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Free Space Representation for Biped Walking Robots

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Motion planning for biped walking robots is a highly demanding task because of the complex kinematics of such machines and the many degrees of freedom involved. One approach to dealing with this problem is to determine a feasible path in a reduced configuration space of the robot and then to perform the motion planning by searching for an appropriate sequence of steps which allows the locomotion along this path. In this work, a novel method for creating a free space representation for biped walking robots is presented. The method rests upon the approximation of the robot by a set of 3D hulls whose shapes allow efficient determination of feasible paths in a 3D configuration space, involving stepping over obstacles and changing the walking level. The robot's environment is partitioned into two regions. In the first region, 2D motion planning can be performed, while the complexity of 3D motion planning in the second region can be significantly reduced by considering only a restricted set of paths sufficient for solving a wide range of locomotion tasks.

Key words: Walking robots, Motion planning, Free space representation

1 INTRODUCTION

HUMANOID robots have drawn special attention from the very beginnings of robotics because of their resemblance to humans and their potential ability to operate in environments created for humans. In comparison to wheeled robots, humanoid robots can move more freely in indoor and outdoor environments. They can climb stairs, step over or upon obstacles. However, motion planning for a biped walking robot is a rather demanding task because of the complex kinematics of such machines and the many degrees of freedom involved.

One approach to this problem is to decompose the motion planning problem into two subproblems. The first subproblem involves generation of feasible stepping actions for handling particular environmental situations while the second considers planning of step-sequences from a starting configuration to a goal configuration by concatenation of appropriate steps. Given a discrete set of walking primitives, step sequence planning can be performed by building a search tree using a dynamic programming technique [1, 2], where each node in this search tree corresponds to a robot configuration defined by its foot placements. The environment and the robot are modeled by 3D polyhedra and the collision checking is performed by a minimum-distance determination method. The method allowed the robot to plan paths which involve striding over obstacles, while changing of the walking level was not considered. The approach was evaluated by simulations in which the robot was given a task to walk from an initial to a goal position in a scenario cluttered by small cuboid obstacles. In order to apply such an approach for guidance of a real robot, a highly advanced perception system which provides a 3D environment model consisting of polygonal objects would be necessary, and the scene reconstruction of that type still represents a great challenge in computer vision.

In the reactive step-sequence planning approach proposed in [3], efficient collision detection is achieved by identifying a box-like boundary around each obstacle such that the obstacle is entirely contained inside it and the robot is capable of stepping over it. Nevertheless, the proposed method assumes that the robot is equipped by a perception system capable of identifying separate obstacles which the system can circumvent from the left and right side as well as to identify a box-like boundary around obstacles. Both are not trivial tasks in the case of an unstructured environment.

A more appropriate environment model which can be created using the state of the art scene reconstruction systems is a 2.5D map. In [2], an im-

proved version of the method proposed in [1] is presented, which uses a 2.5D environment map. The proposed step sequence planner allows stepping over or upon obstacles. The feasibility of a step is evaluated by checking if the robot's foot can safely clear all the cells along the path from its previous location to the new one. The rest of the robot's body is not considered in the collision checking. Efficiency of the collision checking is achieved by using quadtrees. Step sequences are generated using two graph search methods, A*-search and best-first search. A*-search is guided by a heuristic which assigns to each configuration of the robot considered in the search procedure the estimated remaining cost of the path from the current configuration to the goal region. If an *admissible* heuristic is used to estimate the remaining cost to the goal, A*-search provides optimal path.

In order to achieve real-time performance and in the same time to obtain paths with desirable properties, a limited-horizon A*-search is used in [4] that computes the best partial footstep path it can find in a predefined time. Furthermore, instead of Euclidean distance which is commonly used in A*-search for estimation of the remaining cost to the goal, an alternative heuristic is applied. The approach proposed in [4] is to plan backwards from the goal to the currently considered configuration with a standard mobile robot planner as a precomputation step, and to use the length of the obtained path as the estimated remaining cost to the goal.

Another approach is to find a path which is guaranteed to be executable using a set of feasible stepping action. Next, a path-following algorithm is used to select the appropriate step sequence which allows the robot to walk along this path. Planning the step sequence along a given local path can be performed by depth-first search [5] or by some heuristic step adaptation strategy [6]. Several methods for generating a global path for biped walking robots given a 2.5D environment model represented by a 2D grid of square cells have been proposed [7, 8, 9 ,10, 11]. They are based on the robot modeling approach in which the robot is represented by a single or multiple cylindrical solids. This approximation enables a very efficient free space detection and path planning in a 2D or a 3D configuration space.

The path planning approach proposed in [7, 8] is based on a common approach to robot modeling in which the robot is represented by a cylinder whose projection onto the ground plane represents a disk in a 2D workspace. The free space is determined by dilation of the obstacles in the workspace by this disk. This technique used typically in motion planning for wheeled robots is adapted in [7] for walking robots by introducing *unstable regions* representing obstacles whose height is small enough for the robot to step over or upon them. A path planned over such a region is considered to be feasible if the robot does not stay in the region for more than some predefined time. The proposed approach enables efficient 2D motion planning methods to be applied for planning paths which involve stepping over obstacles or changes in walking level, for example by climbing stairs. However, the height of the obstacle which is used as the criterion for distinguishing between the obstacles and the potentially traversable unstable regions is not the only factor to be considered when deciding if the obstacle can be overcome or not. In some cases the shape and arrangement of obstacles doesn't allow the robot to overcome them although their height is sufficiently small. Furthermore, an additional criterion for the path feasibility proposed in [8] is that the robot can not stay in the unstable region more than some predefined time. This prevents the motion planning algorithm to plan paths over larger unstable regions, such as staircases with steps whose surfaces are not large enough to completely contain the disk representing the robot.

A similar approach is proposed in [9]. The environment of the robot is represented by a 2D navigation map computed from a given 2.5D map, where each cell in the navigation map is assigned 2 attributes, the type of the cell and its clearance. The type of a cell can be *floor*, *stairs*, *border*, *obstacle* or *unknown* and the clearance of the cell represents the distance between the robot and the closest obstacle. The space the robot occupies during the execution of a particular stepping action is approximated by multiple cylinders. The feasibility of the action is evaluated by comparing the radius of these cylinders to the clearance assigned to the cells of the map involved in the action. The motion planning is performed in a discretized 3D configuration space, where only 8 orientations of the robot are considered. The position of the robot is discretized with a resolution of 0.04 m which is comparable to the size of the foot of the considered robot. An optimal path, representing a sequence of neighboring cells in the environment map starting with the current position of the robot and terminating in the given goal position, is determined by A*-search. Because of its computational efficiency, the presented approach is applicable for real-time robot motion planning. However, the proposed approach has certain limitations. The resolution of the navigation map used in the

reported experiments is rather low which can prevent finding paths in situations where obstacles are positioned close to one another. In addition, using only 8 orientations can prevent finding paths in certain environments where walking in particular direction is necessary in order to reach the goal, e.g. in a narrow corridor oriented at an angle of 20°. Finally, the used action set and the proposed navigation map do not allow planning of paths which include stepping over obstacles.

The approach to representing the free space of walking robots proposed in this article is based on a concept similar to the method presented in [9]. However, there are some important differences and improvements. A novel model of biped walking robots is introduced which enables a much more precise analysis of contact situations between the robot and its environment, thereby allowing the locomotion abilities of the robot to be exploited to a greater extent. This robot model represents the basis for an efficient free space detection strategy. Because of its modest computational complexity, it can be applied for on-line motion planning.

The remainder of this article is structured as follows. The problem of motion planning for biped walking robots is formally defined in Section 2. In Section 3, a novel model of biped walking robot based on multiple cylindrical solids is proposed. The application of this model for free space detection is described in Section 4. The result of the considered free space detection procedure is a free space representation suitable for efficient robot motion planning. The applicability of the proposed approach has been evaluated by a series of simulations of various walking scenarios. Some evaluation results are presented in Section 5.

2 PROBLEM DEFINITION

In this work, a humanoid biped robot is considered whose task is to walk from an initial position to a goal position in a walking scenario such as the one presented in Fig. 1. The robot is assumed to be capable of walking on horizontal surfaces sufficiently large to completely contain a robot's foot, stepping over small objects as well as changing the walking level, e.g. by climbing stairs. If an object on the walking trail is too large for the robot to step over, a path around the object should be planned, if possible.

