Delay-Throughput Analysis in Distributed Wireless Networks

by

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A thesis

presented to the University of Waterloo

in fulfillment of the

thesis requirement for the degree of

Doctor of Philosophy

in

Electrical and Computer Engineering

Waterloo, Ontario, Canada, 2009

I hereby declare that I am the sole author of this thesis. This is a true copy of the thesis, including any required final revisions, as accepted by my examiners.

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Abstract

A primary challenge in wireless networks is to use available resources efficiently so that the Quality of Service (QoS) is satisfied while maximizing the throughput of the network. Among different resource allocation strategies, power and spectrum allocations have long been regarded as efficient tools to mitigate interference and improve the throughput of the network. Also, achieving a low transmission delay is an important QoS requirement in buffer-limited networks, particularly for users with real-time services. For these networks, too much delay results in dropping some packets. Therefore, the main challenge in networks with real-time services is to utilize an efficient power allocation scheme so that the delay is minimized while achieving a high throughput. This dissertation deals with these problems in distributed wireless networks.

In Chapters 2 and 3, a distributed single-hop wireless network with K links is considered, in which the links are partitioned into a fixed number (M) of clusters, each operating in a subchannel with bandwidth $\frac{W}{M}$. The subchannels are assumed to be orthogonal to each other. A general shadow-fading model, described by parameters (α, ϖ), is considered where α denotes the probability of shadowing, and ϖ ($\varpi \leq 1$) represents the average cross-link gains. The main goal of these chapters is to find the maximum network throughput in the asymptotic regime of $K \to \infty$, which is achieved by: i) proposing a distributed and non-iterative power allocation strategy, in which the objective of each user is to maximize its best estimate (based on its local information, i.e., direct channel gain) of the average throughput of the network, and ii) choosing the optimum value for M. In Chapter 2, the network throughput is defined as the *average sum-rate* of the network, which is shown to scale as $\Theta(\log K)$. Moreover, it is

proved that in the strong interference scenario, the optimum power allocation strategy for each user is a threshold-based on-off scheme. In Chapter 3, the network throughput is defined as the *guaranteed sum-rate*, when the outage probability approaches zero. In this scenario, it is demonstrated that the on-off power allocation scheme maximizes the throughput, which scales as $\frac{W}{\alpha\varpi}\log K$. Moreover, the optimum spectrum sharing for maximizing the average sum-rate and the guaranteed sum-rate is achieved at $M = 1$.

Chapter 4 investigates the delay-throughput tradeoff of the underlying network with $M = 1$ (or equivalently $K = n$). The analysis relies on the distributed *on-off power allocation strategy* for the deterministic and stochastic packet arrival processes. In the first part, the effective throughput maximization of the network is analyzed. It is proved that the effective throughput of the network scales as $\frac{\log n}{\hat{\alpha}}$, with $\hat{\alpha} \triangleq \alpha\varpi$, despite the packet arrival process. Then, the delay characteristics of the underlying network in terms of a packet dropping probability are presented. In addition, the necessary conditions in the asymptotic case of $n \to \infty$ are derived such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network. Finally, the trade-off between the effective throughput of the network and delay-bounds for different packet arrival processes is analyzed. In particular, it is determined how much degradation will be enforced in the throughput by introducing other constraints.

Acknowledgments

This work would not be possible without the assistance, guidance and support received from many individuals who deserve to be recognized.

First and foremost, I am deeply grateful to Professor Amir K. Khandani for providing guidance and inspiration throughout the course of my graduate studies. It was a great opportunity for me to work with such a brilliant, insightful, creative, and knowledgeable supervisor. I would like to thank Professor Khandani for his attention and support, which was not only limited to research work. In fact, he has been a supervisor and a close friend of mine over these years. His positive thinking and encouraging words during difficult times gave me the confidence to master challenging research problems. Professor Khandani was instrumental in developing me into the researcher that I am today, and for this I will always be grateful.

I also wish to thank the members of my dissertation committee, Professors David Gesbert, Ravi Mazumdar, Liang-Liang Xie and Kate Larson for taking the time to carefully review my thesis, providing me with insightful questions and suggestions.

Also acknowledged is the financial support received from the Iranian Ministry of Science, Research and Technology (MSRT) during my Ph.D. program.

I would like to thank all members of the Coding and Signal Transmission (CST) Lab, including Sandra Ramautarsingh, MBA the Department's Office Manager. These persons helped to create a very pleasant and friendly learning environment. During the course of my study, I engaged in several beneficial and useful discussions with them. Of significance as well, I would like to thank Dr. Alireza Bayesteh and Dr. Masoud Ebrahimi for their valuable comments and suggestions.

Finally, I would also like to acknowledge my family, mainly my beloved parents, for their endless love and support, without which I would not have succeeded. My special thanks and sincere appreciation are extended to the love of my life, Tahereh Asheghmadineh, my wife, for her patience during the process of this work. I am thankful for all of her love and support.

Dedications

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To my parents:

Hassan Abouei
Hajar Ghanei

To my beloved wife:

Tahereh Asheghmadineh

To my kids:

Zahra & *Ali*

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Abbreviations

AWGN	Additive White Gaussian Noise
BAP	Bernoulli Arrival Process
CAP	Constant Arrival Process
CDF	Cumulative Distribution Function
FDM	Frequency Division Multiplexing
i.i.d	independent and identically distributed
LOS	Line of Sight
PAP	Poisson Arrival Process
pdf	probability density function
pmf	probability mass function
QoS	Quality of Service
SINR	Signal-to-Interference-plus-Noise Ratio
WLAN	Wireless Local Area Network
WSN	Wireless Sensor Network

Notations

Boldface letters	vectors
$\mathscr{C}^C$	The complement of $\mathscr{C}$
$\mathbb{A}_j$	Set of active links in $\mathbb{C}_j$
β_{ki}	The shadowing factor
$\mathbb{C}_j$	Cluster j
$\mathbb{E}[.]$	expectation operator
$\mathrm{E}_i(\ .\ )$	Exponential-integral function
K	Total number of links in the network
λ	Delay-bound
$\log(.)$	The natural logarithm function
M	Number of clusters in the network
m_j	Number of active links in $\mathbb{C}_j$
N_0	Noise power spectral density
n	Number of links in each cluster
$\mathbb{N}_n$	The set $\{1, 2, ..., n\}$
$\mathbb{P}\{\mathscr{B}\}$	Probability of event $\mathscr{B}$
p_i	Transmit power of user i
$\mathrm{RH}(.)$	Right hand side of the equations
$\mathbf{P}^{(j)}$	The vector of all the user's power in $\mathbb{C}_j$
τ_n	Threshold level
ϖ	Average cross channel gains

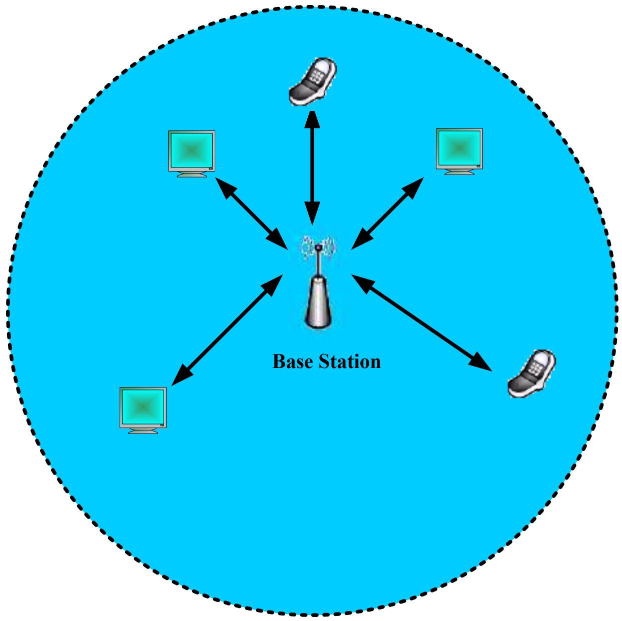
W	Bandwidth
$\bar{X}$	The expected value of X (denoted by $\mathbb{E}[X]$)
$f(n) = o(g(n))$	$\lim_{n\to\infty} \left\lvert \frac{f(n)}{g(n)} \right\rvert = 0$
$f(n) = O(g(n))$	$\lim_{n\to\infty} \left\lvert \frac{f(n)}{g(n)} \right\rvert < \infty$
$f(n) = \omega(g(n))$	$\lim_{n\to\infty} \left\lvert \frac{f(n)}{g(n)} \right\rvert = \infty$
$f(n) = \Omega(g(n))$	$\lim_{n\to\infty} \left\lvert \frac{f(n)}{g(n)} \right\rvert > 0$
$f(n) = \Theta(g(n))$	$\lim_{n\to\infty} \left\lvert \frac{f(n)}{g(n)} \right\rvert = c$, where $0 < c < \infty$
$f(n) \sim g(n)$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} = 1$
$f(n) \lesssim g(n)$	$\lim_{n\to\infty} \frac{f(n)}{g(n)} \leq 1$
$A \approx B$	Approximate equality between A and B such that if one replaces A by B in the equations, the results still hold

Chapter 1

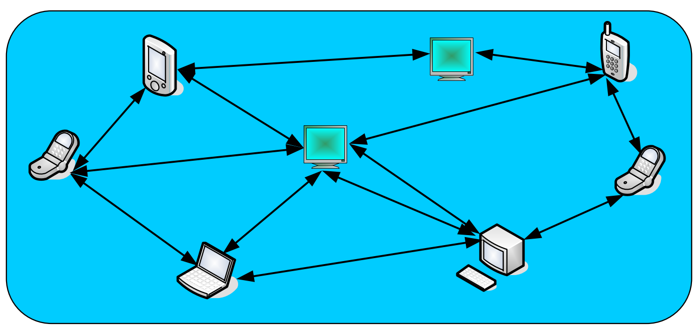
Introduction

1.1 Background and Motivation

A wireless communication network is a system of interconnected facilities designed to transfer information from a variety of telecommunication sources with high reliability. There are currently two variations of wireless networks. The first is known as centralized networks, including cellular telephone systems and Wireless Local Area Networks (WLANs). In this category of networks, users communicate exclusively with the corresponding base stations or central nodes (Fig. 1.1-a). The second type is distributed or decentralized networks, including ad hoc wireless networks. An ad hoc wireless network is a collection of mobile nodes which can be rapidly and flexibly deployed without any inherent infrastructure (Fig. 1.1-b). An important aspect of both centralized and distributed wireless networks is that the nodes can share a single wireless channel. In this case, there is significant interference among simultaneously communicating nodes,



(a)



(b)

Figure 1.1: a) Centralized and b) Distributed wireless networks.

resulting in performance (e.g. network throughput) degradiation as the number of nodes in the network increases. Thus, smart interference management should be implemented to efficiently harvest high data rates. To deal with interference mitigation, a primary challenge is to use resources (e.g., power and spectrum) efficiently so that the throughput of the network is maximized.

On the other hand, achieving a low transmission delay is an important Quality of Service (QoS) requirement in buffer-limited networks [1], particularly for backlogged users[1] with real-time services (e.g., interactive games, live sport videos, etc). For these networks, too much delay results in dropping some packets. Therefore, the main challenge in wireless networks with real-time services is to utilize efficient resource allocation schemes so that the delay is minimized, while achieving a high throughput.

Motivated by the above considerations, this thesis addresses the following specific questions in distributed single-hop wireless networks with many nodes in the network:

- What is the optimum power allocation strategy (based on local information) which maximizes the throughput of the network?
- How does the network's throughput scale with the number of links in the network?
- How does the delay characteristics of the underlying network scale with the number of links in the network?
- What is the tradeoff between delay and throughput?

[1]For each user, there is always a packet available to be transmitted.

1.2 Network Throughput Maximization

Throughput maximization in multi-user wireless networks has been addressed from different perspectives; including resource allocation (e.g., power and bandwidth allocations [2–5]), scheduling [6], routing by using relay nodes [7], exploiting mobility of the nodes [8] and exploiting channel characteristics (e.g., power decay-versus-distance law [9–11], geometric pathloss and fading [12,13] and random connections [14]). Central to the study of network throughput maximization is the problem of resource allocation.

Among different resource allocation strategies, power and spectrum allocation have long been regarded as efficient tools to mitigate interference and improve the throughput of the network. In recent years, various power and spectrum allocation schemes have been extensively studied in cellular and multihop wireless networks [2, 3, 15–21]. In [18], the authors provide a comprehensive survey in the area of resource allocation, in particular, in the context of spectrum assignment. Much of these works rely on centralized and cooperative algorithms. Clearly, centralized resource allocation schemes provide a significant improvement in the throughput of the network over decentralized (distributed) approaches. However, these schemes require extensive knowledge of the network's configuration. In particular, when the number of nodes is large, deploying centralized schemes may not be practically feasible. In addition, in a cooperative wireless network, when the number of nodes becomes large, the amount of exchanged information grows extensively. This is critical for time-varying networks, as the algorithms cannot perfectly track the speed of the channel variations. Due to significant challenges in using centralized approaches, the attention of researchers has been drawn to the decentralized resource allocation schemes [4, 22–26].

The main goal of applying a decentralized mechanism is that operational decisions concerning network parameters (e.g., rate and/or power) are made solely by the individual nodes based on their local information. The local decision parameters that can be used for adjusting the rate are the Signal-to-Interference-plus-Noise Ratio (SINR) and the direct channel[2] gains. Most of the works on decentralized throughput maximization target the SINR parameter by using iterative algorithms [22, 24, 25]. This leads to the use of game theory concepts such as repeated game [27], in which the main challenge is the convergence issue. For instance, Etkin *et al.* [22] develop power and spectrum allocation strategies in multiple wireless systems by using game theory. Under the assumptions of the omniscient nodes and strong interference, the authors show that Frequency-Division Multiplexing (FDM) is the optimal scheme in the sense of throughput maximization. They use an iterative algorithm that converges to the optimum power values. In [24], Huang *et al.* propose an iterative power control algorithm in an ad hoc wireless network, in which the receivers broadcast adjacent channel gains and interference prices to optimize the throughput of the network. However, this algorithm incurs a great deal of overhead in large wireless networks. Deploying iterative power allocation algorithms leads to more power consumption, which is a dominant factor in ad hoc wireless networks with battery life constraints (e.g. Wireless Sensor Networks (WSNs)).

A more practical approach in time-varying networks is to rely on the channel gains

[2]The direct channel is different from the Line-of-Sight (LOS), and is referred to as the link between a transmitter and its corresponding receiver.

as local decision parameters, and to avoid iterative schemes. Motivated by this consideration, the throughput maximization of a distributed single-hop wireless network with K links, operating in a bandwidth W is studied. To mitigate the interference, the links are partitioned into a fixed number (M) of clusters, each operating in a subchannel with bandwidth $\frac{W}{M}$. Throughput maximization of the underlying network is achieved by proposing a distributed and non-iterative power allocation strategy based on the direct channel gains, and then choosing the optimum value for M.

1.3 Delay-Throughput Tradeoff

The throughput maximization problem in cellular and multihop wireless networks has been extensively studied in [8–11, 14]. In these works, delay analysis is not considered. However, it is shown that the high throughput is achieved at the cost of a large delay [28]. This problem has motivated the researchers to study the relation between the delay characteristics and the throughput in wireless networks [29–32]. In particular, in most recent literature [28,33–40], the tradeoff between delay and throughput have been investigated as a key measure of the performance of the network. The first studies on achieving a high throughput along with a low delay in ad hoc wireless networks are framed in [29] and [32]. This line of work is further expanded in [28, 33, 34] by using different mobility models. El Gamal *et al.* [28] analyze the optimal delay-throughput scaling for some wireless network topologies. For a static random network with n nodes, the authors prove that the optimal tradeoff between throughput T_n and delay D_n is given by $D_n = \Theta(nT_n)$. Reference [28] also shows that the same result is achieved in random mobile networks, when $T_n = O(1/\sqrt{n \log n})$. Neely and Modiano [34] consider the delay-throughput tradeoff for mobile ad hoc networks under the assumption of redundant packets transmission through multiple paths. Sharif and Hassibi [36] analyze the delay characteristics and the throughput maximization in a broadcast channel. They propose an algorithm to reduce the delay without too much degradation in the throughput. This line of work is further extended in [37] by demonstrating that it is possible to achieve the maximum throughput and short-term fairness simultaneously in a large-scale broadcast network.

In Chapter 4, the same model in Chapter 2 with $M = 1$ (or equivalently $K = n$) is used. The analysis relies on the distributed on-off power allocation strategy proposed in Chapter 2. In this chapter, the effective throughput maximization of the network is analyzed. Also, the question: "How does the delay characteristics of the network scale with the number of links n" is addressed? In addition, the delay-throughput tradeoff of the underlying network is analyzed.

1.4 Overview of Contributions

In Chapters 2 and 3, a distributed single-hop wireless network with K links is considered, where the links are partitioned into a fixed number (M) of clusters each operating in a subchannel with bandwidth $\frac{W}{M}$. The subchannels are assumed to be orthogonal to each other. A general shadow-fading model, described by parameters (α, ϖ), is considered where α denotes the probability of shadowing, and ϖ $(\varpi \leq 1)$ represents the average cross-link gains. The main goal of these chapters is to find the maximum network throughput in the asymptotic regime of $K \to \infty$, which is achieved by: i) proposing a distributed and non-iterative power allocation strategy, in which the objective of each user is to maximize its best estimate (based on its local information, i.e., direct channel gain) of the average throughput of the network, and ii) choosing the optimum value for M. In Chapter 2, the network throughput is defined as the *average sum-rate* of the network, which is shown to scale as $\Theta(\log K)$. Moreover, it is proved that in the strong interference scenario, the optimum power allocation strategy for each user is a threshold-based on-off scheme. In Chapter 3, the network throughput is defined as the *guaranteed sum-rate*, when the outage probability approaches zero. In this scenario, it is demonstrated that the on-off power allocation scheme maximizes the throughput, which scales as $\frac{W}{\alpha\varpi}\log K$. Moreover, the optimum spectrum sharing for maximizing the average sum-rate and the guaranteed sum-rate is achieved at $M = 1$ [3].

Chapter 4 investigates the delay-throughput tradeoff of the underlying network with $M = 1$ (or equivalently $K = n$). The analysis relies on the distributed *on-off power allocation strategy* for the deterministic and stochastic packet arrival processes. In the first part, the effective throughput maximization of the network is analyzed. It is proved that the effective throughput of the network scales as $\frac{\log n}{\hat{\alpha}}$, with $\hat{\alpha} \triangleq \alpha\varpi$, despite the packet arrival process. It is shown that increasing the number of links gives rise to increasing the throughput of the network, at the cost of increasing the delay. This results in higher packet droppings in real-time applications with limited buffer sizes. In the second part, the delay characteristics of the underlying network in terms of a

[3] The material in Chapters 2 and 3 has been previously presented in CISS'07 and ISIT'08 conferences [41, 42] and has been submitted to IEEE Transactions on Information Theory [43].

packet dropping probability are presented. In addition, the necessary conditions in the asymptotic case of $n \to \infty$ are derived such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network. Finally, the tradeoff between the effective throughput of the network and delay-bounds for different packet arrival processes is analyzed. In particular, it is determined how much degradation will be enforced in the throughput by introducing other constraints[4]. .

Finally, Chapter 5 outlines a summary of the thesis contributions and discusses possible future research directions.

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[4]The material in Chapter 4 has been previously presented in ISIT'08 conference [44] and to be submitted to IEEE Transactions on Information Theory [45].

Chapter 2

Utility-Based Power Allocation Strategy

2.1 Introduction

In this chapter, we study the throughput maximization of a spatially distributed wireless network with K links, where the sources and their corresponding destinations communicate directly with each other without using relay nodes. Wireless networks using unlicensed spectrum (e.g. Wi-Fi systems based on IEEE 802.11b standard [46]) are typical examples of such networks. It is assumed that the links are partitioned into M clusters each operating in subchannels with bandwidth $\frac{W}{M}$. The cross channel gains are assumed to be Rayleigh-distributed with shadow-fading, described by parameters (α, ϖ), where α denotes the probability of shadowing, and ϖ represents the statistical average cross-link gains. This configuration differs from the geometric models proposed in [8–11, 17], in which the signal power decays based on the distance between nodes. Unlike [22–25] which rely on iterative algorithms using SINR, it is assumed that each transmitter adjusts its power solely based on its direct channel gain.

Clearly, if each user maximizes its rate selfishly, the optimum power allocation strategy for all users is to transmit with full power. This strategy results in excessive interference, degrading the network throughput. To prevent this undesirable effect, one should consider the negative impact of each user's power increment on the other links performance. A reasonable approach for each user is to choose a non-iterative power allocation strategy to maximize its best local estimate of the throughput of the network. In this setup, the optimization problem is subject to the power constraint for each individual link, instead of a total power constraint. This assumption is more practical for decentralized wireless networks.

The throughput in this chapter is defined as the *average sum-rate* of the network, which is shown to scale at most as $\Theta(\log K)$ in the asymptotic case of $K \to \infty$. This order is achievable by the distributed *threshold-based on-off scheme* (i.e., links with a direct channel gain above certain threshold transmit at full power and the rest remain silent). Moreover, in the strong interference scenario, the on-off power allocation scheme is the optimal strategy. These results are different from the link activation strategy studied in [47], where the authors use a similar on-off scheme for $M = 1$ and prove its optimality only among all on-off schemes. This work also differs from [14] and [48] in terms of the network model. In this thesis, a distributed power allocation strategy in a single-hop network with M disjoint subchannels is used; while [14] and [48] consider an ad hoc network model with random connections (for $M = 1$) and relay nodes.

The average throughput of the network in terms of the number of clusters (M) is optimized. It is proved that the maximum average sum-rate of the network for every value of α is achieved at $M = 1$. In other words, splitting the bandwidth W into M orthogonal subchannels does not increase the throughput.

The rest of the chapter is organized as follows. In Section 2.2, the network model and objectives are described. The distributed on-off power allocation strategy and the average sum-rate of the network are presented in Section 2.3. In Section 2.4, the numerical results are provided. Finally, in Section 2.5, an overview of the results and some conclusion remarks are indicated.

2.2 Network Model and Description

2.2.1 Network Model

In this work, a single-hop wireless network consisting of K pairs of nodes[1] indexed by $\{1, ..., K\}$, operating in bandwidth W is considered. All the nodes in the network are assumed to have a single antenna. The links are assumed to be randomly divided to M clusters denoted by $\mathbb{C}_j$, $j = 1, ..., M$ so that the number of links in all clusters are the same. Without loss of generality, $\mathbb{C}_j \triangleq \{(j-1)n+1, ..., jn\}$, where $n \triangleq \frac{K}{M}$ denotes the cardinality of the set $\mathbb{C}_j$, and it is assumed to be known to all users[2]. To eliminate the mutual interference among the clusters, an M-dimensional orthogonal coordinate

[1] The term "pair" is used to describe a transmitter and its corresponding receiver, while the term "user" is used only for the transmitter.

[2] It is assumed that K is divisible by M, and hence, $n = \frac{K}{M}$ is an integer number.

system[3] is considered, in which the bandwidth W is split into M disjoint subchannels each with bandwidth $\frac{W}{M}$. It is assumed that the links in $\mathbb{C}_j$ operate in subchannel j. It is also assumed that M is fixed, i.e., it does not scale with K. The power of Additive White Gaussian Noise (AWGN) at each receiver is supposed to be $\frac{N_0 W}{M}$, where N_0 is the noise power spectral density.

The channel model is assumed to be flat Rayleigh fading with shadowing effect. The channel gain[4] between transmitter k and receiver i is represented by the random variable $\mathcal{L}_{ki}$. For $k = i$, the *direct channel gain* is defined as $\mathcal{L}_{ki} \triangleq h_{ii}$ where h_{ii} is exponentially distributed with unit mean (and unit variance). For $k \neq i$, the *cross channel gains* are defined based on a shadowing model as follows[5]:

$$\mathcal{L}_{ki} \triangleq \begin{cases} \beta_{ki} h_{ki}, & \text{with probability} \quad \alpha \\ 0, & \text{with probability} \quad 1-\alpha, \end{cases} \tag{2.1}$$

where h_{ki}'s have the same distribution as h_{ii}'s, $0 \leq \alpha \leq 1$ is a fixed parameter, and the random variable β_{ki}, referred to as the *shadowing factor*, is independent of h_{ki} and satisfies the following conditions:

- $\beta_{\min} \leq \beta_{ki} \leq \beta_{\max}$, where $\beta_{\min} > 0$ and $\beta_{\max}$ is finite,
- $\mathbb{E}\big[\beta_{ki}\big] \triangleq \varpi \leq 1$.

It is also assumed that $\{\mathcal{L}_{ki}\}$ and $\{\beta_{ki}\}$ are mutually independent random variables for different (k, i).

[3]There are several ways to generate M orthogonal subspaces such as M different frequency bands, M different time slots, or M orthogonal codes (e.g., Walsh-Hadamard codes).

[4]In this work, *channel gain* is defined as the square magnitude of the *channel coefficient*.

[5]For more details, the reader is referred to [49] and [50] and references therein.

All channels in the network are assumed to be quasi-static block fading, i.e., the channel gains remain constant during one block, and change independently from block to block. In addition, it is assumed that each transmitter knows its direct channel gain.

In this work, we consider a homogeneous network in the sense that all the links have the same configuration and use the same protocol. Thus, the transmission strategy for all the users are determined in advance. The transmit power of user i is denoted by p_i, where $p_i \in \mathscr{P} \triangleq [0, \mathrm{P}_{max}]$. The non-negative vector $\mathbf{P}^{(j)} = (p_{(j-1)n+1}, ..., p_{jn})$ represents the power vector of the users in $\mathbb{C}_j$. Also, $\mathbf{P}^{(j)}_{-i}$ denotes the vector consisting of elements of $\mathbf{P}^{(j)}$ other than the i^{th} element, $i \in \mathbb{C}_j$. To simplify the notations, it is assumed that the noise power $\frac{N_0 W}{M}$ is normalized by P_{max}. Therefore, without loss of generality, it is assumed that $\mathrm{P}_{max} = 1$. Assuming that the transmitted signals are Gaussian, the interference term seen by link $i \in \mathbb{C}_j$ will be Gaussian with power

$$I_i = \sum_{\substack{k \in \mathbb{C}_j \\ k \neq i}} \mathcal{L}_{ki} p_k. \tag{2.2}$$

Due to the orthogonality of the allocated subchannels, no interference is imposed from links in $\mathbb{C}_k$ on links in $\mathbb{C}_j$, $k \neq j$. Under these assumptions, the achievable data rate of each link $i \in \mathbb{C}_j$ is expressed as

$$R_i(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)}_i) = \frac{W}{M} \log \left(1 + \frac{h_{ii} p_i}{I_i + \frac{N_0 W}{M}} \right), \tag{2.3}$$

where $\boldsymbol{\mathcal{L}}^{(j)}_i \triangleq (\mathcal{L}_{((j-1)n+1)i}, ..., \mathcal{L}_{(jn)i})$. To analyze the performance of the underlying network, the *network average sum-rate* is used and is defined as follows:

$$\bar{R}_{ave} \triangleq \mathbb{E} \left[\sum_{j=1}^{M} \sum_{l \in \mathbb{C}_j} R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}^{(j)}_l) \right], \tag{2.4}$$

where the expectation is computed with respect to $\boldsymbol{\mathcal{L}}^{(j)}_l$.

2.2.2 Objective

The main objective of this chapter is to maximize the average sum-rate of the network when the interference is strong enough, i.e., $\mathbb{E}[I_i] = \omega(1)$. This is achieved by:

- Proposing a distributed and non-iterative power allocation strategy, in which each user maximizes its best estimate (based on its local information, i.e., direct channel gain) of the average sum-rate of the network.

- Choosing the optimum value for M.

