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MODEL PROIZVODNE JEDINICE S UKLJUČENIM UVJETIMA OKOLINE U MODELU POUZDANOSTI I RASPOLOŽIVOSTI ELEKTROENEGETSKOG SUSTAVA

POWER PLANT MODEL WITH ENVIRONMENTAL CONDITIONS INCLUDED IN THE ELECTRIC POWER SYSTEM RELIABILITY AND AVAILABILITY MODEL

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U radu se izlažu rezultati teorijsko-metodološke razrade modela kojim se proiz
jedinice, na čiji rad, odnosno pogonske osobine, a time i raspoloživost i pouzdanos
stanje okoline, najčešće nepovoljne atmosferske prilike, uključuju u modele za
pouzdanosti i raspoloživosti elektroenergetskog sustava pri operativnim plan
njegova rada. Model proizvodne jedinice s uključenim uvjetima okoline pokraj ov
vremenu u izračun pokazatelja pouzdanosti i raspoloživosti elektroenergetskog
uvodi i ovisnost o uvjetima okoline u kojima se jedinica nalazi tijekom vremena izl
tim uv

This work presents the results of the theoretical-methodological elaboration
model by virtue of which the generating units are included in the models for calcu
of reliability and availability of the electric power system in the operative plans o
operation. The operation, that is, the operative features of these generating uni
and therewith also their reliability and availability, are influenced by the state of the environment, mostly by adverse weather conditions. The generating units model with environmental conditions included also introduces into the calculation of indicators of the electric power system reliability and availability, along with the time dependence, the dependence on the environmental conditions which the unit is exposed to during duration of the period of such conditions.

Ključne riječi: dvoparametarski model proizvodne jedinice; model pouzdanosti i raspoloživosti sustava; uvjeti stanja okoline; utjecaj okoline na stanje proizvodne jedinice

Key words: double-parameter generating unit model; system reliability and availability model; environmental state conditions; environmental impact on the state of the generating units

1 UVOD

Opći problem prikupljanja podataka, statističke obrade i izračuna parametara i pokazatelja za uspostavu modela proizvodnih jedinica pri operativnim planiranjima pogona, te načina pristupa i rješenja problema uključivanja proizvodnih jedinica u model pouzdanosti i raspoloživosti elektroenergetskog sustava, do sada su detaljnije obrađeni i izloženi u više radova [1] do [14]. Ovaj rad uvodi uvjete, odnosno stanje okoline kao parametra koji kada prijeđe određenu razinu nepovoljnog utjecaja, ili zbog određenih tehničko-tehnoloških rješenja proizvodnih jedinica, utječe na stanje raspoloživosti i pouzdanosti proizvodne jedinice, a time i raspoloživosti i pouzdanosti elektroenergetskog sustava u cjelini. Uvjeti okoline mogu imati utjecaja na samo jedno proizvodno postrojenje, ali i više njih ako se radi o nepovoljnim atmosferskim prilikama koje zahvaćaju ili djeluju na velikom području. Ipak, pretpostavlja se da je iznimno niska vjerojatnost da nepovoljne atmosferske prilike istodobno zahvate i nepovoljno utječu na veliki broj ili pak sve proizvodne objekte elektroenergetskog sustava.

Pod uvjetima okoline kod elektroenergetskih postrojenja najčešće se podrazumijevaju atmosferske prilike koje ih okružuju. Velik dio opreme, odnosno postrojenja elektroenergetskog sustava, zaštićen je, tj. izoliran od nepovoljnih utjecaja okoline, pogotovo atmosferskih prilika, pa za njih s velikom točnošću vrijedi pretpostavka o konstantnosti učestalosti prijelaza iz stanja u stanje, bez obzira na stanje okoline. Pogotovo to vrijedi za proizvodna i ostala postrojenja smještena u zgradama, podzemnim prostorijama i sl. Međutim, kako su postrojenja pokraj svih zaštita, a osobito kada tih zaštita nema, za vrijeme olujnog vremena, grmljavine, padalina s jakim vjetrom, i sl., vrlo često izložena nepovoljnim utjecajima, pretpostavka o konstantnoj učestalosti kvara nije više održiva kada ti nepovoljni utjecaji prijeđu određenu razinu.

2 MODEL JEDNE JEDINICE S UKLJUČENIM UVJETIMA OKOLINE

2.1 Jednostavni model proizvodne jedinice - model bazne jedinice

Model jedinice s dva stanja (slika 1) detaljno je obrađen i izložen u [2], [10] i [12]. To je ujedno i najčešće korišteni model budući da pokraj jednostavnosti i potrebnog malog broja podataka najbolje aproksimira neprekidan rad jedinice koja pokriva bazni dio dijagrama opterećenja.

1 INTRODUCTION

So far, the general problem of data collection, statistical analyses and calculations of parameters and indicators for the establishment of the generating unit models in operational planning, and the manner of approaching and solving problems of inclusion of generating units into the electric power system reliability and availability model have been analyzed in more detail and presented in several works [1] to [14]. This work introduces conditions, that is, state of the environment as the parameter which, when it passes the level of adverse impact, or, due to certain technical-technological solutions of generating units, impacts the state of availability and reliability of the generating unit, and therewith the availability and reliability of the electric power system as a whole. Environmental conditions can impact only one generating plant, but also more of them if adverse weather conditions are such that they encompass or affect a wide territory. Still, it is presumed that the probability that adverse atmospheric conditions will simultaneously encompass and negatively impact a large number, or possibly even all the production facilities of the electric power system, is quite low.

The environmental conditions in electric power plants mostly mean atmospheric conditions which surround it. A large portion of the equipment, that is, of the electric power system plant, is protected, that is, insulated from adverse environmental impacts, atmospheric conditions especially, so the assumption of the constancy of the rate of transition from state to state applies to that portion with great accuracy, regardless of the state of the environment. This applies especially to production and all other plants located in the buildings, underground rooms, etc. However, as the plants, with all the protections and even when there are no protections, during stormy weather, thunder, rainfall with heavy winds, etc., are very often exposed to adverse conditions, the assumption on the constancy of the failure rate is no longer sustainable when those impacts exceed a certain level.

2 THE MODEL OF A UNIT WITH ENVIRONMENTAL CONDITIONS INCLUDED

2.1 Simple generating unit model - basic unit model

The unit model with two states (Figure 1) is analyzed in detail and presented in [2], [10] and [12]. This is also the most frequently used model because with the simplicity and small amount of data necessary it approximates best the perpetual operation of the unit covering the basic part of the load diagram.

Sustav linearnih diferencijalnih jednadžbi kojima se opisuje Markovljev proces prema tom jednostavnom modelu u općem obliku glasi:

The system of linear differential equations which describes the Markov process according to that simple model in the general form reads:

$$\dot{P}_0(t) = -\lambda P_0(t) + \mu P_1(t), \quad \dot{P}_1(t) = \lambda P_0(t) - \mu P_0(t), \tag{1}$$

gdje je:

$\dot{P}_i(t)$ – $\left(\frac{\mathrm{d}\,P_i(t)}{\mathrm{d}\,t}\right)$ derivacija vjerojatnosti stanja i po vremenu t,

$P_i(t)$ – vjerojatnost stanja i $(i = 0, 1, 2, ..., n)$.

Uz pretpostavku da je u $t = 0$ jedinica u ispravnom stanju, početni uvjeti glase:

where it is as follows:

$\dot{P}_i(t)$ – $\left(\frac{\mathrm{d}\,P_i(t)}{\mathrm{d}\,t}\right)$ derivation of the probability of the state i per time t,

$P_i(t)$ – probability of the state i $(i = 0, 1, 2, ..., n)$

With the assumption that in $t = 0$ the unit is in sound condition, the initial conditions read:

$$P_0(0) = 1\ , \quad P_1(0) = 0. \tag{2}$$

U svakom trenutku mora biti ispunjen uvjet da se proizvodna jedinica mora nalaziti u jednom od stanja u prostoru stanja, tako da jednadžba identiteta glasi:

At all times the condition that the generating unit must be in one of the states in the state space must be fulfilled, so that the identity equation reads:

$$P_0(t) + P_1(t) = 1\ . \tag{3}$$

Gornji sustav linearnih diferencijalnih jednadžbi uz početne uvjete (2) ima sljedeće rješenje:

– raspoloživost $A(t)$:

The above system of linear differential equations with the initial conditions (2) has the following solution:

– availability $A(t)$:

$$A(t) = P_0(t) = \frac{\mu}{\lambda + \mu} + \frac{\lambda}{\lambda + \mu}\mathrm{e}^{-(\lambda+\mu)t}\ , \tag{4}$$

– neraspoloživost $N(t)$:

– unavailability $N(t)$:

$$N(t) = P_1(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda}{\lambda + \mu}\mathrm{e}^{-(\lambda+\mu)t}\ . \tag{5}$$

Prijelazni dio funkcije raspoloživosti vrlo brzo teži k nuli, a vremenski trenutak kada se može zanemariti prijelazni dio funkcije u odnosu na stacionarni

The transient part of the availability function gravitates towards zero very soon, and the moment in time when the transient part of the function can be dio ovisi o veličini učestalosti popravka μ. Općenito, može se uzeti da prijelazno stanje završava nakon vremena $n = t(\lambda + \mu) = 4$, ($t = \frac{4}{\lambda + \mu}$).