The concept of configuration space [12] is used herein in order to formulate the problem of motion planning for biped walking robots. According to

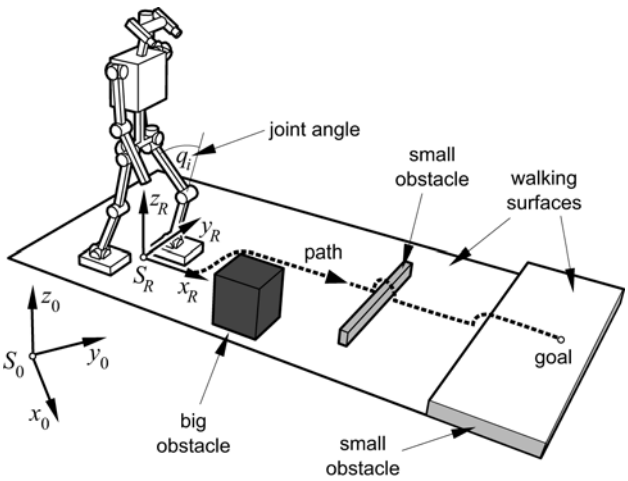

Fig. 1 Typical walking scenario considered in this work

this concept, a *configuration* of an object is defined as a specification of the position of every point in this object relative to a fixed reference frame. The configuration of a walking robot relative to the world reference frame S_0 can be represented by the vector $\mathrm{c}=[\mathrm{q}^T\ \mathrm{p}^T]^T$ where q is the vector of joint angles q_i of the robot and p is the pose of the robot relative to world frame, cf. Fig. 1. The joint angles of the robot define the position of all points of the robot relative to the robot reference frame S_R and $\mathrm{p}=[x\ y\ z\ \alpha]^T$ describes the pose of S_R relative to S_0, where the coordinates x, y and z represent the position of the origin of S_R relative to S_0 and α the orientation of the x-axis of S_R in the xy-plane of S_0. The z-axes of S_R and S_0 are directed anti-parallel to the gravity vector. The space of all configurations of the robot is called *configuration space*, denoted herein by $\mathcal{C}$. The free space $\mathcal{C}_{free}$, as understood herein, represents the subset of all feasible configurations $\mathrm{c}\in\mathcal{C}$. In the case of a biped walking robot, a configuration is considered to be feasible if certain stability conditions are satisfied and if there are no collisions between the mechanical parts of the robot (self-collisions) as well as between the robot and the objects in its environment. In this work, stability is not discussed in detail. The only stability related assumption considered herein is that at any given instant of time at least one foot must rest on a flat horizontal *walking surface*, i.e. the entire sole of the foot must be in contact with this surface.

The task of robot motion planning is to find a *feasible path* Ψ specifying a continuous sequence of feasible configurations $\mathrm{c}\in\mathcal{C}_{free}$ starting at an initial configuration c_{init} and terminating at the goal configuration c_{goal}. The resulting path can be rep-

resented as a continuous function $\Psi: [0, 1] \rightarrow \mathcal{C}$ which assigns a configuration $\Psi(s) = \mathrm{c} \in \mathcal{C}$ to every value $s \in [0, 1]$, with $\Psi(0) = \mathrm{c}_{init}$ and $\Psi(1) = \mathrm{c}_{goal}$.

Because of the high dimensionality of the configuration space $\mathcal{C}$, the search for a feasible path in $\mathcal{C}$ is in general a complex task. In order to reduce the complexity of motion planning, the motion of the walking robot is split up into a sequence of *steps*, thereby dividing the planning problem into two subproblems. The first subproblem involves generation of feasible steps for handling particular environmental situations and the second involves generation of paths from an initial to a goal configuration by concatenation of appropriate steps. A step is considered herein to be a path $\Upsilon: [0, 1] \rightarrow \mathcal{C}$ from a configuration $\mathrm{c}_0 = \Upsilon(0)$ to a configuration $\mathrm{c}_1 = \Upsilon(1)$ with the following properties.

1) A step is executable by the robot in terms of its mechanical abilities. This means that all joint angles along the path Υ are inside the limited workspace of robot's joints q and that there are no self-collisions.
2) Assuming that there are no collisions between the robot and its environment, that the robot's feet do not slip and that at c_0 and c_1 both feet rest on a flat horizontal surface, the robot is stable at every configuration $\Upsilon(s)$, $s \in [0, 1]$.

Two steps can be concatenated only if the final configuration of the preceding step matches the initial configuration of the following step. For two steps to be concatenated, not only the configurations but also the first derivatives of joint angles $\dot{\mathrm{q}}$ have to match [13]. However, robot dynamics is not considered in this work.

Feasible steps can be generated on-line [14] or acquired from a walking primitive database generated off-line [13, 15]. The methods for generation of steps are not discussed herein. It is assumed that the control system of the robot is able to generate an appropriate step for certain arrangements of walking surfaces and obstacles on the walking trail. The focus of this work is determination of a subset of the free space $\mathcal{C}_{free}$ sufficient for solving a wide range of locomotion tasks and representation of this set in a form which allows efficient search for feasible paths. Only those paths are considered which can be obtained by concatenations of steps. According to the definition of the step, such a path Ψ is feasible if the following *feasibility conditions* are satisfied.

1) There are no collisions between the robot and its environment at any configuration $\Psi(s)$, $s \in [0, 1]$.
2) At the initial and final configuration of every step, both feet of the robot rest on a walking surface.

3 MODEL OF BIPED WALKING ROBOT

In this section, a robot model is proposed which enables efficient determination of path feasibility. Let $V(\mathrm{c})$ be a subset of $\mathbb{R}^3$ representing all points occupied by the robot at some configuration c and let M be a subset of $\mathbb{R}^3$ representing the points of all objects in the robot's environment. If

$$V(\mathrm{c}) \cap M = \phi \tag{1}$$

then there is no collision between the robot at configuration c and objects in the robot's environment. In order to perform collision checking by detecting the intersection between $V(\mathrm{c})$ and M, the motion planner must be able to determine $V(\mathrm{c})$ for each relevant configuration $\mathrm{c} = [\mathrm{q}^T\ \mathrm{p}^T]^T$. However, the complex kinematics of walking robots allows them to have many different postures corresponding to $V(\mathrm{c})$ of very complex shapes. Therefore, collision checking using an exact $V(\mathrm{c})$ and M is impractical in most cases. The complexity of collision checking can be significantly reduced by approximating $V(\mathrm{c})$ and M by simpler shapes which still allow analysis of important contact situations between the robot and its environment. For collision-checking it is suitable to approximate the robot by some *bounding approximation*, i.e. a set $V'(\mathrm{c})$ of simpler shape such that $V(\mathrm{c}) \subset V'(\mathrm{c})$ [12].

A. Bounding Approximation of the Robot

In this work, an approximation technique suited to biped walking robots is proposed, which allows efficient handling of contact situations between the robot and its environment. The idea is to define a set of *actions* the robot can execute such as straight ahead walking, changing direction and stepping over or upon an obstacle. The set of all points occupied by the robot during the execution of an action Θ is approximated by a compact set $V_\Theta \subset \mathbb{R}^3$ called the *hull* of the action whose shape allows efficient feasibility checking. An action $\Theta: [0, 1] \rightarrow \mathcal{C}$ is a path representing a sequence of one or several steps. The set V_Θ must satisfy the following condition

$$V_\Theta \supset \bigcup_{s \in [0,1]} V[\Theta(s)] \tag{2}$$

The approximation method proposed in this work consists of modelling the robot by a hull V_Θ from a set $\mathcal{H}$ of 3D solids with the following properties.

1) A solid $V_\Theta \in \mathcal{H}$ can be represented by

$$V_\Theta = \bigcup_{i=1}^{n_\Theta} V_\Theta^{(i)} \tag{3}$$

where each $V_\Theta^{(i)}$ is a cylindrical solid with axis parallel to the gravity axis. The base of $V_\Theta^{(i)}$ is located on the horizontal plane $z = z_\Theta + \Delta z_\Theta^{(i)}$, where z_Θ is the reference z-coordinate of V_Θ and $\Delta z_\Theta^{(i)}$ is the parameter defining the height of $V_\Theta^{(i)}$ relative to z_Θ.