To address this problem, a utility function for link $i \in \mathbb{C}_j$ $(j = 1, ..., M)$ is defined which describes the average sum-rate of the links in cluster $\mathbb{C}_j$ as follows:

$$u_i(p_i, h_{ii}) \triangleq \mathbb{E}\left[\sum_{l \in \mathbb{C}_j} R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right], \tag{2.5}$$

where the expectation is computed with respect to $\{\mathcal{L}_{kl}\}_{k,l \in \mathbb{C}_j}$ excluding $k = l = i$ (namely h_{ii}) [6]. As mentioned earlier, h_{ii} is considered as the local (known) information for link i, however, all the other gains are unknown to user i, which is the reason behind statistical averaging over these parameters in (2.5). User i selects its power using

$$\hat{p}_i = \arg \max_{p_i \in \mathscr{P}} \; u_i(p_i, h_{ii}), \quad i \in \mathbb{C}_j, \quad j = 1, ..., M. \tag{2.6}$$

It will be shown that when the number of links is large and the interference is strong enough, the optimum power allocation strategy for the optimization problem in (2.6) is the on-off power scheme. Assuming that the channel gains change independently

[6] Since, the transmission power p_l depends on the channel gain h_{ll} (i.e., $p_l = g(h_{ll})$), the utility function of link $i \in \mathbb{C}_j$ $(j = 1, ..., M)$ is a function of its direct channel gain and the power p_i.

from block to block, each user updates its on-off decision based on its direct channel gain in each block. Given the optimum power vector $\hat{\mathbf{P}}^{(j)} = (\hat{p}_{(j-1)n+1}, ..., \hat{p}_{jn})$ obtained from (2.6), the average sum-rate of the network is then computed as (2.4). Next, the optimum value of M is chosen so that the average sum-rate of the network is maximized, i.e.:

$$\hat{M} = \arg \max_{M} \bar{R}_{ave}. \tag{2.7}$$

Also, for the moderate and weak interference regimes (i.e., $\mathbb{E}[I_i] = O(1)$), upper bounds for the average sum-rate of the network are obtained.

2.3 Network Average Sum-Rate

2.3.1 Strong Interference Scenario ($\mathbb{E}[I_i] = \omega(1)$)

In order to maximize the average sum-rate of the network, the optimum power allocation policy is first determined. Using (2.5), the utility function of link $i \in \mathbb{C}_j, \quad j = 1, ..., M$, can be expressed as

$$u_i(p_i, h_{ii}) = \bar{R}_i(p_i, h_{ii}) + \sum_{\substack{l \in \mathbb{C}_j \\ l \neq i}} \bar{R}_l(p_i), \tag{2.8}$$

where

$$\bar{R}_i(p_i, h_{ii}) = \mathbb{E}\left[\frac{W}{M} \log\left(1 + \frac{h_{ii} p_i}{I_i + \frac{N_0 W}{M}}\right)\right], \tag{2.9}$$

with the expectation computed with respect to I_i defined in (2.2), and

$$\begin{aligned} \bar{R}_l(p_i) &= \mathbb{E}\left[R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right] &(2.10)\\ &= \mathbb{E}\left[\frac{W}{M}\log\left(1+\frac{h_{ll}p_l}{I_l+\frac{N_0W}{M}}\right)\right] &(2.11)\\ &= \mathbb{E}\left[\frac{W}{M}\log\left(1+\frac{h_{ll}p_l}{\mathcal{L}_{il}p_i+\sum_{k\neq l,i}\mathcal{L}_{kl}p_k+\frac{N_0W}{M}}\right)\right], \ k,l\in\mathbb{C}_j, l\neq i, &(2.12)\end{aligned}$$

with the expectation computed with respect to $\mathbf{P}_{-i}^{(j)}$ and $\{\mathcal{L}_{kl}\}_{k,l\in\mathbb{C}_j}$ excluding $l = i$[7]. It is worth mentioning that the power p_i in (2.12) prevents the i^{th} user from selfishly maximizing its average rate given in (2.9). Using the fact that all users follow the same power allocation policy, and since the channel gains $\mathcal{L}_{kl}$ are random variables with the same distributions, $\bar{R}_l(p_i)$ becomes independent of l. Thus, by dropping the index l from $\bar{R}_l(p_i)$, the utility function of link i can be simplified as

$$u_i(p_i, h_{ii}) = \bar{R}_i(p_i, h_{ii}) + (n-1)\bar{R}(p_i). \tag{2.13}$$

Noting that p_i depends only on the channel gain h_{ii}, in the sequel $p_i = g(h_{ii})$ is used.

Lemma 2.1. *Let us assume $\mathbb{E}[p_k] \triangleq q_n$, $0 < \alpha \leq 1$ is fixed and the interference is strong enough (i.e., $\mathbb{E}[I_i] = \omega(1)$). Then with probability one (w. p. 1), we have:*

$$I_i \sim (n-1)\hat{\alpha}q_n, \tag{2.14}$$

as $K \to \infty$ (or equivalently, $n \to \infty$), where $\hat{\alpha} \triangleq \alpha\varpi$. More precisely, substituting I_i by $(n-1)\hat{\alpha}q_n$ does not change the asymptotic average sum-rate of the network.

[7]Note that the power of the users are random variables, since they are a deterministic function of their corresponding direct channel gains, which are random variables.

Proof. See Appendix A. □

Lemma 2.2. *For large values of n, the links with a direct channel gain above $h_{Th} = c\log n$, where $c > 1$ is a constant, have negligible contribution in the average sum-rate of the network.*

Proof. Denoting $\mathbb{T}_j \triangleq \{l \in \mathbb{C}_j \mid h_{ll} > h_{Th}\}$, the cardinality of the set $\mathbb{T}_j$ is a binomial random variable with the mean $n\mathbb{P}\{h_{ll} > h_{Th}\}$. From (2.4), we have

$$\bar{R}_{ave} = \sum_{j=1}^{M} \mathbb{E}\left[\sum_{l\in\mathbb{C}_j} R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right], \tag{2.15}$$

where

$$\mathbb{E}\left[\sum_{l\in\mathbb{C}_j} R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right] = \mathbb{E}\left[\sum_{l\in\mathbb{T}_j} R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right] + \mathbb{E}\left[\sum_{l\in\mathbb{T}_j^C} R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right],$$

in which $\mathbb{T}_j^C$ denotes the complement of $\mathbb{T}_j$. Note that

$$\begin{aligned}
\mathbb{E}\left[\sum_{l\in\mathbb{T}_j} R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right] &= n\frac{W}{M}\mathbb{E}\left[\log\left(1+\frac{h_{ll}p_l}{I_l+\frac{N_0W}{M}}\right)\middle| h_{ll} > h_{Th}\right]\mathbb{P}\{h_{ll} > h_{Th}\} \\
&\leq n\frac{W}{M}\mathbb{E}\left[\log\left(1+\frac{h_{ll}}{\frac{N_0W}{M}}\right)\middle| h_{ll} > h_{Th}\right]\mathbb{P}\{h_{ll} > h_{Th}\} \\
&\overset{(a)}{\leq} \frac{n}{N_0}e^{-h_{Th}}\mathbb{E}\left[h_{ll} | h_{ll} > h_{Th}\right] \\
&= \frac{n}{N_0}e^{-h_{Th}}(1+h_{Th}),
\end{aligned} \tag{2.16}$$

where (a) follows from $\log(1+x) \leq x$, for $x \geq 0$. It is observed that for $h_{Th} = c\log n$, where $c > 1$, the right hand side of (2.16) tends to zero as $n \to \infty$. Thus,

$$\lim_{n\to\infty} \mathbb{E}\left[\sum_{l\in\mathbb{T}_j} R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right] = 0.$$

Consequently,

$$\lim_{n\to\infty}\sum_{j=1}^{M}\mathbb{E}\left[\sum_{l\in\mathbb{T}_j}R_l(\mathbf{P}^{(j)},\boldsymbol{\mathcal{L}}_l^{(j)})\right]=0,$$

and this completes the proof of the lemma. □

From Lemma 2.2 and for large values of n, we can limit our attention to a subset of links for which the direct channel gain h_{ii} is less than $c\log n$, $c>1$.

Theorem 2.3. *Assuming the strong interference scenario and sufficiently large K, the optimum power allocation policy for (2.6) is $\hat{p}_i = g(h_{ii}) = U(h_{ii}-\tau_n)$, where $\tau_n > 0$ is a threshold level which is a function of n, and $U(.)$ is the unit step function. Also, the maximum average sum-rate of the network in (2.4) is achieved at $M=1$ and is given by*

$$\bar{R}_{ave} \sim \frac{W}{\hat{\alpha}}\log K. \tag{2.17}$$

Proof. The steps of the proof are as follows: First, an upper bound on the utility function given in (2.13) is derived. Then, it is proved that the optimum power allocation strategy that maximizes this upper bound is $\hat{p}_i = g(h_{ii}) = U(h_{ii}-\tau_n)$. Based on this optimum power allocation policy, in Lemma 2.5, the optimum threshold level τ_n is derived. It is then shown that using this optimum threshold value, the maximum value of the utility function in (2.13) becomes asymptotically the same as the maximum value of the upper bound obtained in the first step. Finally, the proof of the theorem is completed by showing that the maximum average sum-rate of the network is achieved at $M=1$.

Step 1: *Upper Bound on the Utility Function*

Assuming $\mathbb{E}[p_k] = q_n$ and using the results of Lemma 2.1, $\bar{R}_i(p_i, h_{ii})$ in (2.13) can be expressed as

$$\begin{aligned}\bar{R}_i(p_i, h_{ii}) &\approx \frac{W}{M}\mathbb{E}\left[\log\left(1 + \frac{h_{ii}p_i}{(n-1)\hat{\alpha}q_n + \frac{N_0W}{M}}\right)\right] && (2.18)\\ &\overset{(a)}{=} \frac{W}{M}\log\left(1 + \frac{h_{ii}p_i}{\lambda}\right), && (2.19)\end{aligned}$$

as $K \to \infty$, where

$$\lambda \triangleq (n-1)\hat{\alpha}q_n + \frac{N_0W}{M}. \tag{2.20}$$

In the above equations, (a) follows from the fact that h_{ii} is a known parameter for user i and $p_i = g(h_{ii})$ is the optimization parameter. In this case, the SINR is equal to $\frac{h_{ii}p_i}{\lambda}$. With a similar argument, (2.12) can be simplified as

$$\begin{aligned}\bar{R}(p_i) &\approx \frac{W}{M}\mathbb{E}\left[\log\left(1 + \frac{h_{ll}p_l}{\mathcal{L}_{il}p_i + (n-2)\hat{\alpha}q_n + \frac{N_0W}{M}}\right)\right], \quad i \neq l\\ &\overset{(a)}{=} \alpha\frac{W}{M}\mathbb{E}\left[\log\left(1 + \frac{h_{ll}p_l}{\beta_{il}h_{il}p_i + (n-2)\hat{\alpha}q_n + \frac{N_0W}{M}}\right)\right] +\\ &\quad (1-\alpha)\frac{W}{M}\mathbb{E}\left[\log\left(1 + \frac{h_{ll}p_l}{(n-2)\hat{\alpha}q_n + \frac{N_0W}{M}}\right)\right] && (2.21)\\ &= \frac{\alpha W}{M}\mathbb{E}\left[\log\left(1 + \frac{h_{ll}p_l}{\beta_{il}h_{il}p_i + \lambda'}\right)\right] + (1-\alpha)\frac{W}{M}\mathbb{E}\left[\log\left(1 + \frac{h_{ll}p_l}{\lambda'}\right)\right] && (2.22)\end{aligned}$$

as $K \to \infty$, where the expectation is computed with respect to h_{ll}, h_{il}, p_l and β_{il}, and $\lambda' \triangleq (n-2)\hat{\alpha}q_n + \frac{N_0W}{M}$. Also, (a) comes from the shadowing model described in (2.1). Using (2.19), (2.22), and the inequality $\log(1+x) \leq x$, for $x \geq 0$, the utility function in (2.13) is upper bounded as[8]

$$u_i(p_i, h_{ii}) \leq \frac{W}{M}\frac{h_{ii}}{\lambda}p_i + n\frac{\alpha W}{M}\mathbb{E}\left[\frac{h_{ll}p_l}{\beta_{il}h_{il}p_i + \lambda'}\right] + n(1-\alpha)\frac{W}{M\lambda'}\mathbb{E}[h_{ll}p_l]. \tag{2.23}$$

[8]Note that the factor $(n-1)$ in (2.13) is replaced by n in (2.23), which does not affect the validity of the equation.

Noting that h_{ll} is independent of h_{il}, $i \neq l$, we have

$$\begin{aligned}
\mathbb{E}\left[\frac{h_{ll}p_l}{\beta_{il}h_{il}p_i+\lambda'}\middle|\beta_{il}\right] &= \mu\int_0^{\infty}\frac{e^{-y}}{y\beta_{il}p_i+\lambda'}dy \\
&= -\frac{\mu}{\beta_{il}p_i}e^{\frac{\lambda'}{\beta_{il}p_i}}\mathrm{Ei}\left(-\frac{\lambda'}{\beta_{il}p_i}\right), \qquad (2.24)
\end{aligned}$$

where

$$\mu \triangleq \mathbb{E}\left[h_{ll}p_l\right] \qquad (2.25)$$

is a constant value, and $\mathrm{Ei}(x) \triangleq -\int_{-x}^{\infty}\frac{e^{-t}}{t}dt$, $x<0$ is the *exponential-integral function* [51]. Thus, the right hand side of (2.23) is simplified as

$$u_i(p_i,h_{ii}) \leq \frac{W}{M}\frac{h_{ii}}{\lambda}p_i - n\frac{\alpha\mu W}{M}\mathbb{E}\left[\frac{1}{\beta_{il}p_i}e^{\frac{\lambda'}{\beta_{il}p_i}}\mathrm{Ei}\left(-\frac{\lambda'}{\beta_{il}p_i}\right)\right] + n(1-\alpha)\frac{W}{M}\frac{\mu}{\lambda'}, \qquad (2.26)$$

where the expectation is computed with respect to β_{il}. An asymptotic expansion of $\mathrm{Ei}(x)$ can be obtained as [51, p. 951]

$$\mathrm{Ei}(x) = \frac{e^x}{x}\left[\sum_{k=0}^{L-1}\frac{k!}{x^k} + O(|x|^{-L})\right]; \quad L = 1,2,..., \qquad (2.27)$$

as $x \to -\infty$. Setting $L=4$, we can rewrite (2.26) as

$$\begin{aligned}
u_i(p_i,h_{ii}) \;&\leq\; \frac{W}{M}\frac{h_{ii}}{\lambda}p_i + n\frac{\alpha W\mu}{M\lambda'}\mathbb{E}\left[\left(1-\frac{\beta_{il}p_i}{\lambda'}+2\left(\frac{\beta_{il}p_i}{\lambda'}\right)^2-6\left(\frac{\beta_{il}p_i}{\lambda'}\right)^3\right)\right]+ \\
&\quad n\frac{\alpha W\mu}{M\lambda'}\mathbb{E}\left[O\left(\left|\frac{\beta_{il}p_i}{\lambda'}\right|^4\right)\right] + n(1-\alpha)\frac{W\mu}{M\lambda'} \qquad (2.28)\\
&\overset{(a)}{\approx}\; \frac{W}{M}\frac{h_{ii}}{\lambda}p_i + n\frac{\alpha W\mu}{M\lambda'}\left(1-\frac{\varpi p_i}{\lambda'}+2\kappa\left(\frac{p_i}{\lambda'}\right)^2-6\eta\left(\frac{p_i}{\lambda'}\right)^3\right)+ \\
&\quad n(1-\alpha)\frac{W\mu}{M\lambda'} \qquad (2.29)\\
&\triangleq\; \Xi_i(p_i,h_{ii}), \qquad (2.30)
\end{aligned}$$

as $\lambda' \to \infty$, where $\kappa \triangleq \mathbb{E}\left[\beta_{il}^2\right]$ and $\eta \triangleq \mathbb{E}\left[\beta_{il}^3\right]$, and (a) follows from the fact that for large values of λ', the term $\mathbb{E}\left[O\left(\left|\frac{\beta_{il}p_i}{\lambda'}\right|^4\right)\right]$ can be ignored.

Step 2: *Optimum Power Allocation Policy for* $\Xi_i(p_i, h_{ii})$

Using the fact that $p_i \in [0,1]$, the second-order derivative of (2.29) in terms of p_i, $\frac{\partial^2 \Xi_i(p_i,h_{ii})}{\partial p_i^2} = n\frac{\alpha W \mu}{M\lambda'}\left(\frac{4\kappa}{\lambda'^2} - \frac{36\eta}{\lambda'^3}p_i\right)$, is positive[9] as $\lambda' \to \infty$. Thus, (2.29) is a *convex* function of p_i. It is known that a convex function attains its maximum at one of the extreme points[10] of its domain [52]. In other words, the optimum power that maximizes (2.29) is $\hat{p}_i \in \{0,1\}$. To show that this optimum power is in the form of a unit step function, it is sufficient to prove that $p_i = g(h_{ii})$ is a monotonically increasing function of h_{ii}.

Suppose the optimum power that maximizes $\Xi_i(p_i, h_{ii})$ is $p_i = 1$. Also, let us define $h_{ii}^{'} \triangleq h_{ii} + \delta$, where $\delta > 0$. From (2.29), it is clear that $\Xi_i(p_i, h_{ii})$ is a monotonically increasing function of h_{ii}, i.e.:

$$\Xi_i(p_i = 1, h_{ii}^{'}) > \Xi_i(p_i = 1, h_{ii}). \tag{2.31}$$

In this case, the optimum power that maximizes $\Xi_i(p_i, h_{ii}^{'})$ is consistently equal to 1. On the other hand, since the optimum power is $p_i = 1$, it is concluded that

$$\Xi_i(p_i = 1, h_{ii}) > \Xi_i(p_i = 0, h_{ii}). \tag{2.32}$$

Using the fact that $\Xi_i(p_i = 0, h_{ii}) = \Xi_i(p_i = 0, h_{ii}^{'})$, the following inequality is obtained:

$$\Xi_i(p_i = 1, h_{ii}^{'}) > \Xi_i(p_i = 0, h_{ii}^{'}). \tag{2.33}$$

[9]It is observed from (2.27) and (2.29) that for any value of $L > 4$, the second-order derivative of (2.29) in terms of p_i is positive too.

[10]A real-valued function f defined on an interval $[a,b]$ is called *convex*, if for any two points $x, y \in [a,b]$ and $t \in [0,1]$, we have $f(tx + (1-t)y) \le tf(x) + (1-t)f(y)$. In this case, the extreme points are a and b.

From (2.31)-(2.33), it is concluded that $g(h_{ii})$ is a monotonically increasing function of h_{ii}. Consequently, the optimum power allocation strategy that maximizes $\Xi_i(p_i, h_{ii})$ is a unit step function, i.e.:

$$\hat{p}_i = \begin{cases} 1, & \text{if} \quad h_{ii} > \tau_n \\ 0, & \text{Otherwise}, \end{cases} \tag{2.34}$$

where τ_n is a threshold level to be determined. This optimum power allocation scheme is termed the *threshold-based on-off power allocation strategy*. It is observed that the optimum power $\hat{p}_i$ is a Bernoulli random variable with parameter q_n, i.e.:

$$f(\hat{p}_i) = \begin{cases} q_n, & \hat{p}_i = 1, \\ 1 - q_n, & \hat{p}_i = 0, \end{cases} \tag{2.35}$$

where $f(.)$ is the probability mass function (pmf) of $\hat{p}_i$. It is concluded from (2.34) and (2.35) that the probability of the link activation in each cluster is $q_n \triangleq \mathbb{P}\{h_{ii} > \tau_n\}$ which is a function of n.

Step 3: *Optimum Threshold Level* τ_n

From Step 1, it is observed that for every value of p_i, we have

$$u_i(p_i, h_{ii}) \leq \Xi_i(p_i, h_{ii}). \tag{2.36}$$

The above inequality is valid for the optimum power $\hat{p}_i$ obtained in Step 2. Thus, using the fact that for $X \leq Y$, $\mathbb{E}[X] \leq \mathbb{E}[Y]$, it is concluded

$$\mathbb{E}[u_i(\hat{p}_i, h_{ii})] \leq \mathbb{E}[\Xi_i(\hat{p}_i, h_{ii})], \tag{2.37}$$

where the expectations are computed with respect to h_{ii}. In the following lemmas, we first derive the optimum threshold level τ_n that maximizes $\mathbb{E}[\Xi_i(\hat{p}_i, h_{ii})]$, and then prove that this quantity is asymptotically the same as the optimum threshold level maximizing[11] $\mathbb{E}[u_i(\hat{p}_i, h_{ii})]$, assuming an on-off power scheme. It is also shown that the maximum value of $\mathbb{E}[u_i(\hat{p}_i, h_{ii})]$ (assuming an on-off power scheme) is the same as the optimum value of $\mathbb{E}[\Xi_i(\hat{p}_i, h_{ii})]$, proving the desired result.

Lemma 2.4. *For large values of n and given $0 < \alpha \leq 1$, the optimum threshold level that maximizes $\mathbb{E}[\Xi_i(\hat{p}_i, h_{ii})]$ is computed as*

$$\hat{\tau}_n \sim \log n. \tag{2.38}$$

Also, the maximum value of $\mathbb{E}[\Xi_i(\hat{p}_i, h_{ii})]$ scales as $\frac{W}{M\hat{\alpha}} \log n$.

Proof. See Appendix B. □

Lemma 2.5. *For large values of n and given $0 < \alpha \leq 1$,*

i) The optimum threshold level that maximizes $\mathbb{E}[u_i(\hat{p}_i, h_{ii})]$ is computed as

$$\hat{\tau}_n = \log n - 2 \log \log n + O(1), \tag{2.39}$$

ii) The probability of the link activation in each cluster is given by

$$q_n = \delta \frac{\log^2 n}{n}, \tag{2.40}$$

where $\delta > 0$ is a constant,

iii) The maximum value of $\mathbb{E}[u_i(\hat{p}_i, h_{ii})]$ scales as $\frac{W}{M\hat{\alpha}} \log n$.

Proof. See Appendix C. □

[11]In fact, since the threshold τ_n is fixed (i.e., it does not depend on a specific realization of h_{ii}), finding the optimum value of τ_n requires averaging the utility function over all realizations of h_{ii}.

Step 4: *Optimum Power Allocation Strategy that Maximize* $u_i(p_i, h_{ii})$

In order to prove that the utility function in (2.13) is asymptotically the same as the upper bound $\Xi_i(p_i, h_{ii})$ obtained in (2.29), it is sufficient to show that the low SINR conditions in (2.19) and (2.22) are satisfied. Using (2.19), (2.20) and (2.40), the SINR is equal to $\frac{h_{ii}p_i}{\lambda}$, where

$$\lambda \approx \hat{\alpha}\delta \log^2 n + \frac{N_0 W}{M}. \tag{2.41}$$

It is observed that λ goes to infinity as $n \to \infty$. On the other hand, since we are limiting our attention to links with $h_{ii} < h_{Th} = c \log n$, $c > 1$, we have

$$\frac{h_{ii}p_i}{\lambda} = O\left(\frac{1}{\log n}\right), \tag{2.42}$$

when $n \to \infty$. Thus, for large values of n, the low SINR condition, $\frac{h_{ii}p_i}{\lambda} \ll 1$, is satisfied. With a similar argument, the low SINR condition for (2.22) is satisfied. Hence, the approximation $\log(1 + x) \approx x$, for $x \ll 1$, can be used to simplify (2.19) and (2.22) as follows:

$$\bar{R}_i(p_i, h_{ii}) \approx \frac{W}{M}\frac{h_{ii}}{\lambda}p_i, \tag{2.43}$$

$$\bar{R}(p_i) \approx \frac{\alpha W}{M}\mathbb{E}\left[\frac{h_{ll}p_l}{\beta_{il}h_{il}p_i + \lambda'}\right] + (1-\alpha)\frac{W}{M\lambda'}\mathbb{E}\left[h_{ll}p_l\right]. \tag{2.44}$$

Consequently, the utility function $u_i(p_i, h_{ii})$ is the same as the upper bound $\Xi_i(p_i, h_{ii})$ obtained in (2.29), when $n \to \infty$. Thus, the optimum power allocation strategy for (2.6) is the same as the optimum power allocation policy that maximizes $\Xi_i(p_i, h_{ii})$.

Step 5: *Maximum Average Sum-rate of the Network*

Using (2.5), the average utility function of each user i, $\mathbb{E}\left[u_i(\hat{p}_i, h_{ii})\right]$, $\; i \in \mathbb{C}_j$, is the same as the average sum-rate of the links in cluster $\mathbb{C}_j$ represented by

$$\bar{R}_{ave}^{(j)} \triangleq \sum_{i \in \mathbb{C}_j} \mathbb{E}\left[R_i(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}_i^{(j)})\right], \quad j = 1, ..., M. \tag{2.45}$$

where $\hat{\mathbf{P}}^{(j)}$ is the on-off powers vector of the links in cluster $\mathbb{C}_j$. In this case, the average sum-rate of the network defined in (2.4) can be written as

$$\bar{R}_{ave} = \sum_{j=1}^{M} \bar{R}_{ave}^{(j)} \tag{2.46}$$

$$\stackrel{(a)}{\approx} \frac{W\hat{\tau}_n}{\hat{\alpha}}, \tag{2.47}$$

where (a) follows from (C-18) of Appendix C. Using (2.39), and noting that $n = \frac{K}{M}$, we have

$$\bar{R}_{ave} \sim \frac{W}{\hat{\alpha}} \log \frac{K}{M}. \tag{2.48}$$

Step 6: *Optimum Spectrum Allocation*

According to (2.47), the average sum-rate of the network is a monotonically increasing function of $\hat{\tau}_n$. Rewriting equation (C-15) of Appendix C, which gives the optimum threshold value for the on-off scheme:

$$-e^{-\hat{\tau}_n} \log\left(1 + \frac{\hat{\tau}_n e^{\hat{\tau}_n}}{n\hat{\alpha}}\right) + \frac{1 + \hat{\tau}_n}{n\hat{\alpha} + \hat{\tau}_n e^{\hat{\tau}_n}} = 0, \tag{2.49}$$

it can be shown that[12]

$$\hat{\tau}_n^2 e^{\hat{\tau}_n} \approx n\hat{\alpha}, \tag{2.50}$$

which implies that $\hat{\tau}_n$ is an increasing function of n. Therefore, the average sum-rate of the network is an increasing function of n and consequently, noting that $n = \frac{K}{M}$, is

[12] In deriving (2.50), we have used the fact that $\frac{\hat{\tau}_n e^{\hat{\tau}_n}}{n\hat{\alpha}} \ll 1$, which is feasible based on the solution given in (2.39).

a decreasing function of M. Hence, the maximum average sum-rate of the network for the strong interference scenario and $0 < \alpha < 1$ is obtained at $M = 1$ and this completes the proof of the theorem. $\square$

Motivated by Theorem 2.3, in the following, the proposed threshold-based on-off power allocation strategy for single-hop wireless networks is described. Based on this scheme, all users perform the following steps during each block:

1- Based on the direct channel gain, the transmission policy is

$$\hat{p}_i = \begin{cases} 1, & \text{if} \quad h_{ii} > \tau_n \\ 0, & \text{Otherwise.} \end{cases}$$

2- Knowing its corresponding direct channel gain, each active user i transmits with full power and rate

$$R_i = \log\left(1 + \frac{h_{ii}}{(n-1)\hat{\alpha}e^{-\tau_n} + \frac{N_0 W}{M}}\right). \tag{2.51}$$

3- Decoding is performed over a sufficiently large number of blocks, yielding the average rate of $\frac{W}{\hat{\alpha}K} \log K$ for each user, and the average sum-rate of $\frac{W}{\hat{\alpha}} \log K$ in the network.