Prijelazno stanje završava i prije ukoliko je $\mu > \lambda$, što je kod proizvodnih jedinica elektroenergetskog sustava redovito ispunjeno. Zbog toga se u svim praktičnim proračunima koriste izrazi za stacionarne vrijednosti raspoloživosti i neraspoloživosti, jer se uz zanemarivu pogrešku postiže znatno pojednostavljenje proračuna. Svi daljnji proračuni u ovom radu bit će vezani i odnosit će se na stacionarne vrijednosti raspoloživosti, odnosno neraspoloživosti općeg oblika:

disregarded in relation to the steady part depends on the repair rate μ. In general, it can be assumed that the transient state ends after the time $n = t(\lambda + \mu) = 4$, ($t = \frac{4}{\lambda + \mu}$).

The transient state ends even before in case of $\mu > \lambda$, which is regularly the case with electric power generating units. Because of that, in all practical calculations, expressions for steady values of availability and unavailability are used because, with negligible error level, a significant simplification of the calculation is obtained. All further calculations in this work will be connected and will relate to the steady values of the availability, that is, unavailability in the general form:

$$P_0 = \lim_{t \to \infty} A(t) = \frac{\mu}{\lambda + \mu} = A(\infty) \quad , \tag{6}$$

$$P_1 = \lim_{t \to \infty} N(t) = \frac{\lambda}{\lambda + \mu} = N(\infty) \; . \tag{7}$$

Pritom u svakom trenutku mora biti ispunjen osnovni uvjet iz jednadžbe identiteta:

Thereat, the basic condition from the identity equation must be fulfilled at all times.

$$A(t) + N(t) = 1 \; . \tag{8}$$

Izraz za raspoloživost jedinice može se napisati i u obliku:

The expression for unit availability can also be written in the form:

$$A(\infty) = \frac{1}{1 + \frac{\lambda}{\mu}} = 1 - \frac{\lambda}{\mu} + \ldots \approx 1 - \frac{\lambda}{\mu} \quad , \tag{9}$$

što kod visokoraspoloživih jedinica predstavlja vrlo dobru aproksimaciju raspoloživosti.

Parametri modela bazne jedinice u stacionarnom stanju mogu se izračunati iz podataka o pogonu jedne ili više jedinica tijekom dužeg vremenskog perioda. Tako, uvažavajući podatke o stanjima i pokazateljima pogonskih stanja proizvodne jedinice ([10] i [12]), procijenjene ili izračunate vrijednosti ulaznih parametara za model bazne jedinice (slika 1) jesu:

and this, in highly-available units, represents very good availability approximation.

Parameters of the steady state basic unit model can be calculated based on the data on the drive of one or more units through a longer period of time. Thus, taking into consideration the data on the states and indicators of the driving states of generating units ([10] i [12]), the estimated or calculated values of input parameters for the basic unit model (Figure 1) are:

$$\hat{\mu} = \frac{N}{SK} \quad , \tag{10}$$

$$\hat{\lambda} = \frac{N}{SP} \quad , \tag{11}$$

$$\hat{P}_0 = \frac{\frac{N}{SK}}{\frac{N}{SK} + \frac{N}{SP}} = \frac{SP}{SK + SP} \quad , \tag{12}$$

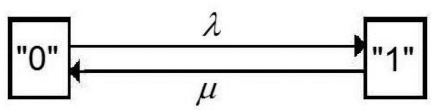
$$\hat{P}_1 = \frac{\frac{N}{SP}}{\frac{N}{SK} + \frac{N}{SP}} = \frac{SK}{SK + SP} \quad , \tag{13}$$

gdje je:

- $\hat{\mu}$ – procijenjena ili izračunata vrijednost učestalosti kvara jedinice,
- $\hat{\lambda}$ – procijenjena ili izračunata vrijednost učestalosti popravka jedinice,
- $\hat{P}_i$ – procijenjena vrijednost vjerojatnosti stanja komponente ili sustava i $(i = 0, 1, 2, ..., n)$,
- N – broj događaja,
- SK – broj sati ispada iz pogona,
- SP – broj sati pogona.

where it is as follows:

- $\hat{\mu}$ – estimated or calculated value of unit failure rate,
- $\hat{\lambda}$ – estimated or calculated value of unit repair rate,
- $\hat{P}_i$ – estimated value of the probability of the state of the component or the system i $(i = 0, 1, 2, ..., n)$,
- N – number of occurrences,
- SK – number of hours of outage,
- SP – number of driving hours.



Slika 1 — Shema jednostavnog modela proizvodne jedinice - model bazne jedinice
Figure 1 — Scheme of the simple production unit model - basic unit model

Oznake:

- "0" – stanje spremnosti jedinice za pogon,
- "1" – stanje nespremnosti jedinice za pogon,
- λ – učestalost kvara jedinice,
- μ – učestalost popravka jedinice.

Očito je da raspoloživost, odnosno neraspoloživost jedinice prema izrazima (12) i (13) ne ovise o broju ispada iz pogona, tako da se njihove vrijednosti mogu procijeniti direktno iz vremena ostajanja ili zadržavanja u pojedinim stanjima tijekom promatranog vremenskog intervala. U modelu dva stanja vrijeme održavanja uvijek se isključuje iz određivanja rizika ispada iz pogona.

Izraz (13) najčešće se naziva rizikom ispada iz pogona (engl. *Forced Outage Rate* – FOR) i predstavlja procjenu vjerojatnosti ispada iz pogona u bilo kojem slučajno odabranom trenutku. Kvaliteta te procjene najviše ovisi o tipu pogona, tako da statistika pogonskih podataka mora uključivati i vrstu pogona. To dalje znači da bi primjena tog jednostavnog modela

Symbols:

- "0" – unit operation condition,
- "1" – unit non-operation condition,
- λ – rate of unit failure,
- μ – rate of unit repair.

It is obvious that availability, that is, unavailability of the unit according to the expressions (12) and (13) do not depend on the number of outages, so that their values can be estimated directly based on the time of abiding or staying in certain states during the observed time interval. In the two-state model, the maintenance time is always excluded from the determination of the outage risk.

Expression (13) is most usually called Forced Outage Rate and it represents the estimate of the probability of outage at any randomly chosen moment. The quality of such estimate mostly depends on the type of drive, so that the statistics of drive data must include the type of drive as kod jedinica koje rade u vršnom dijelu dijagrama opterećenja, jedinica koje se češće uključuju i isključuju iz pogona (ciklički rad), jedinica u rezervi ili pripravnosti za pogon, tj. u svim uvjetima koji ne znače kontinuiran pogon pod opterećenjem, dovela do neadekvatne procjene rizika ispada iz pogona.

Međutim, vrlo često neraspoloživost odgovarajućih podataka o pogonu proizvodnih postrojenja potrebnih da bi se izračunalo parametre složenijih modela uvjetuje primjenu upravo tog jednostavnog modela dva stanja, bez obzira što se pogon znatno razlikuje od konstantnog. Jedan od načina da se smanje problemi i netočnosti primjene modela dva stanja koji pritom nastaju jest da se iz promatranja isključe trajanja pripravnosti jedinice za pogon ili stanja rezerve. U tom slučaju sati ispada iz pogona reduciraju se samo na sate ispada iz pogona tijekom vremena potrebe za pogonom. Parametri prema izrazima (10) i (11) računaju se bez sati rezervnog isključenja. Izraz za rizik ispada iz pogona dobiva novi oblik:

well. Furthermore, this means that the application of that simple model in units working in the peak part of the load diagram, units which go on and off from operation more often (cyclic operation), units in standby or state of operation condition, that is, in all the conditions which do not mean a continued operation under load, would bring about an inadequate assessment of outage risk.