2) The projection $A_\Theta^{(i)}$ of each $V_\Theta^{(i)}$ onto the xy-plane can be obtained by *dilation* of a set $B_\Theta^{(i)}$ of points by a circular structural element, i.e. a disk of radius $r_\Theta^{(i)}$. The dilation of a set of points U by a structural element V is defined as follows

$$U \oplus V = \{\mathrm{u} + \mathrm{v} \mid (\mathrm{u} \in U) \wedge (\mathrm{v} \in \mathrm{V})\} \tag{4}$$

Set $B_\Theta^{(i)}$ is referred to herein as *seed point set* of $V_\Theta^{(i)}$.

An example of modelling a robot executing an action Θ by a hull from the set $\mathcal{H}$ is shown in Fig. 2. The action Θ consists of two steps. At the initial configuration of Θ, the robot stands on the walking surface A ahead of an obstacle with closed feet. The position of the robot in the xy-plane is x_0

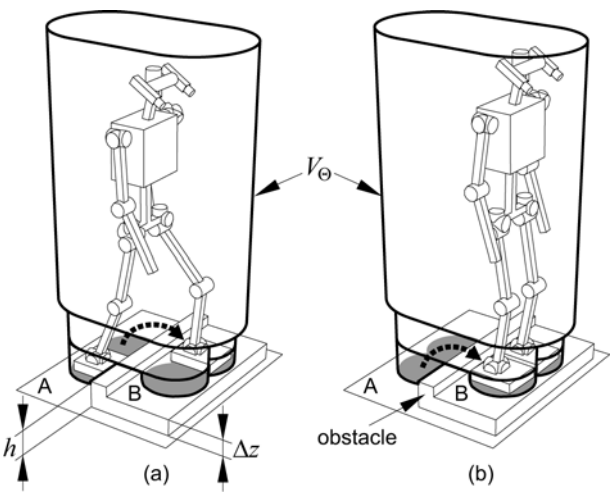

Fig. 2 Biped walking robot represented by a hull containing all points occupied by the robot during the execution of an action comprising two steps for striding over an obstacle

and the distance between the centers of its feet is l_{fd}. In order to step onto the walking surface B, the robot executes an appropriate step, shown in Fig. 2(a), thereby striding over the obstacle. By executing the second step, shown in Fig. 2(b), the robot closes its feet on the walking surface B. The final position of the robot after the execution of the action is x_1 and the distance travelled during the execution of the action is l_Θ. The height of the obstacle considered in the example relative to the walking surface A is h, and the height difference between A and B is Δz. The obstacle represents a combination of two basic types of obstacles which the robot is expected to deal with. Assuming that $h > 0$ and $\Delta z = 0$, the obstacle represents a barrier which the robot can step over. For $h = \Delta z \neq 0$, the obstacle represents a change in the walking level, such as a step of a staircase. The set of points occupied by the robot during the execution of the action Θ is approximated by the hull V_Θ consisting of seven cylindrical solids $V_\Theta^{(i)}$, $i = 1,\ldots,7$ corresponding to different parts of the robot, cf. Fig. 3(a). Solid $V_\Theta^{(1)}$ approximates the robot's upper body and the upper parts of its legs. Solids $V_\Theta^{(2)}$, $V_\Theta^{(4)}$, $V_\Theta^{(6)}$ represent the lower parts of the robot's left leg and $V_\Theta^{(3)}$, $V_\Theta^{(5)}$, $V_\Theta^{(7)}$ the lower parts of the right leg. The bases of the solids $V_\Theta^{(i)}$, $i = 4,\ldots,7$ completely contain the soles of the robot's feet at the initial and final configuration. The reference z-coordinate z_Θ is equal to the height of the bases $A_\Theta^{(4)}$ and $A_\Theta^{(5)}$ of the solids $V_\Theta^{(4)}$ and $V_\Theta^{(5)}$, i.e. $\Delta z_\Theta^{(4)} = \Delta z_\Theta^{(5)} = 0$. The projection of the hull V_Θ onto the xy-plane is shown in Fig. 3(b). The base of each solid $V_\Theta^{(i)}$ can be obtained by dilation of the corresponding seed point set $B_\Theta^{(i)}$ by a disc of radius $r_\Theta^{(i)}$, where $r_\Theta^{(i)} = r_f \geq 0.5\, l_{fd}$, $i = 2,\ldots,7$ and $r_\Theta^{(1)} = r_{rot} = r_f + 0.5\, l_{fd}$. These seed point sets can be defined using the rectangle $R(\mathrm{x}_0, \mathrm{x}_1)$ represented in Fig. 3(b) by dashed lines. The rectangle is defined by the points x_0 and x_1 representing the midpoints of its two opposite sides of length l_{fd}. Let x_i, $i = 4,\ldots,7$ be the vertices of $R(\mathrm{x}_0, \mathrm{x}_1)$. The seed point sets of the solids $V_\Theta^{(i)}$ are

$$B_\Theta^{(1)} = \mathrm{x}_0 \mathrm{x}_1 \tag{5}$$

$$B_\Theta^{(2)} = \mathrm{x}_4 \mathrm{x}_6, \quad B_\Theta^{(3)} = \mathrm{x}_5 \mathrm{x}_7 \tag{6}$$

$$B_\Theta^{(4)} = \mathrm{x}_4, \quad B_\Theta^{(5)} = \mathrm{x}_5, \quad , B_\Theta^{(6)} = \mathrm{x}_6 \quad B_\Theta^{(7)} = \mathrm{x}_7 \tag{7}$$

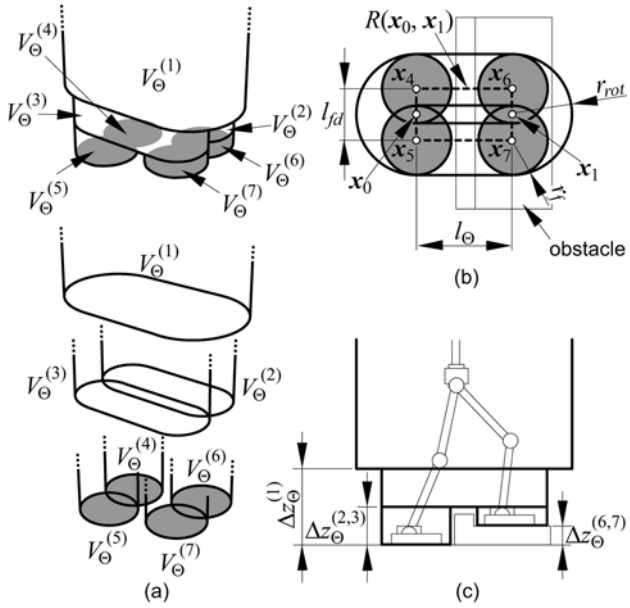

Fig. 3 Hull V_Θ approximating the robot during the execution of an action for stepping over obstacles. (a) Perspective view. (b) Top view. (c) Side view. The components of the hull V_Θ are denoted by full bold lines in all views.

where $\mathrm{x}_i\mathrm{x}_j$ denotes the straight line segment with endpoints x_i and x_j.

B. Determining the Feasibility of Actions using an Extended 2.5D Map

Modelling a walking robot by hulls from the set $\mathcal{H}$ allows efficient collision-checking using a specially designed environment map, as described in this section. The proposed environment representation is based on the 2.5D map. The *2.5D map* of the environment can be defined by a function $z_M: \mathbb{R}^2 \to \mathbb{R}$ which divides the set of all 3D points $\mathrm{w} \in \mathbb{R}^3$ into two subsets, the subset of free points and the subset M of points occupied by some object. Subset M is defined by

$$M = \left\{ \mathrm{w} \in \mathbb{R}^3 \quad | \quad z < z_M(\mathrm{x}) \right\}$$
$$\mathrm{w} = \begin{bmatrix} x & y & z \end{bmatrix}^T, \qquad \mathrm{x} = \begin{bmatrix} x & y \end{bmatrix}^T \tag{8}$$

where x, y and z are the coordinates of a 3D point in the environment of the robot relative to S_0. In further discussion it is assumed that all relevant contact situations between the robot and its environment can be modelled by interactions between a particular robot hull and the set M.

