suitable for managing the resources in a large network. Additionally, all of these methods more or less share the idea of dividing the available resources among the users. In other words, a network of M transmitter-receiver pairs could only offer $\frac{1}{M}$ of the available resources (on average) to each user. While this line of thinking is suitable in an interference-free wired medium, it is not the best that can be done in wireless due to its shareable nature.

Recently, it is shown that under certain conditions each user can utilize one half of all the network resources regardless of the number of users operating in the system [11]. This result is very interesting since the sum of the used resources for the whole network can become much larger than what is available. To this end, it is required to fit the unwanted signals from different users into a small space. In an interesting work in [11], the authors utilize channel variations to remove the unwanted interference by extending the signals over multiple time intervals. Using time-extension the authors show that a DOF of $\frac{M}{2}$ is possible for an M-user Interference Channel (IC).

The idea of Interference Alignment (IA) in multiple antenna systems is first introduced by Maddah-Ali et.al. in [38]. They showed that a MIMO X Channel [1] can offer more DOF than what was previously being speculated. Roughly speaking, the total number of DOF in a single antenna X Channel was thought to be one. Accepting this to be true, it was easy to generalize the result for the case where all the transmitters and receivers have $N > 1$ antennas and although not theoretically proved, the DOF for N-antenna MIMO X Channel was anticipated to be N. In [38], the authors changed the popular consensus and proved that the N-antenna X Channel can offer as much as $\lfloor \frac{4N}{3} \rfloor$ DOF using IA. Various approaches developed after their paper show that the results from MIMO can be extended for single-antenna X Channels as well and prove that the DOF of the single-antenna

[1] X Channel is a wireless network consisting of two transmitters and two receivers where each transmitter has an independent message for each receiver.

X Channel is actually $\frac{4}{3}$ [33]. IA in quasi-static channels is also presented by Motahari et.al. in [41, 42]. Using a lattice-based approach they showed that IA is not limited to N-dimensional Euclidean spaces ($N \geq 2$) and under certain conditions one dimensional systems can also be used.

1.1 Thesis Outline

The current chapter has a brief introduction to some of the concepts and terms used throughout the thesis. It also contains links to a few cornerstone papers whose results are vital for understanding most of the materials presented here.

The second chapter describes the MIMO equivalent model for two popular groups of wireless networks, namely X Channel and IC. The chapter also contains a matrix reformulation of the IA conditions which unifies the alignment problems for the X Channel and IC into one. The maximum achievable DOF is also determined based on the number of free variables and the number of equations. Based on the results, the $M \times N$-user X Channel can have a total DOF of MN if the number of antennas for each transmitter and receiver exceeds $\lceil \frac{MN+1}{2} \rceil$. Similarly, if the number of antennas is more than $\lceil \frac{N+1}{2} \rceil$, the N-user IC can achieve a total DOF of N.

Chapter Three looks into the single-antenna X channel in a quasi-static environment. It is shown that with the introduction of a full-duplex relay, it is possible to exploit the total DOF with a simple linear processing. The relay can be seen as a channel perturbing device which enables the necessary variations to achieve the total DOF in the static medium. The relative power scaling range for the relay is also determined to show that for any number s, the relay output power can vary as $P(\log P)^s$ and all the predicted DOF is guaranteed.

The fourth chapter investigates a similar problem of N-user IC. It is shown first that the approach taken in Chapter Three cannot be applied directly to the single-antenna case with quasi-static channels. A new scheme is then proposed using a half-duplex MIMO relay to perfectly align the interference terms. Instead of a randomizing role, the MIMO relay re-structures the equivalent channel in such a way that perfect alignment is possible. It is shown that a relay with at least $(N-2)(N-1)$ coefficients can be used for achieving a DOF of N in N-user IC. The asymptotic power requirements for the delay are also discussed at the end of this chapter.

Chapter Five considers the N-user IC with the assumption that the transmitters can listen to the signals from other users. This opens an opportunity for transmitter cooperation during subsequent channel uses. It is shown that with the help of full-duplex nodes, the transmitters can take a similar role as the one that relays have in the previous chapters. Studying the number of equations as well as the number of independent variables, it is derived that a DOF of $\sqrt{\frac{N}{2}}$ is possible for the N-user IC.

The last chapter contains a brief summary of the research as well as some future directions. Appendix A, presents a numerically stabilized iterative algorithm for the alignment problem in Chapter Two. An algorithm for IA in IC with full-duplex transmitters (Chapter Five) is described in Appendix B and finally, Appendix C proposes an intuitive method for forcing K eigenvalues of a matrix to zero. The scheme avoids direct calculation of the eigenvalues and thus greatly simplifies the involved complexity.

Chapter 2

MIMO Interference Alignment

Interference alignment (IA) has recently become a hot topic in the area of wireless communication networks. The concept was first introduced in 2006 by Maddah-Ali et.al. in [37, 38] where they determined the Multiplexing Gain (MG) of a 2×2-user MIMO X Channel with different antenna configurations. Applying this idea to the single antenna K-user Interference Channel (IC) in [12, 11], Cadambe and Jafar showed that if the channel coefficients change in every time slot the Degrees of Freedom (DOF) can be as high as $\frac{K}{2}$. The same single-antenna/time-extension approach was then extended to the $M\times N$-user X Channel in [9, 10, 14] to prove that the DOF is upper bounded by $\frac{MN}{M+N-1}$. Despite the fact that these results are quite interesting, the fast fading requirement and the large number of time slots needed for transmissions have kept this approach impractical. A rather appealing development on IA in MIMO networks was investigated in [25] where the authors considered a non-centralized algorithm for aligning the interference terms.

Directed towards existing wireless systems with multiple antennas, signal design for MIMO networks was studied in block fading (quasi-static) environments. A block fading channel is a randomly-selected channel that stays unchanged over the whole transmission period. Since the IA for MIMO often requires global knowledge (knowing the channels between every transmitter-receiver pair in one central location), block fading assumption significantly reduces the amount of Channel State Information (CSI) that needs to be communicated. In [53], the authors adapt IA for usage in cognitive radio and achieve a non-zero rate on a secondary Point to Point (P2P) MIMO link that uses the same resources as the main MIMO link. Another paper in [62] demonstrates a solution based on finding the eigen-values to achieve a DOF of K in the K-user MIMO IC where all the transmitters and receivers have $K-1$ antennas. The method presented in the paper offers a very small DOF per antenna ($\frac{K}{K-1}$ compared with the lower bound of $\frac{K}{2}$). Other related works on this topic that worth mentioning include [56, 23, 31, 65, 54, 64, 57].

As the IA concept has started maturing, the research concentration is shifted towards practical considerations including efficient algorithms for beamformer design. In [55] the authors prove that maximizing the number of total DOF for the K-user IC with an arbitrary number of antennas in each transmitter and receiver is NP-hard. They propose a distributed algorithm to maximize the system throughput based on Minimum Mean Square Error (MMSE) criteria. A related paper looks at MMSE-based throughput maximization as a semi-definite program and considers robustness to imperfect CSI as an extra condition [15]. Further optimization approaches to throughput maximization can be found in [51, 52]. Finally, in [20] the authors report successfully testing a physical implementation of IA for 3-user IC using MIMO-OFDM.

This chapter contains a unified approach for IA in MIMO X Channel and MIMO IC. Assuming a symmetric setup (i.e. all the transmitters and receivers have the same number of antennas), the minimum number of antennas needed for achieving one DOF per each transmitter-receiver pair is determined. Zero-forcing approach is used for deriving the alignment requirements which are then reformulated into a matrix equation. The matrix representation of the IA problem proves to be useful for deriving the iterative algorithm that is used for finding the solutions. The algorithm along with the source codes have been moved to the Appendix A.

2.1 System Model

The next few sections describe two popular network structures whose performance is usually limited by the unwanted interference. The MIMO models that are introduced here will be used in the subsequent chapters as an underlying structure for describing all the signals inside the network. A lot of aspects such as existence of a solution, its derivation and the relation between system parameters remain unanswered at this stage and are delayed until further details are known.

One final note before going into the details of the networks is that the models can be applied to both real and complex transmissions. In other words, if the transmit and receive information are assumed to be real, the channel gains, noise and all the parameters involved for the communication will be real as well. Similarly, complex data transmission is possible if all the signals in the network are assumed to be complex. It is obvious that in this case each complex dimension is equivalent to two real dimensions.

2.1.1 Channel Definition

A network of M transmitters and N receivers is considered. It is assumed that all the transmitters and receivers are equipped with K antennas. Therefore, the communication channel between Transmitter i and Receiver j can be modeled as a $K \times K$ matrix called H_{ij} ($i = 1, 2, \ldots, M$ and $j = 1, 2, \ldots, N$). Unless stated otherwise, the transmission medium is assumed to be block fading. In other words, all the channel matrices are randomly selected based on a matrix distribution, but are kept constant over the whole communication period. This is also called a static environment which is quite popular for modeling the channel between two points. From practical point of view, the channels do change over time due to the physical movements of the transmitter or receiver antennas as well as the objects between them. The rate of such changes is however independent from transmission speed. Therefore, it is always possible to adjust communication parameters in such a way that the channel is practically static for the required amount of time. This is an important property as most communication schemes based on MIMO IA require global knowledge of the channel matrices. The channel information should be delivered to the users before data communication starts. As a result, if the channel variations are too fast, the required frequent updates waste a large portion of the valuable system resources. A slowly varying channel, on the other hand, can be tracked blindly on the receiver side.

2.1.2 Communication Scheme for the X Channel

In the X Channel setup, every transmitter has a separate and independent data to be sent to every receiver making a total of MN transmitter-receiver pairs, as depicted in Figure 2.1. The information from Transmitter i to Receiver j is selected from a Gaussian codebook and defined as a scalar x_{ij}. The one-dimensional data stream, x_{ij}, is sent using the multiple antennas of Transmitter i by scaling a $K \times 1$ column vector named T_{ij}. In other words, the transmit signal from each of the K antennas is equal to one of the elements in $T_{ij}x_{ij}$ vector. In all future references, this method of transmission is shortly noted as sending x_{ij}

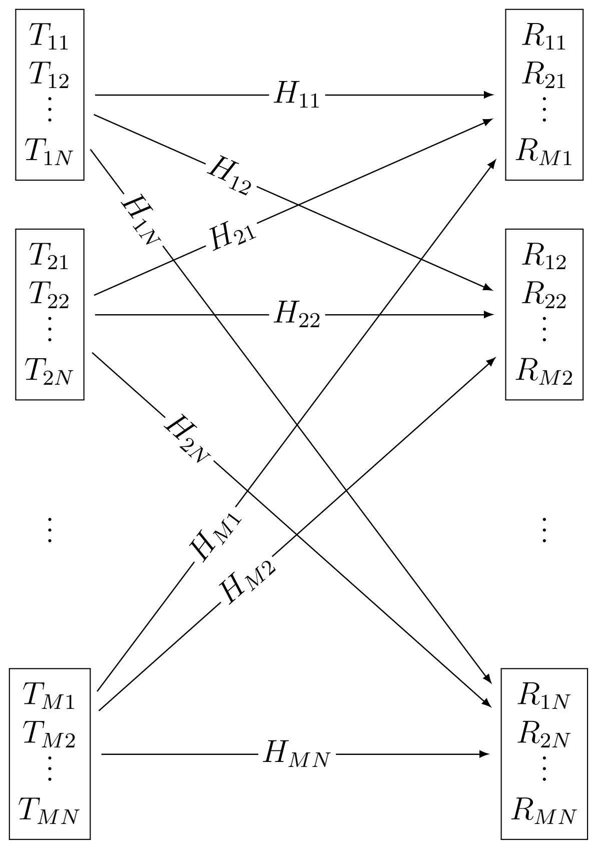

Figure 2.1: $M \times N$-user X Channel along with its transmit and receive directions

in T_{ij} direction and T_{ij} is referred to as the transmit direction. The final transmit signal for Transmitter i or X_i is constructed by adding all the partial signals for different receivers together, yielding to

$$X_i = T_{i1}x_{i1} + T_{i2}x_{i2} + \cdots + T_{iN}x_{iN}. \tag{2.1}$$

The signal received by the multiple antennas of Receiver j or Y_j is a superposition of the X_i's, each transformed by their corresponding channel matrix, as below

$$Y_j = H_{1j}X_1 + H_{2j}X_2 + \cdots + H_{Nj}X_N + Z_j. \tag{2.2}$$

The last term in the equation, Z_j, is the effect of the Additive White Gaussian Noise (AWGN) which corrupts the signal on Receiver j.

In order to extract the portion of the Y_j that contains the information from Transmitter i to Receiver j, a linear combination of the Y_j elements is computed. The resulting scalar is named y_{ij} which corresponds to the reconstructed version of x_{ij}. The weights of the linear combination are also put in a column vector defined as R_{ij} resulting in the following equation

$$y_{ij} = R_{ij}^H Y_j. \tag{2.3}$$

For easier reference, computing this linear combination is referred to as projection of the received vector, Y_j, in R_{ij} direction. The vector R_{ij} is also called the receive direction.

As the final step, the conditions for being able to estimate x_{ij} from y_{ij} are considered. Due to its simpler and more straightforward relations, most of the results in the upcoming sections are formulated based on zero-forcing approach. Using other methods such as MMSE can deliver better performance specially in lower Signal to Noise Ratios (SNR), but as far as the amount of total DOF is concerned both methods are equivalent and the difference between the two schemes is just a constant shift in the throughput vs SNR curves. Based on the zero-forcing method all the unwanted interference terms should be zero as the following ($i, i' = 1, 2, \ldots, M$ and $j, j' = 1, 2, \ldots, N$)

$$R_{i'j}^H H_{ij} T_{ij'} = g_{ij}\delta(i - i')\delta(j - j'), \tag{2.4}$$

where g_{ij}'s are arbitrary non-zero scalars. If (2.4) holds for all the combinations of i, i', j and j' then

$$y_{ij} = g_{ij}x_{ij} + R_{ij}^H Z_j, \tag{2.5}$$

and the MIMO X Channel is decoupled into MN interference-free single-dimensional parallel channels with AWGN.

2.1.3 Communication Scheme for the Interference Channel

The setup for IC is quite similar to the X Channel and can even be considered as a special case. Since it is a better match to real world applications, IC deserves having its own model and is treated as a separate network. Very similar to the $N\times N$-user X Channel, IC setup consists of N transmitters and N receivers all equipped with K antennas. As shown in Figure 2.2, each transmitter has an independent stream of information to send to only one of the receivers, and reversely each receiver expects data from exactly one of the transmitters. For simpler future references, the numbering of the transmitters and receivers is re-ordered in such a way that Transmitter i sends data to Receiver i only ($i = 1, 2, \ldots, N$). The scalar data sample that is sent from Transmitter i to Receiver i is declared as x_i and is selected from a Gaussian codebook. Using multiple antennas on Transmitter i, x_i is sent in T_i direction, where T_i is a $K\times 1$ column vector. Therefore, the transmit signal for Transmitter i is characterized as

$$X_i = T_i x_i. \tag{2.6}$$

The received signal at Receiver j is defined as Y_j and is determined from (2.2). Moreover, the received scalar data stream at Receiver j, y_j, is decoded by computing the projection of the received signal, Y_j in R_j direction according to

$$y_j = R_j^H Y_j. \tag{2.7}$$

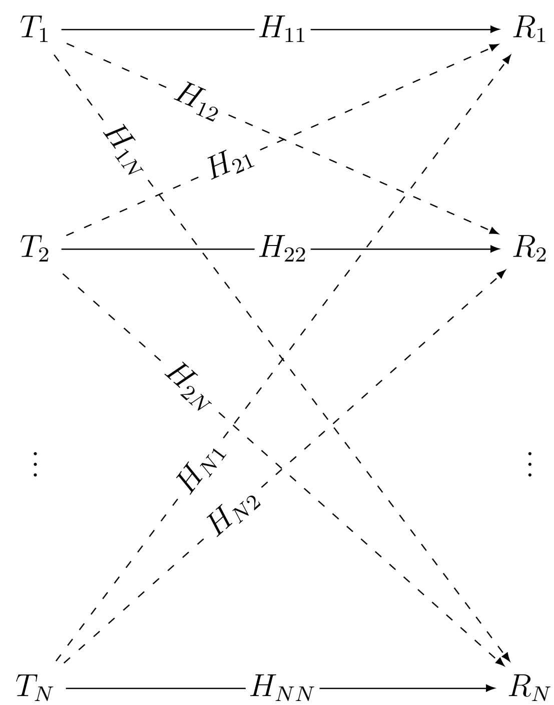

Figure 2.2: N-user Interference Channel, dashed paths carry interference only

Similar to the X Channel, the shared nature of the wireless medium makes the extraction of data from unwanted interference quite challenging. The conditions that should be satisfied in order to perform zero-forcing on the received signals are defined as

$$R_j^H H_{ij} T_i = g_i \delta(i-j), \tag{2.8}$$

with g_i's being non-zero scalars. Finally, satisfying the conditions for $i, j = 1, 2, \ldots, N$, results in

$$y_i = g_i x_i + R_i^H Z_i. \tag{2.9}$$

Therefore, the N-user MIMO IC is converted into N independent one-dimensional AWGN channels that do not impose any interference on each other.