Remark 2.6. *Theorem 2.3 states that the average sum-rate of the network for fixed M depends on the value of $\hat{\alpha} = \alpha\varpi$ and scales as $\frac{W}{\hat{\alpha}} \log \frac{K}{M}$. Also, for values of M such that* $\log M = o(\log K)$, *the average sum-rate of the network scales as $\frac{W}{\hat{\alpha}} \log K$.*

Remark 2.7. *Let m_j denote the number of active links*[13] *in $\mathbb{C}_j$. Lemma 2.5 states that the optimum selection of the threshold value yields $\mathbb{E}[m_j] = nq_n = \Theta\left(\log^2 n\right)$. More*

[13]Note that the number of active links in each cluster is a binomial random variable with parameters (n, q_n).

precisely, it can be shown that the optimum number of active users scales as $\Theta\left(\log^2 n\right)$*, with probability one.*

2.3.2 Moderate and Weak Interference Scenarios ($\mathbb{E}[I_i] = O(1)$)

Theorem 2.8. *Let us assume K is large and M is fixed. Then,*

i) For the moderate interference scenario (i.e., $\mathbb{E}[I_i] = \Theta(1)$), the average sum-rate of the network is bounded by $\bar{R}_{ave} \leq \Theta(\log n)$.

ii) For the weak interference scenario (i.e., $\mathbb{E}[I_i] = o(1)$), the average sum-rate of the network is bounded by $\bar{R}_{ave} \leq o(\log n)$.

Proof. i) From (2.4), we have

$$\begin{aligned}
\bar{R}_{ave} &= \sum_{j=1}^{M} \sum_{l \in \mathbb{C}_j} \mathbb{E}\left[\frac{W}{M} \log\left(1 + \frac{h_{ll}\hat{p}_l}{I_l + \frac{N_0 W}{M}}\right)\right] && (2.52)\\
&\overset{(a)}{\leq} \sum_{j=1}^{M} \sum_{l \in \mathbb{C}_j} \frac{W}{M} \mathbb{E}\left[\log\left(1 + \frac{\hat{p}_l c \log n}{I_l + \frac{N_0 W}{M}}\right)\right] && (2.53)\\
&\leq \sum_{j=1}^{M} \sum_{l \in \mathbb{C}_j} \frac{W}{M} \mathbb{E}\left[\log\left(1 + \frac{\hat{p}_l c \log n}{\frac{N_0 W}{M}}\right)\right] && (2.54)\\
&\overset{(b)}{\leq} \sum_{j=1}^{M} \sum_{l \in \mathbb{C}_j} \frac{W}{M} \log\left(1 + \frac{c q_n \log n}{\frac{N_0 W}{M}}\right) && (2.55)\\
&\overset{(c)}{\leq} \frac{cM}{N_0} n q_n \log n, && (2.56)
\end{aligned}$$

where (a) follows from Lemma 2.2, which implies that the realizations in which $h_{ll} > c \log n$ for some $c > 1$ has negligible contribution in the average sum-rate of the network, (b) results from the *Jensen's inequality*, $\mathbb{E}[\log x] \leq \log(\mathbb{E}[x])$, $x > 0$. Also, (c) follows from the fact that $\log(1 + x) \leq x$, $x \geq 0$. Since for the moderate interference, $\mathbb{E}[I_i] =$

$\hat{\alpha}(n-1)q_n = \Theta(1)$, and using the fact that M is fixed, the following inequality is derived:

$$\begin{aligned}\bar{R}_{ave} &\leq \frac{cM}{\hat{\alpha}N_0}\Theta(1)\log n & (2.57)\\ &= \Theta(\log n). & (2.58)\end{aligned}$$

ii) For the weak interference scenario, where $\mathbb{E}[I_i] = \hat{\alpha}(n-1)q_n = o(1)$, and similar to the arguments in part (i), it is concluded from (2.56) that

$$\begin{aligned}\bar{R}_{ave} &\leq \frac{cM}{\hat{\alpha}N_0}o(1)\log n & (2.59)\\ &= o(\log n). & (2.60)\end{aligned}$$

$\square$

Remark 2.9. *It is concluded from Theorems 2.3 and 2.8 that the maximum average sum-rate of the proposed network is scaled as* $\Theta(\log K)$*.*

2.3.3 M Not Fixed (Scaling With K)

So far, it has assumed that M is fixed, i.e., it does not scale with K. In the following, some results for the case that M scales with K are presented[14]. It should be noted that the results for $M = o(K)$ are the same as the results in Theorem 2.3.

Theorem 2.10. *In the network with the on-off power allocation strategy, if* $M = \Theta(K)$ *and* $0 < \alpha < 1$*, then the maximum average sum-rate of the network in (2.4) is less*

[14]Obviously, the values of M which are in the interval $[1, K]$, i.e., $M = o(K)$ and $M = \Theta(K)$ are considered.

than that of $M = 1$. Consequently, the maximum average sum-rate of the network for every value of $1 \leq M \leq K$ is achieved at $M = 1$.

Proof. Let us define $\mathbb{A}_j$ as the set of active links in cluster j. The random variable m_j denotes the cardinality of the set $\mathbb{A}_j$. Noting that for $M = \Theta(K)$, $\lim_{K\to\infty} \frac{M}{K}$ is constant, it is concluded that n and $m_j \in [1, n]$ do not grow with K. To obtain the network's average sum-rate, it is assumed that among M clusters, Γ clusters have $m_j = 1$ and the rest have $m_j > 1$. First, an upper bound on the average sum-rate in each cluster is obtained when $m_j = 1$, $1 \leq j \leq M$. Clearly, since only one user in each cluster activates its transmitter, $I_i = 0$. Thus, by using (2.45), the maximum achievable average sum-rate of cluster $\mathbb{C}_j$ is computed as

$$\bar{R}_{ave}^{(j)} = \frac{W}{M}\mathbb{E}\left[\log\left(1 + \frac{M}{N_0 W} h_{\max}\right)\right], \tag{2.61}$$

where $h_{\max} \triangleq \max\ \{h_{ii}\}_{i\in\mathbb{C}_j}$ is a random variable. Since $\log x$ is a concave function of x, an upper bound of (2.61) is obtained through *Jensen's inequality*, $\mathbb{E}[\log x] \leq \log(\mathbb{E}[x])$, $x > 0$. Thus,

$$\bar{R}_{ave}^{(j)} \leq \frac{W}{M}\log\left(1 + \frac{M}{N_0 W}\mathbb{E}[h_{\max}]\right). \tag{2.62}$$

Under a Rayleigh fading channel model and noting that $\{h_{ii}\}$ is a set of i.i.d. random variables over $i \in \mathbb{C}_j$, we have

$$\begin{aligned} F_{h_{max}}(y) &= \mathbb{P}\{h_{\max} \leq y\}, \quad y > 0 && (2.63)\\ &= \prod_{i\in\mathbb{C}_j}\mathbb{P}\{h_{ii} \leq y\} && (2.64)\\ &= \left(1 - e^{-y}\right)^n, && (2.65)\end{aligned}$$

where $F_{h_{max}}(.)$ is the Cumulative Distribution Function (CDF) of $h_{\max}$. Hence,

$$\mathbb{E}\left[h_{\max}\right] = \int_0^\infty nye^{-y}\left(1-e^{-y}\right)^{n-1}dy. \tag{2.66}$$

Since $\left(1-e^{-y}\right)^{n-1} \leq 1$, the following inequality is derived:

$$\mathbb{E}\left[h_{\max}\right] \leq \int_0^\infty nye^{-y}dy = n. \tag{2.67}$$

Consequently, the upper bound of (2.62) can be simplified as

$$\bar{R}_{ave}^{(j)} \leq \frac{W}{M}\log\left(1+\frac{K}{N_0W}\right). \tag{2.68}$$

For $m_j > 1$ and due to the shadowing effect with parameters (α, ϖ), the average sum-rate of cluster $\mathbb{C}_j$ can be written as

$$\bar{R}_{ave}^{(j)} = \sum_{i\in\mathbb{A}_j}\frac{W}{M}\mathbb{E}\left[\log\left(1+\frac{h_{ii}}{\sum_{\substack{k\in\mathbb{A}_j\\k\neq i}}u_k\beta_{ki}h_{ki}+\frac{N_0W}{M}}\right)\right], \tag{2.69}$$

where u_k's are Bernoulli random variables with parameter α. Thus,

$$\begin{aligned}
\bar{R}_{ave}^{(j)} &= \frac{W}{M}\sum_{i\in\mathbb{A}_j}\sum_{l=0}^{m_j-1}\binom{m_j-1}{l}\alpha^l(1-\alpha)^{m_j-1-l}\mathbb{E}\left[\log\left(1+\frac{h_{ii}}{\Sigma_l+\frac{N_0W}{M}}\right)\right] && (2.70)\\
&= \frac{W}{M}\sum_{i\in\mathbb{A}_j}(1-\alpha)^{m_j-1}\mathbb{E}\left[\log\left(1+\frac{h_{ii}}{\frac{N_0W}{M}}\right)\right]+\\
&\quad \frac{W}{M}\sum_{i\in\mathbb{A}_j}\sum_{l=1}^{m_j-1}\binom{m_j-1}{l}\alpha^l(1-\alpha)^{m_j-1-l}\mathbb{E}\left[\log\left(1+\frac{h_{ii}}{\Sigma_l+\frac{N_0W}{M}}\right)\right], && (2.71)
\end{aligned}$$

where Σ_l is the sum of l i.i.d random variables $\{Z_i\}_{i=1}^l$, where $Z_i \triangleq \beta_{ki}h_{ki}$, $k \neq i$. For $m_j > 1$, Σ_l is greater or equal than the interference term caused by one interfering link. Thus, an upper bound on the average sum-rate of cluster $\mathbb{C}_j$ is computed as

$$\begin{aligned}
\bar{R}_{ave}^{(j)} &\leq \frac{W}{M}m_j(1-\alpha)^{m_j-1}\mathbb{E}\left[\log\left(1+\frac{Y}{\frac{N_0W}{M}}\right)\right]+\\
&\quad \frac{W}{M}\sum_{i\in\mathbb{A}_j}\sum_{l=1}^{m_j-1}\binom{m_j-1}{l}\alpha^l(1-\alpha)^{m_j-1-l}\mathbb{E}\left[\log\left(1+\frac{Y}{Z_i+\frac{N_0W}{M}}\right)\right], && (2.72)
\end{aligned}$$

where $Y \triangleq h_{\max} = \max\ \{h_{ii}\}_{i\in\mathbb{C}_j}$. According to binomial formula, we have

$$\sum_{l=1}^{m_j-1}\binom{m_j-1}{l}\alpha^l(1-\alpha)^{m_j-1-l} = 1-(1-\alpha)^{m_j-1}. \tag{2.73}$$

Thus,

$$\begin{aligned}\bar{R}_{ave}^{(j)} \leq\ & \frac{W}{M}m_j(1-\alpha)^{m_j-1}\mathbb{E}\left[\log\left(1+\frac{Y}{\frac{N_0W}{M}}\right)\right]+\\ & \frac{W}{M}m_j\left(1-(1-\alpha)^{m_j-1}\right)\mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{ki}h_{ki}+\frac{N_0W}{M}}\right)\right].\end{aligned} \tag{2.74}$$

We have

$$\mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{ki}h_{ki}+\frac{N_0W}{M}}\right)\right] \leq \mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{\min}h_{ki}}\right)\right]. \tag{2.75}$$

Defining $Z \triangleq \beta_{\min}h_{ki}$ and $X \triangleq \frac{Y}{Z}$, the CDF of X can be evaluated as

$$\begin{aligned}F_X(x) &= \mathbb{P}\{X \leq x\}, \quad x>0\\ &= \mathbb{P}\{Y \leq Zx\}\\ &= \int_0^\infty \mathbb{P}\{Y \leq Zx|Z=z\}f_Z(z)dz\\ &= \int_0^\infty \left(1-e^{-zx}\right)^n\frac{1}{\beta_{\min}}e^{-\frac{z}{\beta_{\min}}}dz\\ &= \int_0^\infty \left(1-e^{-t\beta_{\min}x}\right)^n e^{-t}dt.\end{aligned}$$

Thus, the probability density function (pdf) of X can be written as

$$\begin{aligned}f_X(x) &= \frac{dF_X(x)}{dx}\\ &= \beta_{\min}\int_0^\infty nte^{-t(1+\beta_{\min}x)}\left(1-e^{-t\beta_{\min}x}\right)^{n-1}dt\\ &\leq \beta_{\min}\int_0^\infty nte^{-t(1+\beta_{\min}x)}dt\\ &= \frac{n\beta_{\min}}{(1+\beta_{\min}x)^2}.\end{aligned} \tag{2.76}$$

Using (2.76), the right hand side of (2.75) can be upper-bounded as

$$
\begin{aligned}
\mathbb{E}\left[\log\left(1+\frac{Y}{\beta_{\min}h_{ki}}\right)\right] &= \int_0^{\infty} f_X(x)\log(1+x)dx && (2.77)\\
&\leq n\beta_{\min}\int_0^{\infty}\frac{\log(1+x)}{(1+\beta_{\min}x)^2}dx && (2.78)\\
&= \frac{-n\log\beta_{\min}}{1-\beta_{\min}} && (2.79)\\
&= \Theta(1), && (2.80)
\end{aligned}
$$

where the last line follows from the fact that $0<\beta_{\min}\leq 1$, and n does not scale with K. Substituting the above equation in (2.74) yields

$$
\begin{aligned}
\bar{R}_{ave}^{(j)} &\leq \frac{W}{M}m_j(1-\alpha)^{m_j-1}\mathbb{E}\left[\log\left(1+\frac{Y}{\frac{N_0W}{M}}\right)\right]+\\
&\quad \frac{W}{M}m_j\left(1-(1-\alpha)^{m_j-1}\right)\Theta(1) && (2.81)\\
&\overset{(a)}{\leq} \frac{W}{M}m_j(1-\alpha)^{m_j-1}\log\left(1+\frac{K}{N_0W}\right)+\Theta\left(\frac{W}{M}\right) && (2.82)\\
&= \frac{W}{M}m_j(1-\alpha)^{m_j-1}\log\left(1+\frac{K}{N_0W}\right)[1+o(1)], && (2.83)
\end{aligned}
$$

where (a) follows from (2.68) and the fact that $m_j\in\{2,...,n\}$ does not scale with K.

Let us assume that among M clusters, Γ clusters have $m_j=1$ and for the $M-\Gamma$ of the rest, the number of active links in each cluster is greater than one. By using (2.68) and (2.83), an upper bound on the average sum-rate of the network is obtained as

$$
\begin{aligned}
\bar{R}_{ave} &\leq \frac{\Gamma W}{M}\log\left(1+\frac{K}{N_0W}\right)+\\
&\quad (M-\Gamma)\frac{W}{M}m_j(1-\alpha)^{m_j-1}\log\left(1+\frac{K}{N_0W}\right)[1+o(1)]. && (2.84)
\end{aligned}
$$

To compare this upper-bounded with the computed average sum-rate of the network in the case of $M=1$, we note that as $\varpi\leq 1$ and $\alpha<1$, we have $\hat{\alpha}<1$ and consequently,

$$
\frac{\Gamma W}{M}\log\left(1+\frac{K}{N_0W}\right)<\frac{\Gamma W}{M\hat{\alpha}}\log\left(1+\frac{K}{N_0W}\right). \tag{2.85}
$$

To prove that the maximum network's average sum-rate obtained in (2.84) is less than that value obtained for $M = 1$ from (2.17), it is sufficient to show

$$(M-\Gamma)\frac{W}{M}m_j(1-\alpha)^{m_j-1}\log\left(1+\frac{K}{N_0W}\right) < (M-\Gamma)\frac{W}{M\hat{\alpha}}\log\left(1+\frac{K}{N_0W}\right), \quad (2.86)$$

or

$$m_j(1-\alpha)^{m_j-1} < \frac{1}{\hat{\alpha}}.$$

Since $\hat{\alpha} \leq \alpha$, it is sufficient to show that $m_j(1-\alpha)^{m_j-1} < \frac{1}{\alpha}$. Defining $\Lambda(\alpha) = \alpha m_j(1-\alpha)^{m_j-1}$, we have

$$\frac{\partial\Lambda(\alpha)}{\partial\alpha} = m_j(1-\alpha)^{m_j-2}(1-\alpha m_j).$$

Thus, the extremum points of $\Lambda(\alpha)$ are located at $\alpha = 1$ and $\alpha = \frac{1}{m_j}$, where $m_j \in \{2, ..., n\}$. It is observed that

$$\Lambda(1) = 0 < 1,$$

and

$$\Lambda\left(\frac{1}{m_j}\right) = \left(\frac{m_j-1}{m_j}\right)^{m_j-1} < 1.$$

Since $\Lambda(\alpha) < 1$, we conclude (2.86), which implies that the maximum average sum-rate of the network for $M = \Theta(K)$ is less than that of $M = 1$. Knowing the fact that for $M = o(K)$, similar to the result of Theorem 2.3, one can show that the maximum average sum-rate of the network is achieved at $M = 1$, it is concluded that using the on-off allocation scheme, the maximum average sum-rate of the network is achieved at $M = 1$, for all values of $1 \leq M \leq K$. □

So far, we have investigated the average sum-rate of the network for $0 < \alpha < 1$. In the next theorem, $\bar{R}_{ave}$ for $\alpha = 0$ and for every value of $1 \leq M \leq K$ is derived.

Theorem 2.11. *Assuming $\alpha = 0$, the maximum average sum-rate of the network for every value of $1 \leq M \leq K$ is achieved at $M = 1$.*

Proof. According to the shadow-fading model proposed in (2.1), it is seen that for $\alpha = 0$, with probability one, $\mathcal{L}_{ki} = 0,\ k \neq i$. This implies that no interference exists in each cluster. In this case, the maximum average sum-rate of the network is clearly achieved by all users in the network transmitting at full power. Using (2.3) and (2.4) and for every value of $1 \leq M \leq K$ and $\alpha = 0$, the average sum-rate of the network is simplified as

$$\bar{R}_{ave} = \sum_{j=1}^{M} \sum_{i \in \mathbb{C}_j} \mathbb{E}\left[\frac{W}{M} \log\left(1 + \frac{h_{ii}}{\frac{N_0 W}{M}}\right)\right], \tag{2.87}$$

where the expectation is computed with respect to h_{ii}. Under a Rayleigh fading channel condition and using the fact that $n = \frac{K}{M}$, (2.87) can be written as

$$\begin{aligned} \bar{R}_{ave} &= nW \int_0^\infty e^{-x} \log\left(1 + \frac{M}{N_0 W} x\right) dx & (2.88) \\ &= \frac{KW}{M} e^{\frac{N_0 W}{M}} \mathrm{E}_1\left(\frac{N_0 W}{M}\right) & (2.89) \\ &= \frac{KW}{M} e^{\frac{N_0 W}{M}} \int_1^\infty \frac{e^{-t\frac{N_0 W}{M}}}{t} dt, & (2.90) \end{aligned}$$

where $\mathrm{E}_1(x) = -\mathrm{Ei}(-x) = \int_1^\infty \frac{e^{-tx}}{t} dt,\ x > 0$ [51]. Taking the first-order derivative of (2.90) in terms of M yields,

$$\frac{\partial \bar{R}_{ave}}{\partial M} = -\frac{KW}{M^2} e^{\frac{N_0 W}{M}} \left(1 + \frac{N_0 W}{M}\right) \mathrm{E}_1\left(\frac{N_0 W}{M}\right) + \frac{KW}{M^2}. \tag{2.91}$$

Since for every value of $N_0 W$, $\frac{\partial \bar{R}_{ave}}{\partial M}$ is negative, it is concluded that the network's average sum-rate is a monotonically decreasing function of M. Consequently, the maximum average sum-rate of the network for $\alpha = 0$ and every value of $1 \leq M \leq K$ is achieved at $M = 1$, where all the links in the network transmit with full power. □

Theorem 2.12. *Assuming $M = K$ and for every value of $0 \leq \alpha \leq 1$, the average sum-rate of the network is asymptotically obtained as*

$$\bar{R}_{ave} \approx W(\log K - \log N_0 W - \boldsymbol{\gamma}), \tag{2.92}$$

where $\boldsymbol{\gamma}$ is Euler's constant

Proof. Noting that for $M = K$ only one user exists in each cluster, all the users can communicate using an interference free channel, i.e., $I_i = 0$ and $p_i = 1$, $i = 1, ..., K$. From (2.3) and (2.4), the average sum-rate of the network is given by

$$\begin{aligned}\bar{R}_{ave} &= \mathbb{E}\left[\sum_{i=1}^{K} R_i(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}_i^{(j)})\right] \\ &= \frac{W}{K}\sum_{i=1}^{K}\mathbb{E}\left[\log\left(1+\frac{h_{ii}}{\frac{N_0 W}{K}}\right)\right],\end{aligned}$$

where the expectation is computed with respect to h_{ii}. Under a Rayleigh fading channel condition, we have

$$\begin{aligned}\bar{R}_{ave} &= W\int_0^{\infty} e^{-x}\log\left(1+\frac{K}{N_0 W}x\right)dx \\ &= W e^{\frac{N_0 W}{K}} \mathrm{E}_1\left(\frac{N_0 W}{K}\right).\end{aligned} \tag{2.93}$$

To simplify (2.93), the following series representation for $\mathrm{E}_1(x)$ is used:

$$\mathrm{E}_1(x) = -\boldsymbol{\gamma} + \log\left(\frac{1}{x}\right) + \sum_{s=1}^{\infty}\frac{(-1)^{s+1}x^s}{s.s!}, \quad x > 0,$$

where $\boldsymbol{\gamma}$ is Euler's constant and is defined by the limit [51]

$$\boldsymbol{\gamma} \triangleq \lim_{s\to\infty}\left(\sum_{k=1}^{s}\frac{1}{k} - \log s\right) = 0.577215665...$$

Thus, (2.93) can be simplified as

$$\bar{R}_{ave} = We^{\frac{N_0 W}{K}} \left(-\boldsymbol{\gamma} + \log\left(\frac{K}{N_0 W}\right) + \sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{s.s!} \left(\frac{N_0 W}{K}\right)^s \right).$$

In the asymptotic case of $K \to \infty$,

$$e^{\frac{N_0 W}{K}} \approx 1,$$

and

$$\sum_{s=1}^{\infty} \frac{(-1)^{s+1}}{s.s!} \left(\frac{N_0 W}{K}\right)^s \approx 0.$$

Consequently, the average sum-rate of the network for $M = K$ is asymptotically obtained by

$$\bar{R}_{ave} \approx W(\log K - \log N_0 W - \boldsymbol{\gamma}).$$

$\square$

Corollary 2.13. *It is concluded from Theorem 2.12 that for every value of* $0 < \alpha < 1$*, the average sum-rate of the network in (2.92) is less than that of* $M = 1$ *obtained in (2.17).*

Remark 2.14. *Note that for* $M = 1$*, in which the average number of active links scales as* $\Theta(\log^2 K)$ *(in the optimum on-off scheme), we have significant energy saving in the network as compared to the case of* $M = K$*, in which all the users transmit with full power.*

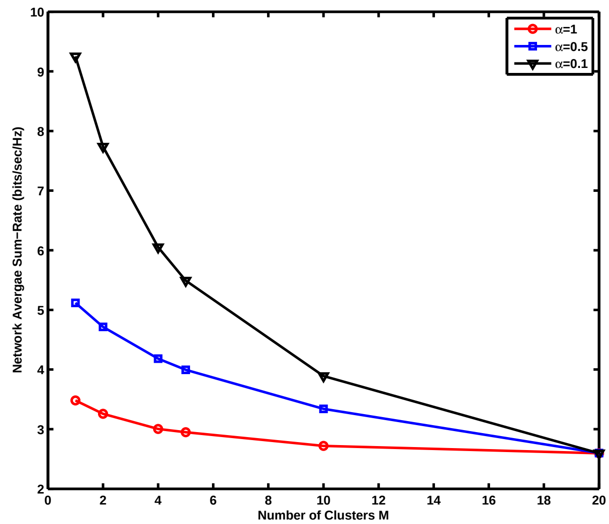
2.4 Numerical Results

So far, we analyzed the average sum-rate of the network in terms of M and $\hat{\alpha}$, and in the asymptotic case of $K \to \infty$. For a finite number of users, the network's average sum-rate versus the number of clusters (M) through simulation is evaluated. For this case, it is assumed that all the users in the network follow the threshold-based on-off power allocation policy, using the optimum threshold value. In addition, the shadowing effect is assumed to be *lognormal* distributed with mean $\varpi \leq 1$ and variance 1. Fig. 2.1 shows the average sum-rate of the network versus M for $K = 20$ and $K = 40$, and different values of α and ϖ. It is observed from this figure that the average sum-rate of the network is a monotonically decreasing function of M for every value of (α, ϖ), which implies that the maximum value of $\bar{R}_{ave}$ is achieved at $M = 1$.

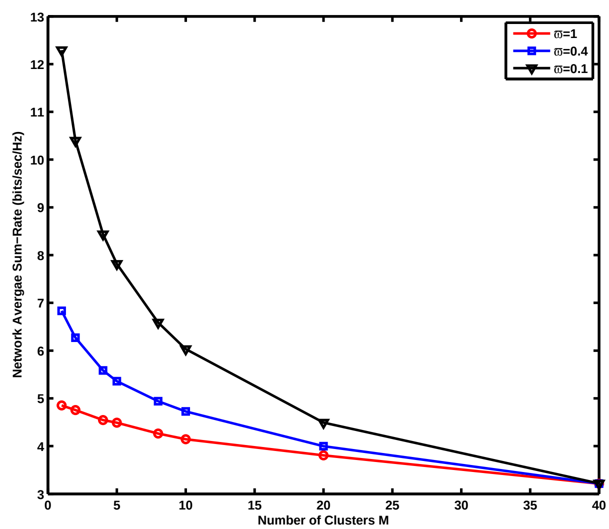
Based on the above arguments, we have plotted the average sum-rate of the network and the optimum threshold level τ_n versus K for $M = 1$ and different values of (α, ϖ). It is observed from figures 2.2 and 2.3 that the network's average sum-rate and τ_n depend strongly on the values of (α, ϖ).