Let Θ be an action consisting of one or several steps and let $V_\Theta \in \mathcal{H}$ be the robot hull defined by (3) such that (2) holds. Furthermore, let $V_\Theta^{(i)}$, $i = i_f, \ldots, n_\Theta$, $i_f \le n_\Theta$ be cylindrical solids whose bases completely contain the soles of robot's feet at the initial and final configuration of all steps of Θ. It can be easily shown that if the following two conditions are satisfied

$$\forall \mathrm{x} \in A_\Theta^{(i)}, \quad i = 1, \ldots, n_\Theta, \quad z_M(\mathrm{x}) \le z_\Theta + \Delta z_\Theta^{(i)} \tag{9}$$

$$\forall \mathrm{x} \in A_\Theta^{(i)}, \quad i = i_f, \ldots, n_\Theta, \quad z_M(\mathrm{x}) = z_\Theta + \Delta z_\Theta^{(i)} \tag{10}$$

the feasibility conditions given in Section 2 are also satisfied. In that case, the action Θ is feasible.

The second property of the hulls from $\mathcal{H}$ can be used for efficient feasibility checking based on the extended 2.5D map of the robot's environment. An *extended 2.5D map* is defined by two functions $f_{ws}: \mathbb{R}^3 \to \{0, 1\}$ and $z_X: \mathbb{R}^3 \to \mathbb{R}$ and it is related to a 2.5D map M by

$$z_X(\mathrm{x}, r) = \max_{\mathrm{x}' \in \Omega(\mathrm{x}, r)} z_M(\mathrm{x}') \tag{11}$$

$$f_{ws}(\mathrm{x}, r) = \begin{cases} 1, \text{ if } \forall \mathrm{x}' \in \Omega(\mathrm{x}, r), z_M(\mathrm{x}') = z_M(\mathrm{x}) \\ 0, \text{ otherwise} \end{cases} \tag{12}$$

$$\Omega(\mathrm{x}, r) = \left\{ \mathrm{x}' \in \mathbb{R}^2 \quad | \quad \|\mathrm{x}' - \mathrm{x}\| \le r \right\} \tag{13}$$

Given an extended 2.5D map of the robot's environment, the feasibility conditions (9) – (10) can be reformulated as follows

$$\forall \mathrm{x} \in B_\Theta^{(i)}, i = 1, \ldots, n_\Theta, z_X\left(\mathrm{x}, r_\Theta^{(i)}\right) \le z_\Theta + \Delta z_\Theta^{(i)} \tag{14}$$

$$\forall \mathrm{x} \in B_\Theta^{(i)}, i = i_f, \ldots, n_\Theta, f_{ws}\left(\mathrm{x}, r_\Theta^{(i)}\right) = 1$$
$$z_X\left(\mathrm{x}, r_\Theta^{(i)}\right) = z_\Theta + \Delta z_\Theta^{(i)} \tag{15}$$

The conditions (14) and (15) have practical importance, because feasibility checking using only the seed points $B_\Theta^{(i)}$ of the regions $A_\Theta^{(i)}$ instead of all points in $A_\Theta^{(i)}$ can significantly reduce the computation time. However, the precondition for application of (14) and (15) is that an efficient method for creating extended 2.5D maps is available.

C. 3D Configuration Space

By restricting the discussion only to steps with properties given in Section 2 and concatenations of such steps, the joint angles of the robot do not have to be considered when searching for feasible paths. Furthermore, because of the feasibility condition (10), the position of the robot in z-direction relative to S_0 is determined by its position in the xy-plane and the 2.5D map. Hence, the pose of the robot can be described by the coordinates x and y defining the position and the angle α defining the robot's orientation in the xy-plane. Motion planning can thus be simplified by projecting the configuration space $\mathcal{C}$ into the 3D space $\mathcal{C}'$ of configurations $\mathrm{c}'=[x\ y\ \alpha]^T$ and searching for a feasible path Ψ' in $\mathcal{C}'$. A path $\Psi':[0,\ 1]\rightarrow\mathcal{C}'$ defined by

$$\Psi'(s)=\left[x'(s)\ \ y'(s)\ \ \alpha'(s)\right]^T,\quad s\in\left[0,1\right] \tag{16}$$

is considered feasible if there exists at least one feasible path $\Psi:[0,\ 1]\rightarrow\mathcal{C}$ defined by

$$\Psi(s)=\left[\mathrm{q}^T(s)\ \ \mathrm{p}^T(s)\right]^T,\quad s\in\left[0,1\right]$$
$$\mathrm{p}(s)=\left[x(s)\ \ y(s)\ \ z(s)\ \ \alpha(s)\right]^T \tag{17}$$

such that $x(s)=x'(s)$, $y(s)=y'(s)$ and $\alpha(s)=\alpha'(s)$ for every $s\in[0,\ 1]$. The path Ψ' can be regarded as the projection of the path Ψ into the 3D configuration space $\mathcal{C}'$. In Section 4, motion planning in $\mathcal{C}'$ is considered.

4 FREE SPACE REPRESENTATION FOR A CLASS OF BIPED WALKING ROBOTS

In this section, a free space representation for a class of biped walking robots is proposed, based on the robot model presented in Section 3. The robot is assumed to be capable of executing a set Ξ of steps which allow it to perform the actions specified in the following.

A. Actions

At the initial and final configuration of each action Θ, the robot stands with the feet closed at a distance l_{fd} as shown in Fig. 2(b). Thus, each action can be concatenated to another action. The points occupied by the robot during the execution of an action Θ are contained inside the corresponding hull $V_\Theta\in\mathcal{H}_\Xi\subset\mathcal{H}$. Every hull $V_\Theta\in\mathcal{H}_\Xi$ consists of n_Θ cylindrical solids $V_\Theta^{(i)}$, where n_Θ is 3 or 7. The reference z-coordinate z_Θ of all hulls is defined by the initial position of the robot relative to S_0. For all hulls in $\mathcal{H}_\Xi$, $r_\Theta^{(i)}=r_{rot}=r_f+0.5\ l_{fd}$, $r_\Theta^{(i)}=r_f$, $i=2,\ldots,\ n_\Theta$, where $r_f\geq 0.5\ l_{fd}$.

1) Stepping over Obstacles: Overcoming the obstacles which represent the transitions between walking surfaces on different heights can be regarded as stepping over them. For example, the stair climbing can be considered as stepping over a sequence of obstacles between walking surfaces. Therefore, overcoming any considered type of obstacle is referred to in the following as stepping over the obstacle. The parameters of an action for stepping over obstacles are the travelled distance l_Θ and change of walking level Δz_Θ. Let $\nu_\Theta=[l_\Theta\ \Delta z_\Theta]^T$ be the vector of action parameters and let $\mathcal{K}_\Theta$ be the set of all parameter combinations the robot can execute defined by

$$0\leq l_\Theta\leq l_{\max} \tag{18}$$

$$-\Delta z_{\max}\leq\Delta z_\Theta\leq\Delta z_{\max} \tag{19}$$

For every vector $\nu_\Theta\in\mathcal{K}_\Theta$, there exists a pair of steps in Ξ which can be concatenated into an action Θ such that all points occupied by the robot during the execution of Θ are contained inside the corresponding hull V_Θ. The hull V_Θ was already described in the example given in Section 3. The parameters $\Delta z_\Theta^{(i)}$ of the hull V_Θ are defined by

$$\Delta z_\Theta^{(1)}=\min\left(0,\Delta z_\Theta\right)+h^{(1)} \tag{20}$$

$$\Delta z_\Theta^{(2)}=\Delta z_\Theta^{(3)}=\min\left(0,\Delta z_\Theta\right)+h^{(2)} \tag{21}$$

$$\Delta z_\Theta^{(6)}=\Delta z_\Theta^{(7)}=\Delta z_\Theta \tag{22}$$

The constants $l_{\max}$, $\Delta z_{\max}$, $h^{(1)}$ and $h^{(2)}$ are determined by the mechanical properties of the robot, and they are constrained by

$$\Delta z_{\max}\leq h^{(2)}\leq h^{(1)} \tag{23}$$

2) Straight ahead Walking on a Flat Surface: For every $l_\Theta\geq 0$ there is a sequence of steps in Ξ which can be concatenated into an action Θ for straight ahead walking on a flat surface whose projection Θ' into $\mathcal{C}'$ is described by

$$\forall s\in\left[0,1\right],\quad \Theta'(s)=\begin{bmatrix} x_0+sl_\Theta\cos\alpha \\ y_0+sl_\Theta\sin\alpha \\ \alpha \end{bmatrix} \tag{24}$$