2.2 Matrix Reformulation of the Interference Alignment Conditions

Based on the results from the previous sections, perfect alignment for the X Channel requires that all the $(MN)^2$ scalar conditions in (2.4) to be satisfied simultaneously. Similarly, on the IC setup, there are N^2 one-dimensional equations in (2.8) that should be true to be able to extract data from all the unwanted interference terms. This section contains a reformulation of these requirements in the matrix form which provides a more intuitive view of the same problem.

The first step in re-defining the conditions is to put the channel matrices in a bigger matrix that corresponds to the transfer function of the whole network. To this end, the $NK \times MK$-element matrix $\mathcal{H}$ is formed as

$$\mathcal{H} = \begin{bmatrix} H_{11} & H_{21} & \cdots & H_{M1} \\ H_{12} & H_{22} & \cdots & H_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ H_{1N} & H_{2N} & \cdots & H_{MN} \end{bmatrix}. \tag{2.10}$$

This matrix relates the signals sent by each of the MK antennas of all the transmitters to the NK antennas of all the receivers. The pre-processing coefficients on the transmitters' side are put in the $MK \times MN$ matrix $\mathcal{T}$ defined as

$$\mathcal{T}_X = \left[\begin{array}{cccc:cccc:c:cccc} T_{11} & T_{12} & \cdots & T_{1N} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & T_{21} & T_{22} & \cdots & T_{2N} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & T_{M1} & T_{M2} & \cdots & T_{MN} \end{array}\right]. \quad (2.11)$$

Using $\mathcal{T}$ the signal that is sent over the transmit antennas can be computed from the MN-dimensional data sent by the transmitters altogether. Finally, the projection of the received signals which extracts the information from the channel output is constructed by combining the receive directions into an $NK \times MN$ matrix named $\mathcal{R}_X$ as the following

$$\mathcal{R}_X = \left[\begin{array}{cccc:cccc:c:cccc} R_{11} & R_{21} & \cdots & R_{M1} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & R_{12} & R_{22} & \cdots & R_{M2} & \cdots & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} & \mathbf{0} & \cdots & \mathbf{0} & \cdots & R_{1N} & R_{2N} & \cdots & R_{MN} \end{array}\right]. \quad (2.12)$$

Based on these definitions, the alignment problem can be restated as

$$\mathcal{R}_X^H \mathcal{H} \mathcal{T}_X = \mathcal{G}_X, \quad (2.13)$$

where $\mathcal{G}_X$ is an $MN \times MN$ matrix defined as

$$\mathcal{G}_X = \left[\begin{array}{cccc:cccc:c:cccc}
g_{11} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & g_{21} & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & g_{M1} & 0 & \cdots & 0 \\
\hdashline
0 & g_{12} & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & g_{22} & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & g_{M2} & \cdots & 0 \\
\hdashline
\vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
\hdashline
0 & 0 & \cdots & g_{1N} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & g_{2N} & \cdots & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & g_{MN}
\end{array}\right]. \tag{2.14}$$

Matrix $\mathcal{G}_X$ has the property that there is only one non-zero element in every row or column. In the case of all the non-zero elements of $\mathcal{G}_X$ being one, the matrix is a special permutation transform which is often called a Vectorized Transpose Matrix (identified as $P_{_{N,M}}$) that permutes the column vector of an $N \times M$ matrix A (denoted by $\text{vec}(A)$) into the column vector of its transpose as below

$$\text{vec}(A^T) = P_{_{N,M}} \cdot \text{vec}(A). \tag{2.15}$$

Similarly, applying the same approach to the N-user MIMO IC, the $NK \times N$ matrices

$\mathcal{T}_{IC}$ and $\mathcal{R}_{IC}$ as well as $N \times N$ matrix $\mathcal{G}_{IC}$ are defined as below

$$\mathcal{T}_{IC} = \begin{bmatrix} T_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & T_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & T_N \end{bmatrix}, \tag{2.16}$$

$$\mathcal{R}_{IC} = \begin{bmatrix} R_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & R_2 & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & R_N \end{bmatrix}, \tag{2.17}$$

$$\mathcal{G}_{IC} = \begin{bmatrix} g_1 & 0 & \cdots & 0 \\ 0 & g_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & g_N \end{bmatrix}. \tag{2.18}$$

These definitions can be used to state the alignment conditions in a more compact form as

$$\mathcal{R}_{IC}^H \mathcal{H} \mathcal{T}_{IC} = \mathcal{G}_{IC}. \tag{2.19}$$

Having all the required definitions and relations, the interference alignment problem can be easily characterized.

Problem Statement. *Choose the non-zero portions of $\mathcal{T}_X$, $\mathcal{R}_X$ and $\mathcal{G}_X$ (or $\mathcal{T}_{IC}$, $\mathcal{R}_{IC}$ and $\mathcal{G}_{IC}$) in such a way that the subset of equations in* (2.13) *(or* (2.19)*) that correspond to the zero elements of $\mathcal{G}_X$ (or $\mathcal{G}_{IC}$) are satisfied*[1].

[1] It is straightforward to see that the equalities that correspond to the non-zero elements of $\mathcal{G}_X$ are

2.3 Number of Antennas (K)

As seen in the previous sections, the interference alignment problem for the $M\times N$-user X Channel can be converted into MN equations that should be satisfied simultaneously. Investigating the structure of $\mathcal{T}_X$ and $\mathcal{R}_X$ reveals that each of them contains KMN non-zero scalars that can be chosen to satisfy the equations in (2.13). Moreover, it is easy to see that if $(\mathcal{T}_X, \mathcal{R}_X)$ is a solution to the problem, for any arbitrary diagonal matrix D with non-zero diagonal entries, $\left(\mathcal{T}_X D^{-1}, \mathcal{R}_X D_\pi^H\right)$ is also another possible solution set for (2.13). D_π is a diagonal matrix whose diagonal entries are a permutation of the diagonal entries of D such that

$$D_\pi \cdot P_{N,M} = P_{N,M} \cdot D. \tag{2.20}$$

Therefore, to ensure that the solutions are unique, additional requirements should be put on the columns of $\mathcal{T}_X$ and $\mathcal{R}_X$. One possible choice is to enforce that the norm of the columns which correspond to the same set of data streams to be equal or in terms of T_{ij} and R_{ij}

$$T_{ij}^H T_{ij} = R_{ij}^H R_{ij}. \tag{2.21}$$

As a result, the total number of equations to have a unique solution is $(MN)^2 + MN$. Comparing the number of equations with the total number of variables that can be changed to satisfy the equations, yields to the following condition on the number of antennas

$$2K \geq MN + 1. \tag{2.22}$$

Unfortunately, since the equations in (2.13) are non-linear, not much can be said regarding

trivially satisfied by scaling the columns of $\mathcal{T}_X$ or $\mathcal{R}_X$. The same concept applies to the IC setup.

				M		
		2	3	4	5	6
	2	**3/3**	**4/4**	**5/5**	**6/6**	**7/7**
	3	**4/4**	**5/5**	6/7	7/8	8/10
N	4	**5/5**	6/7	7/9	8/11	9/13
	5	**6/6**	7/8	8/11	9/13	10/16
	6	**7/7**	8/10	9/13	10/16	11/19

$$M + N - 1 / \left\lceil \frac{MN+1}{2} \right\rceil$$

Table 2.1: Number of antennas, K, in an $M \times N$-user X Channel: Lower bound/Achievable

the existence of a solution for all different choices of $\mathcal{H}$. Numerous recursive algorithms exist, however, for determining the possible solution(s). Please refer to the Appendix A to see a MATLAB listing of one of these algorithms which is based on Newton's Method. Based on the random tests, it is safe to claim that the set of equations are solvable for a subset of $\mathcal{H}$ matrices (actually all the random matrices tested were solvable), but no mathematical proof has been given regarding whether the measure of the non-solvable matrices is zero or not.

Table 2.1 compares the number of antennas based on directly solving the equations as presented here to the lower bound from [10]. As it can be seen, although the direct approach has the minimum number of antennas in $M \times 2$, $2 \times N$ and 3×3 cases, it quickly diverges from the lower bound as M and N increase. It should be however emphasized that despite being inefficient, directly solving the equations offers at least one practical method for interference alignment that can be used in existing MIMO wireless networks.

Similar to the X Channel, the number of antennas for N-user IC can be determined. There are $2KN$ free variables in $\mathcal{T}_{IC}$ and $\mathcal{R}_{IC}$ to be adjusted for solving N^2 equations.

	N						
	2	3	4	5	6	7	8
K	**2/2**	**2/2**	2/3	2/3	2/4	2/4	2/5

$$2/\left\lceil \frac{N+1}{2} \right\rceil$$

Table 2.2: Number of antennas, K, in an N-user IC: Lower bound/Achievable

Moreover, there are N extra conditions on the norms of $\mathcal{T}_{IC}$ and $\mathcal{R}_{IC}$ columns. Therefore, the number of antennas, K, should satisfy the following inequality

$$2K \geq N + 1. \tag{2.23}$$

In other words, using the approach presented in this section, the total DOF per antenna is at most $\frac{2N}{N+1}$. While this value is less than $\frac{N}{2}$, it offers a practical solution for quasi-static MIMO with no need for infinite time-extensions, infinite channel precision or other complex operations (required by other methods for achieving a DOF of $\frac{N}{2}$). Table 2.2 lists the number of antennas for the existing approach as well as the predicted values for the lower bound.

2.4 Conclusion

This chapter describes the IA problem in two popular network setups namely X Channel and IC. Zero-forcing approach is used for determining the alignment equations. The resulting model is then restructured to a form that is suitable for iterative root finding algorithms. Additionally, the minimum number of antennas is specified based on the fact that the equivalent system of non-linear equations should not become over-determined (to ensure that a solution exists for all channel instances). Based on this reasoning, the minimum number of antennas to have a total DOF of MN in an $M \times N$-user X Channel or a total DOF of N in an N-user IC is respectively $\lceil \frac{MN+1}{2} \rceil$ and $\lceil \frac{N+1}{2} \rceil$.

Chapter 3

Relay-aided Interference Alignment for the X Channel

Performance of the wireless networks with finite resources is limited by the amount of noise at the receivers as well as the amount of interference from unwanted transmitters [29]. For a constant noise power, the first limitation can be overcome by increasing the transmit powers which in turn increases the likelihood of the signals being lost due to strong interfering transmitters. Interference Alignment (IA) [38] is one of the possible methods to maximize the performance by trying to achieve both objectives at the same time.

The alignment concept in IA comes from the idea that interference from different transmitters is packed in a small portion of the receive space. Thus, no matter how big the transmit powers are, each receiver can have a fraction in the receive space that is not occupied with interference. That fraction can be used for interference-free data transmission. Majority of the early results depend on extremely large number of dimensions for achieving the theoretical bounds for the total number of Degrees of Freedom (DOF). Increasing the number of antennas is one way to add to the dimensions which simplifies the IA in networks. Gaining benefit from having more antennas has motivated the researchers to investigate IA over multiple time intervals [33, 11]. It is worth noting that a time-extended channel can be regarded as a Multiple-Input Multiple-Output (MIMO) channel. In fact, transmit or receive signals are combined into blocks of K time slots and the equivalent channel between each transmitter-receiver pair is viewed as a diagonal MIMO system. This approach is applicable to time-varying single antenna systems where extension over time results in an equivalent MIMO with non-equal diagonal channel matrices. By using this technique, Cadambe and Jafar in [11] showed that the total DOF in the Interference Channel (IC) scales linearly with the number of users, provided that the channel is time-varying from transmission to transmission.

Results obtained based on the time-extension approach rely on the global knowledge of all the channel gains. Moreover, the time-extension method requires independent channel variations to obtain distinct directions for each transmitter-receiver pair. While the fast fading assumption is not unrealistic by itself, the necessity to know all the channel instances inside a block at the start of the transmission proves to be impractical[1]. One way to avoid the non-causal channel knowledge is suggested in [11] by extending the channel in the frequency domain instead of the time. Replaced with frequency selectivity, the requirement for channel variation still remains. Another approach is suggested in [45], where realizations are paired and signaling is performed over such pairs. This technique sends each transmit signal in two parts and the transmitter delays sending the second part until a specific channel realization occurs (which depends on the channel gains during the first transmission). As a result, depending on the dimensions of the channel and the

[1] All the transmitters and receivers should know the future channel gains in advance if they want to align the interference.

amount of channel variations, the required delay can become very large.

In a static channel, the proposed techniques based on time-extension are not applicable because all of the channel matrices become a constant multiple of the identity matrix. In this case, intended signal and interference occupy the same subspace and therefore they cannot be separated based on the IA. In the recent works by Motahari et.al. in [41, 42], it is proved that the application of IA is not restricted to MIMO systems and the full DOF of single antenna networks can be achieved. In other words, contrary to the belief that N-dimensional Euclidean spaces where $N \geq 2$ are required for achieving the full DOF, [41, 42] shows that one-dimensional systems provide the same foundation for IA, which can result in achieving the full DOF of the system. Unfortunately, although their results are valid for a group of channel conditions with measure one, there is a countable realization of channels, i.e., channels with rational coefficients, to which their approach cannot be applied directly.

The use of relays in a wireless network has been the addressed by many researchers. In [22], the authors investigate the scaling laws for the throughput of a random network with one source, one destination and $n-2$ relays and show that the capacity of such a network can scale with $\log(n)$. The benefits of cooperative use of the relays is also discussed in a number of papers including [36, 34, 48]. On the IA context, using the deterministic channel model in [40], it has been shown that relays are helpful to increase the number of achievable DOF for some specific examples. Interesting results are presented in [13] where the authors have proved that if a network is fully connected, the total DOF does not depend on the number of relays operating in it. The amount of DOF in such a network is only determined by the number of transmitters and receivers and how many antennas each of them possess. In other words, introducing relays to a network might increase the sum-rate, but it does not change the total DOF. This statement is important for the results that are presented in this chapter because it confirms that the achieved total DOF is in fact optimal.

It is shown here[2] that adding a single relay can change a static environment to an equivalent time-varying one. Moreover, relays can impose wanted variations on the channel behavior by processing their input signals over time. Unfortunately, the amount of variations that relays can add to a network is limited. As will be seen in the upcoming sections, by relying on time extensions and a casual linear processing, relays can convert the equivalent MIMO channel into a lower triangular with equal diagonal entries. In this chapter, a novel cooperative scheme is introduced for static channels, which achieves the available DOF of the X Channel. We also determine the required scaling factor for the relay's output power with respect to the power of the main transmitters.

The system model is described in Section 3.1. Section 3.2 presents a closed-form solution for a group of MIMO X Channels. Section 3.3 is devoted to the characterization of the total DOF along with asymptotic results. Finally, the chapter is concluded in Section 3.4.

3.1 Relay-Assisted Interference Alignment

In a quasi-static network, the total DOF is achievable for a set of channel realizations with measure one [42]. However, using the authors' approach, there are infinitely many realizations where the total DOF cannot be achieved. In fact, there is no connected set of channel realizations with achievable total DOF. To overcome this problem, a relay is added to the network. Time extension together with a linear processing relay are enough

[2]The results presented in this chapter have been published in [49].

to convert the quasi-static channel to an equivalent MIMO with a lower triangular channel matrix. MIMO channels with non-identity channel matrices provide feasible IA by careful design of the transmit and receive directions. Being the total DOF as the main criterion, this chapter aims at designing optimum signaling schemes for such networks. To this end, we consider the $M \times 2$-user X Channel.

In this section, we describe the system model and obtain the equivalent MIMO system for this network. In the following sections, we investigate the performance of the X Channel in which IA is used as part of the signaling.

3.1.1 Equivalent MIMO Channel

Consider a network of single antenna users operating in a quasi-static environment along with a full duplex single-antenna relay. The relay listens to the transmit signals over consecutive time slots and sends their linear combinations in the next slots. In the general case, we assume that there are M transmitters and N receivers. It is also assumed that a total of K time slots are used for sending one complex scalar data symbol from each transmitter to its designated receivers.