2.5 Conclusion

In this chapter, a distributed single-hop wireless network with K links was considered, where the links were partitioned into a fixed number (M) of clusters each operating in a subchannel with bandwidth $\frac{W}{M}$. The network's throughput is defined as the *average sum-rate* of the network, which is shown to scale as $\Theta(\log K)$. It was proved that in the strong interference scenario, the optimum power allocation strategy for each user was a threshold-based on-off scheme. Moreover, it was demonstrated that the optimum spectrum sharing for maximizing the average sum-rate is achieved at $M = 1$. In other words, partitioning the bandwidth W into M subchannels has no gain in terms

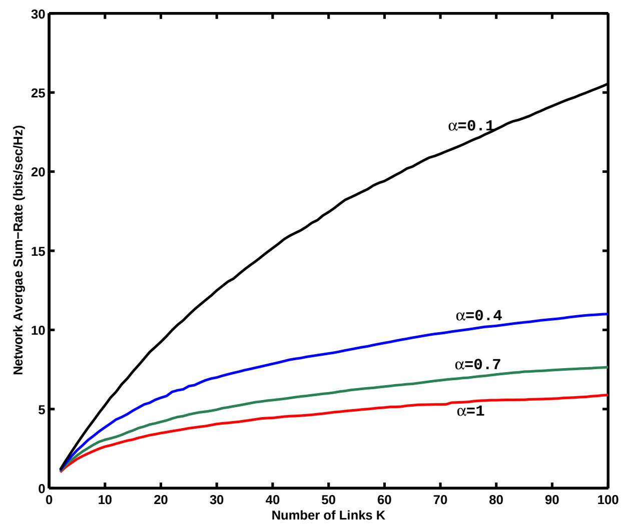


(a)

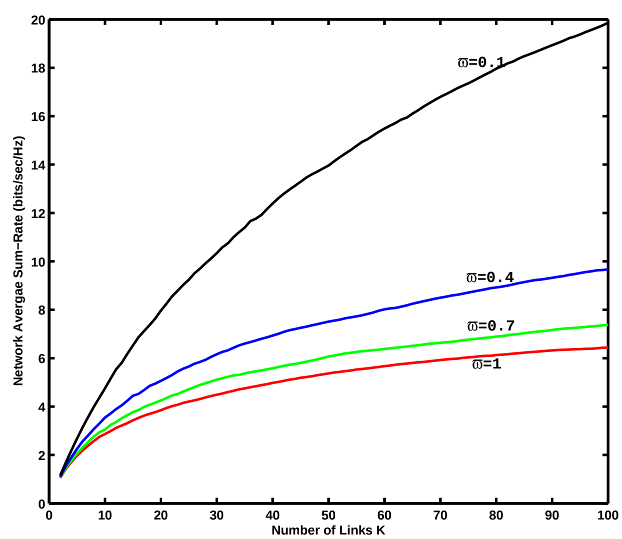


(b)

Figure 2.1: Network's average sum-rate versus M for a) $K = 20$, $\alpha = 1,\ 0.5,\ 0.1$ and shadowing model with $\varpi = 0.5$ and variance 1, and b) $K = 40$, $\alpha = 0.5$ and shadowing model with $\varpi = 1,\ 0.4,\ 0.1$ and variance 1.



(a)



(b)

Figure 2.2: Network's average sum-rate versus K for $M = 1$ and a) shadowing model with $\varpi = 0.5$ and variance 1, and $\alpha = 1,\ 0.7,\ 0.4,\ 0.1$, and b) shadowing model with $\varpi = 1,\ 0.7,\ 0.4,\ 0.1$ and variance 1, and $\alpha = 0.5$.

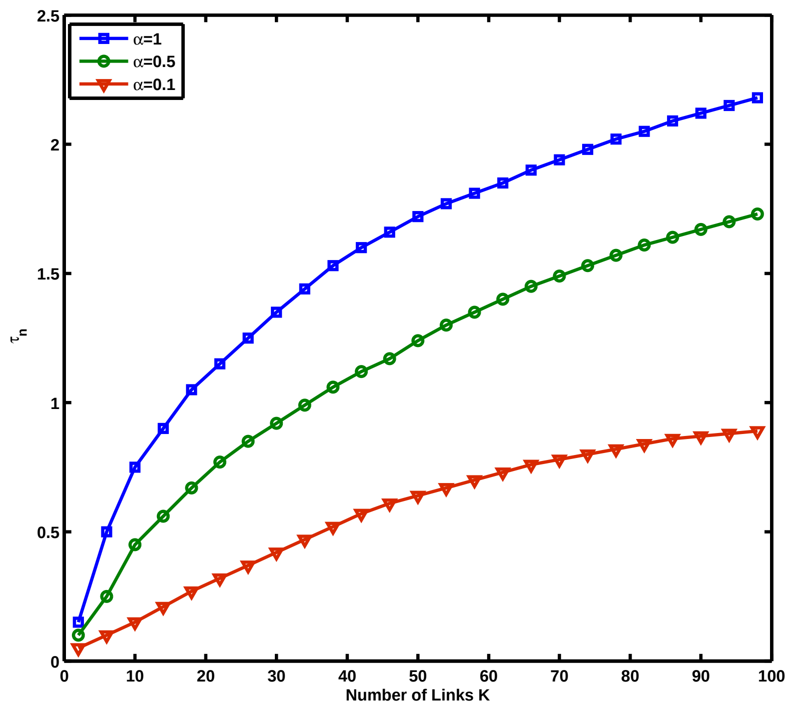


Figure 2.3: Optimum threshold level τ_n versus the number of links K for $M = 1$ and shadowing model with $\varpi = 0.5$, variance 1, and $\alpha = 1,\ 0.5,\ 0.1$.

of enhancing the throughput. The interesting point is that under the on-off power allocation strategy, the total network energy for $M = 1$ is significantly lower when compared to the case that all the users transmit with full power all the time. Also, the proposed on-off scheme has the advantage of not requiring a central controller and is simple to implement in practical time-varying networks.

Chapter 3

Network Guaranteed Sum-Rate

3.1 Introduction

Chapter 2 is centered on maximization of the average sum-rate of the network, by proposing the on-off power scheme. In this chapter, the throughput of the network is defined as the *network's guaranteed sum-rate*, in which decoding is performed over each separate block. This metric is useful when there exists a stringent decoding delay constraint. In the first part of this chapter, an upper-bound and a lower-bound for the network's guaranteed sum-rate is derived. It is then shown that these bounds converge to each other as $K \to \infty$. Also, it is proved that the maximum guaranteed sum-rate is achieved by using the on-off scheme and scales as $\frac{W}{\hat{\alpha}} \log K$. Moreover, the optimum spectrum sharing for maximizing the network's guaranteed sum-rate is the same as the one maximizing the network's average sum-rate ($M = 1$).

3.2 Problem Formulation

In the present chapter, the same network model introduced in Chapter 2 is considered. It is also assumed that M is fixed. The performance metric used in this chapter is different from the network's average sum-rate used in the previous chapter which is defined over arbitrary large number of blocks. We use the *network's guaranteed sum-rate*, denoted by $\bar{R}_g$, as the throughput of the network as follows:

$$\bar{R}_g \triangleq \sum_{j=1}^{M} \sum_{l \in \mathbb{C}_j} \mathbb{E}_{h_{ll}} \left[R^*(h_{ll}) \right], \tag{3.1}$$

in which for all h_{ll}, $l \in \mathbb{C}_j$, we have

$$R^*(h_{ll}) \triangleq \sup \; R(h_{ll}), \tag{3.2}$$

such that

$$\mathbb{P} \left\{ R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)}) < R(h_{ll}) \right\} \to 0. \tag{3.3}$$

This metric is useful when there exists a stringent decoding delay constraint, i.e, decoding must be performed over each separate block. In this case, as the transmitter does not have any information about the interference term, an outage event may occur. Network's guaranteed throughput is the average sum-rate of the network which is guaranteed for all channel realizations.

The main objective of this chapter is to find the maximum achievable network's guaranteed sum-rate in the asymptotic case of $K \to \infty$. For this purpose, a lower bound and an upper-bound on the network's guaranteed sum-rate are presented and shown to converge to each other as $K \to \infty$ (or equivalently $n \to \infty$). Also, the optimum power allocation scheme and the optimum value of M are obtained.

3.3 Network Guaranteed Sum-Rate

In order to compute the guaranteed rate for link $l \in \mathbb{C}_j$, we first define the corresponding outage event as follows:

$$\mathcal{O}_l^{(j)} \equiv \left\{ R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)}) < R(h_{ll}) \right\}. \tag{3.4}$$

In this case, the corresponding outage probability is defined as follows:

$$\begin{aligned} \mathbb{P}\left\{\mathcal{O}_l^{(j)}\right\} &\triangleq \mathbb{P}\left\{R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)}) < R(h_{ll})\right\} && (3.5)\\ &= \mathbb{P}\left\{\log\left(1 + \frac{p_l h_{ll}}{I_l + \frac{N_0 W}{M}}\right) < R(h_{ll})\right\}. && (3.6) \end{aligned}$$

In the next sections, we use (3.6) to obtain an upper-bound and a lower bound on the network's guaranteed sum-rate in the asymptotic case of $K \to \infty$.

3.3.1 Upper-Bound

In the next lemma, an upper-bound on the guaranteed sum-rate by using lower-bounding the outage probability is derived.

Lemma 3.1. *For the strong and moderate interference regimes, the outage probability defined in (3.6) is lower bounded as*

$$\mathbb{P}\left\{\mathcal{O}_l^{(j)}\right\} \geq 1 - e^{-\frac{\gamma N_0 W}{2M\beta_{\max}}\left(1 - \frac{t(h_{ll})}{R(h_{ll})}\right)}, \tag{3.7}$$

where $\gamma \triangleq \min\left(1, \frac{M(n-1)q_n\hat{\alpha}}{N_0 W}\right)$ *and* $t(h_{ll}) = \frac{p_l h_{ll}}{(n-1)\hat{\alpha}q_n + (1-\frac{\gamma}{2})\frac{N_0 W}{M}}$.

Proof. Using (3.6) and $\log(1+x) \leq x$, for $x \geq 0$, we have

$$\begin{aligned} \mathbb{P}\left\{\mathcal{O}_l^{(j)}\right\} &\geq \mathbb{P}\left\{\frac{p_l h_{ll}}{I_l + \frac{N_0 W}{M}} < R(h_{ll})\right\} && (3.8)\\ &= \mathbb{P}\left\{p_l h_{ll} - \frac{N_0 W}{M} R(h_{ll}) < I_l R(h_{ll})\right\}, && (3.9) \end{aligned}$$

Denoting $\nu = h_{ll}$, we can write

$$\mathbb{P}\left\{\mathcal{O}_l^{(j)}\right\} \overset{(a)}{\geq} \mathbb{P}\left\{e^{-I_l \xi(\nu) R(\nu)} \leq e^{\xi(\nu)\left(\frac{N_0 W}{M} R(\nu) - p_l \nu\right)}\right\} \tag{3.10}$$

$$\overset{(b)}{\geq} 1 - e^{-\xi(\nu)\left(\frac{N_0 W}{M} R(\nu) - p_l \nu\right)} \mathbb{E}\left[e^{-I_l \xi(\nu) R(\nu)}\right], \tag{3.11}$$

for some positive $\xi(\nu)$, where the expectation is computed with respect to I_l. In the above equations, (a) results from (3.9), noting that $\xi(\nu) > 0$, and (b) comes from the following *Markov's inequality* [53, p. 77]:

$$\mathbb{P}\{X \geq a\} \leq \frac{\mathbb{E}[X]}{a}, \quad a > 0, \tag{3.12}$$

where X is a nonnegative random variable. The above equations imply that finding an upper-bound for $\mathbb{E}\left[e^{-I_l \xi(\nu) R(\nu)}\right]$ is sufficient for the lower-bounding the outage probability. For this purpose, using (2.2), we can write

$$\begin{aligned}
\mathbb{E}\left[e^{-I_l \xi(\nu) R(\nu)}\right] &= \mathbb{E}\left[e^{-\xi(\nu) R(\nu) \sum_{\substack{k \in \mathbb{C}_j \\ k \neq l}} \mathcal{L}_{kl} p_k}\right] \\
&\overset{(a)}{=} \prod_{\substack{k \in \mathbb{C}_j \\ k \neq l}} \mathbb{E}\left[e^{-\xi(\nu) R(\nu) \mathcal{L}_{kl} p_k}\right] \\
&\overset{(b)}{=} \prod_{\substack{k \in \mathbb{C}_j \\ k \neq l}} \mathbb{E}\left[e^{-\xi(\nu) R(\nu) u_{kl} \beta_{kl} h_{kl} p_k}\right] \\
&\overset{(c)}{=} \left(\mathbb{E}\left[e^{-\xi(\nu) R(\nu) u_{kl} \beta_{kl} h_{kl} p_k}\right]\right)^{n-1}, \ k \neq l.
\end{aligned} \tag{3.13}$$

In the above equations, (a) follows from the fact that $\{\mathcal{L}_{kl}\}_{k \in \mathbb{C}_j}$ with $k \neq l$, and $\{p_k\}_{k \in \mathbb{C}_j}$ are mutually independent random variables, (b) results from writing $\mathcal{L}_{kl}$ as $u_{kl}\beta_{kl}h_{kl}$ (from (2.1)), in which u_{kl} is an indicator variable which takes zero when $\mathcal{L}_{kl} = 0$ and one, otherwise. Also, (c) follows from the symmetry which incurs that all the terms $\mathbb{E}\left[e^{-\xi(\nu) R(\nu) u_{kl} \beta_{kl} h_{kl} p_k}\right]$, $k \in \mathbb{C}_j$, are equal and independent of index k. Noting that u_{kl},

β_{kl}, h_{kl}, and p_k are independent of each other, we have

$$\begin{aligned}
\mathbb{E}\left[e^{-\xi(\nu)R(\nu)u_{kl}\beta_{kl}h_{kl}p_k}\right] &= \mathbb{E}_{\beta_{kl}}\left[\mathbb{E}_{h_{kl}}\left[\mathbb{E}_{u_{kl}}\left[\mathbb{E}_{p_k}\left[e^{-\xi(\nu)R(\nu)u_{kl}\beta_{kl}h_{kl}p_k}\right]\right]\right]\right] \\
&\overset{(a)}{\leq} \mathbb{E}_{\beta_{kl}}\left[\mathbb{E}_{h_{kl}}\left[\mathbb{E}_{u_{kl}}\left[(1-q_n)+q_n e^{-\xi(\nu)R(\nu)u_{kl}\beta_{kl}h_{kl}}\right]\right]\right] \\
&\overset{(b)}{=} \mathbb{E}_{\beta_{kl}}\left[\mathbb{E}_{h_{kl}}\left[(1-q_n)+q_n\left(1-\alpha+\alpha e^{-\xi(\nu)R(\nu)\beta_{kl}h_{kl}}\right)\right]\right] \\
&\overset{(c)}{=} \mathbb{E}_{\beta_{kl}}\left[1-\alpha q_n+\frac{\alpha q_n}{1+\beta_{kl}\xi(\nu)R(\nu)}\right] \\
&= \mathbb{E}_{\beta_{kl}}\left[1-\frac{\alpha q_n\beta_{kl}\xi(\nu)R(\nu)}{1+\beta_{kl}\xi(\nu)R(\nu)}\right]. \qquad (3.14)
\end{aligned}$$

In the above equations, (a) follows from the fact that $e^{-\theta x} \leq (1-x)+xe^{-\theta}$, $\forall\theta \geq 0$ and $0 \leq x \leq 1$, noting that $\mathbb{E}[p_k] = q_n$. (b) results from the definition of u_{kl}, which is an indicator variable taking zero with probability $1-\alpha$ and one, with probability α. (c) follows from the fact that as h_{kl} is exponentially-distributed, we have $\mathbb{E}_{h_{kl}}\left[e^{-\xi(\nu)R(\nu)\beta_{kl}h_{kl}}\right] = \frac{1}{1+\beta_{kl}\xi(\nu)R(\nu)}$. Since $\beta_{kl} \leq \beta_{\max}$ and $\mathbb{E}[\beta_{kl}] = \varpi$, and using the fact that for $X \leq Y$, $\mathbb{E}[X] \leq \mathbb{E}[Y]$, (3.14) can be simplified as

$$\begin{aligned}
\mathbb{E}\left[e^{-\xi(\nu)R(\nu)u_{kl}\beta_{kl}h_{kl}p_k}\right] &\leq 1-\frac{\alpha q_n\varpi\xi(\nu)R(\nu)}{1+\beta_{\max}\xi(\nu)R(\nu)} \\
&\overset{(a)}{\leq} e^{-\frac{\hat{\alpha}q_n\xi(\nu)R(\nu)}{1+\beta_{\max}\xi(\nu)R(\nu)}}, \qquad (3.15)
\end{aligned}$$

where (a) results from the facts $1-x \leq e^{-x}$, $\forall x$ and noting that $\alpha\varpi = \hat{\alpha}$. Combining (3.13) and (3.15) and substituting into (3.11) yields

$$\begin{aligned}
\mathbb{P}\left\{\mathcal{O}_l^{(j)}\right\} &\geq 1-e^{-\xi(\nu)\left(\frac{N_0W}{M}R(\nu)-p_l\nu\right)}e^{-\frac{(n-1)\hat{\alpha}q_n\xi(\nu)R(\nu)}{1+\beta_{\max}\xi(\nu)R(\nu)}} \qquad (3.16) \\
&= 1-e^{-\xi(\nu)R(\nu)\left(\frac{(n-1)\hat{\alpha}q_n}{1+\beta_{\max}\xi(\nu)R(\nu)}+\frac{N_0W}{M}\right)\left(1-\frac{t(\nu)}{R(\nu)}\right)}, \qquad (3.17)
\end{aligned}$$

where

$$t(\nu) \triangleq \frac{p_l\nu}{\frac{(n-1)\hat{\alpha}q_n}{1+\beta_{\max}\xi(\nu)R(\nu)}+\frac{N_0W}{M}}. \qquad (3.18)$$

Let us define $\gamma \triangleq \min\left(1, \frac{M(n-1)q_n\hat{\alpha}}{N_0 W}\right)$ and set $\xi(\nu) \triangleq \frac{\frac{\gamma}{2}\frac{N_0W}{M}}{\beta_{\max}R(\nu)\left((n-1)\hat{\alpha}q_n - \frac{\gamma}{2}\frac{N_0W}{M}\right)}$. Clearly, in the cases of $\mathbb{E}\{I_l\} = (n-1)\hat{\alpha}q_n = \omega(1)$ (strong interference) or $\mathbb{E}\{I_l\} = \Theta(1)$ (moderate interference), we have $\xi(\nu) > 0$. From the above setting, it is concluded that

$$\frac{(n-1)\hat{\alpha}q_n}{1+\beta_{\max}\xi(\nu)R(\nu)} + \frac{N_0W}{M} = (n-1)\hat{\alpha}q_n + (1-\frac{\gamma}{2})\frac{N_0W}{M}, \tag{3.19}$$

and

$$t(\nu) = \frac{p_l\nu}{(n-1)\hat{\alpha}q_n + (1-\frac{\gamma}{2})\frac{N_0W}{M}}. \tag{3.20}$$

Thus, (3.17) can be simplified as

$$\mathbb{P}\left\{\mathcal{O}_l^{(j)}\right\} \geq 1 - e^{-\frac{\frac{\gamma}{2}\frac{N_0W}{M}\left[(n-1)\hat{\alpha}q_n + (1-\frac{\gamma}{2})\frac{N_0W}{M}\right]}{\beta_{\max}\left[(n-1)\hat{\alpha}q_n - \frac{\gamma}{2}\frac{N_0W}{M}\right]}\left(1-\frac{t(\nu)}{R(\nu)}\right)} \tag{3.21}$$

$$\geq 1 - e^{-\frac{\gamma N_0 W}{2M\beta_{\max}}\left(1-\frac{t(\nu)}{R(\nu)}\right)}, \tag{3.22}$$

and this completes the proof of the lemma. □

Lemma 3.2. *The guaranteed sum-rate of the underlying network in the asymptotic case of $K \to \infty$ is upper bounded by*

$$\bar{R}_g \leq \frac{W}{\hat{\alpha}}\log K. \tag{3.23}$$

Proof. For the strong and moderate interference regimes, $\frac{\gamma N_0 W}{2M\beta_{\max}} = \Theta(1)$. Thus, it follows from (3.22) that the necessary condition to have $\mathbb{P}\left\{\mathcal{O}_l^{(j)}\right\} \to 0$ is to have $R(\nu) \lesssim t(\nu) = \frac{p_l\nu}{(n-1)\hat{\alpha}q_n + (1-\frac{\gamma}{2})\frac{N_0W}{M}}$. In other words,

$$R^*(\nu) \lesssim \frac{p_l\nu}{(n-1)\hat{\alpha}q_n + (1-\frac{\gamma}{2})\frac{N_0W}{M}}, \tag{3.24}$$

which implies that $\bar{R}_g$ defined in (3.1) is upper bounded by

$$\bar{R}_g \lesssim nW\mathbb{E}_\nu\left[\frac{p_l\nu}{(n-1)\hat{\alpha}q_n + (1-\frac{\gamma}{2})\frac{N_0W}{M}}\right] \tag{3.25}$$

$$= \frac{nW\mathbb{E}_\nu\left[p_l\nu\right]}{(n-1)\hat{\alpha}q_n + (1-\frac{\gamma}{2})\frac{N_0W}{M}}. \tag{3.26}$$

Assuming $\Psi_n > 0$, we have

$$\mathbb{E}_{\nu}\left[p_l\nu\right] \leq \mathbb{E}\left[p_l\nu|\nu \leq \Psi_n\right]\mathbb{P}\{\nu \leq \Psi_n\} + \mathbb{E}\left[p_l\nu|\nu > \Psi_n\right]\mathbb{P}\{\nu > \Psi_n\}. \tag{3.27}$$

To simplify the first term on the right hand side of (3.27), we use the fact that for $X \leq Y$, $\mathbb{E}[X] \leq \mathbb{E}[Y]$. Thus,

$$\mathbb{E}\left[p_l\nu|\nu \leq \Psi_n\right]\mathbb{P}\{\nu \leq \Psi_n\} \leq \Psi_n\mathbb{E}\left[p_l|\nu \leq \Psi_n\right]\mathbb{P}\{\nu \leq \Psi_n\} \leq \Psi_n\mathbb{E}[p_l] = \Psi_n q_n. \tag{3.28}$$

Also using $0 \leq p_l \leq 1$, the second term on the right hand side of (3.27) can be simplified as

$$\mathbb{E}\left[p_l\nu|\nu > \Psi_n\right]\mathbb{P}\{\nu > \Psi_n\} \leq \mathbb{E}\left[\nu|\nu > \Psi_n\right]\mathbb{P}\{\nu > \Psi_n\}. \tag{3.29}$$

Now, defining $\Psi_n \triangleq \log n + 2\log\log n$, and using (3.28) and (3.29), we have

$$\begin{aligned}
\mathbb{E}_{\nu}\left[p_l\nu\right] &\leq q_n\Psi_n + \mathbb{E}\left[\nu|\nu > \Psi_n\right]\mathbb{P}\{\nu > \Psi_n\} && (3.30)\\
&\overset{(a)}{=} q_n\Psi_n + (\Psi_n + 1)e^{-\Psi_n} && (3.31)\\
&\overset{(b)}{\sim} q_n \log n. && (3.32)
\end{aligned}$$

In the above equations, (a) comes from the fact that ν is exponentially-distributed. Also, (b) follows from the facts that i) as we are considering the strong and moderate interference scenarios, it yields that $(n-1)\hat{\alpha}q_n = \Omega(1)$, or equivalently, $q_n = \Omega(\frac{1}{n})$, and ii) the term $(\Psi_n + 1)e^{-\Psi_n}$ scales as $\frac{1}{n\log n}$ (due to the definition of Ψ_n) which is negligible with respect to the first term $q_n\Psi_n$. Combining (3.26) and (3.32) yields

$$\begin{aligned}
\bar{R}_g &\lesssim \frac{Wnq_n\log n}{(n-1)\hat{\alpha}q_n + (1-\frac{\gamma}{2})\frac{N_0W}{M}} && (3.33)\\
&\overset{(a)}{\lesssim} \frac{W}{\hat{\alpha}}\log n && (3.34)\\
&\lesssim \frac{W}{\hat{\alpha}}\log K, && (3.35)
\end{aligned}$$

where (a) follows from the strong and moderate interference regimes.

In the case of weak interference, we have $\gamma = o(1)$ which results in

$$\begin{aligned}
\bar{R}_g &\leq nW \frac{\mathbb{E}[p_l \nu]}{\frac{N_0 W}{M}} && (3.36)\\
&= \frac{Mn}{N_0} \mathbb{E}[p_l \nu]. && (3.37)
\end{aligned}$$

Rewriting (3.31) and selecting $\Psi_n = \log(q_n^{-2})$, we obtain

$$\begin{aligned}
\mathbb{E}[p_l \nu] &\leq q_n \Psi_n + (\Psi_n + 1) e^{-\Psi_n}, \quad \forall \Psi_n > 0 && (3.38)\\
&\approx q_n \log(q_n^{-2}). && (3.39)
\end{aligned}$$

Defining $\varepsilon \triangleq n q_n$, we have

$$\begin{aligned}
\bar{R}_g &\lesssim \frac{2Mnq_n}{N_0} \log(q_n^{-1}) && (3.40)\\
&= \frac{2M\varepsilon}{N_0} \left(\log n + \log(\varepsilon^{-1}) \right). && (3.41)
\end{aligned}$$

As in the weak interference scenario we have $\varepsilon = o(1)$, it follows from the above equation that $\bar{R}_g = o(W \log n)$ in this scenario. Comparing with (3.35), it follows that

$$\bar{R}_g \lesssim \frac{W}{\hat{\alpha}} \log K. \tag{3.42}$$

□

3.3.2 Lower-Bound

For the lower-bound, the on-off power allocation scheme with the threshold level τ_n is considered. Also, assume that $M = 1$ (or equivalently, $n = K$). In this section, it is proved that the lower bound converges asymptotically to the upper bound obtained in (3.42).

Lemma 3.3. *Let us assume that $M = 1$, where all the users follow the on-off scheme with $\tau_n = \log n - 2\log\log n$. In this case*

$$\bar{R}_g \gtrsim \frac{W}{\hat{\alpha}} \log K. \tag{3.43}$$

Proof. Noting $q_n = e^{-\tau_n}$ and $\tau_n = \log n - 2\log\log n$, we have the strong interference scenario, i.e.:

$$\mathbb{E}[I_l] = (n-1)\hat{\alpha}q_n = \Theta(\log^2 n). \tag{3.44}$$

Therefore, using the result of Lemma 2.1, it is realized that with probability one $(n-1)\hat{\alpha}q_n(1-\epsilon) \leq I_l \leq (n-1)\hat{\alpha}q_n(1+\epsilon)$, for some $\epsilon = o(1)$. In other words, defining

$$\mathit{\Phi}(h_{ll}) \triangleq W \log\left(1 + \frac{p_l h_{ll}}{(n-1)\hat{\alpha}q_n(1+\epsilon) + N_0 W}\right), \tag{3.45}$$

it follows that

$$\begin{aligned}
\mathbb{P}\left\{R_l(\mathbf{P}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)}) < \mathit{\Phi}(h_{ll})\right\} &= \mathbb{P}\left\{W\log\left(1 + \frac{p_l h_{ll}}{I_l + N_0 W}\right) < \mathit{\Phi}(h_{ll})\right\} && (3.46)\\
&= o(1), && (3.47)
\end{aligned}$$

which implies that $R^*(h_{ll}) \geq \mathit{\Phi}(h_{ll})$. Thus from (3.1), we have

$$\begin{aligned}
\bar{R}_g &\geq n\mathbb{E}[\mathit{\Phi}(h_{ll})] && (3.48)\\
&= nW\mathbb{E}\left[\log\left(1 + \frac{p_l h_{ll}}{(n-1)\hat{\alpha}q_n(1+\epsilon) + N_0 W}\right)\right] && (3.49)\\
&\overset{(a)}{=} nW \int_{\tau_n}^{\infty} \log\left(1 + \frac{\nu}{(n-1)\hat{\alpha}q_n(1+\epsilon) + N_0 W}\right) e^{-\nu} d\nu && (3.50)\\
&\geq nW \int_{\tau_n}^{\Psi_n} \log\left(1 + \frac{\nu}{(n-1)\hat{\alpha}q_n(1+\epsilon) + N_0 W}\right) e^{-\nu} d\nu, && (3.51)
\end{aligned}$$

where $\Psi_n \triangleq \log n + 2\log\log n$ and (a) follows from the on-off power allocation assumption. As $(n-1)\hat{\alpha}q_n(1+\epsilon) = \Theta(\log^2 n)$, it follows that $\frac{\nu}{(n-1)\hat{\alpha}q_n(1+\epsilon)+N_0W} = o(1)$ in the interval $[\tau_n, \Psi_n]$, which implies that

$$\log\left(1+\frac{\nu}{(n-1)\hat{\alpha}q_n(1+\epsilon)+N_0W}\right)\sim\frac{\nu}{(n-1)\hat{\alpha}q_n(1+\epsilon)+N_0W}, \tag{3.52}$$

in the interval of integration $[\tau_n, \Psi_n]$. Hence,

$$\begin{aligned}
\bar{R}_g &\gtrsim nW\int_{\tau_n}^{\Psi_n}\frac{\nu}{(n-1)\hat{\alpha}q_n(1+\epsilon)+N_0W}e^{-\nu}d\nu && (3.53)\\
&= \frac{nW}{(n-1)\hat{\alpha}q_n(1+\epsilon)+N_0W}\int_{\tau_n}^{\Psi_n}\nu e^{\nu}d\nu && (3.54)\\
&= \frac{nW}{(n-1)\hat{\alpha}q_n(1+\epsilon)+N_0W}\left((\tau_n+1)e^{-\tau_n}-(\Psi_n+1)e^{-\Psi_n}\right) && (3.55)\\
&\overset{(a)}{\sim} \frac{nW\tau_n q_n}{(n-1)\hat{\alpha}q_n(1+\epsilon)+N_0W} && (3.56)\\
&\sim \frac{W}{\hat{\alpha}}\log n && (3.57)\\
&= \frac{W}{\hat{\alpha}}\log K, && (3.58)
\end{aligned}$$

where (a) results from the facts that $(\Psi_n+1)e^{-\Psi_n}\ll(\tau_n+1)e^{-\tau_n}$ and $e^{-\tau_n}=q_n$. □

Theorem 3.4. *The guaranteed sum-rate of the underlying network in the asymptotic case of $K\to\infty$ is obtained by*

$$\bar{R}_g\sim\frac{W}{\hat{\alpha}}\log K, \tag{3.59}$$

which is achievable by the decentralized on-off power allocation scheme.