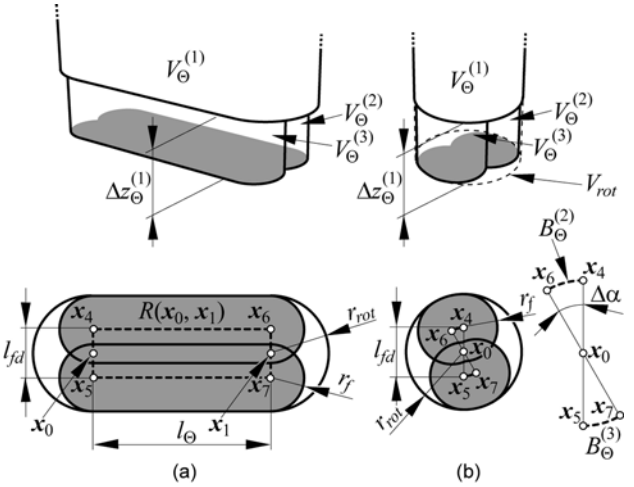

Fig. 4 Hulls V_Θ approximating the robot during the execution of actions for walking on a flat surface and the projections of the hulls onto the xy-plane. The components of the hulls V_Θ are denoted by full bold lines. (a) Straight ahead walking. (b) Changing direction

where $x_0=[x_0\ y_0]^T$ is the initial position of the robot in the *xy*-plane and $x_1=x_0+l_\Theta[\cos\alpha\ \sin\alpha]^T$ its final position. The hull V_Θ of Θ is shown in Fig. 4(a). It represents a special case of the hull used for modelling the actions for stepping over obstacles where $\Delta z_\Theta=h_\Theta=0$. The solids $V_\Theta^{(i)}$, $i=4,\ldots,7$ are completely contained inside $V_\Theta^{(2)}$ and $V_\Theta^{(3)}$, so they can be excluded from the model. The bases of $V_\Theta^{(2)}$ and $V_\Theta^{(3)}$ completely contain the soles of the robot's feet at the initial and final configuration of each step of Θ. The height of $V_\Theta^{(1)}$ relative to z_Θ is $\Delta z_\Theta^{(1)}=h^{(1)}$.

3) Changing Direction on a Flat Surface: For every $-\pi\le\Delta\alpha\le\pi$, there is a sequence of steps in Ξ which can be concatenated into a direction changing action Θ whose projection Θ' into $\mathcal{C}'$ is described by

$$\forall s\in\left[0,1\right],\quad \Theta'(s)=\left[x_0\ y_0\ \alpha+s\Delta\alpha\right]^T \quad (25)$$

where $x_0=[x_0\ y_0]^T$ is the position of the robot in the *xy*-plane, α its initial orientation and $\alpha+\Delta\alpha$ its final orientation. The hull V_Θ of Θ consists of three cylindrical solids $V_\Theta^{(1)}$, $V_\Theta^{(2)}$ and $V_\Theta^{(3)}$ shown in Fig. 4(b), where the bases of $V_\Theta^{(2)}$ and $V_\Theta^{(3)}$ completely contain the soles of the robot's feet at the initial and final configuration of each step of Θ. Let x_4 and x_5 be the positions of the robot feet centers at the initial configuration of Θ' and x_6 and x_7 the foot positions at the final configuration. The point x_0 is the common midpoint of the line segments x_4x_5 and x_6x_7, their length is l_{fd} and the angle between them is $\Delta\alpha$, cf. Fig. 4(b). Let Z be the circle of radius 0.5 l_{fd} with the center in x_0. The arc of Z between x_4 and x_6 represents the seed point set $B_\Theta^{(2)}$ and the arc between x_5 and x_7 the seed point set $B_\Theta^{(3)}$. The height of $V_\Theta^{(1)}$ relative to z_Θ is $\Delta z_\Theta^{(1)}=h^{(1)}$ and its seed point set $B_\Theta^{(1)}$ consists of the single point x_0.

B. Feasibility of Paths in 3D Configuration Space

The motion planning approach considered herein is to partition the configuration space into two subsets, set $\mathcal{C}'_{free,rot}$ and its complement $\bar{\mathcal{C}}'_{free,rot}$ and to apply a different planning strategy in each set. The set $\mathcal{C}'_{free,rot}$ represents the free space of the robot modelled by a cylinder V_{rot} of radius r_{rot}, represented in Fig. 4(b) by thin dashed lines. It can be defined by

$$\mathcal{C}'_{free,rot}=\left\{\left[\mathrm{x}^T\alpha\right]^T\in\mathcal{C}'\middle|f_{ws}\left(\mathrm{x},r_{rot}\right)=1\right\} \quad (26)$$

Since it is determined by the function f_{ws}, it can be obtained directly from the extended 2.5D map. The set $\mathcal{C}'_{free,rot}$ contains configurations at which the robot stands on a walking surface so that the distance to the closest obstacle is sufficient for the robot to change its walking direction by turning in place without collisions. Motion planning in this set can be performed by representing the robot by a point in the *xy*-plane and applying some of the computationally inexpensive 2D motion planning methods proposed in the literature [12]. The set $\bar{\mathcal{C}}'_{free,rot}$ contains configurations at which the robot is in collision with obstacles or relatively close to them. Motion planning in $\bar{\mathcal{C}}'_{free,rot}$ must consider also the orientation of the robot and therefore must be performed in 3D.

Let us first consider motion planning in $\mathcal{C}'_{free,rot}$. Let Ψ' be a path defined by (24). Assuming that the robot walks along this path by executing an action for straight ahead walking, the set of points occupied by the robot can be approximated by the hull presented in Fig. 4(a). The projection of this hull onto the *xy*-plane is completely contained in the region obtained by dilation of the line segment x_0x_1 by a disc of radius r_{rot}. Hence, the conditions (9)–(10) are satisfied for any straight walking path in $\mathcal{C}'_{free,rot}$. Furthermore, since the projection of the hull of any direction changing path is completely contained in the circle of radius r_{rot}, the conditions (9)–(10) are satisfied for any direction changing path in $\mathcal{C}'_{free,rot}$. It follows that any path in $\mathcal{C}'_{free,rot}$

obtained by concatenation of straight walking paths defined by (24) and direction changing paths defined by (25) is feasible. Moreover, since the robot can freely change its orientation in any configuration $c' \in \mathcal{C}'_{free,rot}$, its orientation can be neglected when planning a path in $\mathcal{C}'_{free,rot}$. Hence, motion planning in $\mathcal{C}'_{free,rot}$ can be reduced to planning in the 2D configuration space $\mathcal{C}''_{free,rot}$ representing the projection of $\mathcal{C}'_{free,rot}$ onto the *xy*-plane.