For $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, N$ and $k = 1, 2, \ldots, K$, $X_i(k)$ is the signal that is sent by Transmitter i during the kth time slot. After passing through the channel, the noisy signals that are received by the relay and by the Receiver j are named $Y_r(k)$ and $Y_j(k)$, respectively. The Additive White Gaussian Noise (AWGN) that corrupts the received signals $Y_r(k)$ and $Y_j(k)$ during the kth time slot are denoted by $W_r(k)$ and $W_j(k)$ with variances ω_r^2 and ω_j^2, respectively.

The physical channel between Transmitter i and Receiver j is characterized by a complex scalar gain h_{ij}. Similarly, the complex scalar channel gains from Transmitter i to the relay and also from the relay to Receiver j are denoted by h_{ir} and h_{rj}, respectively. We also assume that all of the channel gains remain unchanged over a signal block whose duration is at least K time intervals. Using the above scheme, we can express the received signals in terms of the transmit signals and the complex channel scalars as

$$Y_r(k) = \sum_{i=1}^{M} h_{ir} X_i(k) + W_r(k), \tag{3.1}$$

$$Y_j(k) = \sum_{i=1}^{M} h_{ij} X_i(k) + W_j(k) + h_{rj} \sum_{l=1}^{k-1} \mathcal{G}(k,l) Y_r(l). \tag{3.2}$$

The complex scalar coefficients $\mathcal{G}(k,l)$ are used for scaling the signal that is received in the time slot l and sent over the kth time slot.

Now if we extend the definition of a symbol to K time slots, the system can be viewed as a network of $K \times K$ MIMO users. To observe this, the signals from each K consecutive time slots are grouped to make a column vector, as is done for $X_i(k)$ in the following:

$$X_i = \begin{bmatrix} X_i(1) \\ X_i(2) \\ \vdots \\ X_i(K) \end{bmatrix}. \tag{3.3}$$

The vectors Y_r, W_r, Y_j and W_j are also defined in the same way. It is now easy to determine H_{ij}, the equivalent channel matrix from Transmitter i to Receiver j. If $H_{ij}(k,l)$ is defined as the complex scalar element in Row k and Column l of H_{ij}, then

$$H_{ij}(k,l) = \begin{cases} 0 & k < l \\ h_{ij} & k = l \\ h_{rj}\mathcal{G}(k,l)h_{ir} & k > l \end{cases}, \tag{3.4}$$

where $k, l = 1, 2, \ldots, K$. Therefore, the equivalent MIMO channel matrix is a lower triangular with equal diagonal entries. It is also noteworthy to emphasize that the relay gains are integrated into H_{ij} and, as it will be seen in the next sections, this integration plays an important role in successfully aligning the interference directions. Similarly, using the same approach, the equivalent noise vector in Receiver j is determined to be

$$Z_j(k) = W_j(k) + \sum_{l=1}^{k-1} h_{rj}\mathcal{G}(k,l)W_r(l). \tag{3.5}$$

To finish the characterization of the equivalent MIMO network, we need to write down the input/output relationships. Using the original model in (3.1) and (3.2) along with definitions for H_{ij} and Z_j (the vectorized form of $Z_j(k)$ as in (3.3)), we have

$$Y_j = H_{1j}X_1 + H_{2j}X_2 + \ldots + H_{Mj}X_M + Z_j. \tag{3.6}$$

Therefore, using a relay with the time-extension scheme for single-antenna networks can transform the time-domain relations into a pseudo MIMO network (with no relay), where all the channel matrices have zeros on their upper triangular entries. The next section describes a specific MIMO X Channel and determines the closed-form solutions for the IA problem.

3.2 $M \times 2$-User MIMO X Channel

We consider the $M \times 2$-user MIMO X Channel in which there are M MIMO transmitters and 2 MIMO receivers where each transmitter wishes to send an independent stream of information to each receiver making a total of $2M$ transmitter-receiver pairs. For $i = 1, 2, \ldots, M$ and $j = 1, 2$, x_{ij} is the complex scalar data stream from Transmitter i to Receiver j, which is precoded in the T_{ij} direction. Therefore, X_i, the output vector for Transmitter i that is sent over its multiple antennas, is

$$X_i = T_{i1}x_{i1} + T_{i2}x_{i2}. \tag{3.7}$$

The channel matrix between Transmitter i and Receiver j is denoted by H_{ij}. Furthermore, Receiver j gets a noisy version of data (mixed with interference) in vector Y_j. Z_j is also the AWGN at the input of Receiver j. The relation between transmit and receive signals can now be written as

$$Y_j = H_{1j}X_1 + H_{2j}X_2 + \ldots + H_{Mj}X_M + Z_j. \tag{3.8}$$

Finally, to extract the specific information that is sent by Transmitter i to Receiver j (x_{ij}), Receiver j computes the projection of its received vector, Y_j, in the R_{ij} direction. The resulting scalar is named y_{ij} and corresponds to the reconstructed version of x_{ij} yielding to

$$y_{ij} = R_{ij}^H Y_j. \tag{3.9}$$

Moreover, we define P_{ij} as the average power for x_{ij} and P_i as the total power in Transmitter i. The covariance matrix for Z_j, the AWGN in Receiver j, is also assumed to be $\sigma_j^2 I$.

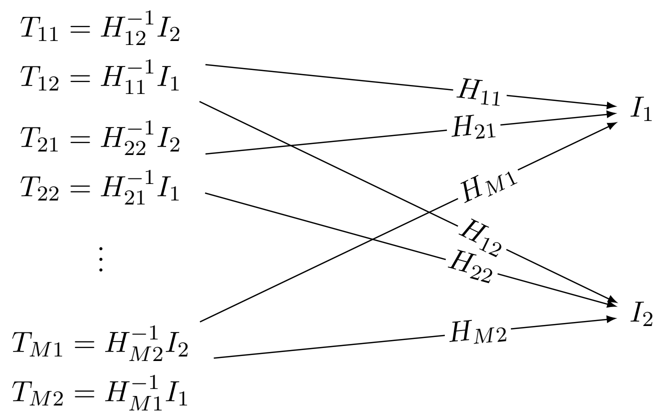

Figure 3.1: Interference Aligned Transmit Directions for $M \times 2$-user X Channel

As will be seen in the rest of this section, such a network can be transformed into $2M$ parallel interference-free channels (one for each transmitter-receiver pair) if all the users of this network are equipped with $M + 1$ antennas. The system in Fig. 3.1 shows this particular network configuration.

To perform IA, let I_1 and I_2 be two arbitrary directions assigned for interference in the first and second receivers. As suggested in [10], the transmit directions to the first receiver should be adjusted so that the signals are received in one direction (I_2) by the second receiver. The same argument is true regarding the transmit directions designed for the second receiver and I_1. Therefore, the transmit directions for Transmitter i are

$$T_{i1} = H_{i2}^{-1} I_2, \tag{3.10}$$

$$T_{i2} = H_{i1}^{-1} I_1. \tag{3.11}$$

The receive directions can also be easily determined by zero-forcing the unwanted signals in either receiver. To obtain the desired directions, we first define the $(M + 1) \times (M + 1)$

matrices B_1 and B_2 as

$$B_1 = \left[H_{11}H_{12}^{-1}I_2 \;\; H_{21}H_{22}^{-1}I_2 \;\; \ldots \;\; H_{M1}H_{M2}^{-1}I_2 \;\; I_1 \right], \tag{3.12}$$

$$B_2 = \left[H_{12}H_{11}^{-1}I_1 \;\; H_{22}H_{21}^{-1}I_1 \;\; \ldots \;\; H_{M2}H_{M1}^{-1}I_1 \;\; I_2 \right]. \tag{3.13}$$

Then, the receive direction R_{ij} that can extract the noisy version of x_{ij} from Y_j is selected as the ith column of B_j^{-H}. To observe this for the first receiver we have

$$R_{i1}^H H_{k1} T_{k1} = R_{i1}^H H_{k1} H_{k2}^{-1} I_2 = \delta(i-k), \tag{3.14}$$

$$R_{i1}^H H_{k1} T_{k2} = R_{i1}^H I_1 = 0. \tag{3.15}$$

Similar results can be obtained for the directions in the second receiver. Therefore, data from Transmitter i to Receiver j can be extracted as

$$\begin{aligned} y_{ij} &= R_{ij}^H Y_j \\ &= R_{ij}^H \left(H_{ij} X_i + Z_j \right) \\ &= R_{ij}^H \left(H_{ij} T_{ij} x_{ij} + Z_j \right) \\ &= x_{ij} + R_{ij}^H Z_j. \end{aligned} \tag{3.16}$$

It should be emphasized that in order to get $2M$ independent paths between all of the transmitters and the two receivers, we need to arrange B_1 and B_2 such that they are both invertible. Such a requirement can be easily satisfied by the appropriate selection of interference directions I_1 and I_2. It is also obvious that even a random selection of the interference directions guarantees the non-singularity condition for matrices in (3.12) and (3.13) almost surely. Therefore, using the transmit and receive directions as specified above, a DOF of $2M$ for $M\times 2$-user X Channel is achieved. The same results exist for $2\times M$-user X Channel by reciprocity[3].

Using the above scheme, we can further analyze the rates from each transmitter to each receiver. Since

$$\mathcal{E}\left\{x_{ij}x_{ij}^*\right\} = P_{ij}, \tag{3.17}$$

$$\mathcal{E}\left\{Z_j Z_j^H\right\} = \sigma_j^2 I, \tag{3.18}$$

using an independent Gaussian codebook for each transmitter-receiver pair, the link from Transmitter i to Receiver j can support an average rate of up to C_{ij} which is defined as below[4]

$$C_{ij} = \mathcal{E}\left\{\log\left(1 + \frac{P_{ij}}{\sigma_j^2 \|R_{ij}\|^2}\right)\right\}. \tag{3.19}$$

Also, assuming that data streams from Transmitter i to the two receivers are uncorrelated ($\mathcal{E}\left\{x_{i1}x_{i2}^*\right\} = 0$), we have

$$\begin{aligned} P_i &= \mathcal{E}\left\{X_i^H X_i\right\} \\ &= P_{i1}\|T_{i1}\|^2 + P_{i2}\|T_{i2}\|^2. \end{aligned} \tag{3.20}$$

To maximize the sum-rate when P_i is limited, the problem can be converted to the classic

[3]To select the directions for $2\times M$-user X Channel, set $H'_{ij} = H_{ji}^H$ and solve the equivalent $M\times 2$-user X Channel to determine T'_{ij} and R'_{ij} as described. Then the set of equations $T_{ji} = R'_{ij}$ and $R_{ji} = T'_{ij}$ will give the desired directions.

[4]Since the additive Gaussian noise values on the equivalent parallel channels are not independent, using separate Gaussian codebooks is not optimal and joint coding schemes are needed. All the discussions about the rate in this section and future sections use ergodic achievable rate for determining the DOF. Interested readers can refer to [46, 32, 3] to get more elaborate results in terms of the ergodic capacity of the wireless networks.

water-filling form [18] and, for example, if

$$P_i > \left| \sigma_1^2 \|T_{i1}\|^2 \|R_{i1}\|^2 - \sigma_2^2 \|T_{i2}\|^2 \|R_{i2}\|^2 \right|, \tag{3.21}$$

then P_{ij}'s can be found from

$$P_{ij}\|T_{ij}\|^2 + \sigma_j^2 \|T_{ij}\|^2 \|R_{ij}\|^2 = \frac{1}{2}\left(P_i + \sigma_1^2 \|T_{i1}\|^2 \|R_{i1}\|^2 + \sigma_2^2 \|T_{i2}\|^2 \|R_{i2}\|^2\right). \tag{3.22}$$

In the next section, we will apply this model to determine the DOF achievable by an X Channel under quasi-static conditions.

3.3 $M\times 2$-User X Channel with a Relay

In [38], a signalling scheme for the 2×2-user X Channel is proposed in which the interfering signals at each receiver are perfectly aligned. This result is later generalized in [10], where it is proved that a single-antenna X Channel consisting of M transmitters and N receivers can asymptotically achieve maximum DOF of $\frac{MN}{M+N-1}$. To this end, the authors extend the transmit signals over multiple time symbols and treat the block as a single MIMO system. To realize the expected DOF, the scalar channel gains must be different during consecutive symbols. But, such a fast changing criteria for physical channels is not an easy requirement to meet. Therefore, in practical situations, the channel variations might not be enough to recover all the potentials of such networks.

In the previous section, we have used some of the results from [10] and restated the signaling for $M\times 2$-user MIMO X Channel. In this section, we construct the transmit and receive directions from the perfectly-aligned $M\times 2$-user MIMO X Channel to determine

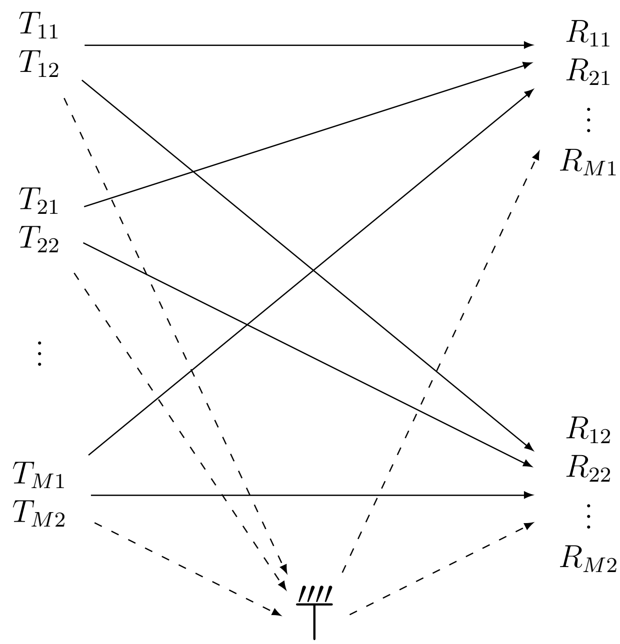

Figure 3.2: Interference alignment in $M \times 2$-user X Channel with the help of a relay

the conditions that ensure achieving the total DOF in our relay-aided setup. As it will be seen, this scheme only uses the relay to randomize the static channel. We also investigate the power requirement of the relay and determine its scaling factor to obtain the maximum DOF. Our approach is equally applicable to frequency-extension scenarios in which most of the results are applicable with minor or no changes.

3.3.1 DOF Achievability Conditions

As depicted in Figure 3.2, in this section we use a single-antenna relay to achieve all the available DOF. In order to conveniently describe the equivalent $K \times K$ channel matrix from Transmitter i to Receiver j, we define a strictly lower triangular square matrix G, which represents the relay coefficients and select $G(k,l)$, its entry in Row k and Column l, as

$$G(k,l) = \begin{cases} 0 & k \leq l \\ \mathcal{G}(k,l) & k > l \end{cases} \quad k,l = 1,2,\ldots,K. \tag{3.23}$$

In addition, the complex scalar parameter α_{ij} that regularly appears in subsequent equations is defined as

$$\alpha_{ij} = -\frac{h_{ir}h_{rj}}{h_{ij}}. \tag{3.24}$$

It should be noted that α_{ij} depends only on the gains of the available paths from Transmitter i to Receiver j. Furthermore, Z_j, the equivalent noise vector in Receiver j, which contains the effects of both W_r and W_j, is

$$Z_j = W_j + h_{rj}GW_r. \tag{3.25}$$

It is evident that the equivalent MIMO noise vector is not uncorrelated anymore and the covariance matrix is

$$\mathcal{E}\left\{Z_j Z_j^H\right\} = \omega_j^2 I + \omega_r^2 |h_{rj}|^2 GG^H. \tag{3.26}$$

The equivalent MIMO channel can now be expressed using G, α_{ij} and h_{ij} as

$$H_{ij} = h_{ij}\left(I - \alpha_{ij}G\right). \tag{3.27}$$

Since the diagonal entries of the lower triangular matrix G are all zeros, it does not contain any non-zero eigenvalues. Therefore, its characteristic polynomial can be written as $f(\lambda) = \lambda^K$ and, considering the fact that every matrix satisfies its own characteristic equation, we can obtain $f(G) = G^K = \mathbf{0}$. Hence, G is a nilpotent matrix and we can easily formulate the inverse of the channel matrix as

$$H_{ij}^{-1} = \frac{1}{h_{ij}} \sum_{k=0}^{K-1} \alpha_{ij}^{k} G^{k}. \tag{3.28}$$

For the remainder of this section, we will limit the discussions to the $M\times 2$ case. This particular network configuration is important because perfect IA can be performed and closed form relations exist. In the previous section we presented the encoding and decoding functions for the $M\times 2$-user X Channel. We also described how the power assignment for transmitters should be adjusted to maximize the sum-rate. The results are used in this section to determine the achievable DOF. Starting from the definitions and the model from the previous section we set $K = M + 1$, therefore, the ith column of B_1 in (3.12) becomes

$$H_{i1} H_{i2}^{-1} I_2 = \frac{h_{i1}}{h_{i2}} I_2 + \left(\frac{h_{i1}}{h_{i2}} - \frac{h_{r1}}{h_{r2}} \right) \sum_{k=1}^{M} \alpha_{i2}^{k} G^{k} I_2. \tag{3.29}$$