Proof. It is concluded from (3.23) and (3.43) that the upper and lower bounds converge to each other as $K\to\infty$. Also, the maximum guaranteed sum-rate of the network is achieved through utilizing the distributed on-off scheme (in Lemma 3.3) and scales as $\frac{W}{\hat{\alpha}}\log K$. □

Remark 3.5. *Similar to the proof steps of Theorem 2.3, it follows that the optimum value of M is equal to one. In fact, since the maximum guaranteed sum-rate of the network is achieved in the strong interference scenario in which the interference term scales as $n\hat{\alpha}q_n$ with probability one, it follows that the maximum network's average sum-rate and the network's guaranteed sum-rate are equal. Therefore, the optimum spectrum sharing for maximizing the network's guaranteed sum-rate is the same as the one maximizing the average sum-rate of the network (i.e., the optimum value of M is equal to one.).*

3.4 Conclusion

In this chapter, we investigated the network's guaranteed sum-rate, a different performance metric of the network with a decoding delay constraint. It was demonstrated that the on-off power allocation scheme maximizes the network's guaranteed sum-rate, which scales as $\frac{W}{\hat{\alpha}} \log K$. Moreover, the optimum spectrum sharing for maximizing the network's guaranteed sum-rate is the same as the one maximizing the average sum-rate of the network $(M = 1)$.

Chapter 4

Delay-Throughput Tradeoff

4.1 Introduction

In Chapters 2 and 3, we addressed the throughput maximization of a distributed single-hop wireless network with K links, where the links are partitioned into a fixed number (M) of clusters each operating in a subchannel with bandwidth $\frac{W}{M}$. It was proved that in the strong interference scenario, the optimum power allocation strategy for each user is a threshold-based on-off scheme. Moreover, it was demonstrated that the optimum spectrum sharing for maximizing the average sum-rate is achieved at $M = 1$. However, the delay related issues were not addressed in Chapters 2 and 3.

In this chapter, we follow the distributed single-hop wireless network model proposed in Chapter 2 with $M = 1$ (which is the case with the maximum throughput) and address the delay-throughput tradeoff of the network. In the first part, we define a new notion of throughput, called *effective throughput*, which denotes the *actual* amount of data transmitted through the links. In order to derive the effective throughput, we obtain the *full buffer probability* of a link for the deterministic and stochastic packet arrival processes. Then, we compute the optimum threshold level τ_n, and the maximum effective throughput of the network, for each packet arrival process. It is proved that the effective throughput of the network scales as $\frac{\log n}{\hat{\alpha}}$, with $\hat{\alpha} \triangleq \alpha\varpi$, despite the packet arrival process.

In the second part, we present the delay characteristics of the underlying network in terms of a packet dropping probability, and for the deterministic and stochastic packet arrival processes. These are quite different from the delay analysis with the ON/OFF Bernoulli scheme in [54]. Primarily, we utilize a distributed approach using local information, i.e., direct channel gains, while [54] relies on a central controller which studies the channel conditions of all the links and decides accordingly. We use a homogeneous network with *quasi-static block fading* without path loss. This differs from the geometric models proposed in [28, 33, 34], which are based on the distance between the source and the destination (i.e., power decay-versus-distance law).

It is shown that increasing the number of links gives rise to increasing the network throughput, at the cost of increasing the delay. This will cause the higher packet droppings in the network with a limited buffer size. We derive the necessary conditions in the asymptotic case of $n \to \infty$ such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network. Finally, we study the tradeoff between the effective throughput of the network and other performance measures, i.e., dropping probability and delay-bounds for different packet arrival processes. In particular, we determine how much degradation will be enforced in the throughput by introducing other constraints, and how much this degradation depends on the packet arrival process.

The rest of the chapter is organized as follows. In Section 4.2, the network model and objectives are described. The throughput maximization of the underlying network is presented in Section 4.3. The delay characteristics in terms of the dropping probability are analyzed in Section 4.4. Section 4.5 establishes the delay-throughput tradeoff for the network. In Section 4.6, the simulation results are presented. Finally, in Section 4.7, an overview of the results and conclusions are presented.

4.2 Network Model and Problem Description

4.2.1 Network Model

In this chapter, we follow the distributed single-hop wireless network model proposed in Chapter 2 with $M = 1$ (or equivalently $K = n$) and $W = 1$ (Fig. 4.1). In addition, it is assumed that each receiver knows its direct channel gain with the corresponding transmitter, as well as the interference power imposed by other users. However, each transmitter is assumed to be only aware of the direct channel gain to its corresponding receiver.

We assume that the time axis is divided into slots with the duration of one transmission block, which is defined as the unit of time. The channel gain between transmitter j and receiver i at time slot t is represented by the random variable $\mathcal{L}_{ji}^{(t)}$ [1]. For $j = i$,

[1] In the sequel, we use the superscript (t) for some events to show that the events occur in time slot t.

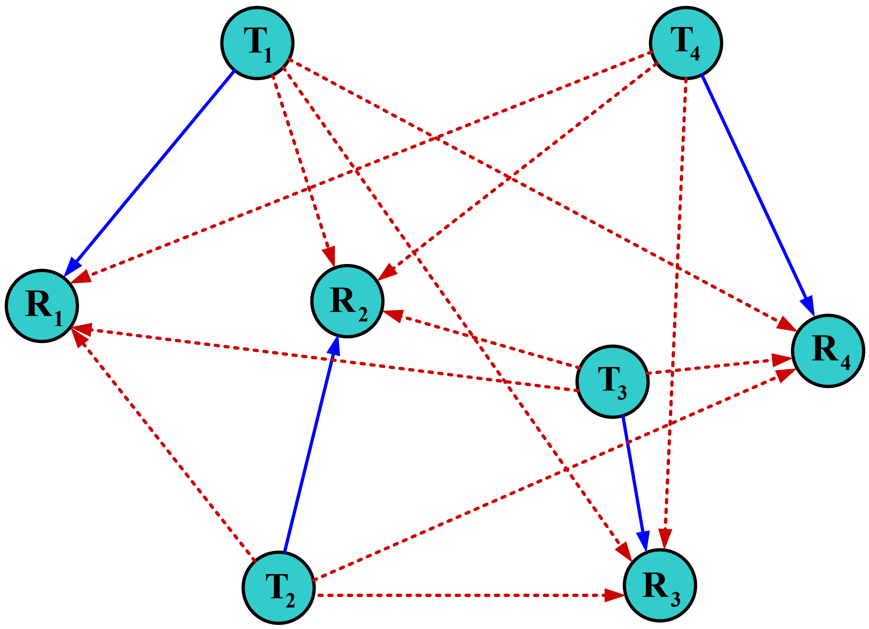


Figure 4.1: A distributed single-hop wireless network with $n = 4$.

the *direct channel gain* is defined as $\mathcal{L}_{ji}^{(t)} \triangleq h_{ii}^{(t)}$, where $h_{ii}^{(t)}$ is exponentially distributed with unit mean (and unit variance). For $j \neq i$, the *cross channel gains* are defined based on a shadowing model as follows:

$$\mathcal{L}_{ji}^{(t)} \triangleq \begin{cases} \beta_{ji}^{(t)} h_{ji}^{(t)}, & \text{with probability} \quad \alpha \\ 0, & \text{with probability} \ \ 1-\alpha, \end{cases} \tag{4.1}$$

where $h_{ji}^{(t)}$s have the same distribution as $h_{ii}^{(t)}$s, and the shadowing factor $\beta_{ji}^{(t)}$ is independent of $h_{ji}^{(t)}$. All the channels in the network are supposed to be quasi-static block fading, where the channel gains remain constant during transmitting one block and change independently from block to block. This model is also used in [37], [36] and [32].

Assuming that the transmitted signals are Gaussian, the interference term seen by receiver $i \in \mathbb{N}_n$ at time slot t will be Gaussian with power

$$I_i^{(t)} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \mathcal{L}_{ji}^{(t)} p_j^{(t)}, \tag{4.2}$$

where $p_j^{(t)}$ is the transmission power of user j at time slot t. Under these assumptions, the achievable data rate of each link $i \in \mathbb{N}_n$ is expressed as

$$R_i^{(t)} = \mathbb{E}_{I_i^{(t)}} \left[\log \left(1 + \frac{h_{ii}^{(t)} p_i^{(t)}}{I_i^{(t)} + N_0} \right) \right] \quad \text{nats/channel use}, \tag{4.3}$$

assuming no constraint on the decoding delay, i.e., decoding can be performed over an arbitrarily large number of blocks.

We assume a limited buffer network, where each link has a buffer size equal to one packet. Also, the transmission blocks of the users are assumed to be synchronous with each other with the same duration. In this chapter, we assume that all the links utilize the threshold-based on-off power allocation strategy proposed in Chapter 2. Unlike most of the works in the literature that assume backlogged users, here we assume a practical model for the packet arrivals in which the buffer of each link is not necessarily full (of packet) all the time. Based on this observation, we adopt the on-off power allocation scheme during each time slot t as follows:

1- Based on the direct channel gain, the transmission policy is[2]

$$p_i^{(t)} = \begin{cases} 1, & \text{if } \ h_{ii}^{(t)} > \tau_n \text{ and the buffer of link } i \text{ is full at time slot } t \\ 0, & \text{Otherwise.} \end{cases} \tag{4.4}$$

2- Knowing its corresponding direct channel gain, each active user i transmits with full power and the rate (4.3).

[2] In fact, if there is no packet in the buffer, it does not make sense for the user to be active, even if its channel is good.

4.2.2 Packet Arrival Process

One of the most important parameters in the network analysis is the model for the packet arrival process. The packet arrival process is a random process which is described by either the arrival time of the packets or the interarrival time between the subsequent packets. These quantities may be modeled by the deterministic or stochastic processes (Fig. 4.2). In this chapter, we consider the following packet arrival processes:

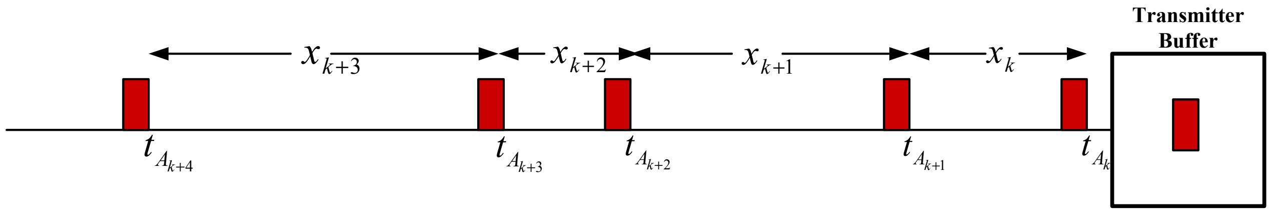
- *Poisson Arrival Process (PAP):* In this process, the number of arrived packets in any interval of unit length is assumed to have a Poisson distribution with the parameter $\frac{1}{\lambda}$. This process is a commonly used model for random and mutually independent packet arrivals in queueing theory [55].

- *Bernoulli Arrival Process (BAP):* In this process, in any given time slot, the probability that a packet arrives is $\rho \triangleq \frac{1}{\lambda}$ [3]. Moreover, the arrival of the packets in different slots occurs independently. This model has been used in many works in the literature such as [34] and [56].

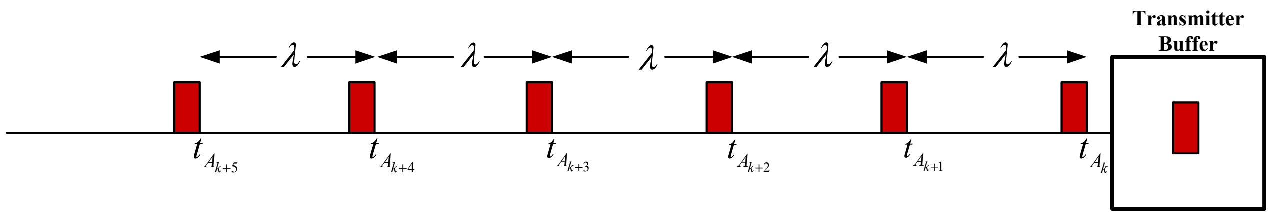
- *Constant Arrival Process (CAP):* In this process, packets arrive continuously with a constant rate of $\frac{1}{\lambda}$ packets per unit length (Fig. 4.2-b) [57].

It is assumed that the packet arrival process for all links is the same. Let us denote $t_{A_k}^{(i)}$ as the time instant of the k^{th} packet arrival into the buffer of link i. It is observed from Fig. 4.2-a that $t_{A_k}^{(i)} = \sum_{j=1}^{k-1} x_j^{(i)} + t_0^{(i)}$ where $t_0^{(i)}$ is the starting time for link i, and

[3] We choose the parameter ρ as $\frac{1}{\lambda}$ to be consistent with other packet arrival processes.



(a)



(b)

Figure 4.2: A schematic figure for a) stochastic packet arrival process, b) constant packet arrival process.

the random variable $x_j^{(i)}$ is the interarrival time defined as

$$x_j^{(i)} \triangleq t_{A_{j+1}}^{(i)} - t_{A_j}^{(i)}, \tag{4.5}$$

with $\mathbb{E}[x_j^{(i)}] = \lambda$. For the CAP, $x_j^{(i)} = \lambda$ and $t_{A_k}^{(i)} = (k-1)\lambda + t_0^{(i)}$[4], while for the PAP, $x_j^{(i)}$'s are independent samples of an exponential random variable x with the probability density function (pdf)

$$f_X(x) = \frac{1}{\lambda} e^{-\frac{1}{\lambda}x}, \qquad x > 0. \tag{4.6}$$

Also for the BAP, $x_j^{(i)}$'s are independent samples of a geometric random variable X with the probability mass function (pmf)

$$p_X(m) \triangleq \mathbb{P}\{X = m\} = (1-\rho)^{m-1}\rho, \qquad m = 1, 2, ..., \tag{4.7}$$

[4]For analysis simplicity, we assume that λ is an integer number.

with $\rho \triangleq \frac{1}{\lambda}$.

We represent $t_{D_k}^{(i)}$ as the time instant at which either the k^{th} arriving packet departs the buffer of link i for the transmission or drops from the buffer. In such configuration, we have the following definition:

Definition 4.1. ***(Delay):*** *The random variable $\mathscr{D}_k^{(i)} \triangleq t_{D_k}^{(i)} - t_{A_k}^{(i)}$ for each link i is defined as the delay between the departure and the arrival time of each packet k, expressed in terms of the number of time slots.*

Due to the finite buffer size and the on-off power allocation strategy, the existing buffered packet may be dropped. The dropping happens when one packet arrives before the previous arrived packet has any chance to be served. Therefore, the event that the dropping of packet k occurs in link $i \in \mathbb{N}_n$ is defined as

$$\begin{aligned} \mathscr{B}_i &\equiv \left\{ \mathscr{D}_k^{(i)} \geq t_{A_{k+1}}^{(i)} - t_{A_k}^{(i)} \right\} && (4.8) \\ &\equiv \left\{ \mathscr{D}_k^{(i)} \geq x_k^{(i)} \right\}. && (4.9) \end{aligned}$$

The packet dropping probability in each link $i \in \mathbb{N}_n$, denoted by $\mathbb{P}\{\mathscr{B}_i\}$, can be obtained as

$$\begin{aligned} \mathbb{P}\{\mathscr{B}_i\} &= \mathbb{P}\left\{ \mathscr{D}_k^{(i)} \geq x_k^{(i)} \right\} && (4.10) \\ &= \int_0^\infty \mathbb{P}\left\{ \mathscr{D}_k^{(i)} \geq x_k^{(i)} \middle| x_k^{(i)} = x \right\} f_X(x)dx, \quad \text{for PAP}, && (4.11) \\ &= \sum_{m=1}^{\infty} \mathbb{P}\left\{ \mathscr{D}_k^{(i)} \geq x_k^{(i)} \middle| x_k^{(i)} = m \right\} p_X(m), \quad \text{for BAP}, && (4.12) \\ &= \mathbb{P}\left\{ \mathscr{D}_k^{(i)} \geq \lambda \right\}, \quad \text{for CAP}. && (4.13) \end{aligned}$$

where $f_X(x)$ and $p_X(m)$ are defined as (4.6) and (4.7), respectively. In Section 4.4, we will obtain $\mathbb{P}\{\mathscr{B}_i\}$ for different packet arrival processes.

4.2.3 Objective

Part I: Throughput Maximization: The main objective of the first part of this chapter is to maximize the throughput of the underlying network. To address this problem, we first define a new notion of throughput, called *effective throughput*, which denotes the *actual* amount of data transmitted through the links. In order to derive the effective throughput, we obtain the *full buffer probability* of a link for the deterministic and stochastic packet arrival processes. Then, we compute the optimum threshold level τ_n, and the maximum effective throughput of the network, for each packet arrival process.

Part II: Delay Characteristics: The main objective of the second part is to analyze the delay characteristics of the underlying network in terms of the number of links (n) and λ. For this purpose, we first formulate the packet dropping probabilities based on the aforementioned packet arrival processes. Then, we derive the necessary conditions in the asymptotic case of $n \to \infty$ such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network.

Part III: Delay-Throughput Tradeoff: The main goal of the third part is to study the tradeoff between the effective throughput of the network and other performance measures, i.e., the dropping probability and the delay-bound (λ) for different packet arrival processes. In particular, we are interested to determine how much degradation will be enforced in the throughput by introducing the other constraints, and how much this degradation depends on the packet arrival process.

4.3 Throughput Maximization

In this section, we aim to derive the maximum throughput of the network with a large number (n) of links, based on using the distributed on-off power allocation strategy. The throughput of the network is defined as the average sum-rate of all links. However, to capture the effect of the packet arrival process, we define a new notion of throughput, called *effective throughput*, which denotes the *actual* amount of data transmitted through the links. In order to derive the effective throughput, we first obtain the *full buffer probability* of each link $i \in \mathbb{N}_n$ for different packet arrival processes. Then, we compute the optimum threshold level τ_n, and the maximum effective throughput of the network, for each packet arrival process.

4.3.1 Effective Throughput

In this section, we present a new performance metric in the network, called *effective throughput*, which is a function of the threshold level τ_n and λ. Let us start with the following definition.

Definition 4.2. ***(Effective Throughput):*** *Under the on-off power allocation strategy, the effective throughput of each link i, $i \in \mathbb{N}_n$, is defined (on a per-block basis) as*

$$\mathfrak{T}_i \triangleq \lim_{L \to \infty} \frac{1}{L} \sum_{t=1}^{L} R_i^{(t)} \mathcal{I}_i^{(t)}, \tag{4.14}$$

where $R_i^{(t)}$ is defined as (4.3) and $\mathcal{I}_i^{(t)}$ is an indicator variable which is equal to 1*, if user i transmits at time slot t, and* 0 *otherwise. Furthermore, the effective throughput* *of the network is defined as*

$$\mathfrak{T}_{\text{eff}} \triangleq \sum_{i=1}^{n} \mathfrak{T}_i. \tag{4.15}$$

The quantity $\mathfrak{T}_i$ represents the average amount of information conveyed through link i in a long period of time. This metric is suitable for real-time applications, where the packets have a certain amount of information and certain arrival rates. It should be noted that $\mathcal{I}_i^{(t)} = 1$ is equivalent to the case in which the buffer is full and the channel gain $h_{ii}^{(t)}$ is greater than the threshold level τ_n at time slot t. Defining the full buffer event as follows

$$\mathscr{C}_i^{(t)} \equiv \{\text{Buffer of link } i \text{ is full at time slot } t\}, \tag{4.16}$$

we have

$$\begin{aligned} \mathbb{P}\left\{\mathcal{I}_i^{(t)} = 1\right\} &= \mathbb{P}\left\{h_{ii}^{(t)} > \tau_n,\ \mathscr{C}_i^{(t)}\right\} && (4.17)\\ &\overset{(a)}{=} \mathbb{P}\left\{h_{ii}^{(t)} > \tau_n\right\} \mathbb{P}\left\{\mathscr{C}_i^{(t)}\right\} && (4.18)\\ &= q_n \Delta_n, && (4.19) \end{aligned}$$

where $q_n \triangleq \mathbb{P}\left\{h_{ii}^{(t)} > \tau_n\right\}$, and $\Delta_n \triangleq \mathbb{P}\left\{\mathscr{C}_i^{(t)}\right\}$ is the *full buffer probability*. In the above equations, (a) follows from the fact that the full buffer event depends on the packet arrival process as well as the direct channel gains $h_{ii}^{(t')}$, for $t' < t$, which is independent of the channel gain $h_{ii}^{(t)}$ (due to the block fading channel model). Thus,

$$\mathcal{I}_i^{(t)} = \begin{cases} 1, & \text{with probability } q_n\Delta_n, \\ 0, & \text{with probability } 1 - q_n\Delta_n. \end{cases} \tag{4.20}$$

It is observed that $\mathcal{I}_i^{(t)}$ is a Bernoulli random variable with parameter $q_n\Delta_n$. In fact, $q_n\Delta_n$ is the probability of the link activation which is a function of n. In the sequel, we derive Δ_n for the aforementioned packet arrival processes.

4.3.2 Full Buffer Probability

Let us denote $t_a^{(i)}$ as the time instant the last packet has arrived in the buffer of link i before or at the same time t. The event $\mathscr{C}_i^{(t)}$ implicitly indicates that during $\mathscr{X}_i^{(t)} \triangleq t - t_a^{(i)}$ time slots, the channel gain of link i is less than the threshold level τ_n. Clearly, $\mathscr{X}_i^{(t)}$ is a random variable which varies from zero to infinity for the stochastic packet arrival processes and is finite for the CAP[5]. Under the on-off power allocation scheme and using the block fading model property, the full buffer probability can be obtained as[6]

$$\Delta_n = \mathbb{E}\left[(1-q_n)^{\mathscr{X}_i^{(t)}}\right], \tag{4.21}$$

where the expectation is computed with respect to $\mathscr{X}_i^{(t)}$, and $q_n \triangleq \mathbb{P}\left\{h_{ii}^{(t)} > \tau_n\right\} = e^{-\tau_n}$.

Lemma 4.3. *Let us denote the full buffer probability of an arbitrary link $i \in \mathbb{N}_n$, for the Poisson, Bernoulli and constant arrival processes as Δ_n^{PAP}, Δ_n^{BAP} and Δ_n^{CAP}, respectively. Then,*

$$\Delta_n^{PAP} = \frac{1}{1+\lambda\log(1-q_n)^{-1}}, \tag{4.22}$$

$$\Delta_n^{BAP} = \frac{1}{1+(\lambda-1)q_n}, \tag{4.23}$$

$$\Delta_n^{CAP} = \frac{1-(1-q_n)^{\lambda}}{\lambda q_n}. \tag{4.24}$$

Proof. For the PAP, since $\mathscr{X}_i^{(t)}$ is an exponential random variable, (4.21) can be sim-

[5] Note that, here we assume that if a packet arrives at time t and the channel gain is greater than τ_n at this time, the packet will be transmitted.

[6] As we will show in Lemma 4.3, Δ_n is independent of index i.

plified as

$$\begin{aligned}
\Delta_n^{PAP} &= \int_0^{\infty} \frac{1}{\lambda}(1-q_n)^x e^{-\frac{1}{\lambda}x} dx && (4.25)\\
&= \frac{1}{1+\lambda \log(1-q_n)^{-1}}. && (4.26)
\end{aligned}$$

Also for the BAP, $\mathscr{X}_i^{(t)}$ is a geometric random variable with parameter $\rho = \frac{1}{\lambda}$. Thus, (4.21) can be simplified as

$$\begin{aligned}
\Delta_n^{BAP} &= \sum_{m=0}^{\infty}(1-q_n)^m \rho(1-\rho)^m && (4.27)\\
&\stackrel{(a)}{=} \frac{1}{1+(\lambda-1)q_n}, && (4.28)
\end{aligned}$$

where (a) follows from the following geometric series:

$$\sum_{m=0}^{\infty} x^m = \frac{1}{1-x}, \quad |x| < 1. \qquad (4.29)$$

For the CAP, the full buffer probability in (4.21) can be written as

$$\begin{aligned}
\Delta_n^{CAP} &\stackrel{(a)}{=} \sum_{m=0}^{\lambda-1}(1-q_n)^m \mathbb{P}\{\mathscr{X}_i^{(t)} = m\} && (4.30)\\
&\stackrel{(b)}{=} \sum_{m=0}^{\lambda-1}(1-q_n)^m \frac{1}{\lambda} && (4.31)\\
&\stackrel{(c)}{=} \frac{1-(1-q_n)^{\lambda}}{\lambda q_n}, && (4.32)
\end{aligned}$$

where (a) follows from Fig. 4.2-b, in which $\mathscr{X}_i^{(t)}$ varies from zero to $\lambda-1$ and (b) follows from the fact that for the deterministic process, $\mathscr{X}_i^{(t)}$ has a uniform distribution. In other words, for every value of $m \in [0, \lambda-1]$, $\mathbb{P}\{\mathscr{X}_i^{(t)} = m\} = \frac{1}{\lambda}$. Also, (c) comes from the following geometric series:

$$\sum_{m=0}^{s} x^m = \frac{1-x^{s+1}}{1-x}. \qquad (4.33)$$

□

Having derived the full buffer probability, we obtain the effective throughput of the network in the following section.

4.3.3 Effective Throughput of the Network

Rewriting (4.14), the effective throughput of link i can be obtained as

$$\begin{aligned}
\mathfrak{T}_i &= \lim_{L\to\infty} \frac{1}{L}\sum_{t=1}^{L} R_i^{(t)}\mathcal{I}_i^{(t)} \\
&\overset{(a)}{=} \mathbb{E}\left[R_i^{(t)}\mathcal{I}_i^{(t)}\right] \\
&= \mathbb{E}\left[R_i^{(t)}\mathcal{I}_i^{(t)}\middle|\mathcal{I}_i^{(t)}=1\right]\mathbb{P}\left\{\mathcal{I}_i^{(t)}=1\right\} + \mathbb{E}\left[R_i^{(t)}\mathcal{I}_i^{(t)}\middle|\mathcal{I}_i^{(t)}=0\right]\mathbb{P}\left\{\mathcal{I}_i^{(t)}=0\right\} \\
&\overset{(b)}{=} q_n\Delta_n\mathbb{E}\left[R_i^{(t)}\middle| h_{ii}^{(t)} > \tau_n, \mathscr{C}_i^{(t)}\right] \\
&\overset{(c)}{=} q_n\Delta_n\mathbb{E}\left[\log\left(1+\frac{h_{ii}^{(t)}}{I_i^{(t)}+N_0}\right)\middle| h_{ii}^{(t)} > \tau_n\right], \qquad (4.34)
\end{aligned}$$

where the expectation is computed with respect to $h_{ii}^{(t)}$ and the interference term $I_i^{(t)}$. In the above equations, (a) follows from the ergodicity of the channels (due to the block fading model), which implies that the average over time is equal to average over realization. (b) results from (4.17)-(4.19) and $\mathbb{E}\left[R_i^{(t)}\mathcal{I}_i^{(t)}|\mathcal{I}_i^{(t)}=0\right]=0$. Finally, (c) results from the fact that the term $\log\left(1+\frac{h_{ii}^{(t)}}{I_i^{(t)}+N_0}\right)$ is independent of $\mathscr{C}_i^{(t)}$.