However, very often the unavailability of adequate data on the drives of the generating plants necessary for the calculation of the parameters of more complex models conditions the application of exactly that simple two-state model, regardless of the fact that the drive is significantly different from the constant one. One of the ways to decrease the extent of the problems and the inaccuracies in the application of the two-state model which occur thereat is to exclude the durations of the unit's readiness or standby for operation from the observation. In that case, the number of hours of outage is reduced only to the hours of outage during the time when operation is required. Parameters according to the expressions (10) and (11) are calculated without hours of standby shutdown. The expression for forced outage rate assumes a new form:

$$\hat{P}_{1,\mathrm{mod}} = \frac{SK(d/24)}{SP + SK(d/24)} \quad , \tag{14}$$

gdje je $\hat{P}_{1,\mathrm{mod}}$ procijenjena vrijednost rizika ispada iz pogona jedinice samo tijekom sati trajanja potrebe za pogonom.

Prethodni izraz podrazumijeva 24-satni ciklus potrebe za pogonom, tj. d sati potrebe za pogonom svaka 24 sata. Općenito, za bilo koji ciklus vrijedit će izraz:

whereat $\hat{P}_{1,\mathrm{mod}}$ is the estimated value of the unit's forced outage rate only during the hours of necessity for operation.

The above expression implies a 24-hour cycle of necessity for operation, that is d hours of necessity for operation every 24 hours. Generally, for any cycle, the following expression will apply:

$$\hat{P}_{1,\mathrm{mod}} = \frac{k \cdot (SK)}{SP + k \cdot (SK)} \quad , \tag{15}$$

gdje je:

$k = d/(d+v)$ – faktor korekcije,
d – sati potrebe za pogonom tijekom ciklusa koji traje $d+v$ sati,
v – vrijeme koje jedinica provede u rezervi između perioda potrebe za pogonom.

Faktor korekcije k uvodi se kako bi se pri proračunu parametara uvažila pretpostavka da ispad jedinice iz pogona, tj. njezin kvar može započeti samo tijekom vremena opterećenja jedinice.

where it is as follows:

$k = d/(d+v)$ – correction factor,
d – hours of necessity for operation during the cycle which lasts $d+v$ hours,
v – time unit spent standby between the periods of necessity for operation.

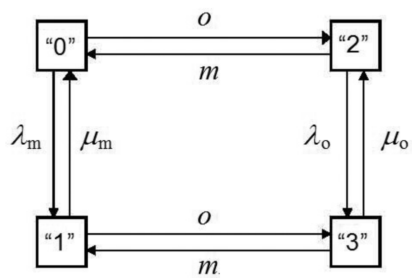
The correction factor k is introduced so as to take into consideration, when estimating the parameters, the assumption that the outage of the unit, that is, its failure may only begin during the time of unit loading.

2.2 Uključivanje utjecaja uvjeta okoline u jednostavni model proizvodne jedinice

Model prikazan na slici 2 dvoparametarski je model proizvodne jedinice kojim se pokraj ovisnosti o vremenu u izračun pokazatelja raspoloživosti i pouzdanosti elektroenergetskog sustava uvodi i ovisnost o uvjetima okoline u kojima se jedinica nalazi tijekom vremena.

2.2 Inclusion of the impact of environmental conditions into the simple generating unit model

The generating unit model shown in Figure 2 is a two-parameter generating unit model by virtue of which, beside the dependency on time, the dependency on environmental conditions of the unit during time is also inserted into the calculation of electric power system availability and reliability indicators.



Slika 2 — Model proizvodne jedinice s uključenim uvjetima okoline
Figure 2 — Generating unit model with environmental conditions included

Oznake:

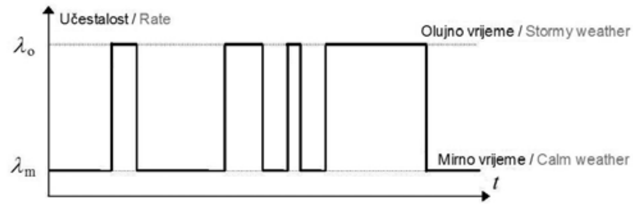
- "0" – stanje pogonske spremnosti jedinice u mirno vrijeme,
- "1" – stanje kvara jedinice u mirno vrijeme,
- "2" – stanje pogonske spremnosti jedinice u olujno vrijeme,
- "3" – stanje kvara jedinice u olujno vrijeme,
- λ_m – učestalost kvara jedinice tijekom mirnog vremena,
- μ_m – učestalost popravka jedinice tijekom mirnog vremena,
- λ_0 – učestalost kvara jedinice tijekom olujnog vremena,
- μ_0 – učestalost popravka jedinice tijekom olujnog vremena,
- m – učestalost prijelaza olujnog vremena u mirno vrijeme,
- 0 – učestalost prijelaza mirnog vremena u olujno vrijeme.

Za jedinice kod kojih učestalosti kvarova rastu s pogoršanjem prilika u okolini nužno je u model uključiti te nepovoljne utjecaje. Općenito, učestalost kvara kontinuirana je funkcija vremena koja se zbog jednostavnosti prikazuje određenim brojem diskretnih stanja. Najjednostavniji je slučaj da se uvedu dvije različite učestalosti kvara jedinice za dva striktno odijeljena stanja okoline, jedna za mirno ili normalno, a druga za olujno vrijeme, koje se tijekom vremena smjenjuju kako se smjenjuju trajanja odnosnih vremenskih uvjeta. Osnovu prikaza vremenskih stanja čini kronološki profil (slika 3). Uvođenjem većeg broja diskretnih stanja modeli postaju praktički nerješivi.

Symbols:

- "0" – unit's operation condition during calm weather,
- "1" – state of unit failure during calm weather,
- "2" – unit's operation condition during stormy weather,
- "3" – state of unit failure during stormy weather,
- λ_m – rate of unit failure during calm weather,
- μ_m – rate of unit repair during calm weather,
- λ_0 – rate of unit failure during stormy weather,
- μ_0 – rate of unit repair during stormy weather,
- m – rate of transition of stormy weather to calm weather,
- 0 – rate of transition of calm weather to stormy weather.

For the units in which failure rate grows with the aggravation of environmental conditions, it is necessary to include those adverse conditions into the model. Generally, the rate of failure is a continued time function which, for the purpose of simplicity, is presented with a certain number of discrete states. The simplest case is to introduce two different rates of unit failure for two strictly separated environmental states, one for calm or normal, the other for stormy weather, and these alternate through time as the durations of the relevant weather conditions alternate. The chronological profile constitutes the basis for the presentation of weather conditions (Figure 3). By introducing a larger number of discrete states, the models become practically unsolvable.



Slika 3 — Dvoparametarski model pogona i kvara proizvodne jedinice
Figure 3 — Two-parameter model of operation and failure of generating unit

Veliki problem predstavlja nemogućnost da se prepoznaju i evidentiraju odjelita stanja vremenskih uvjeta, pogotovo kada ih je više. Naime, trajanja vremenskih uvjeta mogu se promatrati kao slučajni procesi koji imaju svoje očekivane vrijednosti, učestalosti pojave i frekvencije. Osnova za razvrstavanje kvarova jesu prevladavajući vremenski uvjeti tijekom njihova trajanja.

Sljedeća pretpostavka kojom se omogućuje primjena Markovljevog modela u određivanju vjerojatnosti stanja u tom posebnom slučaju jest da su razdoblja trajanja normalnih i olujnih vremenskih prilika slučajne varijable s eksponencijalnom ili približno eksponencijalnom razdiobom, a to dalje znači konstantnom učestalosti pojave mirnog i olujnog vremena. Učestalost prijelaza olujnog u mirno vrijeme, odnosno mirnog u olujno vrijeme može se odrediti iz odgovarajućeg broja prijelaza i trajanja zadržavanja u pojedinom stanju. Dakle:

The impossibility to recognize and record separate states of weather conditions, especially when there are more of them, is a great problem. Namely, the durations of weather conditions can be viewed as random processes which have their expected values, rates of occurrence and frequencies. The bases for failure classification are dominant weather conditions during their abidance.

The next assumption which enables the application of the Markov model in the determination of the probability of the state in that special case is that periods of duration of normal and stormy weather conditions are random variables with exponential or nearly exponential division, and that further means, with a constant rate of occurrence of calm and stormy weather. The rate of transition from stormy to calm weather, that is, calm to stormy weather, can be determined from an adequate number of transitions and durations of abiding in a certain state. Therefore:

$$m = \frac{N_m}{O}; \quad o = \frac{N_o}{M}, \tag{16}$$

gdje je:

N_m – broj prijelaza olujnog u mirno vrijeme,
N_o – broj prijelaza mirnog u olujno vrijeme (uz uvjet $N_m = N_o$),
O – ukupno trajanje olujnog vremena,
M – ukupno trajanje mirnog vremena.

Sustav linearnih diferencijalnih jednadžbi Markovljeva modela jedinice kad su uključeni i uvjeti okoline ima oblik:

where it is as follows:

N_m – number of transitions of stormy to calm weather,
N_o – number of transitions of calm to stormy weather (with the condition $N_m = N_o$),
O – total duration of stormy weather,
M – total duration of calm weather.