We also define the square matrix Φ_2 as

$$\Phi_2 = \left[\begin{array}{cccc} I_2 & GI_2 & \dots & G^{M} I_2 \end{array} \right]. \tag{3.30}$$

Noticing that the first k rows in G^k are all zeros, it is easy to show that Φ_2 is a lower triangular matrix. We also use (3.29) to decompose B_1 into $B_1 = \Phi_2 \Psi_2$, where Ψ_2 is

$$\Psi_2 = \left[\begin{array}{ccc:c} \frac{h_{11}}{h_{12}} & \dots & \frac{h_{M1}}{h_{M2}} & \\ \left(\frac{h_{11}}{h_{12}} - \frac{h_{r1}}{h_{r2}} \right) \alpha_{12} & \dots & \left(\frac{h_{M1}}{h_{M2}} - \frac{h_{r1}}{h_{r2}} \right) \alpha_{M2} & \Phi_2^{-1} I_1 \\ \vdots & \ddots & \vdots & \\ \left(\frac{h_{11}}{h_{12}} - \frac{h_{r1}}{h_{r2}} \right) \alpha_{12}^{M} & \dots & \left(\frac{h_{M1}}{h_{M2}} - \frac{h_{r1}}{h_{r2}} \right) \alpha_{M2}^{M} & \end{array} \right]. \tag{3.31}$$

Similarly, matrices Φ_1 and Ψ_1 can be defined to have $B_2 = \Phi_1\Psi_1$. As previously stated, non-singularity of both B_1 and B_2 is a sufficient condition for getting $2M$ DOF. Therefore, it is necessary to arrange all Φ_1, Φ_2, Ψ_1 and Ψ_2 matrices such that they are invertible. The lower triangular matrices Φ_1 and Φ_2 are independent from the physical channel and are only a function of the interference directions I_1, I_2, and the relay coefficients matrix, G. As a result, it is fairly easy to adjust I_1, I_2 and G such that both Φ_1 and Φ_2 have non-zero diagonal entries to secure their non-singularity prerequisite. It can be easily proved that as long as all the lower triangular elements of G and all the entries in I_1 and I_2 are non-zero, Φ_1 and Φ_2 are guaranteed to be invertible, although this is not a necessary condition. The other two matrices, Ψ_1 and Ψ_2, mainly depend on the random channel gains and are non-singular almost surely (it should be emphasized that each of these matrices contains an $M\times M$ Vandermonde matrix). Care must be taken, however, in the selection of I_1 and I_2 to prevent the last column of Ψ_1 and Ψ_2 from becoming linearly dependent on the first M columns.

The preceding discussion sheds light on the critical role of the relay to provide the additional DOF. Having non-zero elements in the lower triangular section of G results in Φ_j matrices becoming non-singular which in turn leads to achieving the DOF predicted by IA. Moreover, although a random selection of G is sufficient for the validity of the results in this section, a lot more can be done by optimizing the free parameters of the scheme (G and I_j) based on the channel coefficients (h_{ij}, h_{ir} and h_{rj}). Optimization does not offer further increase in DOF, but can be used to maximize the gains of the equivalent parallel channels. In practice, however, there should be a trade-off between the lower power requirements and the extra complexity imposed by searching for the right parameters instead of just randomly choosing them.

For the traditional 2×2-user X Channel, we can also use a half duplex relay, thus making the relay structure much simpler. To this end, we change the behavior of the relay such that it listens and stores the received signals during the first two time slots and then sends their linear combination over the third time slot. Equivalently, another scheme that offers the same results would be a relay that listens to the received signal in the first time slot and sends different scaled versions of its received signal during the next two slots. In both schemes, the relay coefficients matrix is all zeros, except for the last row (or the first column in the latter case). Although both schemes offer the same performance, the former has only one transmission period (which saves power), while the latter requires storing only one signal (which saves memory). In any case, all of the results from the current section are equally applicable to both scenarios. In this part we present a new method for achieving all of the available DOF for the M×2-user X Channel in a quasi-static environment. It is interesting to see that the channel variation requirement for the IA is easily waived by adding a simple randomizing relay. Since global knowledge about all the channel parameters is presumed, block fading assumption (in contrast to fast fading) offers a more feasible approach to implement a practical system using IA.

3.3.2 Asymptotic Power Analysis

After demonstrating that the addition of a relay can help in realizing the available DOF, we also determine how the relay power should scale with respect to the power of the main transmitters. To this end, we define the relay gain, g, as the ratio of the transmit power of the relay to the relay's received power. To find a simple relation between the relay coefficients matrix and the ratio of powers, we assume that the vector received by the relay, Y_r, is white Gaussian[5]. In this case g is obtained from

$$\begin{aligned} g^2 &= \frac{\mathcal{E}\left\{Y_r^H G^H G Y_r\right\}}{\mathcal{E}\left\{Y_r^H Y_r\right\}} \\ &= \frac{1}{M+1}\mathrm{tr}\left(GG^H\right) \\ &= \frac{1}{M+1}\sum_{k=1}^{K}\sum_{l=1}^{k-1}|G(k,l)|^2. \end{aligned} \tag{3.32}$$

We also define the normalized relay coefficients matrix, $\hat{G}$, as

$$\hat{G} = \frac{1}{g}G. \tag{3.33}$$

Using the above definition, we can rewrite Φ_2 from (3.30) as

$$\Phi_2 = \hat{\Phi}_2 \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & g & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & g^M \end{bmatrix}, \tag{3.34}$$

where $\hat{\Phi}_2$ is defined similar to Φ_2 with G replaced by $\hat{G}$. The same relation exists for Φ_1 as well. For future references, we name the diagonal matrix in (3.34) as $D(g)$. Now, it is possible to observe how directions change as the relay gain, g, goes to zero or to infinity.

[5]It should be emphasized that the white Gaussian assumption for Y_r is not really required. If the received signal has a different distribution, the relay gain is still defined by (3.32), but it will not be the ratio of the average powers anymore. In fact, one can choose any other matrix norm that measures the amount of the relay's amplification or attenuation, and the results will still be valid.

Relay-limited Power Scaling

If the power of the transmitters can scale faster than the relay power, it is possible to have situations where g approaches zero and we need to quantify how the powers should scale in order to guarantee achieving all the available DOF. This scenario can happen in practical applications when we want to add a low-cost (for example, solar-powered) relay and need to know about the amount of fluctuation required to randomize a quasi-static network. Using the previous results, the norm of the transmit directions for the small values of g can be written as

$$\lim_{g\to 0} \|T_{i1}\|^2 = \frac{1}{|h_{i2}|^2}\|I_2\|^2. \tag{3.35}$$

Similarly, we have

$$\begin{aligned}\lim_{g\to 0} g^M B_1^{-H} &= \lim_{g\to 0} g^M \hat{\Phi}_2^{-H} D\left(\frac{1}{g}\right)\Psi_2^{-H}\\ &= \hat{\Phi}_2^{-H}\begin{bmatrix} 0 & 0 & \dots & 0\\ 0 & 0 & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & 1\end{bmatrix}\Psi_2^{-H}\\ &\triangleq \hat{R}_1. \end{aligned} \tag{3.36}$$

It is easy to observe that all the entries of $\hat{R}_1$ are non-zero almost certainly. Therefore, if the ith column of $\hat{R}_1$ is denoted by $\hat{R}_{i1}$, we have

$$\lim_{g\to 0} g^M\|R_{i1}\|^2 = \|\hat{R}_{i1}\|^2 \neq 0. \tag{3.37}$$

Also σ_j^2, the equivalent noise variance in Receiver j, can be obtained from (3.26) as

$$\begin{aligned}\lim_{g\to 0}\sigma_j^2 &= \lim_{g\to 0}\frac{1}{M+1}\mathrm{tr}\Big(\omega_j^2 I + g^2\omega_r^2|h_{rj}|^2\hat{G}\hat{G}^H\Big)\\ &= \omega_j^2. \end{aligned} \tag{3.38}$$

Substituting (3.35), (3.37) and (3.38) in (3.22), as g goes to zero, the limit of P_{ij} is written as

$$\lim_{g\to 0} P_{ij} = \lim_{g\to 0} a_{ij}P_i + b_{ij}g^{-M}, \tag{3.39}$$

where a_{ij} and b_{ij} are non-zero real constant scalars that are independent from g. Therefore, using (3.19), the asymptotic rate between Transmitter i and Receiver 1 can be derived as

$$\begin{aligned}\lim_{g\to 0} C_{i1} &= \lim_{g\to 0}\mathcal{E}\Bigg\{\log\left(1+\frac{P_{i1}}{\sigma_j^2\|R_{i1}\|^2}\right)\Bigg\}\\ &\stackrel{a}{=} \mathcal{E}\Bigg\{\lim_{g\to 0}\log\left(1+\frac{P_{i1}}{\sigma_j^2\|R_{i1}\|^2}\right)\Bigg\}\\ &\stackrel{b}{=} \mathcal{E}\left\{\lim_{g\to 0}\log\left(1+a'_{i1}g^M P_i + b'_{i1}\right)\right\}\\ &\stackrel{c}{=} \lim_{g\to 0}\mathcal{E}\Big\{\log\left(1+1+a'_{i1}g^M P_i + b'_{i1}\right)\Big\}\end{aligned} \tag{3.40}$$

where a'_{i1} and b'_{i1} are two constants that do not depend on g. In the derivations, (a) and (c) are based on Lebesques's dominated convergence theorem[6] and (b) is written using (3.37) and (3.39). The asymptotic rates from Transmitter i to Receiver 2 can be determined in a similar way.

Now let us assume $P_1 = P_2 = \ldots = P_M = P$ and $g = (\log P)^{-s}$ for an arbitrary positive

[6]Dominated convergence theorem provides sufficient conditions under which the limit and integral operators can be swapped, please refer to [4] for more information.

constant s, then

$$\lim_{P\to\infty} \frac{C_{ij}}{\log\frac{P}{\sigma_j^2}} = \lim_{P\to\infty} \frac{\log\left(P\left(\log P\right)^{-sM}\right)}{\log P}$$
$$= 1 - \lim_{P\to\infty} \frac{sM\log\log P}{\log P}$$
$$= 1. \tag{3.41}$$

and the output power of the relay scales as

$$P_r \propto g^2 P = \frac{P}{\left(\log P\right)^{2s}}. \tag{3.42}$$

We can control the growth rate for the output power of the relay through parameter s and make it as slow as desired, bearing in mind that as long as s is positive all the available DOF can be utilized. Figure 3.3 shows the slope of the sum-rate as a function of transmit power P and the relay gain g for the 2×2-user X Channel. DOF is defined as the value of the slope when the power tends to infinity.

Transmitter-limited Power Scaling

For the applications in which there are more constraints on the output power of the main transmitters than the relay's power, scenarios might arise where the relay gain, g, becomes very large. A popular example includes a network of battery operated mobile users with channels that have very slow fluctuations over time. Using a similar approach, we deploy a fixed relay that can add enough perturbations to the equivalent MIMO channels to achieve higher DOF. To investigate the behavior of this scheme, we start by determining the asymptotic limits of the transmit and receive directions for a large g. Starting with

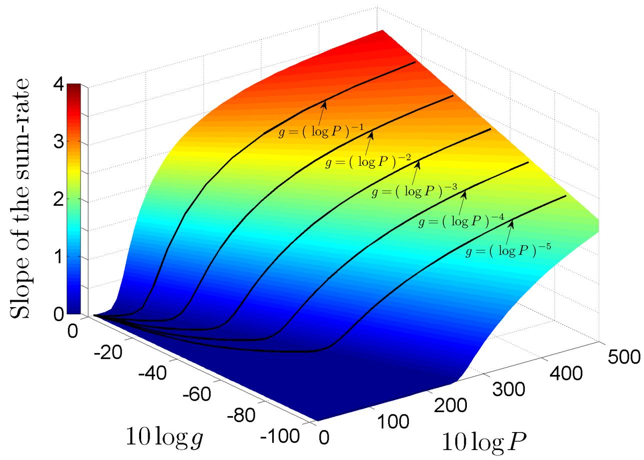

Figure 3.3: Simulation results for slope of the sum-rate in 2×2-user X Channel in a relay-limited case

the transmit directions to the first receiver, T_{i1}, we have

$$\lim_{g\to\infty} g^{-M}\|T_{i1}\|^2 = \frac{|\alpha_{i2}|^{2M}}{|h_{i2}|^2}\|\hat{G}^M I_2\|^2. \tag{3.43}$$

Similarly, as the relay gain goes to infinity, B_1^{-H}, the matrix that contains the receive directions in its columns becomes

$$\begin{aligned}\lim_{g\to\infty} B_1^{-H} &= \lim_{g\to\infty} \hat{\Phi}_2^{-H} D\left(\frac{1}{g}\right)\Psi_2^{-H} \\ &= \hat{\Phi}_2^{-H}\begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{bmatrix}\Psi_2^{-H} \\ &\triangleq \check{R}_1. \end{aligned} \tag{3.44}$$

Checking the square matrix $\check{R}_1$ shows that all of the entries in the first row are non-zero almost surely, therefore

$$\lim_{g\to\infty} \|R_{i1}\|^2 = \|\check{R}_{i1}\|^2 \neq 0, \tag{3.45}$$

where $\check{R}_{i1}$ is the ith column of $\check{R}_1$. Moreover, the noise variance in Receiver j can be demonstrated as

$$\lim_{g\to\infty} g^{-2}\sigma_j^2 = \frac{\omega_r^2 |h_{rj}|^2}{M+1} \mathrm{tr}\Big(\hat{G}\hat{G}^H\Big). \tag{3.46}$$

And finally, after substituting the results we can determine that

$$\lim_{g\to\infty} P_{ij} = \lim_{g\to\infty} c_{ij} g^{-M} P_i + d_{ij} g^2, \tag{3.47}$$

for nonzero constants c_{ij} and d_{ij} that are independent from g. Consequently, the rate from Transmitter i to Receiver 1 is

$$\lim_{g\to\infty} C_{i1} = \lim_{g\to\infty} \mathcal{E}\Big\{\log\left(1 + c'_{i1} g^{-(M+2)} P_i + d'_{i1}\right)\Big\}, \tag{3.48}$$

where c'_{i1} and d'_{i1} are constants. Therefore, if we set $g = (\log P)^t$ for $t > 0$ and use the equal power assumption for all transmitters, we can write the DOF for the link from Transmitter i to Receiver j as

$$\begin{aligned} \lim_{P\to\infty} \frac{C_{ij}}{\log \frac{P}{\sigma_j^2}} &= \lim_{P\to\infty} \frac{\log\Big(P\left(\log P\right)^{-t(M+2)}\Big)}{\log P} \\ &= 1. \end{aligned} \tag{3.49}$$

Hence, if the output power of the relay scales with $P\left(\log P\right)^{2t}$, the use of all the available DOF is ensured.

3.4 Conclusion

In this chapter we analyzed the benefits of adding a single-antenna relay to a $M \times 2$-user X Channel. We showed that inserting a simple relay into a quasi static environment can help the network to achieve a higher DOF. We also derived the scaling relation between the relay output power and the output power of the main transmitters in order to guarantee achieving all of the available DOF.

An advantage of the results presented in this chapter is that they can be easily applied to a network of single-antenna mobile users who cannot easily benefit from the IA technique in a static medium by themselves.

Chapter 4

Relay-aided Interference Alignment for Interference Channel

Despite the recent advances in Interference Alignment (IA) [11, 33, 43], it is still far from being applicable to a practical system as there are difficult conditions on the channels that must be met. First and foremost, the required number of dimensions (number of antennas or amount of time/frequency extensions) are very large. Moreover the achievable Degrees of Freedom (DOF) depends on the precision of the channel parameters. Extremely precise channel gains are needed in order to get all the available DOF. There is a parallel line of work directed towards adding extra elements to the network in order to make IA much easier instead of further increase in DOF. The result is having more DOF than what traditional approaches can offer, but the inherent complexity of the IA is traded for the cost of additional nodes in the system. In [49], a relay is added to randomize the quasi-static $M\times 2$-user X Channel which induces the needed time variations by changing the relay gains over time. In another work the authors combine network coding with IA to provide a bidirectional link in M-user Interference Channel (IC) with the help of a relay [24].