In order to derive the effective throughput, we need to find the statistical behavior of $I_i^{(t)}$ which is performed in the following lemmas:

Lemma 4.4. *Under the on-off power scheme, we have*

$$\mathbb{E}\left[I_i^{(t)}\right] = (n-1)\hat{\alpha}q_n\Delta_n, \qquad (4.35)$$

$$Var\left[I_i^{(t)}\right] \le (n-1)(2\alpha\kappa q_n\Delta_n), \qquad (4.36)$$

where $\hat{\alpha} \triangleq \alpha\varpi$ *and* $\kappa \triangleq \mathbb{E}\left[\left(\beta_{ji}^{(t)}\right)^2\right]$.

Proof. See Appendix D. $\square$

Lemma 4.5. *The maximum effective throughput is achieved at* $\lambda = o(n)$ *and the strong interference regime which is defined as* $\mathbb{E}[I_i^{(t)}] = \omega(1)$, $i \in \mathbb{N}_n$.

Proof. Suppose that $\lambda \neq o(n)$ which implies that $\lambda = \Omega(n)$. Using (4.34), we have

$$\begin{aligned} \mathfrak{T}_i &\leq q_n\Delta_n\mathbb{E}\left[\log\left(1+\frac{h_{ii}^{(t)}}{N_0}\right)\middle| h_{ii}^{(t)} > \tau_n\right] && (4.37)\\ &\overset{(a)}{\leq} q_n\Delta_n\log\left(1+\frac{\mathbb{E}\left[h_{ii}^{(t)}\middle| h_{ii}^{(t)} > \tau_n\right]}{N_0}\right) && (4.38)\\ &= q_n\Delta_n\log\left(1+\frac{\tau_n+1}{N_0}\right), && (4.39)\end{aligned}$$

where (a) comes from the concavity of $\log(.)$ function and *Jensen's inequality*, $\mathbb{E}[\log x] \leq \log(\mathbb{E}[x])$, $x > 0$. Following (4.22) - (4.24), it is revealed that $\Delta_n \leq \min\left(1, \frac{1}{\lambda q_n}\right)$ for all packet arrival processes. Substituting in (4.39), we have

$$\begin{aligned} \mathfrak{T}_i &\leq \frac{1}{\lambda}\log\left(1+\frac{\log\lambda+1}{N_0}\right)\\ &\sim \frac{\log\log\lambda}{\lambda}, && (4.40)\end{aligned}$$

which follows from the fact that the maximum value of $q_n\Delta_n\log\left(1+\frac{\tau_n+1}{N_0}\right)$ with the condition of $\Delta_n \leq \min\left(1, \frac{1}{\lambda q_n}\right)$ is attained at $q_n = \frac{1}{\lambda}$. Noting that $\lambda = \Omega(n)$, we have $\mathfrak{T}_i \leq \Theta\left(\frac{\log\log n}{n}\right)$.

Now, suppose that $\lambda = o(n)$ but $\mathbb{E}[I_i^{(t)}] \neq \omega(1)$, or equivalently, $\mathbb{E}[I_i^{(t)}] = O(1)$ for some i. Since $\mathbb{E}[I_i^{(t)}] = (n-1)\hat{\alpha}q_n\Delta_n$, the condition $\mathbb{E}[I_i^{(t)}] = O(1)$ implies that there exists a constant c such that $q_n\Delta_n \leq \frac{c}{n}$. Noting (4.22) - (4.24), it follows that either

$\Delta_n \sim \frac{1}{\lambda q_n}$ or $\Delta_n = \Theta(1)$. In the first case, the condition $q_n\Delta_n \leq \frac{c}{n}$ implies that $n \leq c\lambda$ which cannot hold due to the assumption of $\lambda = o(n)$. Therefore, we must have $q_n \leq \frac{c'}{n}$, for some constant c'. Substituting in (4.39) yields

$$\begin{aligned}
\mathfrak{T}_i &\leq \frac{c'}{n}\log\left(1+\frac{\tau_n+1}{N_0}\right)\\
&\overset{(a)}{\leq} \frac{c'}{n}\log\left(1+\frac{\log(n/c')+1}{N_0}\right)\\
&= \Theta\left(\frac{\log\log n}{n}\right),
\end{aligned} \tag{4.41}$$

where (a) results from the fact that $q_n \log\left(1+\frac{\tau_n+1}{N_0}\right)$ is an increasing function of q_n and reaches its maximum at the boundary which is $\frac{c'}{n}$.

In the sequel, we present a lower-bound on the effective throughput of link i in the region $\lambda = o(n)$ and $\mathbb{E}[I_i^{(t)}] = \omega(1)$ and show that this lower-bound beats the upper-bounds derived in the other regions, proving the desired result. For this purpose, using (4.34), we write

$$\begin{aligned}
\mathfrak{T}_i &\overset{(a)}{\geq} q_n\Delta_n\log\left(1+\frac{\tau_n}{\mathbb{E}\left[I_i^{(t)}\middle| h_{ii}^{(t)}>\tau_n\right]+N_o}\right)\\
&\overset{(b)}{=} q_n\Delta_n\log\left(1+\frac{\tau_n}{(n-1)\hat{\alpha}q_n\Delta_n+N_o}\right)\\
&\overset{(c)}{\approx} q_n\Delta_n\log\left(1+\frac{\tau_n}{(n-1)\hat{\alpha}q_n\Delta_n}\right),
\end{aligned} \tag{4.42}$$

where (a) follows from the convexity of the function $\log(1+\frac{b}{x+a})$ with respect to x and Jensen's inequality, (b) results from the independency of $I_i^{(t)}$ from $h_{ii}^{(t)}$, and (c) follows from neglecting the term N_0 with respect to $(n-1)\hat{\alpha}q_n\Delta_n$ due to the strong interference assumption. Setting $q_n = \frac{\log^2 n}{n}$ and $\lambda = \frac{n}{\log^2 n}$, it is easy to check that $\frac{\tau_n}{(n-1)\hat{\alpha}q_n\Delta_n} = o(1)$ and hence, $\log\left(1+\frac{\tau_n}{(n-1)\hat{\alpha}q_n\Delta_n}\right) \approx \frac{\tau_n}{(n-1)\hat{\alpha}q_n\Delta_n}$ which gives the effective throughput as $\frac{\tau_n}{(n-1)\hat{\alpha}} = \Theta\left(\frac{\log n}{n}\right)$ which is greater than the throughput obtained in the other regimes. $\square$

Due to the result of Lemma 4.5, we restrict ourselves to the case of $\lambda = o(n)$ and the strong interference regime in the rest of the chapter.

Lemma 4.6. *Let us assume* $0 < \alpha \leq 1$ *is fixed and we are in the strong interference regime (i.e.,* $\mathbb{E}\left[I_i^{(t)}\right] = \omega(1)$*). Then with probability one (w. p. 1), we have*

$$I_i^{(t)} \;\sim\; (n-1)\hat{\alpha}q_n\Delta_n, \tag{4.43}$$

as $n \to \infty$*. More precisely, substituting* $I_i^{(t)}$ *by* $(n-1)\hat{\alpha}q_n\Delta_n$ *does not change the asymptotic effective throughput of the network.*

Proof. Proof follows along the same line as the proof for Lemma 2.1. $\square$

Lemma 4.7. *The effective throughput of the network for large values of* n *can be obtained as*

$$\mathfrak{T}_{\text{eff}} \approx nq_n\Delta_n \log\left(1 + \frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}\right). \tag{4.44}$$

Proof. Using (4.34), the effective throughput of the network in the asymptotic case of $n \to \infty$ is obtained as

$$\begin{aligned}
\mathfrak{T}_{\text{eff}} &= \sum_{i=1}^{n} \mathfrak{T}_i && (4.45)\\
&\overset{(a)}{\approx} nq_n\Delta_n\mathbb{E}\left[\log\left(1 + \frac{h_{ii}^{(t)}}{(n-1)\hat{\alpha}q_n\Delta_n + N_0}\right)\middle| h_{ii}^{(t)} > \tau_n\right] && (4.46)\\
&\overset{(b)}{\approx} nq_n\Delta_n\mathbb{E}\left[\log\left(1 + \frac{h_{ii}^{(t)}}{n\hat{\alpha}q_n\Delta_n}\right)\middle| h_{ii}^{(t)} > \tau_n\right], && (4.47)
\end{aligned}$$

where (a) results from the strong interference assumption and Lemma 4.6, and (b) follows from approximating $(n-1)\hat{\alpha} q_n \Delta_n + N_0$ by $n\hat{\alpha} q_n \Delta_n$ due to the strong interference assumption and large values of n. A lower-bound on (4.47) can be written as

$$\mathfrak{T}_{\text{eff}}^{l} = nq_n\Delta_n \log\left(1 + \frac{\tau_n}{n\hat{\alpha} q_n \Delta_n}\right). \tag{4.48}$$

Furthermore, due to the concavity of log(.) function and Jensen's inequality, an upper-bound on $\mathfrak{T}_{\text{eff}}$ can be given as

$$\begin{aligned} \mathfrak{T}_{\text{eff}}^{u} &= nq_n\Delta_n \log\left(1 + \frac{\mathbb{E}\left[h_{ii}^{(t)} \middle| h_{ii}^{(t)} > \tau_n\right]}{n\hat{\alpha} q_n \Delta_n}\right) \\ &= nq_n\Delta_n \log\left(1 + \frac{\tau_n + 1}{n\hat{\alpha} q_n \Delta_n}\right). \end{aligned} \tag{4.49}$$

In order to prove that the above upper and lower bounds have the same scaling, it is sufficient to show that the optimum threshold value (τ_n) is much larger than one. For this purpose, we note that if $\tau_n = O(1)$, then the effective throughput of the network will be upper-bounded by

$$\mathfrak{T}_{\text{eff}} \overset{(a)}{\leq} \frac{\tau_n + 1}{\hat{\alpha}} \tag{4.50}$$

$$= O(1), \tag{4.51}$$

where (a) follows from $\log(1+x) \leq x$. In other words, the effective throughput of the network does not scale with n, while the throughput of $\Theta(\log n)$, as will be shown later, is achievable. This suggests that the optimum threshold value must grow with n, and hence, the bounds given in (4.48) and (4.49) are asymptotically equal to $nq_n\Delta_n \log\left(1 + \frac{\tau_n}{n\hat{\alpha} q_n \Delta_n}\right)$ and this completes the proof of the lemma. □

Lemma 4.8. *The maximum effective throughput of the network is obtained in the region that* $\tau_n = o\left(n\hat{\alpha}q_n\Delta_n\right)$.

Proof. Rewriting the expression of the effective throughput of the network from (4.44) and noting the fact that $\log(1+x) \leq x$, for $x \geq 0$, we have

$$\begin{aligned} \mathfrak{T}_{\text{eff}} &\approx nq_n\Delta_n \log\left(1 + \frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}\right) \\ &\leq \frac{\tau_n}{\hat{\alpha}}. \end{aligned} \tag{4.52}$$

It can be shown that if the condition $\tau_n = o\left(n\hat{\alpha}q_n\Delta_n\right)$ is not satisfied, the ratio $\frac{\log\left(1+\frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}\right)}{\frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}}$ is strictly less than one. Having $\tau_n = o\left(n\hat{\alpha}q_n\Delta_n\right)$ results in

$$\log\left(1 + \frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}\right) \approx \frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}, \tag{4.53}$$

yielding the upper-bound $\frac{\tau_n}{\hat{\alpha}}$. This means that to achieve the maximum throughput, the interference should not only be strong but also be much larger than τ_n. □

Having the expression for the effective throughput of the network in (4.44), in the next theorem, we find the optimum value of q_n (or equivalently τ_n) in terms of n and λ for the aforementioned packet arrival processes, i.e.:

$$\hat{q}_n = \arg \max_{q_n} \mathfrak{T}_{\text{eff}}. \tag{4.54}$$

As shown in the proof of Lemma 4.7, since the optimum threshold value is much larger than one, the optimizer $\hat{q}_n$ is sufficiently small, i.e., $\hat{q}_n = o(1)$.

Theorem 4.9. *Assuming the Poisson packet arrival process and large values of* n*, the optimum solution for (4.54) is obtained as*

$$q_n^{PAP} = \delta \frac{\log^2 n}{n} \tag{4.55}$$

for some constant δ. Furthermore, the maximum effective throughput of the network asymptotically scales as $\frac{\log n}{\hat{\alpha}}$, for $\lambda = o\left(\frac{n}{\log n}\right)$.

Proof. Taking the first-order derivative of (4.44) with respect to τ_n yields

$$\begin{aligned}
\frac{\partial \mathfrak{T}_{\text{eff}}}{\partial \tau_n} &\overset{(a)}{=} nq_n\left[\frac{\partial \Delta_n}{\partial \tau_n} - \Delta_n\right]\log\left(1+\frac{\tau_n}{n\hat{\alpha}q_n\Delta_n}\right) + nq_n\frac{(1+\tau_n)\Delta_n - \tau_n\frac{\partial \Delta_n}{\partial \tau_n}}{n\hat{\alpha}q_n\Delta_n + \tau_n} \\
&\overset{(b)}{\approx} nq_n\left[\frac{\partial \Delta_n}{\partial \tau_n} - \Delta_n\right]\frac{\tau_n}{n\hat{\alpha}q_n\Delta_n} + nq_n\frac{(1+\tau_n)\Delta_n - \tau_n\frac{\partial \Delta_n}{\partial \tau_n}}{n\hat{\alpha}q_n\Delta_n + \tau_n}, \qquad (4.56)
\end{aligned}$$

where (a) comes from $q_n = e^{-\tau_n}$ and $\frac{\partial q_n}{\partial \tau_n} = -q_n$. Also, (b) follows from Lemma 4.8 and using the approximation $\log(1+x) \approx x$, for $x \ll 1$. Setting (4.56) equal to zero yields

$$n\hat{\alpha}q_n\Delta_n^2 = \left(\Delta_n - \frac{\partial \Delta_n}{\partial \tau_n}\right)\tau_n^2. \qquad (4.57)$$

It should be noted that (4.57) is valid for every packet arrival process. Recalling from (4.22), the full buffer probability for the PAP is given by

$$\begin{aligned}
\Delta_n^{PAP} &= \frac{1}{1+\lambda\log(1-q_n)^{-1}} \qquad (4.58)\\
&\overset{(a)}{\approx} \frac{1}{1+\lambda q_n}, \qquad (4.59)
\end{aligned}$$

where (a) follows from the fact that for $q_n = o(1)$, $\log(1-q_n)^{-1} \approx q_n$. In this case, $\frac{\partial \Delta_n^{PAP}}{\partial \tau_n} = \frac{\partial \Delta_n^{PAP}}{\partial q_n}\frac{\partial q_n}{\partial \tau_n} = \frac{\lambda q_n}{(1+\lambda q_n)^2}$, which results in

$$\Delta_n^{PAP} - \frac{\partial \Delta_n^{PAP}}{\partial \tau_n} \approx \frac{1}{(1+\lambda q_n)^2} = \left(\Delta_n^{PAP}\right)^2. \qquad (4.60)$$

Thus for the Poisson arrival process, (4.57) can be simplified as

$$n\hat{\alpha}q_n = \tau_n^2. \qquad (4.61)$$

It can be verified that the solution for (4.61) is

$$\tau_n^{PAP} = \log n - 2\log\log n + O(1). \qquad (4.62)$$

Using $q_n = e^{-\tau_n}$, we conclude that

$$q_n^{PAP} = \delta \frac{\log^2 n}{n}, \tag{4.63}$$

for some constant δ.

To satisfy the condition of lemma 4.8, we should have

$$\frac{\tau_n}{n\hat{\alpha} q_n \Delta_n^{PAP}} \ll 1, \tag{4.64}$$

Using (4.59), (4.62), and (4.63), it yields

$$\lambda^{PAP} = o\left(\frac{n}{\log n}\right). \tag{4.65}$$

Thus, the maximum effective throughput of the network obtained in (4.44) can be written as

$$\mathfrak{T}_{\text{eff}} \approx \frac{\tau_n}{\hat{\alpha}}. \tag{4.66}$$

□

Theorem 4.10. *Assuming the Bernoulli packet arrival process and large values of n, the optimum solution for (4.54) is obtained as*

$$q_n^{BAP} = \delta \frac{\log^2 n}{n} \tag{4.67}$$

for some constant δ. Furthermore, the maximum effective throughput of the network asymptotically scales as $\frac{\log n}{\hat{\alpha}}$, for $\lambda = o\left(\frac{n}{\log n}\right)$.

Proof. Using (4.23), we have $\frac{\partial \Delta_n^{BAP}}{\partial \tau_n} = \frac{\partial \Delta_n^{BAP}}{\partial q_n}\frac{\partial q_n}{\partial \tau_n} = -q_n \frac{\partial \Delta_n^{BAP}}{\partial q_n} = \frac{q_n(\lambda-1)}{(1+(\lambda-1)q_n)^2}$. In this case,

$$\Delta_n^{BAP} - \frac{\partial \Delta_n^{BAP}}{\partial \tau_n} = \frac{1}{\left(1+(\lambda-1)q_n\right)^2} = \left(\Delta_n^{BAP}\right)^2. \tag{4.68}$$

Thus for the Bernoulli arrival process, (4.57) can be simplified as

$$n\hat{\alpha}q_n = \tau_n^2. \tag{4.69}$$

It can be observed that (4.69) is exactly equal to (4.61) and hence, its solution can be written as

$$\tau_n^{BAP} = \log n - 2\log\log n + O(1), \tag{4.70}$$

and

$$q_n^{BAP} = \delta \frac{\log^2 n}{n}, \tag{4.71}$$

for some constants δ. Similarly, the maximum effective throughput of the network for the BAP is obtained as

$$\mathfrak{T}_{\text{eff}} \approx \frac{\tau_n}{\hat{\alpha}}, \tag{4.72}$$

which is achieved under the condition

$$\lambda^{BAP} = o\left(\frac{n}{\log n}\right). \tag{4.73}$$

□

Theorem 4.11. *Assuming a deterministic packet arrival process, the optimum solution of (4.54) and the corresponding maximum effective throughput of the network are asymptotically obtained as*

i) $q_n^{CAP} = \delta \frac{\log^2 n}{n}$ *and* $\mathfrak{T}_{\text{eff}} \approx \frac{\log n}{\hat{\alpha}}$, *for* $\lambda = o\left(\frac{n}{\log^2 n}\right)$,

ii) $q_n^{CAP} = \delta' \frac{\log^2 n}{n}$ *and* $\mathfrak{T}_{\text{eff}} \approx \frac{\log n}{\hat{\alpha}}$, *for* $\lambda = \Theta\left(\frac{n}{\log^2 n}\right)$,

iii) $q_n^{CAP} = \frac{\log\left(\frac{\lambda \log^2 \lambda}{n\hat{\alpha}}\right)}{\lambda}$ *and* $\mathfrak{T}_{\text{eff}} \approx \frac{\log n}{\hat{\alpha}}$, *for* $\lambda = \omega\left(\frac{n}{\log^2 n}\right)$ *and* $\lambda = o\left(\frac{n}{\log n}\right)$,

for some constants δ *and* δ'.

Proof. Using (4.24), we have

$$\begin{aligned}
\frac{\partial \Delta_n^{CAP}}{\partial \tau_n} &= \frac{\partial \Delta_n^{CAP}}{\partial q_n}\frac{\partial q_n}{\partial \tau_n} && (4.74)\\
&= -q_n \frac{\partial \Delta_n^{CAP}}{\partial q_n} && (4.75)\\
&= \frac{1-(1-q_n)^\lambda}{\lambda q_n} - (1-q_n)^{\lambda-1} && (4.76)\\
&= \Delta_n^{CAP} - (1-q_n)^{\lambda-1}. && (4.77)
\end{aligned}$$

Hence, $\Delta_n^{CAP} - \frac{\partial \Delta_n^{CAP}}{\partial \tau_n} = (1-q_n)^{\lambda-1}$. In this case, (4.57) can be simplifies as

$$n\hat{\alpha} q_n \frac{\left[1-(1-q_n)^\lambda\right]^2}{(\lambda q_n)^2} = (1-q_n)^{\lambda-1}\tau_n^2. \tag{4.78}$$

or

$$n\hat{\alpha} = \frac{\tau_n^2 \lambda^2 q_n (1-q_n)^{\lambda-1}}{[1-(1-q_n)^\lambda]^2}. \tag{4.79}$$

Since $q_n = o(1)$, we have $(1-q_n)^{\lambda-1} = e^{(\lambda-1)\log(1-q_n)} \overset{(a)}{\approx} e^{-\lambda q_n}$, and $1-(1-q_n)^\lambda \overset{(b)}{\approx} 1-e^{-\lambda q_n}$. It should be noted that (a) and (b) are valid under the condition $\frac{\lambda q_n^2}{2} = o(1)$[7]. Thus, (4.79) can be simplified as

$$n\hat{\alpha} = \frac{\tau_n^2 \lambda^2 q_n e^{-\lambda q_n}}{[1-e^{-\lambda q_n}]^2}, \tag{4.80}$$

or

$$\frac{\nu \log \nu^{-1}}{(1-\nu)^2} = \Psi, \tag{4.81}$$

where $\nu \triangleq e^{-\lambda q_n}$ and $\Psi \triangleq \dfrac{n\hat{\alpha}}{\tau_n^2 \lambda}$. For this setup, we have the following cases:

Case 1: $\Psi \gg 1$

[7] As we will show the condition $\frac{\lambda q_n^2}{2} = o(1)$ is satisfied for the optimum q_n and the corresponding λ.

It is realized from (4.81) that for $\Psi \gg 1$, $\nu = 1-\epsilon$, where $\epsilon = o(1)$. Thus, (4.81) can be simplified as

$$\Psi \approx \frac{\log(1-\epsilon)^{-1}}{\epsilon^2} \tag{4.82}$$
$$\stackrel{(a)}{\approx} \frac{\epsilon}{\epsilon^2} \tag{4.83}$$
$$= \frac{1}{\epsilon}, \tag{4.84}$$

where (a) follows from the Taylor series expansion $\log(1-z) = -\sum_{k=1}^{\infty} \dfrac{z^k}{k} \approx -z, \ \ |z| \ll 1$. Since $\nu \triangleq e^{-\lambda q_n}$ and $\nu = 1-\epsilon$, we have

$$e^{-\lambda q_n} = 1 - \frac{1}{\Psi}, \tag{4.85}$$
$$\Longrightarrow q_n \stackrel{(a)}{\approx} \frac{1}{\Psi\lambda} = \frac{\tau_n^2}{n\hat{\alpha}}, \tag{4.86}$$

where (a) follows from the fact that as $\lambda q_n = o(1)$, we have $e^{-\lambda q_n} \approx 1 - \lambda q_n$. It can be verified that the solution for (4.86) is

$$\tau_n^{CAP} = \log n - 2\log\log n + O(1). \tag{4.87}$$

Using $q_n = e^{-\tau_n}$, we conclude that

$$q_n^{CAP} = \delta \frac{\log^2 n}{n}, \tag{4.88}$$

for some constant δ.

The above results are valid for $\Psi \triangleq \frac{n\hat{\alpha}}{\tau_n^2\lambda} \gg 1$ or $\lambda = o\left(\frac{n}{\log^2 n}\right)$. Also, it can be verified that $\frac{\lambda q_n^2}{2} = o(1)$, and therefore the approximations $(1-q_n)^{\lambda-1} \approx e^{-\lambda q_n}$ and $1-(1-q_n)^{\lambda} \approx 1 - e^{-\lambda q_n}$ are valid in this region.

To satisfy the condition of Lemma 4.8, we must have

$$\frac{\tau_n}{n\hat{\alpha} q_n^{CAP} \Delta_n^{CAP}} \ll 1. \tag{4.89}$$

From (4.24), (4.87) and noting that as $\lambda = o\left(\frac{n}{\log^2 n}\right)$, $\left[1-(1-q_n)^\lambda\right] \approx 1-e^{-\lambda q_n} \approx \lambda q_n$, we can write

$$
\begin{aligned}
\frac{\tau_n}{n\hat{\alpha} q_n^{CAP} \Delta_n^{CAP}} &\approx \frac{\lambda \log n}{n\hat{\alpha}\left[1-(1-q_n)^\lambda\right]} && (4.90)\\
&\approx \frac{\log n}{n\hat{\alpha} q_n}\\
&= O\left(\frac{1}{\log n}\right), && (4.91)
\end{aligned}
$$

which means that the condition of Lemma 4.8 is automatically satisfied in this region. Thus, the maximum effective throughput of the network obtained in (4.44) can be simplified as

$$\mathfrak{T}_{\text{eff}} \approx \frac{\tau_n}{\hat{\alpha}} \approx \frac{\log n}{\hat{\alpha}}. \tag{4.92}$$

Case 2: $\Psi = \Theta(1)$

From (4.81) which gives $\frac{\nu \log \nu^{-1}}{(1-\nu)^2} = \Psi = \Theta(1)$, we conclude that $\nu \triangleq e^{-\lambda q_n} = \Theta(1)$. Thus,

$$
\begin{aligned}
q_n &= \frac{c_1}{\lambda} && (4.93)\\
&\overset{(a)}{=} \frac{c_2 \tau_n^2}{n\hat{\alpha}} && (4.94)
\end{aligned}
$$

where c_1 and c_2 are constants and (a) follows from $\Psi \triangleq \frac{n\hat{\alpha}}{\tau_n^2 \lambda} = \Theta(1)$. It can be verified that the solution for (4.94) is

$$
\begin{aligned}
\tau_n^{CAP} &= \log n - 2\log\log n + O(1). && (4.95)\\
q_n^{CAP} &= \delta' \frac{\log^2 n}{n}, && (4.96)
\end{aligned}
$$

for some constant δ'.

The above results are valid for $\Psi \triangleq \frac{n\hat{\alpha}}{\tau_n^2 \lambda} = \Theta(1)$ or $\lambda = \Theta\left(\frac{n}{\log^2 n}\right)$. Also, it can be verified that $\frac{\lambda q_n^2}{2} = o(1)$, and therefore, the approximations $(1-q_n)^{\lambda-1} \approx e^{-\lambda q_n}$ and $1-(1-q_n)^{\lambda} \approx 1-e^{-\lambda q_n}$ are valid in this region.