Motion planning in $\mathcal{C}'_{free,rot}$ involves striding over obstacles and moving on walking surfaces in the vicinity of obstacles. Let Ψ' be a straight path starting at the position x_0 in the *xy*-plane and terminating at the position x_1. Furthermore, let $R(x_0, x_1)$ be the rectangle defined by x_0 and x_1 representing the midpoints of its two opposite sides of length l_{fd} and let x_i, $i=4,\ldots,7$ be the vertices of the rectangle, cf. Fig. 3(b). Such a rectangle is referred to in the following as *path frame*. Given an extended 2.5D map of the robot's environment, the feasibility of Ψ' can be determined using the specifications of the hulls modelling the actions for stepping over obstacles and straight ahead walking on a flat surface given in Section 4-A together with the conditions (14) and (15). When applying the conditions (14) and (15) to determine the feasibility of the actions for stepping over obstacles, $i_f=4$ and $n_\Theta=7$, while when checking the feasibility of actions for straight ahead walking, $i_f=2$ and $n_\Theta=3$. It follows that if

$$f_{ws}\left(x_i, r_f\right)=1, \quad i=4\ldots,7 \tag{27}$$

$$\left|z_X\left(x_6, r_f\right)-z_X\left(x_4, r_f\right)\right| \le \Delta z_{\max} \tag{28}$$

$$\forall x \in x_4x_6 \cup x_5x_7, \quad z_X\left(x, r_f\right) \le z_{\min}+h^{(2)} \tag{29}$$

$$\forall x \in \{x_0x_1\}, \quad z_X\left(x, r_{rot}\right) \le z_{\min}+h^{(1)} \tag{30}$$

$$z_{\min}=\min_{i=4,6} z_X\left(x_i, r_f\right) \tag{31}$$

$$\left\|x_1-x_0\right\| \le l_{\max} \tag{32}$$

then there exists a feasible action for stepping over obstacles which enables the robot to move along the path Ψ'. Furthermore, if

$$\forall x \in x_4x_6 \cup x_5x_7, \quad f_{ws}\left(x, r_f\right)=1 \tag{33}$$

$$\forall x \in \{x_0x_1\}, \quad z_X\left(x, r_{rot}\right) \le z_X\left(x_4, r_f\right)+h^{(1)} \tag{34}$$

then there exists a feasible action for straight ahead walking on a flat surface which enables the robot to move along Ψ'. In both cases, the path Ψ' is feasible. The feasibility conditions for direction changing paths similar to (33)–(34) can be derived using the conditions (14)–(15) and the specification of the corresponding hull. It can be shown that checking the condition (15) for all points of the arcs $B_\Theta^{(2)}$ and $B_\Theta^{(3)}$ can be avoided in cases where the direction changing path connects two feasible straight paths with certain properties. Details are presented in [11].

C. Free Space Representation

From the discussion in Section IV-B it follows that $\mathcal{C}'_{free,rot} \subseteq \mathcal{C}'_{free}$. Furthermore, a configuration $c' \in \bar{\mathcal{C}}'_{free,rot}$ is an element of the free space if there exists a feasible path Ψ' such that $\exists s \in [0, 1]$, $c'=\Psi'(s)$. However, determining the free space by checking all possible paths in $\bar{\mathcal{C}}'_{free,rot}$ is not feasible. The approach proposed herein is to determine a set $\mathcal{P}$ of feasible paths in $\bar{\mathcal{C}}'_{free,rot}$ which together with the configuration set $\bar{\mathcal{C}}'_{free,rot}$ represents a subset of the total free space $\mathcal{C}'_{free}$ sufficient for handling a wide range of obstacle situations. Motion planning can then be performed by searching for paths in 2D configuration space $\mathcal{C}''_{free,rot}$ connected by paths from $\mathcal{P}$.

The set $\mathcal{P}$ is determined using the shape and pose of obstacles. Assuming that the sole of the robot's foot is represented by a disk of radius r_f, the set defined by

$$\mathcal{C}''_{ob}=\left\{x \in \mathbb{R}^2 \middle| f_{ws}\left(x, r_f\right)=0\right\} \tag{35}$$

can be interpreted as the set of points in the *xy*-plane where the center of a robot's foot can not be placed in such a way that its sole lies entirely on a flat surface. This set is referred to in the following as *obstacle region*. An example of an obstacle region is presented in Fig. 5. An obstacle region $\mathcal{C}''_{ob}$ can be represented by the union of its maximal balls. A *maximal ball* is an open ball inside the boundary of a subset X of the n-dimensional Euclidean space not contained in any other open ball inside that boundary. Examples of maximal balls of $\mathcal{C}''_{ob}$ are shown in Fig. 5. The strategy proposed herein is to consider only those paths over obstacles whose directions are defined by

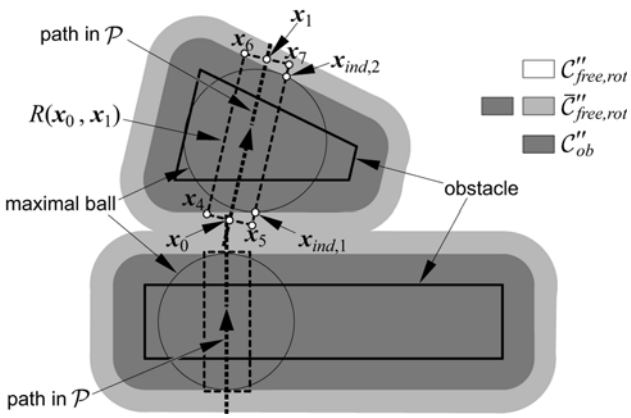

Fig. 5 Regions $\mathcal{C}''_{free,rot}$, $\bar{\mathcal{C}}''_{free,rot}$ and $\mathcal{C}''_{ob}$ of the xy-plane and two paths over $\mathcal{C}''_{ob}$ determined using the maximal balls of $\mathcal{C}''_{ob}$.

pairs of points in which the maximal balls of $\mathcal{C}''_{ob}$ touch the boundary of $\mathcal{C}''_{ob}$. A pair of points $x_{ind,1}$ and $x_{ind,2}$ in which a maximal ball of $\mathcal{C}''_{ob}$ touches the boundary of $\mathcal{C}''_{ob}$ represents an indicator for two paths over an obstacle. First, the search for the smallest path frame $R(x_0, x_1)$ which satisfies the conditions (27)–(32) and whose side x_5x_7 contains $x_{ind,1}$ and $x_{ind,2}$ is performed, cf. Fig. 5. If such a path frame exists, the straight path Ψ' from x_0 to x_1 is feasible. Then, the obtained path is extended by straight walking paths in both directions until reaching $\mathcal{C}''_{free,rot}$ or $\mathcal{C}''_{ob}$. These paths are required to satisfy (33)–(34). The same procedure is repeated by searching for the corresponding path frame whose side x_4x_6 contains $x_{ind,1}$ and $x_{ind,2}$. The result is a set of feasible straight paths over obstacles, such as those shown in Fig. 5, whose initial and final positions are contained in $\mathcal{C}''_{free,rot}$ or are close to the boundary of $\mathcal{C}''_{ob}$. Two such paths whose projections onto the *xy*-plane intersect can be connected by a direction changing path if there is enough space for the robot to change its walking direction, i.e. if the corresponding feasibility condition given in [11] is satisfied. The obtained straight paths over obstacles together with the direction changing paths form the set $\mathcal{P}$.

The free space representation consisting of the configuration set $\mathcal{C}''_{free,rot}$ and the set $\mathcal{P}$ of paths in $\bar{\mathcal{C}}'_{free,rot}$ can be efficiently computed by discretizing the *xy*-plane into small square cells and creating a graph whose nodes correspond to the cells containing feasible configurations. First, given a discretized 2.5D map of the robot's environment, the values of the functions f_{ws} and z_X defining the corresponding extended 2.5D map are computed for the radii r_f and r_{rot}. The 2.5D map consists of cells representing small square regions of the *xy*-plane. Each cell is assigned a height z_M defined in Section III-B. A suitable approximation of the extended 2.5D map can be computed very efficiently by dilation of the objects and surfaces in the 2.5D map by discs of radii r_f and r_{rot}. The extended 2.5D map is then used to determine the regions $\mathcal{C}''_{free,rot}$ and $\mathcal{C}''_{ob}$. Finally, a graph is created whose nodes correspond to the cells in $\mathcal{C}''_{free,rot}$ and cells lying on the projections of the paths $\Psi' \in \mathcal{P}$ onto the *xy*-plane up to the discretization accuracy. The set $\mathcal{P}$ is determined using the proposed approach based on the maximal balls of $\mathcal{C}''_{ob}$. Two nodes η_1 and η_2 of the graph corresponding to the cells Γ_1 and Γ_2 respectively are connected only if the following two conditions are satisfied.

1) $\Gamma_1 \equiv \Gamma_2$ or Γ_1 and Γ_2 are neighboring cells.
2) Γ_1 and Γ_2 are contained in $\mathcal{C}''_{free,rot}$ or their centers lie on the projection of a path $\Psi' \in \mathcal{P}$ onto the *xy*-plane up to the discretization accuracy.

An algorithm for creating the discussed graph representation is given in [11].