This chapter presents a similar idea[1]. It considers the M-user IC along with a half duplex MIMO relay. The relay stores the received signal vector during the first phase of its operation. It then transforms the vector into another one by multiplying it by a matrix and sends the new signal during the second phase. It is shown that carefully adjusting the transformation matrix results in a feasible zero-forcing solution in Euclidean space. In other words, the resulting linear transform changes the channel structure in such a way that signal and interference are easily separable[2]. After determining the required relay gains, the scaling range for relay's output power is also presented. This range is important as it describes how fast/slow the relay power should change (with respect to the power of the main transmitters), in order to guarantee achieving a DOF of $\frac{M}{2}$ in this network.

The system model along with the relay scheme is described in Section 4.1. The relay's transformation matrix that provides zero-forcing solutions for IC is fully characterized in Section 4.2. Section 4.3 analyzes the asymptotic behavior of the relay gains and finally, the chapter is concluded in Section 4.4.

4.1 Relay Assisted Time Extension

In this section, we describe a relay with store-transform-forward scheme. Operation of such a relay in the network of single-antenna users has an overall effect of modifying the equivalent channel between each transmitter and receiver. By extending the definition of a symbol to multiple time-slots, it is shown that the channel becomes a lower triangular

[1]The results presented in this chapter have been published in [50].

[2]As will be revealed in Section 4.2, using the randomizing relay approach in [49, 50] cannot help IA for M-user IC. Therefore the relay gains are adjusted instead of being selected randomly.

matrix. Since the relay gains control some of the elements of the equivalent channel, they can be adjusted to help aligning the interference directions.

Although the system model described in this section uses an arbitrary full duplex MIMO relay with K time extensions, in the next section we will show that $K = 2$ is enough to achieve $\frac{M}{2}$ DOF for M-user IC. Therefore the operation of a half duplex relay is sufficient for our scheme. Moreover, in Section 4.3, we will show that as long as the product of the number of relay's transmit antennas by the number of relay's receive antennas is more than $(M-1)(M-2)$ a DOF of $\frac{M}{2}$ is guaranteed.

4.1.1 System Model

Consider a network of single antenna users operating in a quasi-static environment along with a MIMO relay (as depicted in Figure 4.1). The relay listens to the transmit signals over consecutive time slots and sends their linear combinations during the next slots. In this chapter, the study is limited to M-user IC. Therefore, there are M transmitter-receiver pairs and each transmitter wishes to send an independent data stream to its corresponding receiver. The relay uses U antennas for reception and V antennas for transmission. It is also assumed that K time slots are used for sending one complex scalar data symbol from each transmitter to its designated receiver.

For $i, j = 1, 2, \ldots, M$, $u = 1, 2, \ldots, U$, $v = 1, 2, \ldots, V$ and $k = 1, 2, \ldots, K$, $X_i(k)$ is the signal that is sent by Transmitter i during the kth time slot. After passing through the channel, the noisy signals that are received by the uth receive antenna of the relay and Receiver j are named $Y_{r_u}(k)$ and $Y_j(k)$, respectively. The Additive White Gaussian Noise (AWGN) that corrupts the received signals $Y_{r_u}(k)$ and $Y_j(k)$ during the kth time slot is

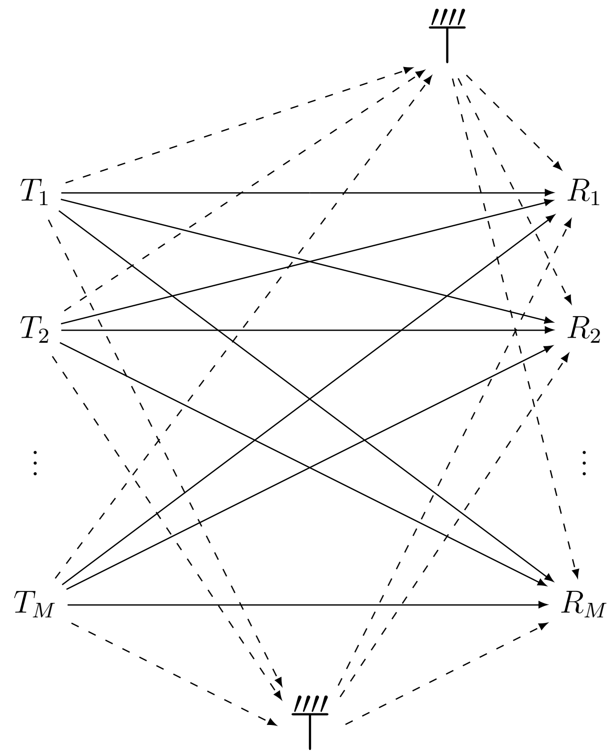

Figure 4.1: Using relays to assist interference alignment in M-user IC

denoted by $W_{r_u}(k)$ and $W_j(k)$ with variances $\omega_{r_u}^2$ and ω_j^2, respectively.

The physical channel between Transmitter i and Receiver j is characterized by a complex scalar gain h_{ij}. Similarly the complex scalar channel gains from Transmitter i to the uth receive antenna of the relay and also from the vth transmit antenna of the relay to Receiver j are denoted by h_{ir_u} and h_{r_vj}, respectively. We also assume that all the channel gains remain unchanged over a signal block whose duration is at least K time intervals. Using the above scheme, we can express the received signals in terms of the transmit signals and the complex channel scalars as

$$Y_{r_u}(k) = \sum_{i=1}^{M} h_{ir_u} X_i(k) + W_{r_u}(k), \tag{4.1}$$

$$Y_j(k) = \sum_{i=1}^{M} h_{ij} X_i(k) + W_j(k) + \sum_{l=1}^{k-1} \sum_{v=1}^{V} \sum_{u=1}^{U} h_{r_v j} \mathcal{G}_{uv}(k,l) Y_{r_u}(l). \tag{4.2}$$

The complex scalar coefficients $\mathcal{G}_{uv}(k,l)$ are used for scaling the signal that is received in the time slot l from the uth receive antenna of the relay and sent by the vth transmit antenna of the relay over the kth time slot.

Now if we extend the definition of a symbol to K time slots, the system can be viewed as a network of $K \times K$ MIMO users. To observe this, the signals from each K consecutive time slots are grouped to make a column vector, as is done for $X_i(k)$ in the following

$$X_i = \begin{bmatrix} X_i(1) \\ X_i(2) \\ \vdots \\ X_i(K) \end{bmatrix}. \tag{4.3}$$

The vectors Y_{r_u}, W_{r_u}, Y_j and W_j are also defined in the same way. It is now easy to determine H_{ij}, the equivalent channel matrix from Transmitter i to Receiver j. If $H_{ij}(k,l)$

is defined as the complex scalar element in Row k and Column l of H_{ij} then

$$H_{ij}(k,l) = \begin{cases} 0 & k < l \\ h_{ij} & k = l \\ \sum_{v=1}^{V}\sum_{u=1}^{U} h_{r_v j}\mathcal{G}_{uv}(k,l)h_{ir_u} & k > l \end{cases}, \tag{4.4}$$

where $k, l = 1, 2, \ldots, K$. Therefore, the equivalent MIMO channel matrix is lower triangular with equal diagonal entries. It is also noteworthy to emphasize that the relay gains are integrated into H_{ij}, and as it will be seen in the next sections, this involvement plays an important role in successfully solving the alignment equations. Using a similar approach, the equivalent noise vector in Receiver j can be determined to be

$$Z_j(k) = W_j(k) + \sum_{l=1}^{k-1}\sum_{v=1}^{V}\sum_{u=1}^{U} h_{r_v j}\mathcal{G}_{uv}(k,l)W_{r_u}(l). \tag{4.5}$$

To finish the characterization of the equivalent MIMO network, the input/output relations along with the encoding/decoding functions should be determined. Using the original model in (4.1) and (4.2) along with definitions for H_{ij} and Z_j (the vectorized form of $Z_j(k)$ as in (4.3)), we have

$$Y_j = H_{1j}X_1 + H_{2j}X_2 + \ldots + H_{Mj}X_M + Z_j. \tag{4.6}$$

Therefore, the time-extension scheme basically transforms the one-dimensional relations of the single-antenna network into a pseudo MIMO model. Moreover, the store-transform-forward relay converts the equivalent channels from purely diagonals into lower-triangular matrices.

The encoding-decoding functions are the same as the MIMO interference alignment schemes. The scalar transmit data stream, x_i, is encoded in T_i direction to form the signal for Transmitter i as below

$$X_i = T_i x_i. \tag{4.7}$$

Similarly, the received scalar data stream, y_i, is decoded by computing the projection of the received signal in R_i direction according to

$$y_i = R_i^H Y_i. \tag{4.8}$$

In the next sections, we will apply this model to determine the amount of achievable DOF for IC under quasi-static conditions.

4.2 Interference Alignment for M-User IC

Following the MIMO system model defined in the previous section, it is straightforward to determine and solve the interference alignment relations for M-user IC. To this end, we start from the 3-user IC whose closed-form solution has already been presented by other authors. The transmit and receive directions should be selected such that the following set of equations are satisfied

$$\text{Receiver 1:} \qquad R_1 \perp H_{21}T_2 \parallel H_{31}T_3, \tag{4.9}$$

$$\text{Receiver 2:} \qquad R_2 \perp H_{32}T_3 \parallel H_{12}T_1, \tag{4.10}$$

$$\text{Receiver 3:} \qquad R_3 \perp H_{13}T_1 \parallel H_{23}T_2. \tag{4.11}$$

Assuming that the channel matrices are non-singular, solving the equations simultaneously results in

$$T_1 \parallel LT_1, \tag{4.12}$$

where the loop gain matrix L is defined as

$$L \triangleq H_{13}^{-1}H_{23}H_{21}^{-1}H_{31}H_{32}^{-1}H_{12}. \tag{4.13}$$

This means that T_1 should be the eigenvector of L. Other directions can be easily determined from

$$T_2 \parallel H_{23}^{-1}H_{13}T_1, \tag{4.14}$$

$$T_3 \parallel H_{32}^{-1}H_{12}T_1, \tag{4.15}$$

$$R_1 \perp H_{21}H_{23}^{-1}H_{13}T_1, \tag{4.16}$$

$$R_2 \perp H_{12}T_1, \tag{4.17}$$

$$R_3 \perp H_{13}T_1. \tag{4.18}$$

Since all the channel matrices take the same form as in (4.4), L is a lower triangular with equal diagonal entries. It can be readily verified that choosing T_1 as the eigenvector of L results in all data and interference being in the same direction and thus non-separable[3]. This is where the importance of having a relay comes into the picture. Since the relay gains can change arbitrarily, they can be selected to form the equivalent channel structures in a

[3]A $K \times K$ lower triangular matrix with equal diagonal entries and non-zero elements in the lower triangle has K equal eigenvalues and only one linearly independent eigenvector. The first $K-1$ entries of the only eigenvector are all zeros. As a result, any data that is sent in the eigenvector direction ends up in the same one-dimensional space which is not enough for separating the intended data from interference at the receivers.

way that aligning the interference directions becomes feasible. To this end, the relay gains are adjusted such that L becomes a multiple of identity matrix. Using this approach, any arbitrarily chosen T_1 satisfies (4.12). The other directions are computed from T_1 as before and it can be seen that the resulting scheme is able to achieve a DOF of $\frac{3}{2}$.

A good thing about this approach is that it can be easily extended. For example, in a 4-user IC, alignment of the interference directions dictates that

$$\text{Receiver 1:} \qquad R_1 \perp H_{21}T_2 \parallel H_{31}T_3 \parallel H_{41}T_4, \tag{4.19}$$

$$\text{Receiver 2:} \qquad R_2 \perp H_{12}T_1 \parallel H_{32}T_3 \parallel H_{42}T_4, \tag{4.20}$$

$$\text{Receiver 3:} \qquad R_3 \perp H_{13}T_1 \parallel H_{23}T_2 \parallel H_{43}T_4, \tag{4.21}$$

$$\text{Receiver 4:} \qquad R_4 \perp H_{14}T_1 \parallel H_{24}T_2 \parallel H_{34}T_3. \tag{4.22}$$

If all the channel matrices are invertible, the set of equations for transmit directions in (4.19) to (4.22) are equivalent to

$$T_1 \parallel H_{13}^{-1}H_{23}T_2 \parallel H_{12}^{-1}H_{32}T_3 \parallel H_{12}^{-1}H_{42}T_4, \tag{4.23}$$

$$T_2 \parallel H_{23}^{-1}H_{13}H_{14}^{-1}H_{24}T_2, \tag{4.24}$$

$$T_3 \parallel H_{32}^{-1}H_{12}H_{14}^{-1}H_{34}T_3 \parallel H_{34}^{-1}H_{24}H_{21}^{-1}H_{31}T_3, \tag{4.25}$$

$$T_4 \parallel H_{42}^{-1}H_{12}H_{13}^{-1}H_{43}T_4 \parallel H_{43}^{-1}H_{23}H_{21}^{-1}H_{41}T_4. \tag{4.26}$$

Unlike the three-user IC in the traditional MIMO setting, it is not possible to satisfy all the equations simultaneously, as the number of equations is more than the available variables. If a MIMO relay is added to this time-extended scenario, it increases the number of free parameters needed to solve the equations altogether. To this end, T_1 is selected randomly and T_2, T_3 and T_4 are determined from the equations in (4.23). Next, the relay gains are adjusted such that the rest of equations in (4.24) to (4.26) are satisfied by forcing the involved matrices to become multiples of identity matrix. Finally, when all the transmit directions satisfying the alignment equations are found, the receive direction in each receiver can be easily determined by computing the vector that is orthogonal to the interference direction in that receiver.

This approach can be easily generalized for more users. In M-user IC there are $M(M-2)$ alignment equations which can be divided into two categories. The first $M-1$ equations determine all the transmit directions based on T_1. The rest are $M(M-2)-(M-1)$ equations which need to be satisfied using something other than choosing the transmit directions (for example relays in the existing context). In the next section, we will determine the amount of required complexity to achieve this.

4.2.1 DOF Achievability Equations

To ensure that there is enough freedom in the selection of the relay gains, we need to have more flexibility than what one single-antenna relay can provide. To this end, we will use a MIMO relay with time-extension of two ($K = 2$). In this scheme, during the first time slot, the main transmitters send data while the receivers as well as the relays are listening. In the second time slot, both main transmitters and relays send information to the receivers. Therefore, using $K = 2$ assumption results in a half duplex relay which has much less implementation complexity. Using this approach, the indices k and l can be dropped from the relay coefficients, $\mathcal{G}_{uv}(k, l)$. The equivalent MIMO channel matrix from Transmitter i

to Receiver j can be easily determined as

$$H_{ij} = h_{ij}\left(I - \epsilon_{ij} Q\right), \tag{4.27}$$

where Q is

$$Q = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \tag{4.28}$$

and ϵ_{ij} is defined as below

$$\epsilon_{ij} = \frac{-1}{h_{ij}} \sum_{v=1}^{V} \sum_{u=1}^{U} h_{r_v j} \mathcal{G}_{uv} h_{i r_u}. \tag{4.29}$$

Therefore, noticing that $Q^2 = \mathbf{0}$, the loop-gain matrix for the 3-user IC in (4.13) can be rewritten as

$$L = \frac{h_{23} h_{31} h_{12}}{h_{13} h_{21} h_{32}} \Big(I + \left(\epsilon_{13} - \epsilon_{23} + \epsilon_{21} - \epsilon_{31} + \epsilon_{32} - \epsilon_{12}\right) Q \Big). \tag{4.30}$$

As a result, in order to make L a multiple of identity matrix, it is sufficient to force the complex scalar factor before Q to become zero as below

$$\epsilon_{13} - \epsilon_{23} + \epsilon_{21} - \epsilon_{31} + \epsilon_{32} - \epsilon_{12} = 0. \tag{4.31}$$

Based on (4.29), ϵ_{ij} is a linear combination of the relay gains and the condition in (4.31) can be further simplified into a weighted sum of $\mathcal{G}_{uv}$. The equivalent equation has non-zero solutions as long as the number of relay gains is more than one. Similarly, in the 4-user IC, the required conditions in (4.24) to (4.26) can be easily converted into five linear equations in terms of the relay gains, thus requiring at least six independent variables to ensure non-zero solutions.

Using the same approach it can be shown that the number of relay gains required to satisfy all the aligning relations for M-user IC is

$$M(M-2)-(M-1)+1=(M-1)(M-2). \tag{4.32}$$

For a MIMO relay, the number of relay gains would be the product of the number of antennas for transmit and receive (or UV). Therefore, as long as $UV \geq (M-1)(M-2)$, the directions that satisfy all the equations can be found.

Additionally, multiple single-antenna relays can be used for this scheme. In this case, $U = V$ and $\mathcal{G}_{uv}$ is non-zero for $u = v$ only ($v = 1, 2, \ldots, V$). This means that at least $(M-1)(M-2)$ single antenna relays are needed to provide M DOF in a double time slot frame.