The system of linear differential equations of the Markov unit model when environmental conditions are included has the form:

$$\begin{aligned}
\dot{P}_0(t) &= -(\lambda_m + o)P_0(t) + \mu_m P_1(t) + mP_2(t),\\
\dot{P}_1(t) &= \lambda_m P_0(t) - (\mu_m + o)P_1(t) + mP_3(t),\\
\dot{P}_2(t) &= oP_0(t) - (\lambda_o + m)P_2(t) + \mu_o P_3(t),\\
\dot{P}_3(t) &= oP_1(t) + \lambda_o P_2(t) - (\mu_o + m)P_3(t).
\end{aligned} \tag{17}$$

Početne uvjete određuje pretpostavka da je u trenutku $t=0$ jedinica u stanju pogonske spremnosti i da je okružuju uvjeti mirnog vremena:

Initial conditions are determined by the assumption that at the moment of $t=0$ the unit is in the state of operation condition and it is surrounded by calm weather conditions.

$$\begin{aligned} P_0(0) &= 1, \\ P_1(0) &= 0, \\ P_2(0) &= 0, \\ P_3(0) &= 0. \end{aligned} \tag{18}$$

Stacionarno rješenje određuje se uz uvjet:

Steady solution is determined under the condition:

$$\begin{aligned} &\dot{P}_n(t) = 0 \\ &n = 0,\ 1,\ 2,\ 3. \end{aligned} \tag{19}$$

Budući da se traži stacionarno rješenje, sustav jednadžbi (17) uz (19) poprima sljedeći oblik:

Since a steady solution is searched for, the equation system (17) and (19) assumes the following form:

$$\begin{aligned} 0 &= -(\lambda_m + o)P_0(t) + \mu_m P_1(t) + mP_2(t), \\ 0 &= \lambda_m P_0(t) - (\mu_m + o)P_1(t) + mP_3(t), \\ 0 &= oP_0(t) - (\lambda_o + m)P_2(t) + \mu_o P_3(t), \\ 0 &= oP_1(t) + \lambda_o P_2(t) - (\mu_o + m)P_3(t). \end{aligned} \tag{20}$$

Da bi se sustav riješio, kao dodatna jednadžba u sustav se uključuje jednadžba identiteta:

In order to solve the system, the identity equation is introduced into the system as an additional equation:

$$P_0 + P_1 + P_2 + P_3 = 1\ . \tag{21}$$

Stacionarno rješenje sustava (20) i (21) jest:

Steady system solution (20) and (21) is:

$$\begin{aligned} P_0 &= \frac{m\mu_m(\mu_o + m) + m(\lambda_o\mu_m + \mu_o o)}{\Delta}, \\ P_1 &= \frac{m\lambda_m(\mu_o + m) + m\lambda_o(\lambda_m + o)}{\Delta}, \\ P_2 &= \frac{o\mu_m(\mu_o + m) + o\mu_o(\lambda_m + o)}{\Delta}, \\ P_3 &= \frac{o\lambda_o(\lambda_m + o) + o(m\lambda_m + \lambda_o\mu_m)}{\Delta}, \\ \Delta &= (m+o)\big[(\lambda_o + \mu_o)(\lambda_m + \mu_m) + m(\lambda_m + \mu_m) + o(\lambda_o + \mu_o)\big]\ . \end{aligned} \tag{22}$$

Stacionarne vrijednosti raspoloživosti i neraspoloživosti jedinice određene su sumom vjerojatnosti stanja "0" i "2", odnosno "1" i "3":

Steady values of unit availability and unavailability are determined by the sum of state probability "0" and "2", that is, "1" and "3".

$$A = P_0 + P_2 = \frac{(m+o)(m\mu_m + o\mu_o + \mu_o\mu_m) + m\mu_m\lambda_o + o\mu_o\lambda_m}{\Delta},$$
$$N = P_1 + P_3 = \frac{(m+o)(m\lambda_m + o\lambda_o + \lambda_m\lambda_o) + m\lambda_m\mu_o + o\lambda_o\mu_o}{\Delta}, \qquad (23)$$

gdje je:

Δ – kao u izrazu (22).

Kad bi se zanemarili uvjeti olujnog vremena, što znači da se vjerojatnost izlaska iz mirnog vremena uzima jednaka nuli ($o\Delta t = 0$), model bi se sveo samo na dva stanja kao na slici 1, dakle uz vrijednosti parametara $m = 1$, $o = 0$ i $\lambda_o = 0$, izrazi za vjerojatnosti stanja, te raspoloživost i neraspoloživost bili bi približno jednaki izrazima (6) i (7). Dakle:

where it is as follows:

Δ – as in the expression (22).

If stormy weather conditions were to be disregarded, which means that the probability of coming out of calm weather is taken as equal to zero ($o\Delta t = 0$), the model would be reduced to only two states as in Figure 1, so, with the values of the parameters $m = 1$, $o = 0$ and $\lambda_o = 0$, the expressions for the state probability of the state, and the availability and unavailability would be equal to the expressions (6) and (7). Therefore:

$$P_0 \approx \frac{\mu_m}{\mu_m + \lambda_m} \approx A,$$
$$P_1 \approx \frac{\lambda_m}{\mu_m + \lambda_m} \approx N,$$
$$P_2 \approx 0,$$
$$P_3 \approx 0 \,. \qquad (24)$$

Budući da se pretpostavlja eksponencijalna razdioba vremena trajanja popravaka, kako za mirno tako i za olujno vrijeme, te da se srednja očekivana vremena trajanja popravaka malo razlikuju, može se s dovoljnom točnošću uzeti da je funkcija učestalosti popravka neovisna o uvjetima okoline ($\mu_o = \mu_m = \mu$), što predstavlja znatno pojednostavljenje modela. Popravci jedinice zapravo se poduzimaju samo tijekom mirnog vremena, odnosno započinju završetkom olujnog vremena koje traje znatno kraće od mirnog vremena.

Za analizu ostaje, dakle, najvažniji parametar modela, a to je učestalost kvara jedinice. Da bi se odredila učestalost kvara jedinice koja vrijedi i za mirno i za olujno vrijeme, dakle prosječna vrijednost učestalosti kvara potrebno je najprije odrediti srednje vrijeme do kvara jedinice. Dakle, potrebno je poći od matrice učestalosti prijelaza stanja u prostoru stanja koja ima sljedeći oblik:

Since the exponential division of time of repair duration is assumed, both for calm and stormy weather, and since the average times of expected repair duration differ only slightly, it can be taken with sufficient accuracy that the function of repair rate is independent of the environmental conditions ($\mu_o = \mu_m = \mu$), which means significant model simplification. Unit repairs are actually undertaken only during calm weather, that is, they start after the end of stormy weather which lasts significantly less than calm weather.

The analysis thus remains to be performed only on the most significant model parameter, and that is the unit failure rate. In order to determine the unit failure rate which applies both for calm and stormy weather, that is, the average failure rate value, it is necessary to determine the average time to unit failure first. Therefore, it is necessary to start from the matrix of the state transition rate in the state space which has the following form:

$$\boldsymbol{A} = \begin{bmatrix} -(\lambda_m + o) & \lambda_m & o & 0 \\ \mu_m & -(\mu_m + o) & 0 & o \\ m & 0 & -(\lambda_o + m) & \lambda_o \\ 0 & m & \mu_o & -(\mu_o + m) \end{bmatrix} \qquad (25)$$

Budući da su stanja "1" i "3" stanja kvara, dakle i apsorbirajuća stanja, ispušta ih se iz matrice učestalosti prijelaza koja poprima novi oblik:

Since "1" and "3" are failure states, and thus also absorbing states, they are released from the transition rate matrix which assumes a new form:

$$\boldsymbol{A} = \begin{bmatrix} -(\lambda_{\mathrm{m}} + o) & o \\ m & -(\lambda_{\mathrm{o}} + m) \end{bmatrix}. \tag{26}$$

Sada je potrebno odrediti inverznu matricu matrice (26), a ona glasi:

Now it is necessary to determine the inverse matrix of the matrix (26) and it reads:

$$\boldsymbol{A}^{-1} = \begin{bmatrix} -(\lambda_{\mathrm{m}} + o) & o \\ m & -(\lambda_{\mathrm{o}} + m) \end{bmatrix}^{-1} = \frac{-1}{\lambda_{\mathrm{m}}\lambda_{\mathrm{o}} + o\lambda_{\mathrm{o}} + m\lambda_{\mathrm{m}}} \begin{bmatrix} (\lambda_{\mathrm{o}} + m) & o \\ m & (\lambda_{\mathrm{m}} + o) \end{bmatrix}. \tag{27}$$