Similar to the argument in Case 1, the condition of Lemma 4.8 is satisfied, and therefore, the maximum effective throughput of the network is obtained as

$$\mathfrak{T}_{\text{eff}} \approx \frac{\tau_n}{\hat{\alpha}} \approx \frac{\log n}{\hat{\alpha}}. \tag{4.97}$$

Case 3: $\Psi \ll 1$

It is concluded from (4.81) that $\frac{\nu \log \nu^{-1}}{(1-\nu)^2} = \Psi$, where $\Psi = o(1)$. In this case, $\nu = o(1)$, and therefore, $\nu \log \nu^{-1} \approx \Psi$. The solution for this equation is $\nu \approx \frac{\Psi}{\log(\Psi)^{-1}}$. In other words,

$$e^{-\lambda q_n} \approx \frac{\frac{n\hat{\alpha}}{\lambda \tau_n^2}}{\log\left(\frac{\lambda \tau_n^2}{n\hat{\alpha}}\right)}. \tag{4.98}$$

Thus,

$$\begin{aligned} \lambda q_n &\approx \log\left(\frac{\lambda \tau_n^2}{n\hat{\alpha}}\right) + \log\log\left(\frac{\lambda \tau_n^2}{n\hat{\alpha}}\right) && (4.99)\\ &\overset{(a)}{\approx} \log\left(\frac{\lambda \tau_n^2}{n\hat{\alpha}}\right), && (4.100) \end{aligned}$$

where (a) follows from $\lambda q_n = \omega(1)$ which comes from $\nu = o(1)$. The solution for the above equation can be written as $\tau_n = \log \lambda - f(\lambda)$ or $q_n = \frac{e^{f(\lambda)}}{\lambda} = o(1)$, where we assume $f(\lambda) = o(\log \lambda)$. Substituting in (4.100), we obtain

$$\begin{aligned} e^{f(\lambda)} &= \log\left(\frac{\lambda(\log \lambda - f(\lambda))^2}{n\hat{\alpha}}\right) && (4.101)\\ &= \log\left(\frac{\lambda \log^2 \lambda}{n\hat{\alpha}}\right) + 2\log\left(1 - \frac{f(\lambda)}{\log \lambda}\right) && (4.102)\\ &\overset{(a)}{\approx} \log\left(\frac{\lambda \log^2 \lambda}{n\hat{\alpha}}\right), && (4.103) \end{aligned}$$

where (a) follows from the fact $f(\lambda) = o(\log \lambda)$. Thus, using $\tau_n = \log \lambda - f(\lambda)$, it yields

$$\tau_n^{CAP} = \log \lambda - \log \log \left(\frac{\lambda \log^2 \lambda}{n\hat{\alpha}} \right). \tag{4.104}$$

It should be noted that (4.104) is derived from (4.98) for $\Psi \triangleq \frac{n\hat{\alpha}}{\tau_n^2 \lambda} \ll 1$. This translates the condition $\frac{n\hat{\alpha}}{\tau_n^2 \lambda} \ll 1$ to $\frac{n\hat{\alpha}}{\lambda \log^2 \lambda} \ll 1$, which incurs that $\lambda = \omega\left(\frac{n}{\log^2 n}\right)$.

Also, in the following we show that the condition $\frac{\lambda q_n^2}{2} = o(1)$ is satisfied. It follows from (4.100) that

$$\begin{aligned} \lambda q_n^2 &= \frac{\log^2\left(\frac{\lambda \tau_n^2}{n\hat{\alpha}}\right)}{\lambda} && (4.105) \\ &\overset{(a)}{\leq} \frac{\log^2\left(\frac{\lambda \log^2 \lambda}{n\hat{\alpha}}\right)}{\lambda} && (4.106) \\ &\overset{(b)}{=} o(1), && (4.107) \end{aligned}$$

where (a) follows from (4.104) and (b) comes from $\lambda = \omega\left(\frac{n}{\log^2 n}\right)$.

To satisfy the condition of Lemma 4.8, we must have

$$\frac{\tau_n}{n\hat{\alpha} q_n^{CAP} \Delta_n^{CAP}} \ll 1. \tag{4.108}$$

From (4.24) and (4.104), we can write

$$\begin{aligned} \frac{\tau_n}{n\hat{\alpha} q_n^{CAP} \Delta_n^{CAP}} &\approx \frac{\lambda \log \lambda}{n\hat{\alpha}\left[1 - e^{-\lambda q_n}\right]} && (4.109) \\ &\overset{(a)}{\approx} \frac{\lambda \log \lambda}{n\hat{\alpha}}, && (4.110) \end{aligned}$$

where (a) follows from $e^{-\lambda q_n} = o(1)$. In order to have $\frac{\lambda \log \lambda}{n\hat{\alpha}} = o(1)$, one must have $\lambda = o\left(\frac{n}{\log n}\right)$. In this case, the maximum effective throughput of the network can be simplified as

$$\mathfrak{T}_{\text{eff}} \approx \frac{\tau_n}{\hat{\alpha}} \approx \frac{\log \lambda}{\hat{\alpha}}. \tag{4.111}$$

Noting that λ satisfies $\lambda = \omega\left(\frac{n}{\log^2 n}\right)$ and $\lambda = o\left(\frac{n}{\log n}\right)$, it follows that $\log\lambda \sim \log n$. In other words, $\mathfrak{T}_{\text{eff}} \approx \frac{\log n}{\hat{\alpha}}$.

$\square$

The above theorems imply that the effective throughput of the network scales as $\frac{\log n}{\hat{\alpha}}$, despite the packet arrival process. Note that this value is the same as the sum-rate scaling of the same network with backlogged users (Theorem 2.3), which is an upper-bound on the effective throughput of the current setup. In other words, the effect of the real-time traffic in the throughput (which is captured in the full buffer probability) is asymptotically negligible. However, we did not consider the effect of dropping on the calculations. In the subsequent section, we include the dropping probability in the analysis and find the maximum effective throughput of the network such that the dropping probability approaches zero.

4.4 Delay Analysis

In this section, we analyze the delay characteristics of the underlying network in terms of the number of links (n) and λ. First, we formulate the packet dropping probabilities based on the aforementioned packet arrival processes. Then, we derive the necessary conditions in the asymptotic case of $n \to \infty$ such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network.

Lemma 4.12. *Let us denote the packet dropping probability of a link i, $i \in \mathbb{N}_n$, for the Poisson, Bernoulli and constant arrival processes as $\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\}$, $\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\}$ and*

$\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\}$, *respectively. Then,*

$$\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \frac{1}{1+\lambda\log(1-q_n)^{-1}}, \tag{4.112}$$

$$\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = \frac{(1-q_n)(\lambda q_n)^{-1}}{1+(1-q_n)(\lambda q_n)^{-1}}, \tag{4.113}$$

$$\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = (1-q_n)^{\lambda}. \tag{4.114}$$

Proof. Recalling $t_{A_k}^{(i)}$ as the time instant of the k^{th} packet arrival into the buffer of link i, each user i is active at time slot $t \geq t_{A_k}^{(i)}$ only when $h_{ii}^{(t)} > \tau_n$. In other words, assuming the buffer is full, no transmission (or no service) occurs in each slot with probability $1-q_n$. From (4.5) and (4.8)-(4.12), since the time duration between subsequent packet arrivals is $x_k^{(i)}$, the packet dropping probability for a link i is obtained as

$$\mathbb{P}\left\{\mathscr{B}_i\right\} = \mathbb{E}\left[(1-q_n)^{x_k^{(i)}}\right], \tag{4.115}$$

where the expectation is computed with respect to $x_k^{(i)}$. For the PAP, since $x_k^{(i)}$ is an exponential random variable, (4.115) can be simplified as

$$\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \int_0^\infty \frac{1}{\lambda}(1-q_n)^x e^{-\frac{1}{\lambda}x} dx \tag{4.116}$$

$$= \frac{1}{1+\lambda\log(1-q_n)^{-1}}. \tag{4.117}$$

Also for the BAP, $x_k^{(i)}$ is a geometric random variable with parameter $\rho = \dfrac{1}{\lambda}$. Thus, (4.115) can be simplified as

$$\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = \sum_{m=1}^{\infty}(1-q_n)^m \rho(1-\rho)^{m-1} \tag{4.118}$$

$$= \frac{\rho}{1-\rho}\sum_{m=1}^{\infty}\left[(1-q_n)(1-\rho)\right]^m \tag{4.119}$$

$$\stackrel{(a)}{=} \frac{(1-q_n)(\lambda q_n)^{-1}}{1+(1-q_n)(\lambda q_n)^{-1}}, \tag{4.120}$$

where (a) comes from the following geometric series:

$$\sum_{m=1}^{\infty} x^m = \frac{x}{1-x}, \qquad |x| < 1. \tag{4.121}$$

According to Fig. 4.2-a, $x_k^{(i)}$ for the CAP is a deterministic quantity and is equal to λ. Thus, we have

$$\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = (1-q_n)^{\lambda}. \tag{4.122}$$

It should be noted that (4.117), (4.120) and (4.122) are valid for every value of $q_n \in [0,1]$. In particular, in the extreme case of $q_n = 1$, $\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = \mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = 0$. $\square$

We are now ready to prove the main result of this section. In the next theorem, we derive the necessary conditions on λ, such that the corresponding packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network.

Theorem 4.13. *For the optimum q_n obtained in Theorems 4.9-4.11 resulting in the maximum effective throughput of the network,*

i) $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = 0$, *if* $\lambda^{PAP} = \omega\left(\frac{n}{\log^2 n}\right)$ *and* $\lambda^{PAP} = o\left(\frac{n}{\log n}\right)$,

ii) $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = 0$, *if* $\lambda^{BAP} = \omega\left(\frac{n}{\log^2 n}\right)$ *and* $\lambda^{BAP} = o\left(\frac{n}{\log n}\right)$,

iii) $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = 0$, *if* $\lambda^{CAP} = \omega\left(\frac{n}{\log^2 n}\right)$ *and* $\lambda^{CAP} = o\left(\frac{n}{\log n}\right)$.

Proof. i) From (4.112), we have

$$\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} \;=\; \frac{1}{1-\lambda^{PAP}\log(1-q_n^{PAP})}. \tag{4.123}$$

It follows from (4.123) that achieving $\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \epsilon$ results in

$$\begin{aligned} \lambda_\epsilon^{PAP} &= \frac{1-\epsilon^{-1}}{\log(1-q_n^{PAP})} \\ &\stackrel{(a)}{\approx} \frac{\epsilon^{-1}-1}{q_n^{PAP}}, \end{aligned} \tag{4.124}$$

where (a) comes from $q_n^{PAP} = o(1)$ and the following approximation:

$$\log(1-z) \approx -z, \quad |z| \ll 1. \tag{4.125}$$

Noting the fact that the optimum value of q_n^{PAP} scales as $\Theta\left(\frac{\log^2 n}{n}\right)$, having $\lambda^{PAP} = \omega\left(\frac{n}{\log^2 n}\right)$ results in $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = 0$. On the other hand, from Theorem 4.9, the condition $\lambda^{PAP} = o\left(\frac{n}{\log n}\right)$ is required to achieve the maximum $\mathfrak{T}_{\text{eff}}$, and this completes the proof of the first part of the Theorem.

ii) It is realized from (4.113) that achieving $\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = \epsilon$ results in

$$\begin{aligned} \lambda_\epsilon^{BAP} &= \frac{1}{q_n^{BAP}}\left[(1-q_n^{BAP})\epsilon^{-1} - (1-q_n^{BAP})\right] \\ &\approx \frac{\epsilon^{-1}}{q_n^{BAP}}, \end{aligned} \tag{4.126}$$

for small enough ϵ. Noting the fact that the optimum value of q_n^{BAP} scales as $\Theta\left(\frac{\log^2 n}{n}\right)$, having $\lambda^{BAP} = \omega\left(\frac{n}{\log^2 n}\right)$ results in $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} = 0$. On the other hand, from Theorem 4.10, $\lambda^{BAP} = o\left(\frac{n}{\log n}\right)$ guarantees achieving the maximum effective throughput of the network.

iii) From (4.114), we have

$$\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = e^{\lambda^{CAP}\log(1-q_n^{CAP})} \tag{4.127}$$

$$\stackrel{(a)}{\approx} e^{-q_n^{CAP}\lambda^{CAP}} \tag{4.128}$$

where (a) follows from (4.125) for $q_n^{CAP} = o(1)$. To achieve $\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = \epsilon$, we must have

$$\lambda_\epsilon^{CAP} = \frac{1}{q_n^{CAP}} \log \epsilon^{-1}. \tag{4.129}$$

It follows from (4.128) that setting $q_n^{CAP}\lambda^{CAP} = \omega(1)$ makes $e^{-q_n^{CAP}\lambda^{CAP}} \to 0$. Using part (iii) in Theorem 4.11, it turns out that choosing $\lambda^{CAP} = \omega\left(\frac{n}{\log^2 n}\right)$ satisfies $q_n^{CAP}\lambda^{CAP} = \omega(1)$ which yields $\lim_{n\to\infty} \mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\} = 0$. We also need the condition $\lambda^{CAP} = o\left(\frac{n}{\log n}\right)$ to ensure achieving the maximum effective throughput of the network. $\square$

Remark 1- It is worth mentioning that the delay-bound (λ) in each link for the CAP scales the same as that of the PAP and the BAP. However, $\mathbb{P}\left\{\mathscr{B}_i^{CAP}\right\}$ decays faster than $\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\}$ and $\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\}$, when n tends to infinity.

An interesting conclusion of Theorem 4.13 is the possibility of achieving the maximum effective throughput of the network while making the dropping probability approach zero. More precisely, there exists some $\epsilon \ll 1$ such that $\mathbb{P}\left\{\mathscr{B}_i\right\} \leq \epsilon$, $\forall i \in \mathbb{N}_n$, while achieving the maximum $\mathfrak{T}_{\text{eff}}$ of $\frac{\log n}{\hat{\alpha}}$. This is true for all arrival processes. However, for arbitrary values of ϵ, there is a tradeoff between increasing the throughput, and decreasing the dropping probability and the delay-bound (λ). This tradeoff is studied in the next section.

4.5 Delay-Throughput Tradeoff

In this section, we study the tradeoff between the effective throughput of the network and other performance measures, i.e., the dropping probability and the delay-bound (λ)

for different packet arrival processes. In particular, we are interested to know how much degradation will be enforced in the throughput by introducing the other constraints, and how much this degradation depends on the packet arrival process.

4.5.1 Tradeoff Between Throughput and Dropping Probability

In this section, we assume that a constraint $\mathbb{P}\{\mathscr{B}_i\} \leq \epsilon$ must be satisfied for the dropping probability. It can be easily shown that the constraint $\mathbb{P}\{\mathscr{B}_i\} \leq \epsilon$ is equivalent to $\mathbb{P}\{\mathscr{B}_i\} = \epsilon$. The aim is to characterize the degradation on the effective throughput of the network in terms of ϵ for different packet arrival processes. First, we consider PAP.

Looking at the equations (4.22) and (4.112), it turns out that $\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \Delta_n^{PAP}$. Hence, the condition $\mathbb{P}\left\{\mathscr{B}_i^{PAP}\right\} = \epsilon$ is translated to $\Delta_n^{PAP} = \epsilon$. Therefore, using (4.44), the effective throughput of the network can be written as

$$\mathfrak{T}_{\text{eff}} \approx nq_n\epsilon \log\left(1 + \frac{\tau_n}{n\hat{\alpha}q_n\epsilon}\right). \tag{4.130}$$

From the above equation, it can be realized that the effective throughput of the network is equal to the average sum-rate of the network with $n\epsilon$ users in the case of backlogged users, which is given in Theorem 2.3 as $\frac{\log(n\epsilon)}{\hat{\alpha}}$ for the case of $n\epsilon \gg 1$ or $\epsilon = \omega(\frac{1}{n})$. Also, the optimum value of q_n is shown to scale as $\delta\frac{\log^2(n\epsilon)}{n\epsilon}$ for some constant δ and hence, the optimum value of λ is given as $\frac{\epsilon^{-1}}{q_n} = \frac{n}{\delta\log^2(n\epsilon)}$. Let us denote $\Delta\mathfrak{T}_{\text{eff}}$ as the degradation in the effective throughput of the network, which is defined as the difference between the maximum effective throughput in the case of no constraint on $\mathbb{P}\{\mathscr{B}_i\}$ (Theorem

4.9-4.11) and the case with constraint on $\mathbb{P}\{\mathscr{B}_i\}$. Using Theorem 4.9, $\Delta\mathfrak{T}_{\text{eff}}$ for the PAP can be written as

$$\begin{aligned}\Delta\mathfrak{T}_{\text{eff}} &\approx \frac{\log n}{\hat{\alpha}} - \frac{\log(n\epsilon)}{\hat{\alpha}} \\ &= \frac{\log(\epsilon^{-1})}{\hat{\alpha}}, \end{aligned} \tag{4.131}$$

for $\epsilon = \omega\left(\frac{1}{n}\right)$[8]. Moreover, for values of ϵ such that $\log(\epsilon^{-1}) = o(\log n)$, it can be shown that the scaling of the effective throughput of the network is not changed, i.e., $\mathfrak{T}_{\text{eff}} \sim \frac{\log n}{\hat{\alpha}}$.

For the BAP, and using (4.23) and (4.113), we have

$$\begin{aligned}\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} &= \frac{1-q_n}{1+(\lambda-1)q_n} \\ &\overset{(a)}{\approx} \frac{1}{1+(\lambda-1)q_n} \\ &= \Delta_n^{BAP}, \end{aligned} \tag{4.132}$$

where (a) follows from the fact that $q_n = o(1)$. Therefore, similar to the case of the PAP, we have $\mathbb{P}\left\{\mathscr{B}_i^{BAP}\right\} \approx \Delta_n^{BAP} = \epsilon$ and as a result, the rest of the arguments hold. In particular,

$$\Delta\mathfrak{T}_{\text{eff}} \approx \frac{\log(\epsilon^{-1})}{\hat{\alpha}}. \tag{4.133}$$

For the CAP, and using (4.24) and (4.114), we have

$$(1-q_n)^{\lambda} = \epsilon \quad \Longrightarrow \lambda q_n \approx \log(\epsilon^{-1}), \tag{4.134}$$

[8] In the case of $\epsilon = O(\frac{1}{n})$, it is easy to see that the effective throughput of the network does not scale with n.

which gives

$$\begin{aligned} \Delta_n^{CAP} &= \frac{1-(1-q_n)^\lambda}{\lambda q_n} && (4.135) \\ &\approx \frac{1}{\log(\epsilon^{-1})}. && (4.136) \end{aligned}$$

Hence, using (4.44), the effective throughput of the network can be written as

$$\mathfrak{T}_{\text{eff}} \approx \frac{n}{\log(\epsilon^{-1})} q_n \log\left(1 + \frac{\tau_n}{\frac{n}{\log(\epsilon^{-1})}\hat{\alpha} q_n}\right), \quad (4.137)$$

which is equal to the average sum-rate of a network with $\frac{n}{\log(\epsilon^{-1})}$ backlogged users and is asymptotically equal to $\frac{\log\left(\frac{n}{\log(\epsilon^{-1})}\right)}{\hat{\alpha}}$, for values of ϵ satisfying $\log(\epsilon^{-1}) = o(n)$. Therefore, the degradation in the effective throughput of the network for the CAP can be expressed as

$$\begin{aligned} \Delta\mathfrak{T}_{\text{eff}} &\approx \frac{\log n}{\hat{\alpha}} - \frac{\log\left(\frac{n}{\log(\epsilon^{-1})}\right)}{\hat{\alpha}} \\ &= \frac{\log\log(\epsilon^{-1})}{\hat{\alpha}}. && (4.138) \end{aligned}$$

Comparing the expressions of $\Delta\mathfrak{T}_{\text{eff}}$ for the Poisson, Bernoulli and constant packet arrival processes, it follows that the degradation in the effective throughput of the network in the cases of PAP and BAP both grow logarithmically with ϵ^{-1}, while in the case of CAP it grows double logarithmically. In other words, the degradation in the throughput in the cases of the PAP and BAP is much more substantial compared to the CAP. This fact is also observed in the simulation results in the next sections.

4.5.2 Tradeoff Between Throughput and Delay

In this section, we aim to find the tradeoff between the effective throughput of the network and the delay-bound (λ), for a given constraint on the dropping probability, i.e., $\mathbb{P}\{\mathscr{B}_i\} \leq \epsilon$.

1-PAP: Using (4.22) and (4.112), it follows that for a given λ and $\epsilon \ll 1$, we have

$$\begin{aligned} q_n &\approx \frac{\epsilon^{-1}}{\lambda}, \\ \Longrightarrow \tau_n &\approx \log(\lambda\epsilon), \end{aligned} \tag{4.139}$$

and

$$q_n \Delta_n \approx \frac{1}{\lambda}. \tag{4.140}$$

Substituting $q_n \Delta_n$ and τ_n from the above equations in (4.44) yields

$$\mathfrak{T}_{\text{eff}} \approx \frac{n}{\lambda} \log\left(1 + \frac{\lambda \log(\lambda\epsilon)}{n\hat{\alpha}}\right). \tag{4.141}$$

Taking the first-order derivative of (4.141) yields

$$\frac{\partial \mathfrak{T}_{\text{eff}}}{\partial \lambda} = -\frac{n}{\lambda^2} \log\left(1 + \frac{\lambda \log(\lambda\epsilon)}{n\hat{\alpha}}\right) + \frac{n}{\lambda} \frac{1 + \log(\lambda\epsilon)}{n\hat{\alpha} + \lambda \log(\lambda\epsilon)}. \tag{4.142}$$

Setting (4.142) equal to zero yields

$$\lambda \log^2(\lambda\epsilon) \approx n\hat{\alpha}. \tag{4.143}$$

It can be verified from (4.143) that $\mathfrak{T}_{\text{eff}}$ has a global maximum at $\lambda_{opt}^{PAP} \approx \frac{n\hat{\alpha}}{\log^2(n\hat{\alpha}\epsilon^{-1})}$. In other words, for $\lambda < \lambda_{opt}^{PAP}$, there is a tradeoff between the throughput and delay, meaning that increasing λ results in increasing both the throughput and delay. However, the increase in the throughput is logarithmic while the delay increases linearly with λ. It should be noted that the region $\lambda > \lambda_{opt}^{PAP}$ is not of interest, since increasing λ from λ_{opt}^{PAP} results in decreasing the throughput and increasing the delay which is not desired.

2-BAP: Due to the similarity between the values of $\mathbb{P}\{\mathscr{B}_i\}$ and Δ_n for the PAP and the BAP, the results obtained for the PAP are also valid for the BAP.

3-CAP: Using (4.24) and (4.114), it follows that for a given λ and $\epsilon \ll 1$, we have

$$\begin{aligned} q_n &\approx \frac{\log(\epsilon^{-1})}{\lambda}, \\ \Longrightarrow \tau_n &\approx \log\left(\frac{\lambda}{\log(\epsilon^{-1})}\right), \end{aligned} \tag{4.144}$$

and

$$q_n \Delta_n \approx \frac{1}{\lambda}. \tag{4.145}$$

As can be observed, all the results for the cases of PAP and BAP are extendable to the case of CAP by substituting ϵ^{-1} with $\log(\epsilon^{-1})$. In particular, the optimum value for λ can be written as $\lambda_{opt}^{CAP} \approx \frac{n\hat{\alpha}}{\log^2(n\hat{\alpha}\log(\epsilon^{-1}))}$, and for $\lambda < \lambda_{opt}^{CAP}$, the effective throughput of the network can be given as $\mathfrak{T}_{\text{eff}} \approx \frac{1}{\hat{\alpha}} \log\left(\frac{\lambda}{\log(\epsilon^{-1})}\right)$. This means that in the region $\lambda < \lambda_{opt}^{CAP}$, which is the region of interest, there is a tradeoff between the throughput and delay such that by increasing λ, $\mathfrak{T}_{\text{eff}}$ increases logarithmically, while the delay increases linearly with λ. Furthermore, comparing the value of λ_{opt} for the PAP and BAP with the CAP, it is realized that $\lambda_{opt}^{CAP} > \lambda_{opt}^{PAP}$. This fact is also observed in the simulations.

4.6 Numerical Results

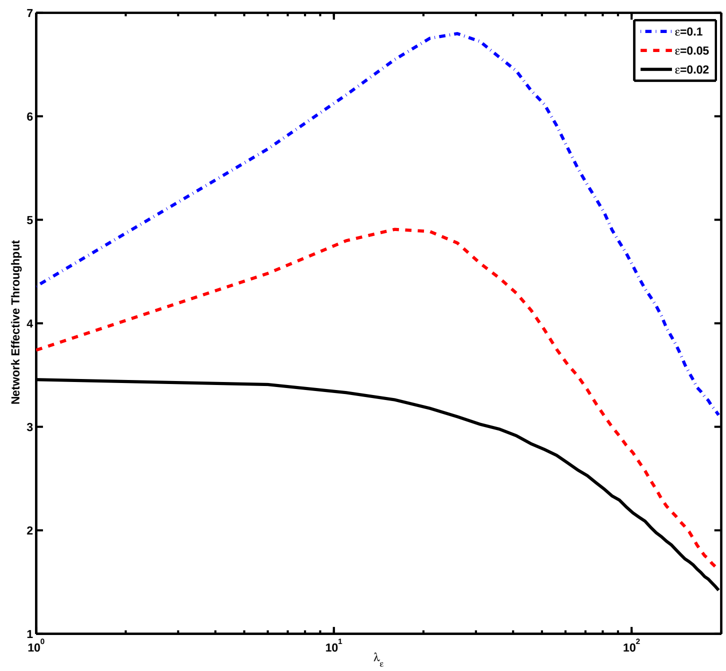
In this section, we present some numerical results to evaluate the tradoff between the effective throughput of the network and other performance measures, i.e., dropping probability and the delay-bound (λ) for different packet arrival processes. For this purpose, we assume that all users in the network follow the threshold-based on-off power allocation policy. In addition, the shadowing effect is assumed to be *lognormal* distributed with mean $\varpi = 0.5$, variance 1 and $\alpha = 0.4$. Furthermore, we assume that $n = 500$ and $N_0 = 1$.

Figures 4.3 and 4.4 show the effective throughput of the network versus λ_ϵ for the PAP, BAP and CAP and different values of ϵ. It is observed from these figures that for a given constraint on the dropping probability (e.g., $\epsilon = 0.05$), and for $\lambda < \lambda_{opt}$, increasing λ results in increasing both the throughput and delay. However, the increase in the throughput is logarithmic while the delay increases linearly with λ as expected. Also, increasing λ from λ_{opt} results in decreasing the throughput and increasing the delay which is not desired. Furthermore, comparing the value of λ_{opt} for the PAP and BAP with the CAP, it is realized that $\lambda_{opt}^{CAP} > \lambda_{opt}^{PAP}$ and $\lambda_{opt}^{CAP} > \lambda_{opt}^{BAP}$, as expected.

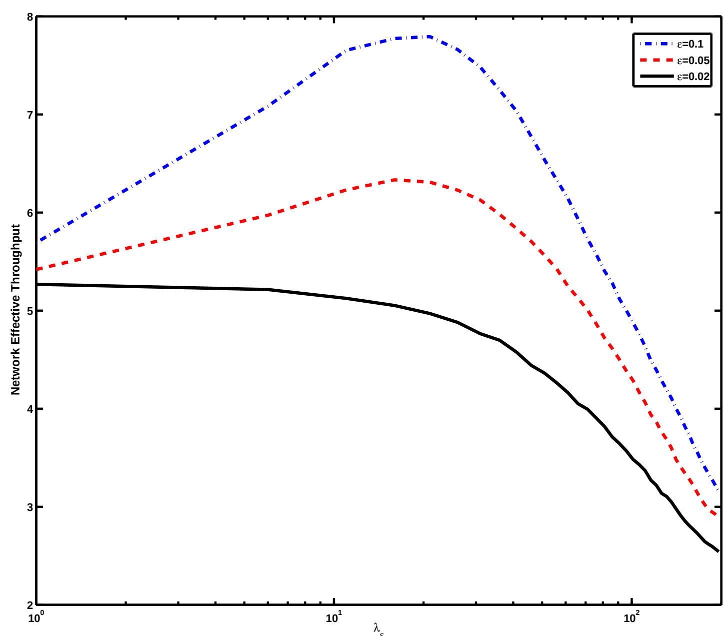
To evaluate the degradation in the effective throughput of the network in terms of dropping probability, we plot $\mathfrak{T}_{\text{eff}}$ versus $\log \epsilon^{-1}$ for different packet arrival processes in Fig. 4.5. It can be seen that the degradation in the throughput in the cases of the PAP and BAP is much more substantial compared to the CAP, as expected. Hence, the performance of the underlying network with the CAP is better than that of the PAP and BAP from the delay-throughput tradeoff points of view.