4.3 Asymptotic Analysis of the Relay Gains in IC

Previous sections showed that the M-user IC can easily provide $\frac{M}{2}$ DOF as long as a MIMO relay is added to the network. The relay operates in two phases. During the first phase it captures the signals from the main transmitters, and during the second phase it sends the stored information to the receivers. An aspect that might be missed during this process is the amount of power that is needed by the relay in order to ensure DOF achievability. To this end, we assume that all the transmit and receive directions are scaled such that their norm is one. Moreover, it is obvious that if $\mathcal{G}_{uv}$ is a solution, any scaled version of the relay coefficients such as $g\mathcal{G}_{uv}$ also satisfies all the relations. As a result, if the latter solution for the relay coefficients is used, ϵ_{ij} in (4.29) is replaced with $g\epsilon_{ij}$ resulting in the new family of channel matrices as below

$$\hat{H}_{ij} = h_{ij} \left(I - g\epsilon_{ij} Q \right), \tag{4.33}$$

Therefore, g can be used as the relay gain and is a measure of the ratio between the relay's transmit power to the receive power of the relay. The equivalent scalar gain from Transmitter i to Receiver i can be written as (j and k are two arbitrarily selected numbers such that $j \neq i$ and $k \neq i, j$)

$$\begin{aligned} R_i^H \hat{H}_{ii} T_i &\overset{a}{=} R_i^H \hat{H}_{ji} \left(\hat{H}_{ji}^{-1} \hat{H}_{ii} \hat{H}_{ik}^{-1} \hat{H}_{jk} \right) T_j \\ &= R_i^H \hat{H}_{ji} \frac{h_{ii} h_{jk}}{h_{ji} h_{ik}} \Big(I + g \left(\epsilon_{ji} - \epsilon_{ii} + \epsilon_{ik} - \epsilon_{jk} \right) Q \Big) T_j \\ &\overset{b}{=} g \frac{h_{ii} h_{jk}}{h_{ji} h_{ik}} \left(\epsilon_{ji} - \epsilon_{ii} + \epsilon_{ik} - \epsilon_{jk} \right) R_i^H \hat{H}_{ji} Q T_j \\ &\overset{c}{=} g \frac{h_{ii} h_{jk}}{h_{ik}} \left(\epsilon_{ji} - \epsilon_{ii} + \epsilon_{ik} - \epsilon_{jk} \right) R_i^H T_j \\ &\triangleq g \gamma_i, \end{aligned} \tag{4.34}$$

where (a) is due to the alignment property $\hat{H}_{ik} T_i = \hat{H}_{jk} T_j$, ($b$) uses the relation $R_i^H \hat{H}_{ji} T_j = 0$ and (c) is because of $Q^2 = \mathbf{0}$. γ_i is also a constant that does not depend on g. Similarly, σ_j, the equivalent noise variance in Receiver j can be written as

$$\begin{aligned} \sigma_i^2 &= \mathcal{E}\{ Z_i^H R_i R_i^H Z_i \} \\ &= R_i^H \begin{bmatrix} \omega_i^2 & 0 \\ 0 & \omega_i^2 + g^2 \sum_{v=1}^{V} \sum_{u=1}^{U} |h_{r_v j}|^2 |\mathcal{G}_{uv}|^2 \omega_{r_u}^2 \end{bmatrix} R_i \\ &\triangleq \alpha_i + g^2 \beta_i, \end{aligned} \tag{4.35}$$

where α_i and β_i are positive real constants not depending on g. Finally, using (4.34) and (4.35) the asymptotic rate between Transmitter i and Receiver i as a function of relay gain is determined to be

$$R_i(g) = \log\left(1 + \frac{g^2|\gamma_i|^2 P}{\alpha_i + g^2\beta_i}\right), \tag{4.36}$$

where P is the transmit power which is assumed to be equal among all transmitters[4]. A necessary condition for this step to be valid is that the rate as a function of g should satisfy the conditions of the Lebesgue's dominated convergence theorem. Since we are looking into the sum-capacity of parallel Gaussian channels with single-antenna nodes it can be verified that the derivations are valid.

The DOF-achieving relay gains are a group of functions $g = f(P)$ such that

$$\lim_{P\to\infty} \frac{R_i\left(f(P)\right)}{\log P} = 1. \tag{4.37}$$

Moreover if $g = f(P) = \frac{1}{(\log P)^t}$ we have

$$\begin{aligned}\lim_{P\to\infty} \frac{R_i\left(f(P)\right)}{\log P} &= \lim_{P\to\infty} \frac{\log\left(1 + \frac{(\log P)^{-2t}|\gamma_i|^2 P}{\alpha_i + (\log P)^{-2t}\beta_i}\right)}{\log P}\\ &= \lim_{P\to\infty} \frac{\log\left(\alpha_i + (\log P)^{-2t}\beta_i + (\log P)^{-2t}|\gamma_i|^2 P\right) - \log\left(\alpha_i + (\log P)^{-2t}\beta_i\right)}{\log P}\\ &= 1. \end{aligned} \tag{4.38}$$

As it can be seen the dominant terms for a large P do NOT depend on the parameter t and therefore setting $g = \frac{1}{(\log P)^t}$ can ensure achieving the DOF for both positive and negative t's. This means that the relay gain can go to zero (for $t < 0$) or infinity (for $t > 0$) and

[4]Optimizing the output powers for each transmitter individually can only enhance the sum-rate by a constant amount and does not offer any benefits from DOF point of view.

DOF is still achieved. The output power of the relay in this case, can be easily determined to scale as $\frac{P}{(\log P)^t}$. Therefore, depending on the sign of t, the output power of the relay can rise at a slower/faster rate than the power of the main transmitters and the DOF of the system is not affected.

4.4 Conclusion

In this chapter we analyzed the benefits of adding relays to a fully connected M-user IC. We showed that adding a relay to a network with quasi-static channels can increase the achievable DOF by providing additional freedom to solve zero-forcing equations. We also derived the required scaling for relay gains to guarantee achieving the DOF for M-user IC.

Chapter 5

Interference Alignment for Full-Duplex Networks

As shown in the previous chapters, Interference Alignment (IA) becomes easier with the introduction of a few relays to the network. The relays add free dimensions that can be utilized during the signal design. The results from the last two chapters show that in order to perfectly align the directions, the number of required single-antenna relays (or the number of antennas in the MIMO relay) should increase with the number of users in the system. The additional hardware costs and complexity can limit the performance benefits offered by the relays. As a result, it is always favorable to minimize the number of elements in a network.

A question that arises from the study of the relay-aided schemes is whether the transmitters or receivers can act as relays or not. Examining the effects that a relay has when operating in a network, it is shown that relays can be easily impersonated by a full duplex transmitter or receiver. According to the transmission schemes of the relay-aided alignments, the relays send a linear combination of what they receive. The relay transmit data is a weighted sum of all the transmit information which is un-decodable by itself. The signal that a full-duplex transmitter hears is exactly the same thing. On the relay-aided schemes, the relay chooses a particular transform that makes the alignment much easier. The full-duplex transmitters could do the same thing. Finally, the relays add cooperation to the (otherwise blind) transmitters. The full-duplex transmitters can also support co-operation to beamform the signals at subsequent transmissions. All of these hints suggest that the full-duplex property in a network can replace the relay tasks. This chapter will show that this is in fact true.

The chapter describes a new method for IA which takes advantage of the full-duplex property of the transmitters. Using the proposed scheme, it is shown that the total Degrees of Freedom (DOF) can be as high as $\sqrt{\frac{N}{2}}$ for N-user Interference Channel (IC). This amount is less than the well-known upper limit of $\frac{N}{2}$, but it is remarkably easier to achieve. This chapter is organized as follows. Section 5.1, describes the system model. The transmission scheme and alignment requirements are presented in Sections 5.2 and 5.3 respectively. Section 5.4, studies two similar network structures that can benefit from the same ideas. Achievability of the results is discussed in Section 5.5 and finally, the summary is presented in Section 5.6.

5.1 System Model

A fully connected network of N transmitters along with N receivers is considered. Transmitter i wishes to send an independent data stream to Receiver i for $i = 1, 2, \ldots, N$. It is also assumed that all the transmitters are full-duplex and have the capability of listening to the signals sent by others during their own transmission period. All the transmitters and receivers are single antenna.

The communication medium between all the nodes is assumed to be quasi-static. The scalar channel coefficients from Transmitter j to Transmitter i and from Transmitter j to Receiver i are denoted by g_{ij} and h_{ij} respectively ($i, j = 1, 2, \ldots, N$). The $N \times N$ transmitter-to-transmitter channel gain matrix, G, is constructed from these scalar coefficients by putting g_{ij} in the ith row of the jth column. The transmission scheme presented in the subsequent sections does not make any use of the reciprocity property of the channel. Therefore, the results of this chapter are valid regardless of g_{ij} being equal to g_{ji} or not. The $N \times N$ transmitter-to-receiver channel gain matrix, H, is also built from h_{ij} in a similar way.

The signal sent by Transmitter j in time slot k is denoted by $t_k(j)$ ($k = 1, 2, \ldots, K$). After going through the channel, the corrupted signal that is received by the Transmitter i is named $q_k(i)$. Similarly, Receiver i collects $r_k(i)$ in time slot k. Finally, the corresponding Additive White Gaussian Noise (AWGN) that is picked up by the ith transmitter as well as Receiver i are defined as $u_k(i)$ and $v_k(i)$ respectively. Using these definitions, the signals received during time slot k by the Transmitter i and ith Receiver can be described as

$$q_k(i) = \sum_{j=1}^{N} g_{ij} t_k(j) + u_k(i) \tag{5.1}$$

$$r_k(i) = \sum_{j=1}^{N} h_{ij} t_k(j) + v_k(i). \tag{5.2}$$

In order to characterize the equivalent channel model in the matrix form, the column vector

T_k is defined as below

$$T_k = \begin{bmatrix} t_k(1) \\ t_k(2) \\ \vdots \\ t_k(N) \end{bmatrix}. \tag{5.3}$$

Column vectors Q_k, R_k, U_k and V_k are also defined accordingly. Using these definitions, the channel input/output model in vectorized form can be written as

$$Q_k = G{\cdot}T_k + U_k \tag{5.4}$$

$$R_k = H{\cdot}T_k + V_k. \tag{5.5}$$

Next section describes the transmission scheme to be used with this model.

5.2 Communication Scheme

The fully connected network of N transmitter-receiver pairs with the input-output channel model described in (5.4) and (5.5) can be perfectly decoupled into N independent parallel channels by forcing the unwanted interference terms to zero. As explained in the system model, there are N independent scalar data streams to be sent from Transmitter i to Receiver i (one data stream for each transmitter-receiver pair). Data for each user is generated from an independent Gaussian source and is represented by $x(i)$ for ith transmitter. Similar to (5.3), the column vector X is defined based on individual data samples from different transmitters.

The vectorized transmit signal at time slot k is generated as a linear combination of X

(for simplicity of representation $Q_0 \triangleq X$), and all the received signals from previous time slots as below

$$T_k = \sum_{m=0}^{k-1} D_{k,m} \cdot Q_m, \tag{5.6}$$

where $D_{k,m}$ represents a diagonal matrix whose ith diagonal entry is the coefficient that the ith transmitter uses for scaling the received signal from time slot m for transmission at kth time slot. It should be emphasized that the diagonal property of $D_{k,m}$ is enforced by the fact that Transmitter i can only have access to its own data ($x(i)$) and previous received signals ($q_m(i)$ for $m = 1, 2, \ldots, k-1$). Therefore the scaling coefficients for $q_m(i')$ ($i' \neq i$) have to be zero.

After K uses of the channel, the receivers have collected K signals as defined in (5.5). The original transmit signal, X, will be decoded in the receivers by computing a linear combination of the R_k's as below

$$\hat{X} = \sum_{m=1}^{K} D_{K+1,m} \cdot R_m. \tag{5.7}$$

Similar to (5.6), the involved matrices, $D_{K+1,m}$, are diagonal ($m = 1, 2, \ldots, K$). The ith diagonal entry in $D_{K+1,m}$ determines how the signal from time slot m on the Receiver i should be scaled in the linear combination for decoding the original transmit information or $x(i)$.

The following lemma shows that the decoded signal, $\hat{X}$, is actually a noisy linear transformation of X. This result can be used for removing the unwanted interference terms. The linear transformation depends on the scaling coefficients, $D_{k,m}$. Therefore a suitable set of these coefficients should be selected in order to recover the original transmit information contained in X. In other words, the recursive use of the equations in (5.4), (5.5) and (5.6) along with (5.7) for the last stage could yield to a structure that helps removing the unwanted interference terms.

Lemma. *The term T_k in the following set of equations*

$$
\begin{aligned}
Q_0 &= X \\
T_1 &= D_{1,0} \cdot Q_0 \\
Q_1 &= G \cdot T_1 + U_1 \\
T_2 &= D_{2,0} \cdot Q_0 + D_{2,1} \cdot Q_1 \\
Q_2 &= G \cdot T_2 + U_2 \\
T_3 &= D_{3,0} \cdot Q_0 + D_{3,1} \cdot Q_1 + D_{3,2} \cdot Q_2 \\
&\vdots \\
Q_{k-1} &= G \cdot T_{k-1} + U_{k-1} \\
T_k &= D_{k,0} \cdot Q_0 + D_{k,1} \cdot Q_1 + \ldots + D_{k,k-1} \cdot Q_{k-1},
\end{aligned}
$$

can always be expressed as

$$T_k = P_k \cdot X + W_k, \tag{5.8}$$

regardless of the values of X, G and $D_{k,m}$. Additionally, P_k and W_k can be recursively determined from the following

$$
\begin{aligned}
P_k &= D_{k,0} + \sum_{m=1}^{k-1} D_{k,m} \cdot G \cdot P_m \\
W_k &= \sum_{m=1}^{k-1} D_{k,m} \cdot \Big(G \cdot W_m + U_m \Big).
\end{aligned} \tag{5.9}
$$

Finally, after K uses of the channel, the decoded sequence, $\hat{X}$, as defined in (5.7) *is related* *to X through the following equation*

$$\hat{X} = \mathcal{P}\cdot X + \mathcal{W}, \tag{5.10}$$

where $\mathcal{P}$ and $\mathcal{W}$ are characterized by

$$\begin{aligned} \mathcal{P} &= \sum_{m=1}^{K} D_{K+1,m}\cdot H\cdot P_m \\ \mathcal{W} &= \sum_{m=1}^{K} D_{K+1,m}\cdot\Big(H\cdot W_m + V_m\Big). \end{aligned} \tag{5.11}$$

Proof. The proof is easily done by induction. For $k = 1$, T_1 can be written as

$$\begin{aligned} T_1 &= D_{1,0}\cdot Q_0 \\ &= P_1\cdot X + W_1. \end{aligned}$$

Moreover, assuming that the lemma holds for all the values up to $k-1$ (inclusive), T_k can be expressed as

$$\begin{aligned} T_k &= \sum_{m=0}^{k-1} D_{k,m}\cdot Q_m \\ &= D_{k,0}\cdot X + \sum_{m=1}^{k-1} D_{k,m}\cdot\Big(G\cdot T_m + U_m\Big) \\ &= D_{k,0}\cdot X + \sum_{m=1}^{k-1} D_{k,m}\cdot\left(G\cdot\Big(P_m\cdot X + W_m\Big) + U_m\right) \\ &= \left(D_{k,0} + \sum_{m=1}^{k-1} D_{k,m}\cdot G\cdot P_m\right)\cdot X + \left(\sum_{m=1}^{k-1} D_{k,m}\cdot\Big(G\cdot W_m + U_m\Big)\right) \\ &= P_k\cdot X + W_k. \end{aligned}$$

The second part of this lemma which determines the relation between $\hat{X}$ and X follows directly from the definitions in (5.5) and (5.7) applied to (5.8) and (5.9). □

According to the lemma, the described scheme relates $\hat{X}$ to X through the matrix $\mathcal{P}$. If the Signal to Noise Ratio (SNR) is high, the effect of $\mathcal{W}$, the additive Gaussian noise part of $\hat{X}$, can be neglected and it is possible to use the zero-forcing method. To this end, $D_{k,m}$ diagonal matrices are chosen such that all the entries in $\mathcal{P}$ that correspond to interference terms are zero. As a result, estimating X from $\hat{X}$ is possible by forcing $\mathcal{P}$ to become a diagonal matrix. If this condition can be met, the reconstructed signal in Receiver i solely depends on the original data send by Transmitter i. Since this is true for all the transmitter-receiver pairs, N independent units of information are transmitted simultaneously in K time slots, yielding to a total DOF of $\frac{N}{K}$.

Next section looks into the requirements on $D_{k,m}$ that ensure being able to make $\mathcal{P}$ a diagonal matrix.