Elementi matrice $\boldsymbol{A}^{-1}$ jesu vremena boravka u pojedinim stanjima prije ulaska u apsorbirajuća stanja. Budući da je ukupno vrijeme do kvara određeno izrazom:

Matrix elements $\boldsymbol{A}^{-1}$ are periods of abiding in certain states before entry into absorbing states. Since the total time to failure is determined by the expression:

$$T_{\Psi} = \sum_{i=1}^{n} T(i)\ , \tag{28}$$

a da izraz za određivanje srednjeg očekivanog vremena T do kvara ima oblik:

and since the expression for determination of the average expected time T until failure has the form:

$$T = -P_{\Psi}(0)\boldsymbol{A}_{\Psi}^{-1}\ , \tag{29}$$

gdje $\boldsymbol{A}_{\Psi}^{-1}$ predstavlja inverznu matricu učestalosti prijelaza između stanja,

srednje vrijeme do kvara, određeno kao suma vremena boravaka u svim stanjima osim apsorbirajućih, određuje se kao produkt vektora početnih vjerojatnost stanja (18) i inverzne matrice učestalosti prijelaza između stanja (27), te iznosi:

where $\boldsymbol{A}_{\Psi}^{-1}$ represents the inverse matrix of rate of transition between states,

average time to failure, defined as the sum of times of abiding in all states except for the absorbing states, is determined as the product of the vectors of initial probabilities of the states (18) and of the inverse matrix of rate of transition between states (27) and it amounts to:

$$T_0 = \frac{\lambda_{\mathrm{o}} + m + o}{\lambda_{\mathrm{m}}\lambda_{\mathrm{o}} + o\lambda_{\mathrm{o}} + m\lambda_{\mathrm{m}}}\ , \tag{30}$$

Promatra li se samo mirno vrijeme, srednje vrijeme do kvara jest:

If only calm weather is considered, average time to failure is:

$$T_0 = \frac{1}{\lambda_{\mathrm{m}}}\ . \tag{31}$$

Učestalost kvara jedinice ako se zanemari vrijeme trajanja popravaka, koje je znatno kraće od sred-

If the duration of repairs, which is significantly shorter than the average time to component failunjeg vremena do kvara komponente, što podrazumijeva učestalost kvara ne uključujući utjecaj popravaka, može se odrediti inverzijom izraza (31). Dakle:

re, is disregarded, which implies the rate of repair not including the influence of repairs, unit failure rate can be determined by inversion of the expression (31). Therefore:

$$\lambda = \frac{1}{T_0} = \frac{\lambda_m \lambda_o + o\lambda_o + m\lambda_m}{\lambda_o + m + o} , \tag{32}$$

Budući da općenito vrijedi:

Since the following generally applies:

$$\lambda_o \lambda_m << o\lambda_o + m\lambda_m$$

i / and (33)

$$\lambda_o << m + o ,$$

približna vrijednost prosječne učestalosti kvara jedinice bit će:

approximate average unit failure rate will be:

$$\lambda \approx \frac{o\lambda_o}{m+o} + \frac{m\lambda_m}{m+o} = \frac{O\lambda_o}{M+O} + \frac{M\lambda_m}{M+O} = \frac{O\lambda_o + M\lambda_m}{M+O} , \tag{34}$$

gdje je:

M – ukupno trajanje mirnog vremena,
O – ukupno trajanje olujnog vremena.

Tako izračunate učestalosti odgovarale bi vrijednostima koje bi se dobile iz podataka o pogonu kada se ne bi vodilo računa o uvjetima okoline.

Parametri modela računaju se iz sljedećih podataka pogonske statistike:

c – broj kvarova u razdoblju mirnog vremena,
c' – broj kvarova u razdoblju olujnog vremena,
p – ukupni broj popravaka za vrijeme trajanja mirnog vremena,
p' – ukupni broj popravaka za vrijeme trajanja olujnog vremena,
T_{pm} – ukupno trajanje popravaka tijekom mirnog vremena,
T_{po} – ukupno trajanje popravaka tijekom olujnog vremena.

Naravno, ukupan broj kvarova treba biti jednak ukupnom broju popravaka ($c + c' = p + p'$), trajanje ciklusa jednako je sumi ukupnih vremena trajanja mirnog i olujnog vremena ($M + O$), a ukupno trajanje svih popravaka jednako je sumi popravaka tijekom mirnog i olujnog vremena ($T_p = T_{pm} + T_{po}$). Procijenjene vrijednosti odgovarajućih učestalosti promjena stanja jesu:

where it is as follows:

M – total duration of calm weather,
O – total duration of stormy weather.

Rates calculated in such way would fit the values which might be obtained from the data on the drive if the environmental conditions were not taken into account.

Model parameters are calculated from the following drive statistics data:

c – number of failures in calm weather period,
c' – number of failures in stormy weather period,
p – total number of failures for the duration of calm weather,
p' – total number of failures for the duration of stormy weather,
T_{pm} – total duration of repairs for the duration of calm weather,
T_{po} – total duration of repairs for the duration of stormy weather,

Of course, total number of failures must be equal to the total number of repairs ($c + c' = p + p'$), duration of the cycle is equal to the sum of total durations of abidance of calm and stormy weather ($M + O$), and the total duration of all repairs equals the sum of repairs during calm and stormy weather ($T_p = T_{pm} + T_{po}$). Estimated values of relevant state transition rates are:

$$\hat{\lambda}_m = \frac{c}{M} \quad ,$$
$$\hat{\lambda}_o = \frac{c'}{O} \quad ,$$
$$\hat{\lambda} = \frac{c + c'}{M + O} = \frac{M\hat{\lambda}_m + O\hat{\lambda}_o}{M + O} \quad ,$$
$$\hat{\mu}_m = \frac{p}{T_{pm}} \quad , \tag{35}$$
$$\hat{\mu}_o = \frac{p'}{T_{po}} \quad ,$$
$$\hat{\mu} = \frac{p + p'}{T_p} = \frac{p + p'}{T_{pm} + T_{po}} = \frac{p + p'}{\frac{p}{\hat{\mu}_m} + \frac{p}{\hat{\mu}_o}} \approx \frac{M\hat{\lambda}_m + O\hat{\lambda}_o}{\frac{M\hat{\lambda}_m}{\hat{\mu}_m} + \frac{O\hat{\lambda}_o}{\hat{\mu}_o}} .$$

S druge strane, ako su poznati podaci o ukupnom broju kvarova ($c + c'$), trajanja razdoblja mirnog i olujnog vremena (M i O) i udio kvarova koji se javljaju tijekom olujnog vremena ($\zeta = c'/(c + c')$), tada se mogu izračunati učestalosti kvarova za mirno i olujno vrijeme:

On the other hand, if data on the total number of failures ($c + c'$), durations of the periods of calm and stormy weather (M and O) and the share of failures which occur during stormy weather ($\zeta = c'/(c + c')$) are known, then the rates of failures for calm and stormy weather can be calculated.

$$\hat{\lambda}_m = (1 - \zeta)\frac{M + O}{M}\hat{\lambda} \; ,$$
$$\hat{\lambda}_o = \zeta \frac{M + O}{O}\hat{\lambda} \; . \tag{36}$$

Tako izračunate učestalosti pojave iskazuju se kao broj kvarova u godini po vremenskom uvjetu u kojem su se dogodili, a ne kao broj kvarova u kalendarskoj godini.

Tablica 1 prikazuje primjer utvrđenih ulaznih parametara modela stanja proizvodne jedinice, s uključenim uvjetima okoline, za mirno i olujno vrijeme, odnosno samo mirno vrijeme. Tablica 2 prikazuje rezultat izračuna, tj. vjerojatnosti stanja u modelu stanja proizvodne jedinice za ulazne parametre iz tablice 1.

Thus calculated rates of occurrence are presented as number of failures in the year per weather condition in which they occurred, and not as the number of failures in the calendar year.

Table 1 shows the example of determined input parameters of the generating unit state model, with environmental conditions included, for calm, that is, for stormy weather only. Table 2 shows the calculation result, that is, the probabilities of the state in the generating unit state model for input parameters from Table 1.