4.7 Conclusion

In this chapter, the delay-throughput tradeoff of a single-hop wireless network in terms of the number of links (n), and under the shadowing effect with parameters (α, ϖ) was



(a)



(b)

Figure 4.3: Effective throughput of the network versus λ_ϵ for $N_0 = 1$, $n = 500$, $\alpha = 0.4$, and different values of ϵ a) PAP and b) BAP.

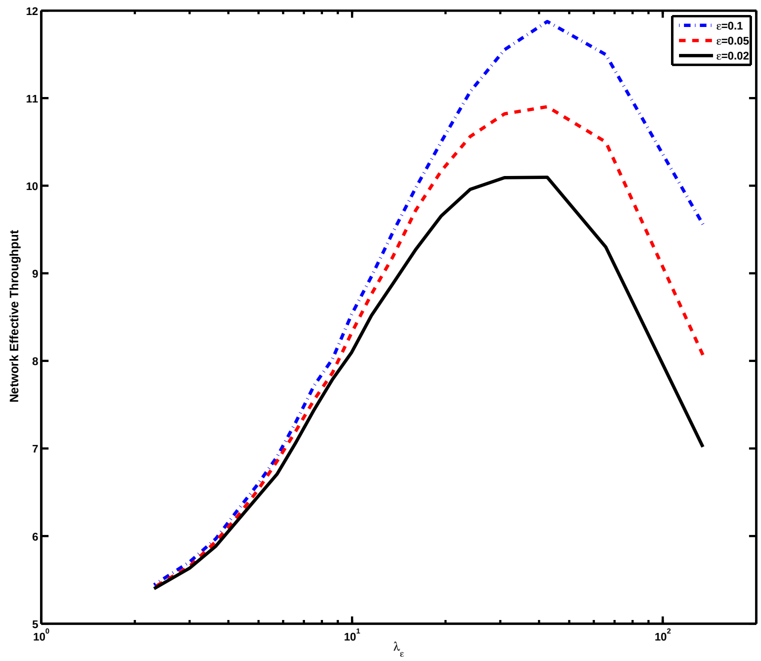


Figure 4.4: Effective throughput of the network versus λ_ϵ for the CAP and $N_0 = 1$, $n = 500$, $\alpha = 0.4$, and different values of ϵ.

analyzed. It was proved that the effective throughput of the network scales as $\frac{\log n}{\hat{\alpha}}$, with $\hat{\alpha} \triangleq \alpha\varpi$, despite the packet arrival process. Then, the delay characteristics of the underlying network in terms of a packet dropping probability was presented. Also, the necessary conditions in the asymptotic case of $n \to \infty$ was derived such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network. Finally, the tradeoff between the effective throughput of the network and delay-bounds for different packet arrival processes was studied. It was shown from the numerical results that the performance of the deterministic packet arrival process is better than that of the Poisson and the Bernoulli packet arrival processes, from the

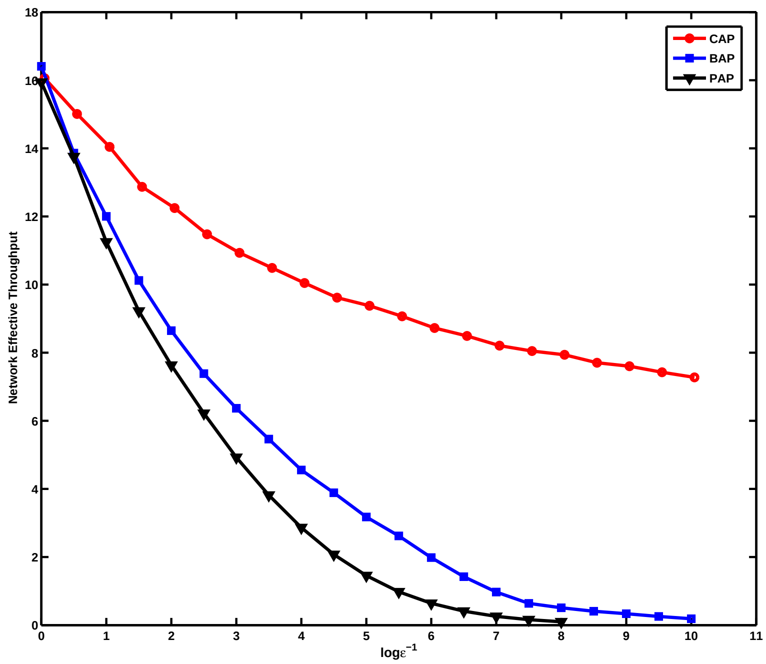


Figure 4.5: Effective throughput of the network versus $\log \epsilon^{-1}$ for different packet arrival processes and $N_0 = 1$, $n = 500$, $\alpha = 0.4$.

delay-throughput tradoff points of view.

Chapter 5

Conclusions and Future Works

5.1 Conclusions

In Chapter 2, a distributed single-hop wireless network with K links was considered, where the links were partitioned into a fixed number (M) of clusters each operating in a subchannel with bandwidth $\frac{W}{M}$. The network throughput is defined as the *average sum-rate* of the network, which is shown to scale as $\Theta(\log K)$. It was proved that in the strong interference scenario, the optimum power allocation strategy for each user was a threshold-based on-off scheme. Moreover, it was demonstrated that the optimum spectrum sharing for maximizing the average sum-rate is achieved at $M = 1$. In other words, partitioning the bandwidth W into M subchannels has no gain in terms of enhancing the throughput. The interesting point is that under the on-off power allocation strategy, the total network energy for $M = 1$ is significantly lower as compared to where all the users transmit with full power all the time. Also, the proposed on-off scheme has the advantage of not requiring a central controller and is simple for implementation in practical time-varying networks.

In Chapter 3, we investigated the network guaranteed sum-rate, a different performance metric of the network with a decoding delay constraint. It was demonstrated that the on-off power allocation scheme maximizes the network's guaranteed sum-rate, which scales as $\frac{W}{\hat{\alpha}} \log K$. Moreover, the optimum spectrum sharing for maximizing the network's guaranteed sum-rate is the same as the one maximizing the average sum-rate of the network ($M = 1$).

In Chapter 4, the delay-throughput tradeoff of a single-hop wireless network in terms of the number of links (n), and under the shadowing effect with parameters (α, ϖ) was analyzed. It was proved that the effective throughput of the network scales as $\frac{\log n}{\hat{\alpha}}$, with $\hat{\alpha} \triangleq \alpha\varpi$, despite the packet arrival process. Then, the delay characteristics of the underlying network in terms of a packet dropping probability was presented. Also, the necessary conditions in the asymptotic case of $n \to \infty$ was derived such that the packet dropping probabilities tend to zero, while achieving the maximum effective throughput of the network. Finally, the tradeoff between the effective throughput of the network and delay-bounds for different packet arrival processes was studied. It was shown from the numerical results that the performance of the deterministic packet arrival process is better than that of the Poisson and the Bernoulli packet arrival processes, from the delay-throughput tradoff points of view.

5.2 Proposal Summary

The dissertation can be continued in several directions as briefly explained in what follows.

The results of Chapters 2 and 3 are based on the assumption that perfect channel knowledge is available at the receiver. In practice, however, the channel estimation at the receiver is often imperfect. Thus, the performance of the system is degraded due to the channel estimation error. A natural extension of these works is to consider the effect of imperfect channel estimation on the results.

In Chapters 2-4, it is assumed that the channels are block fading, i.e., there is no correlation between the channel gains in the consecutive blocks. Investigating the effect of the temporal correlation on the results of these chapters is an interesting direction for future research. Also, an extension of the results in these chapters is to derive the average network's throughput for Rician fading channel model. Rician channel model is one of the most widely-used models for wireless links, in particular, when there is a line of sight (LOS) between the transmitter and the receiver.

In Chapter 4, we investigated the delay-throughput tradeoff where all the nodes in the network are equipped with a single antenna. A fruitful future work is to investigate the effect of increasing the number of antenna on the delay characteristics.

Appendix A

Proof of Lemma 2.1

Let us define $\chi_k \triangleq \mathcal{L}_{ki} p_k$, where $\mathcal{L}_{ki}$ is independent of p_k, for $k \neq i$. Under a quasi-static Rayleigh fading channel model, it is concluded that χ_k's are independent and identically distributed (i.i.d.) random variables with

$$\begin{aligned}
\mathbb{E}\left[\chi_k\right] &= \mathbb{E}\left[\mathcal{L}_{ki} p_k\right] = \hat{\alpha} q_n, && \text{(A.1)} \\
\mathrm{Var}\left[\chi_k\right] &= \mathbb{E}\left[\chi_k^2\right] - \mathbb{E}^2\left[\chi_k\right] && \text{(A.2)} \\
&\stackrel{(a)}{\leq} 2\alpha\kappa q_n - (\hat{\alpha} q_n)^2, && \text{(A.3)}
\end{aligned}$$

where $\mathbb{E}\left[h_{ki}^2\right] = 2$ and $\hat{\alpha} \triangleq \alpha\varpi$. Also, (a) follows from the fact that $p_k^2 \leq p_k$. Thus, $\mathbb{E}[p_k^2] \leq \mathbb{E}[p_k] = q_n$. The interference $I_i = \sum_{\substack{k \in \mathbb{C}_j \\ k \neq i}} \chi_k$ is a random variable with mean μ_n and variance ϑ_n^2, where

$$\begin{aligned}
\mu_n &\triangleq \mathbb{E}\left[I_i\right] = (n-1)\hat{\alpha} q_n, && \text{(A.4)} \\
\vartheta_n^2 &\triangleq \mathrm{Var}\left[I_i\right] \leq (n-1)(2\alpha\kappa q_n - (\hat{\alpha} q_n)^2) \leq (n-1)(2\alpha\kappa q_n). && \text{(A.5)}
\end{aligned}$$

Using the *Central Limit Theorem* [58, p. 183], we obtain

$$\mathbb{P}\{|I_i - \mu_n| < \psi_n\} \approx 1 - Q\left(\frac{\psi_n}{\vartheta_n}\right) \tag{A.6}$$

$$\stackrel{(a)}{\geq} 1 - e^{-\frac{\psi_n^2}{2\vartheta_n^2}}, \tag{A.7}$$

for all $\psi_n > 0$ such that $\psi_n = o\left(n^{\frac{1}{6}}\vartheta_n\right)$. In the above equations, the $Q(.)$ function is defined as $Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-u^2/2} du$, and (a) follows from the fact that $Q(x) \leq e^{-\frac{x^2}{2}}$, $\forall x > 0$. Selecting $\psi_n = (nq_n)^{\frac{1}{8}} \sqrt{2}\vartheta_n$, we obtain

$$\mathbb{P}\{|I_i - \mu_n| < \psi_n\} \geq 1 - e^{-(nq_n)^{\frac{1}{4}}}. \tag{A.8}$$

Therefore, defining $\varepsilon \triangleq \frac{\psi_n}{\mu_n} = O\left((nq_n)^{-\frac{3}{8}}\right)$, we have

$$\mathbb{P}\{\mu_n (1 - \varepsilon) \leq I_i \leq \mu_n (1 + \varepsilon)\} \geq 1 - e^{-(nq_n)^{\frac{1}{4}}}. \tag{A.9}$$

Noting that $nq_n \to \infty$, it follows that $I_i \sim \mu_n$, with probability one. Now, we show a stronger statement, which is, the contribution of the realizations in which $|I_i - \mu_n| > \psi_n$ in the average sum-rate of the network is negligible. For this purpose, we give a lower-bound and an upper-bound for the average sum-rate of the network and show that these bounds converge to each other in the strong interference regime, when $nq_n \to \infty$. A lower-bound denoted by $\bar{R}_{ave}^{(L)}$, can be given by

$$\bar{R}_{ave}^{(L)} \triangleq nW\mathbb{E}\left[\log\left(1 + \frac{\hat{p}_i h_{ii}}{I_i + \frac{N_0 W}{M}}\right) \middle| |I_i - \mu_n| < \psi_n\right] \mathbb{P}\{|I_i - \mu_n| < \psi_n\} \tag{A.10}$$

$$\geq nW\mathbb{E}\left[\log\left(1 + \frac{\hat{p}_i h_{ii}}{\mu_n(1+\varepsilon) + \frac{N_0 W}{M}}\right)\right]\left[1 - e^{-(nq_n)^{\frac{1}{4}}}\right], \tag{A.11}$$

which scales as $\frac{W}{\hat{\alpha}} \log n$ (as shown in the proof of Theorem 2.3, by optimizing the power allocation function). An upper-bound for the average sum-rate of the network, denoted by $\bar{R}_{ave}^{(U)}$, can be given as

$$
\begin{aligned}
\bar{R}_{ave}^{(U)} &= nW\mathbb{E}\left[\log\left(1+\frac{\hat{p}_i h_{ii}}{I_i+\frac{N_0 W}{M}}\right)\middle| |I_i-\mu_n|<\psi_n\right]\mathbb{P}\{|I_i-\mu_n|<\psi_n\}+ \\
& \quad nW\mathbb{E}\left[\log\left(1+\frac{\hat{p}_i h_{ii}}{I_i+\frac{N_0 W}{M}}\right)\middle| |I_i-\mu_n|\geq\psi_n\right]\mathbb{P}\{|I_i-\mu_n|\geq\psi_n\} \quad \text{(A.12)} \\
&\leq \bar{R}_{ave}^{(L)} + nW\mathbb{E}\left[\log\left(1+\frac{\hat{p}_i h_{ii}}{\frac{N_0 W}{M}}\right)\right]e^{-(nq_n)^{\frac{1}{4}}} \quad \text{(A.13)} \\
&\stackrel{(a)}{\leq} \bar{R}_{ave}^{(L)} + nW\mathbb{E}\left[\frac{\hat{p}_i h_{ii}}{\frac{N_0 W}{M}}\right]e^{-(nq_n)^{\frac{1}{4}}} \quad \text{(A.14)} \\
&\stackrel{(b)}{=} \bar{R}_{ave}^{(L)} + WO(nq_n\log n)e^{-(nq_n)^{\frac{1}{4}}} \quad \text{(A.15)} \\
&\stackrel{(c)}{\sim} \bar{R}_{ave}^{(L)}. \quad \text{(A.16)}
\end{aligned}
$$

In the above equations, (a) follows from the fact that $\log(1+x)\leq x$, for $x\geq 0$, (b) comes from the facts that $\mathbb{E}\{p_i h_{ii}\}\lesssim q_n\log n$ (this is shown in the proof of Lemma 3.2) and $\frac{N_0 W}{M}$ is fixed, and finally, (c) results from the fact that as $nq_n\to\infty$, $nq_n e^{-(nq_n)^{\frac{1}{4}}}\to 0$. The above equations implies that substituting I_i by its mean $((n-1)\hat{\alpha}q_n)$ does not affect the analysis of the average sum-rate of the network in the asymptotic case of $K\to\infty$.

Appendix B

Proof of Lemma 2.4

Using (2.29), we have

$$
\begin{aligned}
\mathbb{E}[\Xi_i(\hat{p}_i, h_{ii})] &\approx \frac{W}{M\lambda}\mathbb{E}[h_{ii}\hat{p}_i] + n\frac{\alpha W\mu}{M\lambda'}\left(1 - \frac{\varpi}{\lambda'}\mathbb{E}[\hat{p}_i] + \frac{2\kappa}{\lambda'^2}\mathbb{E}[\hat{p}_i^2] - \frac{6\eta}{\lambda'^3}\mathbb{E}[\hat{p}_i^3]\right) + \\
&\quad n(1-\alpha)\frac{W\mu}{M\lambda'} \qquad \text{(B-1)} \\
&\overset{(a)}{=} \frac{W}{M\lambda}(1+\tau_n)q_n - \frac{n\hat{\alpha}W}{M\lambda'^2}(1+\tau_n)q_n^2 + \frac{n\alpha W 2\kappa}{M\lambda'^3}(1+\tau_n)q_n^2 - \\
&\quad \frac{n\alpha W 6\eta}{M\lambda'^4}(1+\tau_n)q_n^2 + \frac{nW}{M\lambda'}(1+\tau_n)q_n \qquad \text{(B-2)} \\
&\overset{(b)}{\approx} \frac{W}{M\hat{\alpha}}\left(1 + \tau_n + \frac{\xi_1}{n^2}(1+\tau_n)e^{\tau_n} - \frac{\xi_2}{n^3}(1+\tau_n)e^{2\tau_n}\right), \qquad \text{(B-3)}
\end{aligned}
$$

where $\xi_1 \triangleq \dfrac{2\kappa}{\varpi\hat{\alpha}}$ and $\xi_2 \triangleq \dfrac{6\eta}{\varpi\hat{\alpha}^2}$. In the above equations, (a) follows from the fact that $\mathbb{E}[h_{ii}\hat{p}_i] = \mu = (1+\tau_n)q_n$, and (b) results from i) $\lambda = (n-1)\hat{\alpha}q_n + \frac{N_0 W}{M} \approx n\hat{\alpha}q_n$ and $\lambda' \approx n\hat{\alpha}q_n$ incurred by the fact that $\lambda \gg 1$, and ii) $q_n = e^{-\tau_n}$. Since $n\hat{\alpha}q_n \to \infty$, it follows that the right hand side of (B-3) is a monotonically increasing function of τ_n, which attains its maximum when τ_n takes its maximum feasible value. The maximum feasible value of τ_n, denoted as $\hat{\tau}_n$, can be obtained as

$$n\hat{\alpha}e^{-\tau_n} \to \infty \Longrightarrow \hat{\tau}_n \sim \log n. \tag{B-4}$$

Thus, the maximum achievable value for $\mathbb{E}[\Xi_i(\hat{p}_i, h_{ii})]$ scales as $\frac{W}{M\hat{\alpha}} \log n$.

Appendix C

Proof of Lemma 2.5

i) Using (2.5) and assuming that all users follow the on-off power allocation policy, $\mathbb{E}[u_i(\hat{p}_i, h_{ii})]$ can be expressed as

$$\mathbb{E}[u_i(\hat{p}_i, h_{ii})] = \sum_{l \in \mathbb{C}_j} \mathbb{E}\left[R_l(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right], \quad j = 1, ..., M, \tag{C-1}$$

where the expectation is computed with respect to h_{ll} and I_l. Noting that $q_n = \mathbb{P}\{h_{ll} > \tau_n\}$, we have

$$\begin{aligned}
\mathbb{E}\left[R_l(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right] &= \mathbb{E}\left[R_l(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)}) \middle| h_{ll} > \tau_n\right] \mathbb{P}\{h_{ll} > \tau_n\} + \\
&\quad \mathbb{E}\left[R_l(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)}) \middle| h_{ll} \le \tau_n\right] \mathbb{P}\{h_{ll} \le \tau_n\} \qquad \text{(C-2)} \\
&= q_n \mathbb{E}\left[R_l(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)}) \middle| h_{ll} > \tau_n\right] + (1 - q_n) \mathbb{E}\left[R_l(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)}) \middle| h_{ll} \le \tau_n\right].
\end{aligned}$$

Since for $h_{ll} \le \tau_n$, $\hat{p}_l = 0$, it is concluded that

$$\mathbb{E}\left[R_l(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right] = \frac{q_n W}{M} \mathbb{E}\left[\log\left(1 + \frac{h_{ll}}{I_l + \frac{N_0 W}{M}}\right) \middle| h_{ll} > \tau_n\right]. \tag{C-3}$$

For large values of K, we can apply Lemma 2.1 to obtain

$$\begin{aligned}
\mathbb{E}\left[R_l(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right] &\approx \frac{q_n W}{M}\mathbb{E}\left[\log\left(1+\frac{h_{ll}}{(n-1)\hat{\alpha}q_n + \frac{N_0 W}{M}}\right)\middle| h_{ll} > \tau_n\right] && \text{(C-4)}\\
&= \frac{q_n W}{M}\mathbb{E}\left[\log\left(1+\frac{h_{ll}}{\lambda}\right)\middle| h_{ll} > \tau_n\right], && \text{(C-5)}
\end{aligned}$$

where the expectation is computed with respect to h_{ll}. Using the Taylor series $\log(1+x) = \sum_{k=1}^{\infty}\frac{(-1)^{k-1}x^k}{k}$, $-1 < x \le 1$, (C-5) can be written as

$$\begin{aligned}
\mathbb{E}\left[R_l(\hat{\mathbf{P}}^{(j)}, \boldsymbol{\mathcal{L}}_l^{(j)})\right] &\approx \frac{q_n W}{M}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k\lambda^k}\mathbb{E}\left[h_{ll}^k\middle| h_{ll} > \tau_n\right] && \text{(C-6)}\\
&\overset{(a)}{\approx} \frac{q_n W}{M}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}}{k(n\hat{\alpha}q_n)^k}\mathbb{E}\left[h_{ll}^k\middle| h_{ll} > \tau_n\right] && \text{(C-7)}\\
&\overset{(b)}{\approx} \frac{q_n W}{M}\sum_{k=1}^{\infty}\frac{(-1)^{k-1}\tau_n^k}{k(n\hat{\alpha}q_n)^k} && \text{(C-8)}\\
&= \frac{q_n W}{M}\log\left(1+\frac{\tau_n}{n\hat{\alpha}q_n}\right) && \text{(C-9)}\\
&\overset{(c)}{=} \frac{e^{-\tau_n} W}{M}\log\left(1+\frac{\tau_n e^{\tau_n}}{n\hat{\alpha}}\right), && \text{(C-10)}
\end{aligned}$$

where (a) follows from the fact that for large values of n, $\lambda \approx n\hat{\alpha}q_n$. Also, (b) results from the fact that under a Rayleigh fading channel model,

$$\mathbb{E}\left[h_{ll}\middle| h_{ll} > \tau_n\right] = 1 + \tau_n, \qquad \text{(C-11)}$$

$$\mathbb{E}\left[h_{ll}^k\middle| h_{ll} > \tau_n\right] = \tau_n^k + k\mathbb{E}\left[h_{ll}^{k-1}\middle| h_{ll} > \tau_n\right]. \qquad \text{(C-12)}$$

Since $\lambda \gg 1$, the term $\frac{\mathbb{E}\left[h_{ll}^{k-1}\middle| h_{ll}>\tau_n\right]}{\lambda^k} \ll \frac{\mathbb{E}\left[h_{ll}^{k-1}\middle| h_{ll}>\tau_n\right]}{\lambda^{k-1}}$, which implies that we can neglect this term and simply write $\mathbb{E}\left[h_{ll}^k\middle| h_{ll} > \tau_n\right] \approx \tau_n^k$. (c) results from $q_n = e^{-\tau_n}$. Thus, (C-1) can be simplified as

$$\mathbb{E}[u_i(\hat{p}_i, h_{ii})] \approx \frac{ne^{-\tau_n}W}{M}\log\left(1+\frac{\tau_n e^{\tau_n}}{n\hat{\alpha}}\right). \qquad \text{(C-13)}$$

In order to find the optimum threshold value:

$$\hat{\tau}_n = \arg \max_{\tau_n} \mathbb{E}[u_i(\hat{p}_i, h_{ii})], \tag{C-14}$$

we set the derivative of the right hand side of (C-13) with respect to τ_n to zero:

$$-e^{-\hat{\tau}_n} \log\left(1 + \frac{\hat{\tau}_n e^{\hat{\tau}_n}}{n\hat{\alpha}}\right) + \frac{1 + \hat{\tau}_n}{n\hat{\alpha} + \hat{\tau}_n e^{\hat{\tau}_n}} = 0, \tag{C-15}$$

which after some manipulations yields

$$\hat{\tau}_n = \log n - 2\log\log n + O(1). \tag{C-16}$$

ii) Using (C-16), it is concluded that

$$\begin{aligned} q_n &= e^{-\tau_n} \\ &= \delta \frac{\log^2 n}{n}, \end{aligned}$$

where $\delta = e^{-O(1)}$ is a constant.

iii) Using (C-16), we have

$$\frac{\hat{\tau}_n e^{\hat{\tau}_n}}{n\hat{\alpha}} = \Theta\left(\frac{1}{\log n}\right), \tag{C-17}$$

which implies that the right hand side of (C-13) can be written as

$$\text{RH (C-13)} \approx \frac{W\hat{\tau}_n}{M\hat{\alpha}}. \tag{C-18}$$

Thus, the maximum value for $\mathbb{E}[u_i(\hat{p}_i, h_{ii})]$ in (C-13) scales as $\frac{W}{M\hat{\alpha}} \log n$.

Appendix D

Proof of Lemma 4.4

Let us define $\chi_j^{(t)} \triangleq \mathcal{L}_{ji}^{(t)} p_j^{(t)}$, where $\mathcal{L}_{ji}^{(t)}$ is independent of $p_j^{(t)}$, for $j \neq i$. Note that

$$\begin{aligned} \mathbb{P}\left\{p_j^{(t)} = 1\right\} &= \mathbb{P}\left\{h_{jj}^{(t)} > \tau_n,\ \mathscr{C}_j^{(t)}\right\} && \text{(D-1)} \\ &\overset{(a)}{=} q_n \Delta_n, && \text{(D-2)} \end{aligned}$$

where (a) follows from (4.19). Thus for the on-off power scheme, we have

$$\mathbb{E}\left[p_j^{(t)}\right] = q_n \Delta_n. \quad \text{(D-3)}$$

Under a quasi-static Rayleigh fading channel model, it is concluded that $\chi_j^{(t)}$s are independent and identically distributed (i.i.d.) random variables with

$$\begin{aligned} \mathbb{E}\left[\chi_j^{(t)}\right] &= \mathbb{E}\left[\mathcal{L}_{ji}^{(t)} p_j^{(t)}\right] = \hat{\alpha} q_n \Delta_n, && \text{(D-4)} \\ \mathrm{Var}\left[\chi_j^{(t)}\right] &= \mathbb{E}\left[\left(\chi_j^{(t)}\right)^2\right] - \mathbb{E}^2\left[\chi_j^{(t)}\right] && \text{(D-5)} \\ &\overset{(a)}{\leq} 2\alpha\kappa q_n \Delta_n - (\hat{\alpha} q_n \Delta_n)^2, && \text{(D-6)} \end{aligned}$$

where $\mathbb{E}\left[\left(h_{ji}^{(t)}\right)^2\right] = 2$, $\mathbb{E}\left[\left(\beta_{ji}^{(t)}\right)^2\right] \triangleq \kappa$ and $\hat{\alpha} \triangleq \alpha\varpi$. Also, (a) follows from the fact that $\left(p_j^{(t)}\right)^2 \leq p_j^{(t)}$. Thus, $\mathbb{E}\left[\left(p_j^{(t)}\right)^2\right] \leq \mathbb{E}\left[p_j^{(t)}\right] = q_n\Delta_n$. The interference $I_i^{(t)} = \sum_{\substack{j=1 \\ j\neq i}}^{n} \chi_j^{(t)}$ is a random variable with mean μ_n and variance ϑ_n^2, where

$$\mu_n \triangleq \mathbb{E}\left[I_i^{(t)}\right] = (n-1)\hat{\alpha}q_n\Delta_n, \tag{D-7}$$

$$\vartheta_n^2 \triangleq \mathrm{Var}\left[I_i^{(t)}\right] \leq (n-1)(2\alpha\kappa q_n\Delta_n - (\hat{\alpha}q_n\Delta_n)^2) \leq (n-1)(2\alpha\kappa q_n\Delta_n). \tag{D-8}$$

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