5.3 Alignment Coefficients

The set of equations to diagonalize $\mathcal{P}$ depend on the diagonal entries of $D_{k,m}$ matrices. It is, however, not clear how many coefficients can be chosen independently and yield a different solution (i.e. a solution which is not obtainable by scaling of another one). To this end, the next three lemmas can be used for determining the requirements on selection of the coefficients.

Lemma. *If a set of $D_{k,m}$ coefficients correspond to $\mathcal{P}$ through the equations in* (5.9) *and* (5.11) *then for any arbitrary matrix such as* A*,*

$$D'_{k,m} = \begin{cases} D_{k,m} \cdot A & k = 1, 2, \ldots, K \textit{ and } m = 0 \\ D_{k,m} & \textit{otherwise} \end{cases}$$

correspond to $P'_k = P_k \cdot A$ *and* $\mathcal{P}' = \mathcal{P} \cdot A$*.*

Proof. Similar to the previous lemma, proof is done by induction. Lemma holds for P'_1 as

$$\begin{aligned} P'_1 &= D'_{1,0} \\ &= D_{1,0} \cdot A \\ &= P_1 \cdot A. \end{aligned}$$

Also if the lemma statement is true for all the values up to $k-1$ then

$$\begin{aligned} P'_k &= D'_{k,0} + \sum_{m=1}^{k-1} D'_{k,m} \cdot G \cdot P'_m \\ &= D_{k,0} \cdot A + \sum_{m=1}^{k-1} D_{k,m} \cdot G \cdot P_m \cdot A \\ &= \left(D_{k,0} + \sum_{m=1}^{k-1} D_{k,m} \cdot G \cdot P_m \right) \cdot A \\ &= P_k \cdot A. \end{aligned}$$

Proving the relation for $\mathcal{P}$ follows directly from the definition in (5.11). □

Lemma. *If* P_k *matrices are generated by* $D_{k,m}$ *coefficients, then for any arbitrary matrix*

B, the new coefficients $D''_{k,m}$ defined as

$$D''_{k,m} = \begin{cases} B{\cdot}D_{k,m} & k = K+1 \textit{ and } m = 1, 2, \ldots, K \\ D_{k,m} & \textit{otherwise} \end{cases}$$

can be used to generate $\mathcal{P}'' = B{\cdot}\mathcal{P}$.

Proof. It directly follows from the definition of matrix $\mathcal{P}$ in (5.11). □

Lemma. *If $D_{k,m}$ coefficients construct P_k matrices, then for any arbitrary scalar c and any integer index $l = 2, 3, \ldots, K$*

$$D'''_{k,m} = \begin{cases} cD_{k,m} & k = l \textit{ and } m = 0, 1, \ldots, l-1 \\ \frac{1}{c}D_{k,m} & m = l \textit{ and } k = l{+}1, l{+}2, \ldots, K{+}1 \\ D_{k,m} & \textit{otherwise} \end{cases}$$

can construct P'''_k and $\mathcal{P}'''$ matrices such that

$$P'''_k = \begin{cases} cP_k & k = l \\ P_k & \textit{otherwise} \end{cases}$$

$$\mathcal{P}''' = \mathcal{P}$$

Proof. It is obvious that for $k = 1, 2, \ldots, l-1$, $P'''_k = P_k$. Also

$$\begin{aligned} P'''_l &= D'''_{l,0} + \sum_{m=1}^{l-1} D'''_{l,m} \cdot G \cdot P'''_m \\ &= cD_{l,0} + \sum_{m=1}^{l-1} cD_{l,m} \cdot G \cdot P_m \\ &= cP_l \end{aligned}$$

and for $k = l+1, l+2, \ldots, K$,

$$\begin{aligned} P'''_k &= D'''_{k,0} + \sum_{m=1}^{k-1} D'''_{k,m} \cdot G \cdot P'''_m \\ &= D_{k,0} + \sum_{\substack{m=1 \\ m \neq l}}^{k-1} D_{k,m} \cdot G \cdot P_m + cD_{k,l} \cdot G \cdot \frac{1}{c} P_l \\ &= P_k \end{aligned}$$

The proof for $\mathcal{P}$ is done in the same way. □

According to the lemmas, if $A = D_{1,0}^{-1}$ and $B = D_{K+1,K}^{-1}$, a set of coefficients that correspond to a diagonal $\mathcal{P}$ can be converted into another set, $D^*_{k,m}$, where $D^*_{1,0} = D^*_{K+1,K} = I$ and $\mathcal{P}^* = B \cdot \mathcal{P} \cdot A$ which is also diagonal (considering the fact that A and B are both diagonal matrices).

Therefore, without any loss in generality, it is assumed that three out of all the $D_{k,m}$ coefficients ($k = 1, 2, \ldots, K+1$ and $m = 0, 1, \ldots, k-1$) are preset matrices whose values can not be changed ($D_{1,0}$ and $D_{K+1,K}$ are Identity matrices and $D_{K+1,0}$ that is never used in the equations is an all-zero matrix). The rest of the coefficients (i.e. a total of $\frac{(K+1)(K+2)}{2} - 3$ diagonal matrices) can be selected (almost) freely to satisfy the alignment conditions.

The last lemma shows that there is one more internal dependency between the coefficients that should be taken out. Defining the scalars $c_l = \sqrt{\frac{N}{\text{Trace}(D_{l,0} \cdot D_{l,0}^H)}}$ for $l = 2, 3, \ldots, K$, and applying the last lemma repeatedly results in a new set of coefficients named $D_{k,m}^{\dagger}$ that correspond to the same $\mathcal{P}$ but are forced to have the following additional relations

$$\text{Trace}(D_{l,0}^{\dagger} \cdot D_{l,0}^{\dagger^H}) = N. \tag{5.12}$$

Having all the missing pieces together, it is now possible to characterize the total DOF with respect to N, the number of users. In order to make $\mathcal{P}$ diagonal, $N^2 - N$ equations need to be satisfied. There are also $K - 1$ additional conditions in (5.12) which make the coefficients unique. Considering the fact that each diagonal coefficient matrix, $D_{k,m}$, has N independent scalar variables that can be independently modified to satisfy the equations, a necessary condition to have a feasible solution is

$$N^2 - N + K - 1 < N\left(\frac{(K+1)(K+2)}{2} - 3\right). \tag{5.13}$$

Reordering the terms yields to

$$\left(K + \frac{3}{2} - \frac{1}{N}\right)^2 > 2N + \frac{17}{4} - \frac{5}{N} + \frac{1}{N^2}. \tag{5.14}$$

Therefore, $\frac{N}{K}$, the number of total DOF achievable by this scheme, can be upper bounded by

$$\frac{N}{K} < \frac{N\sqrt{\frac{N}{2} + \frac{17}{16} - \frac{5}{4N} + \frac{1}{4N^2}} + \frac{3N}{4} - \frac{1}{2}}{N + 1 - \frac{1}{N}}. \tag{5.15}$$

Reiterating this result, any N-user IC, can transfer N independent data streams (one for each transmitter-receiver pair) in K time slots as long as N and K satisfy the inequality in (5.15). The limit on the total achievable DOF scales with $\sqrt{\frac{N}{2}}$ for large number of users. The theoretical limit on the total DOF is $\frac{N}{2}$ and although a few methods have already been proposed to achieve that limit, they are still too complex and require a large number of dimensions to be close enough to $\frac{N}{2}$. The scheme presented here has a smaller upper limit but is much more efficient for getting close to that bound. Table 5.1 contains a comparison between the proposed scheme and two of these methods.

5.4 Other Network Topologies

The benefits offered by the full-duplex property of the nodes are not limited to the exact structure to which the proposed scheme has been applied. The next two sections show that the same communication scheme can be utilized in other networks.

5.4.1 N-User IC with Full-Duplex Transmitters and Receivers

If the receivers are able to send and listen simultaneously, they can play the same role as the full-duplex transmitters and help to achieve even a higher DOF. In this case the network is treated as having $2N$ nodes (N transmitters and N receivers). The $2N \times 2N$ matrix G is defined from the scalar channel coefficients between every two nodes. The $2N$ dimensional column vector T_k is defined as the transmit signal during the kth time slot. The first N elements of T_k correspond to the signals sent by the N transmitters and last N elements correspond to that of the N receivers. The same definition applies to Q_k which contains the received signals from $2N$ nodes at time slot k. The N independent scalar

Table 5.1: A comparison between three methods for N-user IC, $N = 7$ is used for numerical values.

	Parameter n^a which controls how close DOF can get to $\frac{N}{2}$		1	2	4	9	27
Cadambe et.al. [12]	Number of freq. slots	$(n+1)^{(N-1)(N-2)-1} + n^{(N-1)(N-2)-1}$	5.4×10^{8}	6.9×10^{13}	1.9×10^{20}	1.0×10^{29}	1.3×10^{42}
	Achievable DOF	$\frac{(n+1)^{(N-1)(N-2)-1}+(N-1)n^{(N-1)(N-2)-1}}{(n+1)^{(N-1)(N-2)-1}+n^{(N-1)(N-2)-1}}$	1.00	1.00	1.01	1.22	2.29
Motahari et.al. [43]	Number of dimensions	$n^{N-1}(n+1)^{(N-1)^2} + (n+1)^{N(N-1)}$	4.5×10^{12}	1.2×10^{20}	2.9×10^{29}	1.5×10^{42}	1.1×10^{61}
	Achievable DOF	$\frac{N}{1+\left(\frac{n+1}{n}\right)^{N-1}+\frac{1}{n^{N-1}(n+1)^{(N-1)^2}}}$	0.11	0.56	1.45	2.43	3.12
Proposed Scheme	Number of time slots	$\left\lceil \sqrt{2N+\frac{17}{4}-\frac{5}{N}+\frac{1}{N^2}}-\frac{3}{2}+\frac{1}{N} \right\rceil$			3		
	Achievable DOF	$\frac{N}{\left\lceil \sqrt{2N+\frac{17}{4}-\frac{5}{N}+\frac{1}{N^2}}-\frac{3}{2}+\frac{1}{N} \right\rceil}$			2.33		

[a] Parameter n determines the trade off between the complexity of the solution and closeness to the optimum DOF. Larger values for n correspond to higher number of dimensions that lead to better alignment opportunities as well as a closer DOF to the upper limit.

information that should be sent are also put in the first N elements of the $2N$ dimensional column vector X and finally, the last N elements of X are set to zero. Based on these definitions, the channel input-output model can be described as in (5.4).

Similar to (5.6), on each time slot, all the $2N$ nodes send a weighted sum of the signals received in the previous time slots. Since the receiver nodes do not have the original information, their corresponding scales (the last N elements on the main diagonal of D_{k0}) should be zero. After K time slots, $\hat{X} \triangleq T_{K+1}$ and X are related through (5.8) with P_{K+1} characterized by (5.9). Therefore, in order to recover the original information, the $N \times N$ sub-matrix in the bottom left corner of P_{K+1} has to be diagonal.

Working through the number of variables as well as the number of equations the following inequality can be written as

$$N^2 - N + K - 1 < 2N\frac{K(K-1)}{2} + 2N(K-1). \tag{5.16}$$

After simplification, the $\frac{N}{K}$ which is the total achieved DOF for this scheme is upper bounded by

$$\frac{N}{K} < \frac{N\sqrt{N + \frac{5}{4} - \frac{3}{2N} + \frac{1}{4N^2}} + \frac{N}{2} - \frac{1}{2}}{N + 1 - \frac{1}{N}}, \tag{5.17}$$

which states that the bound on the DOF scales with $\sqrt{N}$ for large number of users (an improvement by a factor of $\sqrt{2}$ with respect to the original scheme).

5.4.2 N-User Wireless Ring with Full-Duplex Nodes

The wireless ring is a fully connected network with N numbered nodes. It is in fact a special IC where transmitters and receivers are actually the same nodes and the indexes of the intended receivers are a derangement[1] of the indexes for their corresponding transmitters. It is always possible to re-label the nodes in such a way that for $i = 1, 2, \ldots, N-1$, User i shares a piece of information with user $i+1$ and User N sends its data to the first indexed user. Throughout this section, the latter description for the data flow is used for determining the alignment requirements.

Investigating the equations for the IC with full-duplex transmitters reveals the similarities between the two models. The channel input-output relation for wireless ring is represented by (5.4). Additionally, the same equation in (5.6) is used for describing the transmission scheme. Finally, after K time slots, all the nodes have a linear combination of their original transmit signals characterized by P_{K+1} in (5.8) and (5.9). In order to make the information from the transmitters decodable at their intended receivers, $(P_{K+1})_{ij}$, the element at Row i and Column j of P_{K+1} should satisfy the following conditions

$$(P_{K+1})_{ij} = \begin{cases} \text{Any Value} & j = i \\ \neq 0 & j = i - 1 \bmod N \\ 0 & \text{otherwise} \end{cases} . \tag{5.18}$$

As a result, the inequality that makes the underlying system of equations under-determined can be written as

$$N^2 - 2N + K - 1 < N\left(\frac{(K+1)(K+2)}{2} - 3\right), \tag{5.19}$$

which is very similar to (5.13) and yields to the following upper bound for the achievable

[1] A derangement is a permutation in which none of the objects appear in their 'natural' (i.e., ordered) place. For example, the only derangements of $(1, 2, 3)$ are $(2, 3, 1)$ and $(3, 1, 2)$.

DOF

$$\frac{N}{K} < \frac{N\sqrt{\frac{N}{2} + \frac{9}{16} - \frac{5}{4N} + \frac{1}{4N^2}} + \frac{3N}{4} - \frac{1}{2}}{N - \frac{1}{N}}. \tag{5.20}$$

5.5 Solution Availability

The previous sections described the equations relating the transmit coefficients to the alignment requirements. A necessary condition in (5.13) was also presented which was then used to find an upper bound for the DOF using the proposed scheme. Proving that the upper bound is in fact achievable is tied to establishing whether the underlying system of equations is solvable or not. Each of the equations is a multivariate polynomial whose degree is at most $K-1$. As a result if $K > 2$, the set of equations becomes non-linear and the proof for existence of a solution is not straightforward. In [65], the authors use results from Algebraic Geometry to prove that a similar set of non-linear equations are solvable almost surely. In their proof, every equation is treated as an Algebraic Curve and therefore determining a solution for the set of equations leads to finding the intersection of all the corresponding curves. Finally, Bézout theorem and its extensions [30, 19] are applied to prove that there are a minimum number of points that lie on all the curves.

It is believed that a mathematical proof similar to that of [65] is possible for the set of equations developed in this chapter. It should also be noted that the condition in (5.13) actually makes the underlying system of equations under-determined (the number of unknowns is more than the number of equations) and an under-determined system has infinite number of solutions most of the time. Assuming that a solution actually exists, there are a number of methods (mostly iterative) to find an approximate set of coefficients that is close to the solution with respect to any given resolution. Since all the equations are polynomials, the Jacobian Matrix is well-defined for all values. Therefore, both Newton and Secant methods [5] can be used for finding the coefficients. Appendix B describes a reformulation of the recursive equations in (5.9) and (5.11) which is better suited for iterative algorithms. A MATLAB source code is also included that solves the system of equations using Newton approach.

5.6 Conclusion

This chapter studies the benefits offered by the use of full-duplex nodes in the context of IA. If the transmitters can listen to what others are sending, they can use that information cooperatively for interference removal. Using the zero-forcing method, it is shown that the total DOF for N-user IC can scale with $\sqrt{\frac{N}{2}}$.

Chapter 6

Summary and Future Work

In the last few chapters the benefits of introducing a relay to a wireless network has been studied. It is shown that a simple relay that just records what it receives and then sends a linear combination of those signals can be very helpful for achieving a higher Degrees of Freedom (DOF) with a much easier scheme. What makes the scheme even more useful is the fact that the output power of the relay can be adjusted based on the applications. In other words, it is possible to setup a wireless network with battery operated mobile users and add one or more fixed relays that are permanently plugged into power sources to induce the changes needed in the system in order to easily perform Interference Alignment (IA). Reversely, a network of fixed base stations in a quasi-static medium can benefit from a simple battery-operating relay that enables the network to offer more DOF. In all of these examples the linear coefficients in the relays become a part of the equivalent channel structure and thus can be adjusted such that easier and more efficient IA becomes possible.

Using the insight from the first few chapters, it was shown in Chapter Five that full-duplex transmitters can act as relays. Full-duplex transmission actually provides cooperation between transmitters which is then used for suppressing the interference terms. Although not proved explicitly, the same cooperation is possible if receivers are assumed to have the full-duplex transmission capability. The communication scheme for the full-duplex nodes is very similar to that of the relays. They should listen to what is being communicated and then send linear combinations of the stored signals (along with the original signals) over the future transmissions. The positive side of the proposed method is the amount of total DOF that can be achieved ($\sqrt{\frac{N}{2}}$ for N-user IC), which is more than the other methods based on MIMO zero-forcing. Unfortunately the underlying non-linear system of equations that should be solved does not have a closed-form solution and thus numerical methods must be used.