3 MODEL DVIJE JEDINICE S UKLJUČENIM UVJETIMA OKOLINE

3.1 Model sustava s dvije jedinice

Model prikazan na slici 4 jednostavan je model prostora stanja dvije različite jedinice, od kojih svaka može biti ili u stanju spremnosti za pogon, dakle ispravna, ili u stanju nespremnosti za pogon, dakle u kvaru. Model je obrađen radi ilustracije i utvrđivanja načela izgradnje modela pouzdanosti najjednostavnijih sustava. Osnovna pretpostavka svih razmatranja jest da se isključuje mogućnost

3 TWO-UNIT MODEL WITH ENVIRONMENT CONDITIONS INCLUDED

3.1 Two-unit system model

The model shown in Figure 4 is a simple state space model of two different units, of which each can be either in operation condition, that is, in operative condition, or in non-operation condition, that is, in the state of failure. The model is analyzed for the purpose of illustration and determination of the principles for the construction of the reliability model of the simplest systems. The basic

Tablica 1 – Parametri modela stanja proizvodne jedinice, s uključenim uvjetima okoline
Table 1 - Parameters of the generating unit state model, with environmental conditions included

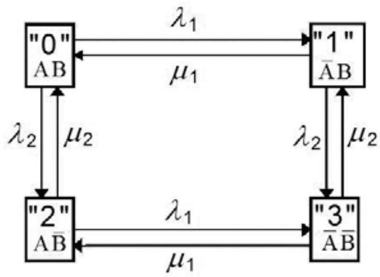
	Slučaj 1 / Case 1 – Mirno i olujno vrijeme / Calm and stormy weather	Slučaj 2 / Case 2 – Samo mirno vrijeme / Calm weather only
M [h]	8 493	8 493
N_m	26	26
m [1/h]	0,097 38	1
O [h]	267	267
N_O	26	26
o [1/h]	0,003 06	0
c	14	
c'	8	
p	20	
p'	2	
T_{pm}	247	
T_{po}	45	
λ_m (procjena) / (estimation)	0,001 65	0,001 65
μ_m (procjena) / (estimation)	0,080 97	0,080 97
λ_o (procjena) / (estimation)	0,029 96	0
μ_o (procjena) / (estimation)	0,044 44	0
T_o [h], izraz / expression (30)	432,32	
λ, izraz / expression (32)	0,002 31	
λ, procjena po izrazu / estimation by the expression (35)	0,002 51	
μ, procjena po izrazu / estimation by the expression (35)	0,075 34	

Tablica 2 – Vjerojatnosti stanja proizvodne jedinice, s uključenim uvjetima okoline
Table 2 - Probabilities of the production unit states, with environmental conditions included

	Slučaj 1 / Case 1 – Olujno i mirno vrijeme / Calm and stormy weather	Slučaj 2 / Case 2 – Samo mirno vrijeme / Calm weather only
P_O	0,944 32	0,980 05
P_1	0,025 20	0,019 95
P_2	0,024 71	
P_3	0,005 77	
Raspoloživost jedinice / Unit availability, $A = P_O + P_2$	0,969 03	0,980 05
Neraspoloživost jedinice / Unit unavailability, $N = P_1 + P_3$	0,030 97	0,019 95

istodobnog zbivanja dvaju ili više događaja, zbog čega se u modelu isključuju mogućnosti direktnih prijelaza između stanja "0" i "3", odnosno "1" i "2".

assumption of all the considerations is the exclusion of the possibility of simultaneous occurrence of two or more events, and because of that the model excludes the possibilities of direct transitions between the states "0" and "3", that is, "1" and "2".



Slika 4 — Model sustava koji čine dvije proizvodne jedinice

Figure 4 — Two-unit system model

Oznake:

- A – prva jedinica,
- B – druga jedinica,
- "0" – jedinice A i B u stanju pogonske spremnosti,
- "1" – jedinica A u kvaru, a B spremna za pogon,
- "2" – jedinica A spremna za pogon, a B u kvaru,
- "3" – jedinice A i B u kvaru,
- λ_1 – učestalost kvara jedinice A,
- μ_1 – učestalost popravka jedinice A,
- λ_2 – učestalost kvara jedinice B,
- μ_2 – učestalost popravka jedinice B.

Symbols:

- A – first unit,
- B – second unit,
- "0" – units A and B in operation condition,
- "1" – failure of unit A, unit B ready for operation,
- "2" – unit A ready for operation, failure of unit B,
- "3" – failure of units A and B,
- λ_1 – unit A failure rate,
- μ_1 – unit A repair rate,
- λ_2 – unit B failure rate,
- μ_2 – unit B repair rate.

Sustav linearnih diferencijalnih jednadžbi modela prema slici 4 ima sljedeći oblik:

System of linear differential model equations according to Figure 4 has the following form:

$$\begin{aligned}
\dot{P}_0(t) &= -(\lambda_1+\lambda_2)P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t)\ ,\\
\dot{P}_1(t) &= \lambda_1 P_0(t) - (\mu_1+\lambda_2)P_1(t) + \mu_2 P_3(t)\ ,\\
\dot{P}_2(t) &= \lambda_2 P_0(t) - (\lambda_1+\mu_2)P_2(t) + \mu_1 P_3(t)\ ,\\
\dot{P}_3(t &= \lambda_2 P_1(t) + \lambda_1 P_2(t) - (\mu_1+\mu_2)P_3(t)\ .
\end{aligned} \tag{37}$$

Početne uvjete određuje pretpostavka da su u trenutku $t=0$ obje jedinice u stanju pogonske spremnosti, dakle da je sustav na početku u stanju "0". Dakle:

Initial conditions are determined by the assumption that at the moment of $t=0$ both units are in operation condition, that is, that at the beginning the system is in "0" state. Therefore:

$$\begin{aligned}
P_0(0) &= 1,\\
P_1(0) &= 0,\\
P_2(0) &= 0,\\
P_3(0) &= 0.
\end{aligned} \tag{38}$$

Kao i u svim prethodnim slučajevima, traži se stacionarno rješenje sustava (37) koje je određeno uvjetom:

As in all other previous cases, a steady system solution (37), determined by the following condition, is searched for:

$$\dot{P}_n(t) = 0 \quad n = 0,\ 1,\ 2,\ 3. \tag{39}$$

Dakle, sustav (37) poprima oblik.

So, system (37) assumes the form:

$$\begin{aligned}
0 &= -(\lambda_1+\lambda_2)P_0(t) + \mu_1 P_1(t) + \mu_2 P_2(t)\ , \\
0 &= \lambda_1 P_0(t) - (\mu_1+\lambda_2)P_1(t) + \mu_2 P_3(t)\ , \\
0 &= \lambda_2 P_0(t) - (\lambda_1+\mu_2)P_2(t) + \mu_1 P_3(t)\ , \\
0 &= \lambda_2 P_1(t) + \lambda_1 P_2(t) - (\mu_1+\mu_2)P_3(t)\ .
\end{aligned} \tag{40}$$

Kao dodatna jednadžba u sustav se uključuje jednadžba identiteta:

As an additional equation, an identity equation is introduced into the system:

$$P_0 + P_1 + P_2 + P_3 = 1 \tag{41}$$

Stacionarno rješenje sustava (40) i (41), tj. stacionarne vjerojatnosti stanja jesu:

Steady system solutions (40) and (41), that is, steady state values, are:

$$\begin{aligned}
P_0 &= \frac{\mu_1\mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} \\
P_1 &= \frac{\lambda_1\mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} \\
P_2 &= \frac{\mu_1\lambda_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} \\
P_3 &= \frac{\lambda_1\lambda_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)}
\end{aligned}\ . \tag{42}$$

Budući da je već ranije pokazano da izrazi:

As it has been shown earlier that the expressions:

$$\begin{aligned}
\lim_{t\to\infty} A(t) &= \frac{\mu}{\lambda+\mu} = A(\infty) = A \\
\lim_{t\to\infty} N(t) &= \frac{\lambda}{\lambda+\mu} = N(\infty) = N
\end{aligned}\ , \tag{43}$$

predstavljaju stacionarnu raspoloživost, odnosno neraspoloživost jedne jedinice, to se stacionarne vjerojatnosti stanja u slučaju sustava koji čine dvije jedinice mogu napisati kako slijedi:

represent steady availability, that is, unavailability of one unit, then the steady probabilities of the state in case of a two-unit system can be written as follows:

$$\begin{aligned}
P_0 &= A_1(\infty)A_2(\infty) = A_1A_2 \\
P_1 &= N_1(\infty)A_2(\infty) = N_1A_2 \\
P_2 &= A_1(\infty)N_2(\infty) = A_1N_2 \\
P_3 &= N_1(\infty)N_2(\infty) = N_1N_2
\end{aligned}\ . \tag{44}$$

Iz toga se izvodi bitno načelo za daljnju razradu modela pouzdanosti. Ono glasi: ako se zna općeniti izraz za stacionarne vrijednosti raspoloživosti i neraspoloživosti svake jedinice u sustavu pojedinačno, može se odmah napisati izraz za stacionarne vjerojatnosti stanja, tj. stacionarne vrijednosti raspoloživosti i neraspoloživosti sustava koristeći pritom osnovne modele pouzdanosti [1], [2] i [10]. Radi provjere, u razmatranje se uzimaju dva najjednostavnija modela, serijski i paralelni.