Motion planning can be performed using the obtained graph by searching for a sequence of connected nodes starting with the node corresponding to an initial position of the robot and terminating with the node corresponding to a given goal position.

Table 1 **Hull parameters used in case study**

Parameter	Value	Parameter	Value
r_f	0.180 m	$\Delta z_{\max}$	0.175 m
l_{fd}	0.260 m	$h^{(1)}$	0.350 m
$l_{\max}$	0.500 m	$h^{(2)}$	0.175 m

5 CASE STUDY

The applicability of the proposed approach has been evaluated by using synthetic walking scenarios. An algorithm for creating graph representations of the free space based on the strategy presented in Section 4-C has been applied to the 2.5D environment maps created from a set of prototypical walking scenarios. Then a simple motion planning algorithm has been used to generate feasible paths between given initial and goal positions in the scenarios. Two sample results are presented in this section. More examples are given in [11].

The parameters of the hulls approximating the robot during the execution of the actions specified in Section 4-A are given in Table 1.

A. Example 1

One of the 3D walking scenarios used for evaluation of the discussed approach is presented in Fig. 6(a). All surfaces in the scenario are either horizontal or vertical. In the top view of the scenario shown in Fig. 6(b), surface boundaries are represented by full black lines. The 3D scenario is transformed into a 2.5D map of size 5 m × 5 m consisting of square cells 10 mm × 10 mm. This map is used to create a graph representation of a subset of the free space consisting of the configuration set $\mathcal{C}'_{free,rot}$ and the set $\mathcal{P}$ of paths in $\bar{\mathcal{C}}'_{free,rot}$. The result of the free space detection is shown in Fig. 6(b), where white region represents $\mathcal{C}''_{free,rot}$, i.e. the projection of $\mathcal{C}'_{free,rot}$ onto the *xy*-plane and light gray lines indicate the projections of the paths $\Psi' \in \mathcal{P}$ to the *xy*-plane. The light gray regions which can be seen in Fig. 6(b) are formed by parallel paths located close to one another. Given a start or initial position x_{init} and a goal position x_{goal} in the *xy*-plane, a feasible path from x_{init} to x_{goal} can be found by searching for a sequence of connected nodes in the graph starting with the node corresponding to x_{init} and terminating with the node corresponding to x_{goal}. The motion planning method applied herein is based on a numerical navigation function computed by a wavefront propagation algorithm similar to NF1 presented in [12]. Every node in the graph is assigned a value of the numerical navigation function representing an estimate of the minimal cost of moving from the position corresponding to the node to the goal position. Then, a path from the initial to the goal position is determined by following the steepest descent of the numerical navigation function. A sample path obtained by this method is presented in Fig. 6(b) by bold dashed lines.

The obstacles in the considered scenario can be divided into three classes according to the action the walking robot must execute in order to overcome them. The first obstacle P on the path between the initial and goal configuration is too large for the robot to step over and therefore must be circumvented by walking around it. The second obstacle R represents a change in walking level and the third obstacle S is a barrier which the robot can step over. The set of small obstacles T appearing on the walking trail after the barrier S demonstrates how an object itself can not be classified as an obstacle in the context of robot walking without considering the arrangement of other objects in its neighborhood. Although the robot could step over each of these small obstacles if they were located apart from each other, their arrangement in the scenario prevents the robot from striding over them. A part of the barrier S is covered by the feasible paths represented in Fig. 6(b) by light gray stripes, which can be regarded as classifying this part as an obstacle over which the robot can step. The rest of the barrier together with the set of small obstacles T can be regarded as one large pseudo obstacle which must be circumvented by planning a path around it.

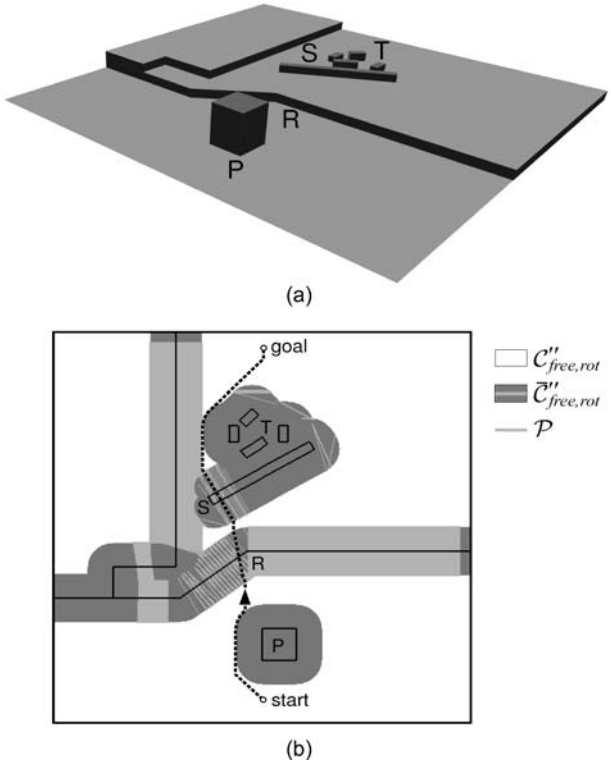

Fig. 6 a) Synthetic 3D scenario with different types of obstacles. b) Result of free space detection and path planning

B. Example 2

Another walking scenario used for evaluation of the discussed approach is presented in Fig. 7(a). The approach proposed herein determines the free space by searching for paths perpendicular to the obstacles. Hence, the method is primarily designed for scenarios with obstacles whose projections onto the *xy*-plane represent regions bounded by two parallel lines, such as those appearing in the previous example. In order to demonstrate that the proposed method can be applied not only to scenarios with rectangular obstacles, it is tested on the scenario

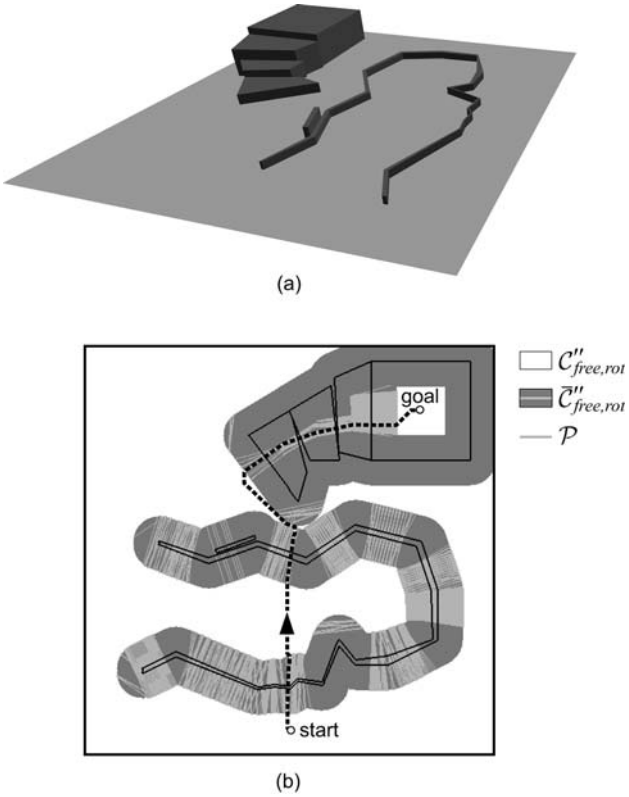

Fig. 7 a) Synthetic 3D scenario with non-cuboid obstacles. b) Result of free space detection and path planning

shown in Fig. 7 containing obstacles of more irregular shape. One of the paths obtained by the considered motion planning approach is shown in Fig. 7. The computation time needed for creating the graph from the 3D model of the scene and path planning was 0.35 seconds on a standard PC.

6 CONCLUSION

In this work, a novel method for creating a free space representation for biped walking robots given a 2.5D environment model is presented. Assuming that a biped robot is capable of executing a certain set of actions for walking on a flat surface and striding over obstacles, the high dimensionality of its configuration space can be reduced to three dimensions, which simplifies the motion planning problem. A further simplification can be achieved by considering only a certain subset of all feasible paths. The result of these two simplifications is a significant reduction of the computation time, but with a certain impact on walking capabilities. The proposed free space detection approach provides a subset of the total free space. Consequently, in certain situations, this method could fail to find a solution to a given motion planning problem, although a feasible path which solves the problem may exist in the total free space. Nevertheless, the proposed method can still be used to solve a wide range of biped locomotion tasks.