6.1 Future Research Directions

All the results in this thesis have been developed under infinite Signal to Noise Ratio (SNR) assumption. Large SNR values simplify the throughput equations greatly and along with zero-forcing method provide simple closed-form answers for the DOF. In reality however, existing wireless systems do not operate with infinite SNR. Depending on the application, the operational range of the SNR can be quite low. Under these conditions, assigning equal power to different transmitters will not be optimal anymore and further optimization is necessary for maximizing the sum-rate. Additionally, in the low SNR region, using zero-forcing approach results in amplifying the noise power which in turn shuts down some of the links. In these cases, it is much better to optimize the total throughput based on the Minimum Mean Square Error (MMSE) approach.

Most of the schemes presented here assume that all the channel coefficients are known in one central location. A number of conditions must be met in order to satisfy this requirement. Firstly, the receivers should be able to determine high-resolution estimates of the channel coefficients. This is not true most of the time, as the receivers can only have noisy versions of the transmit signals and even if the original transmit vectors are known, the reliability of the channel coefficients still depends on the noise power. Secondly, since the receivers should report back the channel coefficients, the available estimates used for IA suffer from channel aging due to channel variations over time. Therefore there is always residual interference terms due to imperfect alignment which add to the noise power on the receivers. An efficient channel estimation and tracking algorithm is needed to minimize these two effects in a distributed manner. It might be even possible to integrate alignment with tracking and thus waive the requirement for a central processing/alignment engine.

Perfect relays have been used in some of the schemes presented in the thesis. In reality, it is very hard to synchronize the time between all the nodes. Additionally, the signal sent by a relay is received in different times by the receivers due to the difference in the distance. Therefore, there will be timing mismatches between the signals originating from different transmitters and/or relays. The effect of such timing errors can be modeled as a phase shift which if known in advance can be integrated in the alignment phase. In the case of moving nodes, the impact of timing errors is much more severe and its dependence on the throughput should be studied. In the mobile environments, alignment algorithms can be adapted based on the space-time schemes that are resistant to such issues.

Interference alignment is still in its early stages of development. The current alignment schemes are either pure theoretical or lacking important details for practical implementation. It is still a far-fetched goal to achieve the theoretical bounds in larger networks. A longer term research topic could be looking into this problem from simplicity standpoint.

APPENDICES

Appendix A

An Iterative Algorithm for MIMO Interference Alignment

This appendix describes the implementation of an algorithm that can be used for finding the solutions to the equation sets in (2.13) and (2.19). A few definitions are needed before describing the algorithm. The matrix-valued function $\mathcal{F}$ is defined as

$$\mathcal{F}\left(\mathcal{T},\mathcal{R}\right)=\mathcal{R}^{H}\mathcal{H}\mathcal{T}-\mathcal{G}. \tag{A.1}$$

The algorithm starts from a randomly selected initial point and on each step computes the incremental changes to the transmit and receive matrices defined as $\Delta\mathcal{T}$ and $\Delta\mathcal{R}$ in such a way that the new matrix output of the function $\mathcal{F}$ becomes closer to the all zero matrix (thus solving the equation $\mathcal{F}\left(\mathcal{T}+\Delta\mathcal{T},\mathcal{R}+\Delta\mathcal{R}\right)=\mathbf{0}$). To this end the incremental steps should satisfy the following equation

$$\mathcal{F}\left(\mathcal{T},\mathcal{R}\right)+\mathcal{R}^{H}H\Delta\mathcal{T}+\Delta\mathcal{R}^{H}H\mathcal{T}=\mathbf{0}. \tag{A.2}$$

Moreover, the following condition should also be satisfied in order to ensure that the norms of the corresponding columns of $\mathcal{T}$ and $\mathcal{R}$ are equal[1]

$$\operatorname{diag}\left(\mathcal{R}^H\mathcal{R} + 2\Delta\mathcal{R}^H\mathcal{R}\right) = P_{N,M} \cdot \operatorname{diag}\left(\mathcal{T}^H\mathcal{T} + 2\mathcal{T}^H\Delta\mathcal{T}\right), \tag{A.3}$$

where, the function diag(A) returns the diagonal entries of the square matrix A as a column vector and $P_{N,M}$ is the Vectorization Transpose Matrix as defined in (2.15) which converts the column vector of a matrix into the column vector of its transpose (please refer to the codes for more information).

The equations in (A.2) and (A.3) are linear in terms of $\Delta\mathcal{T}$ and $\Delta\mathcal{R}^H$ and thus can be solved through matrix inversion. The resulting incremental updates are then applied to $\mathcal{T}$ and $\mathcal{R}$ and the algorithm is repeated until a sufficiently close solution is found or is failed due to non-convergence,

The following codes are written in MATLAB scripting language and can be used for solving the alignment problems for IC as well as the X channel. The codes heavily utilize the properties of Kronecker products and column vector operators which can be found in classical matrix calculus textbooks. The first code (mimo_ic_solver.m) is for the N-User IC and the second code (mimo_x_solver.m) is for the $M \times N$-User X channel.

Source codes are moved to here. Alternatively, they can be obtained by sending an email to behzad@nourani.net.

[1] This is a sufficient condition, conveniently modified from the original definition of the norm to make sure that the relations are kept linear for both real and complex coefficients.

Appendix B

An Iterative Algorithm for Full-Duplex Interference Alignment

Checking the the recursive equations in (5.9) and (5.11) reveals that it is very complex to compute the gradients of P_k and $\mathcal{P}$ matrices with respect to $D_{k,m}$ coefficients. An alternative matrix decomposition for the $\mathcal{P}$ is presented in this appendix which can greatly simplify the gradient computation. A new kN dimensional column vector A_k is defined as

$$A_k = \begin{bmatrix} T_1 \\ T_2 \\ \vdots \\ T_k \end{bmatrix}, \tag{B.1}$$

where T_k's are the transmit vectors as defined in (5.6). Limiting the study to the cases were $D_{1,0} = I$[1], the matrix B_k that generates A_k from A_{k-1} can be easily determined from

[1]This assumption bears no loss in generality, please refer to the lemmas in Chapter Five

below

$$
\begin{aligned}
A_k &= \begin{bmatrix} I & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & I & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & I \\ D_{k,0} + D_{k,1}G & D_{k,2}G & \cdots & D_{k,k-1}G \end{bmatrix} A_{k-1} \\
&\triangleq B_k A_{k-1}.
\end{aligned}
\tag{B.2}
$$

Similarly, the equation in (5.7) is used to describe the recovered information as a function of A_K

$$
\begin{aligned}
\hat{X} &= \begin{bmatrix} D_{K+1,1}H & D_{K+1,2}H & \cdots & D_{K+1,K}H \end{bmatrix} A_K \\
&\triangleq C_{K+1} A_K.
\end{aligned}
\tag{B.3}
$$

Finally, using the fact that $A_1 = T_1 = X$ and combining the last two relations together results in

$$
\begin{aligned}
\hat{X} &= C_{K+1} B_K B_{K-1} \cdots B_2 X \\
&= \mathcal{P} X.
\end{aligned}
\tag{B.4}
$$

This matrix decomposition of $\mathcal{P}$ is very helpful because each of the $D_{k,m}$ coefficients appears in one of the partial matrices only. This property greatly simplifies describing the gradient of $\mathcal{P}$ with respect to each of the coefficient matrices. The following code uses MATLAB scripting language to find a set of coefficients that diagonalize $\mathcal{P}$ for $K = 3$.

Source codes are moved to here. Alternatively, they can be obtained by sending an email to behzad@nourani.net.

Appendix C

Eigenvalue Placement Algorithm

A lot of matrix-based interference alignment problems end up in adjusting the elements of an $N \times N$ matrix $A(x)$ in such a way that an $N \times K$ vector $V(x)$ $(K < N)$ can be found to satisfy

$$A(x) \cdot V(x) = \mathbf{0}, \tag{C.1}$$

where $\mathbf{0}$ is an all zero matrix. Matrices $A(x)$ and $V(x)$ are both declared as functions of the vector x to emphasize their dependence on a set of scalars. The dimension of x is often needed to be as small as possible and is determined from N and K. In the alignment context, matrix $A(x)$ is usually the equivalent transfer function from one of the transmitters to a receiver which depends on the channel gains as well as a few free variables such as relay coefficients, pre-coding scales or the linear factors operating on the received signals. Regardless of their exact definition or role, the main purpose of these free variables which are defined as x is to provide the required freedom to satisfy the conditions needed for (C.1). The transmitter-receiver pair whose equivalent transfer function, $A(x)$, satisfies these conditions can use the subspace generated by the column vectors of $V(x)$ for sending data to other receivers without posing any unwanted interference on the intended receiver.

Assuming that the column vectors of $V(x)$ are linearly independent, the rank of this matrix should be K. Therefore, solving the problem in (C.1), is equivalent to finding the vector x such that $A(x)$ has K zero eigenvalues. In this case, the columns of $V(x)$ are the eigenvectors that correspond to the zero eigenvalues. To this end, Newton's Method as well as Quasi-Newton (Secant) Methods can be used in order to solve for vector x. This family of iterative algorithms are based on the function gradients and require approximating Jacobian or Hessian matrices. Derivatives of eigenvalues with respect to matrix elements have been addressed extensively in the literature [47, 39, 44, 1]. But unfortunately, the derivatives become too complex as soon as the matrix has equal eigenvalues. Besides complexity issues, the use of this approach to solve the problem in (C.1) quickly results in a numerically unstable algorithm, as it is hard to label various eigenvalues that are very close to each other (refer to [1] for more information).

The approach presented here is quite efficient, as it tries to push all the K eigenvalues towards zero at the same time. To this end, using an intuitive indirect method, the algorithm avoids all the complexities that are inherent to finding solutions of eigensystems. Before describing the algorithm, a very useful set of equations is defined as below.

C.1 Newton's Identities

Newton's Identities also known as the Newton-Girard formulae relate two types of symmetric functions. Using these identities, one can determine the power sum series from the elementary symmetric polynomials and vice versa. As a result, the sum of kth power of all roots of a polynomial can be expressed in terms of the polynomial coefficients without directly finding those roots.

Let $x_1, x_2, \ldots, x_N$ be an arbitrary set of variables, the kth power sum for this set is defined as ($k \geq 1$)

$$p_k(x_1, x_2, \ldots, x_N) = x_1^k + x_2^k + \ldots + x_N^k. \tag{C.2}$$

Moreover for the same set of variables, the elementary symmetric polynomial (the sum of all distinct products of k distinct variables) is expressed as

$$\begin{aligned}
e_0(x_1, x_2, \ldots, x_N) &= 1 \\
e_1(x_1, x_2, \ldots, x_N) &= x_1 + x_2 + \ldots + x_N \\
e_2(x_1, x_2, \ldots, x_N) &= \sum_{\substack{i,j=1 \\ i<j}}^{N} x_i x_j \\
&\vdots \\
e_N(x_1, x_2, \ldots, x_N) &= x_1 x_2 \ldots x_N \\
e_k(x_1, x_2, \ldots, x_N) &= 0 \qquad \text{for } k > N.
\end{aligned}$$

Using these definitions, the Newton's Identities can be stated as (for $k \geq 1$)

$$k e_k(x_1, x_2, \ldots, x_N) = \sum_{i=1}^{k} (-1)^{i-1} e_{k-i}(x_1, x_2, \ldots, x_N) \cdot p_i(x_1, x_2, \ldots, x_N). \tag{C.3}$$

C.2 Application to the Eigenvalues of a Matrix

Recalling from linear algebra, $c(\lambda)$, the characteristic polynomial of the square matrix A, is a polynomial whose roots are eigenvalues of that matrix. If the eigenvalues of A are defined as λ_i

$$c(\lambda) = \lambda^N + c_1\lambda^{N-1} + \ldots + c_{N-1}\lambda + c_N \tag{C.4}$$

$$= (\lambda - \lambda_1)(\lambda - \lambda_2)\ldots(\lambda - \lambda_N) \tag{C.5}$$

$$= \sum_{i=0}^{N} (-1)^i e_i(\lambda_1, \lambda_2, \ldots, \lambda_N)\lambda^i. \tag{C.6}$$

Therefore, the coefficients of the characteristic polynomial can be expressed by the elementary symmetric polynomials. Moreover, using the basic properties of the trace operator on A

$$\text{trace}(A^k) = \lambda_1^k + \lambda_2^k + \ldots + \lambda_N^k \tag{C.7}$$

$$= p_k(\lambda_1, \lambda_2, \ldots, \lambda_N). \tag{C.8}$$

and the trace of a matrix power is equal to the power sum over the matrix eigenvalues. Finally, the Newton's Identities provide the following relation between the traces of A^k and the coefficients of the characteristics polynomial

$$kc_k + \sum_{i=1}^{k} \text{trace}(A^i)\cdot c_{k-i} = 0, \tag{C.9}$$

where $c_0 = 1$ and $k = 1, 2, \ldots, N$.

C.3 Gradient-based Eigenvalue Alignment

Assuming that $(A)_{ij}$ is the element at row i and column j of the matrix A, the gradient of a scalar function $x(A)$ with respect to the $N \times N$ matrix A is another $N \times N$ matrix which is usually referred to as $\nabla_A x$ and defined as

$$(\nabla_A x)_{ij} = \frac{\partial x}{\partial (A)_{ji}}. \tag{C.10}$$

Using this definition for the gradient, the value of the function after a small change in A can be approximated as

$$x(A + \delta A) \approx x(A) + \sum_{i,j} \frac{\partial x}{\partial (A)_{ij}} (\delta A)_{ij} \tag{C.11}$$

$$\approx x(A) + \operatorname{trace}(\nabla_A x \cdot \delta A). \tag{C.12}$$

Evaluating the relation in (C.9) for $A+\delta A$ and noting the fact that it holds for every small δA, the following recursive matrix equation can be written

$$k\nabla_A c_k + \sum_{i=1}^{k} \Big(\operatorname{trace}(A^i) \cdot \nabla_A c_{k-i} + iA^{i-1} \cdot c_{k-i}\Big) = \mathbf{0}, \tag{C.13}$$

where $\nabla_A c_0 = \mathbf{0}$ and $k = 1, 2, \ldots, N$. Using the relation in (C.13), $\nabla_A c_k$ the derivative of kth coefficient of the characteristic polynomial can be recursively determined.

Back to the problem of putting the eigenvalues on origin, if a matrix has K zero eigenvalues, the last K coefficients of the characteristic polynomial $(c_{N-K+1}, c_{N-K+2}, \ldots, c_N)$ should be zero. Therefore, the algorithm has to adjust the free variables in the matrix in such a way that over the iterations, those coefficients are moved towards zero. To this end, δA is selected to satisfy the following set of linear equations $(i = 0, 1, \ldots, K-1)$

$$c_{N-i}(A + \delta A) \approx c_{N-i}(A) + \operatorname{trace}(\nabla_A c_{N-i} \cdot \delta A) = 0. \tag{C.14}$$

A closed form solution for δA is possible with the help of vec operator. The operator, vec(A), creates a column vector from its matrix parameter, A, by stacking the column vectors of A below one another. Using this definition δA can be computed from

$$\operatorname{vec}(\delta A) = -\begin{bmatrix} \operatorname{vec}^T\Big((\nabla_{\!A} c_{N-K+1})^T\Big) \\ \operatorname{vec}^T\Big((\nabla_{\!A} c_{N-K+2})^T\Big) \\ \vdots \\ \operatorname{vec}^T\Big((\nabla_{\!A} c_{N})^T\Big) \end{bmatrix}^{-1} \begin{bmatrix} c_{N-K+1} \\ c_{N-K+2} \\ \vdots \\ c_N \end{bmatrix}. \tag{C.15}$$

Finally the eigenvalue placement algorithm is actually the Newton method (equivalent to the Steepest Descent but in multi-dimensions) that uses the computed gradient for determining the best direction towards the solution. The steps of this algorithm are detailed below (μ is a positive real scalar).

1. Select a random starting point for A
2. Compute c_k using (C.9)
3. Compute $\nabla_{\!A} c_k$ using (C.13)
4. Compute δA using (C.15)
5. Update A using $A_{(\text{new})} = A_{(\text{old})} + \mu \cdot \delta A$
6. Check c_k's, if the last K are close enough to zero stop
7. Goto step 2

The following example uses MATLAB scripting language to adjust the first three elements on the main diagonal entries of a matrix such that the resulting matrix has three zero eigenvalues.

Source codes are moved to here. Alternatively, they can be obtained by sending an email to behzad@nourani.net.

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