From that, an important principle for further elaboration of the reliability model is derived. It reads: if the general term for steady values of availability and unavailability of each unit separately in the system is known, the term for steady state values, that is, for steady values of availability and unavailability of the system can immediately be written, using thereat the elementary reliability models [1], [2] and [10]. For the purpose of a test, two simplest models are taken into consideration, the serial and the parallel ones.

Kod paralelnog modela sustava, u slučaju kada sustav čine neovisne jedinice ili komponente, stacionarna raspoloživost sustava određena je sljedećim izrazom:

In the parallel system model, in the case when the system is composed of independent units or components, steady system availability is determined by the following expression:

$$A_S(\infty) = A_1(\infty)A_2(\infty)A_3(\infty)...A_i(\infty)...A_n(\infty) = \prod_{i=1}^{n} A_i(\infty) \quad , \tag{45}$$

iz čega slijedi:

and from this it follows:

$$A_S(\infty) = A_1(\infty)A_2(\infty) = P_0 = \frac{\mu_1\mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} \quad . \tag{46}$$

Po definiciji serijski sustav je neraspoloživ ako je neraspoloživa bilo koja od jedinica koje ga čine. To znači da u prostoru stanja treba odrediti vjerojatnost unije događaja koji znače neraspoloživost bilo koje od jedinica, dakle stanja "1", "2" i "3". Nakon toga, raspoloživost sustava određuje se na sljedeći način:

By definition, a serial system is unavailable if any of the units it is composed of are unavailable. That means that in the state space the probability should be determined of the union of events which mean unavailability of any of the units, that means states "1", "2" and "3". After that, the availability of the system is determined in the following way:

$$\begin{aligned} A_S(\infty) &= 1 - N_S(\infty) = 1-(P_1+P_2+P_3) = 1-(N_1A_2+A_1N_2+N_1N_2) = \\ &= 1-\frac{\lambda_1\mu_2+\mu_1\lambda_2+\lambda_1\lambda_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = \frac{\mu_1\mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} = A_1A_2 \end{aligned} \quad . \tag{47}$$

Izrazi (46) i (47) identični su. Frekvencija kvara serijskog sustava jednaka je frekvenciji napuštanja stanja "0", odnosno iznosi:

Expressions (46) and (47) are identical. Serial system failure frequency equals the frequency of leaving the "0" state, that is, it amounts to:

$$f_s = \sum_{i=1}^{2} f_i \prod_{\substack{j=1 \\ j\neq i}}^{2} A_j(\infty) = f_1A_2 + f_2A_1 \quad , \tag{48}$$

gdje je:

f_1, f_2 – frekvencije kvara jedinica 1 i 2.

where it is as follows:

f_1, f_2 – frequency of failures of units 1 and 2.

Srednje vrijeme trajanja kvara serijskog sustava određeno je sljedećim izrazom:

Average time of serial system failure duration is determined by the following expression:

$$\bar{p}_s = \frac{N_S(\infty)}{f_s} = \frac{N_1 A_2 + A_1 N_2 + N_1 N_2}{f_1 A_2 + f_2 A_1} \quad . \tag{49}$$

Kod paralelnog modela sustava, kojeg čine neovisne jedinice ili komponente, stacionarna raspoloživost sustava određena je sljedećim izrazom:

In the parallel system model, composed of independent units or components, steady system availability is determined by the following expression:

$$\begin{aligned} A_p(\infty) &= 1 - N_1(\infty)N_2(\infty)N_3(\infty)\ldots N_i(\infty)\ldots N_n(\infty) = 1 - \prod_{i=1}^{n} N_i(\infty) = 1 - \prod_{i=1}^{n}(1 - A_i(\infty)) = \\ & A_1(\infty) + A_2(\infty) + A_3(\infty) + \ldots + A_i(\infty) + \ldots + A_n(\infty) - \sum_{\substack{i=1 \\ j=i+1}}^{\substack{j=n \\ i=n-1}} A_i(\infty)A_j(\infty) + \ldots \\ & + (-1)^{n-1}\left[A_1(\infty)A_2(\infty)A_3(\infty)\ldots A_i(\infty)\ldots A_n(\infty)\right] \end{aligned} \quad . \tag{50}$$

Neraspoloživost paralelnog sustava određena je izrazom:

Unavailability of the parallel system is determined by the expression:

$$N_p(\infty) = N_1(\infty)N_2(\infty)N_3(\infty)\ldots N_i(\infty)\ldots N_n(\infty) = \prod_{i=1}^{n} N_i(\infty) \, , \tag{51}$$

iz čega slijedi:

and from this it follows:

$$\begin{aligned} A_p(\infty) &= 1 - N_1(\infty)N_2(\infty) = 1 - P_3 = 1 - \frac{\lambda_1\lambda_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} \\ &= \frac{\mu_1\mu_2 + \lambda_1\mu_2 + \mu_1\lambda_2}{(\lambda_1+\mu_2)(\lambda_2+\mu_2)} \end{aligned} \quad . \tag{52}$$

S druge strane, sustav je raspoloživ ako je raspoloživa barem jedna jedinica, te će vjerojatnost toga događaja prema modelu na slici 4 biti jednaka sumi vjerojatnosti stanja "0", "1" i "2", tj.:

On the other hand, the system is available if at least one unit is available, and the probability of that event, according to the model in Figure 4, will be equal to the sum of the probability of the states "0", "1" and "2", that is:

$$A_p(\infty) = P_0 + P_1 + P_2 = A_1 A_2 + N_1 A_2 + A_1 N_2 = \frac{\mu_1\mu_2 + \lambda_1\mu_2 + \mu_1\lambda_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} \, , \tag{53}$$

Izrazi (52) i (53) identični su. Frekvencija kvara određena je sljedećim izrazom:

Expressions (52) and (53) are identical. Failure frequency is determined by the following expression:

$$\begin{aligned} f_p &= N_p(\infty)\sum_{i=1}^{2}\mu_i = \prod_{i=1}^{2} N_i(\infty)\sum_{i=1}^{2}\mu_i = P_3(\mu_1+\mu_2) \\ &= N_1 N_2(\mu_1+\mu_2) = f_1 N_2 + f_2 N_1 \end{aligned} \quad . \tag{54}$$

Srednje vrijeme trajanja kvara paralelnog sustava, odnosno popravka jedinice iznosi:

Average time of parallel system failure duration, that is, duration of unit repair, amounts to:

$$\overline{p}_{\mathrm{p}} = \frac{N_{\mathrm{p}}(\infty)}{f_{\mathrm{p}}} = \frac{N_1 N_2}{f_1 N_2 + f_2 N_1} = \frac{1}{\frac{f_1}{N_1} + \frac{f_2}{N_2}} = \frac{1}{\frac{1}{p_1} + \frac{1}{p_2}} = \frac{p_1 p_2}{p_1 + p_2} = \frac{1}{\mu_1 + \mu_2} \quad , \qquad (55)$$

gdje je P_1, P_2 - trajanja kvara jedinica 1 i 2.

Prema modelu sustava s dvije različite jedinice frekvencija pojedinog stanja može se odrediti bilo kao produkt vjerojatnosti stanja i sume učestalosti napuštanja tog istog stanja, bilo kao produkt sume učestalosti ulazaka u stanje i vjerojatnosti stanja iz kojeg se dolazi. Frekvencije su iste promatrane od strane izlazaka, kao i od strane ulazaka. Prema prvome frekvencije stanja jesu:

where P_1, P_2 is the duration of failures of units 1 and 2.