The proposed free space representation consists of two sets, a subset of the configuration space in which 2D motion planning can be performed and a set of feasible paths in the complement of this subset. A graph representing a discretized approximation of these two sets can be computed from a 2.5D map of a walking scenario given in the form of a 2D rectangular grid, where each cell of the grid is assigned a particular height. Motion planning can be performed using the obtained graph by searching for a sequence of connected nodes starting with the node corresponding to an initial position of the robot and terminating with the node corresponding to a given goal position. Assuming that the discretization effects can be neglected, this sequence of nodes corresponds to a set of feasible step sequences each representing a solution to the given locomotion task.

The applicability of the proposed approach has been evaluated using a set of synthetic 3D models of different walking scenarios. The 2.5D map obtained from each 3D model was used to compute the proposed free space representation in the form of a graph. The obtained graph was then used for motion planning. The motion planning was performed by an algorithm based on a numerical navigation function. For all considered scenarios, the computation time needed for free space detection and path planning was less than 0.4 seconds on a standard PC, which indicates that the proposed strategy can be applied on-line for navigation of a walking robot.

The path obtained using the proposed free space representation can be used for planning a step sequence which allows the robot to walk along the path. This is one possible application of the proposed approach. Another possible application is for estimation of the remaining cost to the goal in a step sequence planning algorithm based on A*-search such as the one proposed in [2, 4].

The heuristic based on a standard mobile robot planner used in [4] for estimation of the remaining cost to the goal takes into account more information about the environment than a Euclidean distance metric. It is demonstrated in [4] that it can significantly speed up the planning process, but only insofar as the optimal path for the considered walking robot can be followed by a wheeled robot. Because of the abilities of a walking robot to step

over small obstacles, the cost of the path which can take the robot to the goal position can in some cases be much smaller than the cost estimated by planning a path for a wheeled robot. Let us, for example, consider the case where a thin and very long obstacle is located between the current and the goal state of the robot. Although the walking robot can reach the goal with one or two steps by stepping over the obstacle, a standard mobile robot planner would plan a path all around the obstacle. The length of this rather long path would be taken as the estimated remaining cost to the goal, thus resulting in an overestimate. Furthermore, in a case where the only way a given goal state can be reached from a current state of the robot is to climb stairs, there is no path for a wheeled robot which can be used for estimating the remaining cost to the goal. In general, the more an environment requires the biped's capabilities to step over or onto obstacles, the less informed the proposed heuristic will be. A planner which would use the free space representation proposed in this paper is expected to provide much better estimation of the remaining cost to the goal since it takes into account the capabilities of walking robot to step over or upon obstacles.

Nevertheless, the applicability of the proposed method is restricted to a class of robots which are capable of performing a set of actions with certain properties. Although the assumptions about the locomotion abilities of the robot used as the basis for this work are not too restrictive, their validity should be evaluated by the experiments with a real biped robot.

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REFERENCES

[1] J. J. Kuffner, K. Nishiwaki, S. Kagami, M. Inaba and H. Inoue, **Footstep Planning Among Obstacles for Biped Robots**, in Proc. (IEEE/RSJ) International Conference on Intelligent Robots and Systems (IROS), Maui, Hawaii, 2001, pp. 500–505.

[2] J. Chestnutt, J. Kuffner, K. Nishiwaki and S. Kagami, **Planning Biped Navigation Strategies in Complex Environments**, in Proc. (IEEE/RAS) International Conference on Humanoid Robots (Humanoids), Karlsruhe/München, Germany, October 2003.

[3] Y. Ayaz, K. Munawar, M. B. Malik and M. U. A. Konno, **Human-Like Approach to Footstep Planning Among Obstacles for Humanoid Robots**, in Proc. (IEEE/RSJ) International Conference on Intelligent Robots and Systems (IROS), Beijing, China, 2006, pp. 5490–5495.

[4] J. Chestnutt and J. Kuffner, **A Tiered Planning Strategy for Biped Navigation**, in Proc. (IEEE/RAS) International Conference on Humanoid Robots (Humanoids), Los Angeles, CA, USA, November 2004.

[5] R. Cupec, J. Denk and G. Schmidt, **Practical Experience with Vision-Based Biped Walking**, in Proc. International Symposium on Experimental Robotics (ISER), Sant'Angelo d'Ischia, Italy, July 2002.

[6] R. Cupec, O. Lorch and G. Schmidt, **Vision-Guided Humanoid Walking – Concepts and Experiments**, in Proc. International Workshop on Robotics in Alpe-Adria-Danube Region (RAAD), Cassino, Italy, May 2003.

[7] T. Y. Li, P. F. Chen and P. Z. Huang, **Motion Planning for Humanoid Walking in a Layered Environment**, in Proc. (IEEE) International Conference on Robotics and Automation (ICRA), Taipei, Taiwan, 2003, pp. 3421–3427.

[8] T. Y. Li and P. Z. Huang, **Planning Humanoid Motions with Striding Ability in a Virtual Environment**, in Proc. (IEEE) International Conference on Robotics and Automation (ICRA), New Orleans, Lousiana, 2004, pp. 3195–3200.

[9] J. S. Gutmann, M. Fukuchi and M. Fujita, **Real-Time Path Planning for Humanoid Robot Navigation**, in Proceedings of the International Joint Conference on Artificial Intelligence, Edinburgh, Scotland, UK, 2005.

[10] R. Cupec and G. Schmidt, **An Approach to Environment Modelling for Biped Walking Robots**, in Proc. (IEEE/RSJ) International Conference on Intelligent Robots and Systems (IROS), Edmonton, Canada, 2005, pp. 3089–3094.

[11] R. Cupec, **Scene Reconstruction and Free Space Representation for Biped Walking Robots**, Doctoral dissertation, TU München, Fortschritt-Berichte VDI, Series 8, No. 1066. Düsseldorf: VDI Verlag, 2005.

[12] J. C. Latombe, **Robot Motion Planning**. Norwell, Massachusetts, USA: Kluwer Academic Publishers, 1991.

[13] J. Denk and G. Schmidt, **Synthesis of Walking Primitive Databases for Biped Robots in 3D--Environments**, in Proc. (IEEE) International Conference on Robotics and Automation (ICRA), Taipei, Taiwan, 2003, pp. 1343–1349.

[14] M. Gienger, K. Löffler and F. Pfeiffer, **Towards the Design of a Biped Jogging Robot**, in Proc. (IEEE) International Conference on Robotics and Automation (ICRA), Seoul, Korea, 2001, pp. 4140–4145.

[15] J. Denk and G. Schmidt, **Walking Primitive Databases for Perception-Based Guidance Control of Biped Robots**, European Journal of Control, vol. 13, pp. 171–181, 2007.

Prikaz slobodnog prostora za dvonožne hodajuće robote. Planiranje kretanja dvonožnih hodajućih robota predstavlja iznimno zahtjevan zadatak zbog složenosti kinematike takvih strojeva i velikog broja stupnjeva slobode gibanja. Jedan pristup tom problemu je da se prvo pronađe izvediva staza u reduciranom konfiguracijskom prostoru robota te da se zatim traži odgovarajući niz koraka koji omogućuje kretanje tom stazom. U ovom radu predstavljena je nova metoda stvaranja prikaza slobodnog prostora za dvonožne hodajuće robote. Metoda se temelji na aproksimaciji robota skupom jednostavnih trodimenzionalnih geometrijskih tijela čiji oblici omogućuju učinkovito određivanje izvedivih staza u 3D konfiguracijskom prostoru, koje mogu uključivati prekoračivanje prepreka te prelazak između hodnih površina različitih visina. Okolina robota dijeli se na dva područja. U prvom području može se primijeniti 2D planiranje koraka, dok se složenost 3D planiranja koraka u drugom području može značajno smanjiti tako što se pri planiranju uzima u obzir samo jedan reducirani skup staza, koji je pak dostatan za rješavanje velikog broja praktičnih zadataka.

Ključne riječi: hodajući roboti, planiranje gibanja, prikaz slobodnog prostora

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