According to the system model with two different units the frequency of certain state can be determined either as a product of the probability of the state and of the sum of the rate of leaving such state, or as the product of the sum of the rate of entry into the state and the probability of the state of origin. The frequencies are the same observed from the exit side as from the entry side. According to the first, the frequencies of the state are:

$$\begin{aligned} f_{"0"} &= P_0(\lambda_1 + \lambda_2) = A_1 A_2(\lambda_1 + \lambda_2) = f_1 A_2 + f_2 A_1 \\ f_{"1"} &= P_1(\mu_1 + \lambda_2) = N_1 A_2(\mu_1 + \lambda_2) = f_1 A_2 + f_2 N_1 \\ f_{"2"} &= P_2(\lambda_1 + \mu_2) = A_1 N_2(\lambda_1 + \mu_2) = f_1 N_2 + f_2 A_1 \\ f_{"3"} &= P_3(\mu_1 + \mu_2) = N_1 N_2(\mu_1 + \mu_2) = f_1 N_2 + f_2 N_1 \end{aligned} \quad . \qquad (56)$$

Vrijeme boravka u svakom pojedinom stanju jednako je recipročnoj vrijednosti sume učestalosti napuštanja tog istog stanja. Prema tome, odgovarajući izrazi jesu:

Period of abidance in each particular state is equal to the reciprocal value of the sum of the rate of leaving that same state. Therefore, the relevant expressions are:

$$\begin{aligned} T_{"0"} &= \frac{1}{\lambda_1 + \lambda_2} \\ T_{"1"} &= \frac{1}{\mu_1 + \lambda_2} \\ T_{"2"} &= \frac{1}{\lambda_1 + \mu_2} \\ T_{"3"} &= \frac{1}{\mu_1 + \mu_2} \end{aligned} \quad . \qquad (57)$$

Slična razmatranja mogu se primijeniti na bilo koji broj neovisnih jedinica koje su povezane u bilo kakav funkcionalni sustav poznate strukture, čiji se rad u potpunosti može prikazati nizom stanja prostora stanja, odnosno modelom stanja i prijelaza između tih stanja. Uz navedeno, jedini je još uvjet da su poznate stacionarne raspoloživosti i neraspoloživosti jedinica, ili, točnije, stacionarne vjerojatnosti stanja svake od jedinica. Stacionarne vjerojatnosti stanja, raspoloživosti ili neraspoloživosti sustava izvode se direktno iz stacionarnih vjerojatnosti stanja, raspoloživosti i neraspoloživosti jedinica, već prema stanjima jedinica, te kombinaciji ili grupi stanja sustava koja u prostoru stanja

Similar observations can be applied to any number of independent units connected in any kind of functional system of a familiar structure, the operation of which can be fully depicted with a series of states of the state space, that is, by the model of the state and transition between those states. Apart from the above, the only other condition is to be familiar with steady availabilities and non-availabilities of the units, or, more precisely, steady probabilities of the states of each of those units. Steady probabilities of the states, availabilities or non-availabilities of the system are derived directly from steady probabilities of the state, availability or unavailability of the units, according to znači raspoloživost ili neraspoloživost sustava. To znači da se uvijek mora voditi računa o specifičnoj građi sustava i njegovim funkcionalnim karakteristikama, jer je samo tako moguće u prostoru svih mogućih stanja iste izdvajati i grupirati prema određenim kriterijima, npr. kriterijima uspješnog rada.

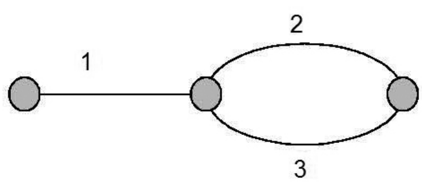
Kako je to već ranije utvrđeno, izrazi za stacionarne raspoloživosti i neraspoloživosti sustava ekvivalentni su onima za pouzdanost i nepouzdanost, što znači da se mogu dobiti tako da se u odgovarajućim izrazima jednostavno zamijene varijable pouzdanosti varijablama raspoloživosti, a varijable nepouzdanosti varijablama neraspoloživosti, kako sustava tako i jedinica.

Kao ilustracija svega što je prethodno navedeno poslužit će jednostavan primjer sustava od tri jedinice čiji je model pouzdanosti prikazan na slici 5.

the states of the units, and the combination or group of the system states which means system availability or unavailability in the state space. That means that what should always be taken into consideration is the specific structure of the system and its functional characteristics because in the space of all possible states it is the only possible way of sorting out and grouping the above according to certain criteria, for example, criteria of successful operation.

As has been determined earlier, expressions for steady system availabilities and non-availabilities are equivalent to those for reliability and non-reliability, which means that they can be obtained so as to simply exchange the variables of reliability with variables of availability in the relevant expressions, and to exchange the variables of unreliability with the variables of unavailability both of the system and the units.

The simple example of the three-unit system, the reliability model of which is shown in Figure 5 will serve as an illustration of all that has been mentioned above.



Slika 5 — Serijsko-paralelni model sustava od tri jedinice
Figure 5 — The three-unit system

Stacionarne vrijednosti raspoloživosti i neraspoloživosti jedinica jesu:

Steady values of the units' availability and unavailability are:

$$\begin{array}{ccc} \text{Jedinica 1} & \text{Jedinica 2} & \text{Jedinica 3} \\ A_1(\infty)=\dfrac{\mu_1}{\lambda_1+\mu_1} & A_2(\infty)=\dfrac{\mu_2}{\lambda_2+\mu_2} & A_3(\infty)=\dfrac{\mu_3}{\lambda_3+\mu_3} \\ N_1(\infty)=\dfrac{\lambda_1}{\lambda_1+\mu_1} & N_2(\infty)=\dfrac{\lambda_2}{\lambda_2+\mu_2} & N_3(\infty)=\dfrac{\lambda_3}{\lambda_3+\mu_3} \end{array} \quad . \tag{58}$$

Vjerojatnosti stanja sustava u prostoru stanja dana su u tablici 3.

Probabilities of the states of the system in the state space are given in Table 3.

Tablica 3 – Stanja sustava, stanja jedinica, vjerojatnosti i frekvencije stanja sustava prema slici 5
Table 3 – System states, units' states, probabilities and frequencies of system states according to Figure 5.

Redni broj / Number	Stanje sustava / System state	Stanja jedinica / Units' states (*) 1	2	3	Vjerojatnosti sustava / Probabilities of system states	Frekvencije stanja sustava / System state frequencies
1.	"0"	1	1	1	$P_{"0"} = A_1A_2A_3$	$f_{"0"} = f_1A_2A_3 + f_2A_1A_3 + f_3A_1A_2$
2.	"1"	0	1	1	$P_{"1"} = N_1A_2A_3$	$f_{"1"} = f_1A_2A_3 + f_2N_1A_3 + f_3N_1A_2$
3.	"2"	1	0	1	$P_{"2"} = A_1N_2A_3$	$f_{"2"} = f_1N_2A_3 + f_2A_1A_3 + f_3A_1N_2$
4.	"3"	1	1	0	$P_{"3"} = A_1A_2N_3$	$f_{"3"} = f_1A_2N_3 + f_2A_1N_3 + f_3A_1A_2$
5.	"4"	0	0	1	$P_{"4"} = N_1N_2A_3$	$f_{"4"} = f_1N_2A_3 + f_2N_1A_3 + f_3N_1N_2$
6.	"5"	0	1	0	$P_{"5"} = N_1A_2N_3$	$f_{"5"} = f_1A_2N_3 + f_2N_1N_3 + f_3N_1A_2$
7.	"6"	1	0	0	$P_{"6"} = A_1N_2N_3$	$f_{"6"} = f_1N_2N_3 + f_2A_1N_3 + f_3A_1N_2$
8.	"7"	0	0	0	$P_{"7"} = N_1N_2N_3$	$f_{"7"} = f_1N_2N_3 + f_2N_1N_3 + f_3N_1N_2$

* 1 – jedinica u stanju pogonske spremnosti / unit in the state of operation condition
0 – jedinica u stanju kvara / unit in the state of failure

Budući da se radi o sustavu čije su komponente neovisne, izraz za pouzdanost sustava glasi:

As the matter at hand is a system whose components are independent, the expression for system reliability reads:

$$\begin{aligned} R_S(t) &= P[x_1(x_2+x_3)] = P(x_1x_2+x_1x_3) = P(x_1x_2)+P(x_1x_3)-P(x_1x_2x_3) \\ &= P(x_1)P(x_2)+P(x_1)P(x_3)-P(x_1)P(x_2)P(x_3) \end{aligned} \quad (59)$$

Prema tome, za uspješan rad sustava nužni su uspješan rad prve jedinice i uspješan rad barem jedne od dviju preostalih jedinica. Jednostavnom zamjenom oznaka za pouzdanost oznakama za raspoloživost u izrazu (59) dobije se izraz za stacionarnu raspoloživost sustava:

According to that, successful operation of the system requires successful operation of the first unit and successful operation of at least one of the other two units. By simple exchange of the symbols for reliability with symbols for availability in the expression (59), the expression for steady availability of the system is obtained:

$$\begin{aligned} A_S(\infty) &= A_1(\infty)A_2(\infty)+A_1(\infty)A_3(\infty)-A_1(\infty)A_2(\infty)A_3(\infty) = \\ &= \frac{\mu_1\mu_2}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)} + \frac{\mu_1\mu_3}{(\lambda_1+\mu_1)(\lambda_3+\mu_3)} - \frac{\mu_1\mu_2\mu_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)} = \\ &= \frac{\mu_1\mu_2\mu_3+\mu_1\lambda_2\mu_3+\mu_1\mu_2\lambda_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)} \end{aligned} \quad (60)$$

S druge strane, analiza prostora stanja (tablica 1) pokazuje da je za uspješan rad sustava potrebno da se ostvari barem jedan od događaja, odnosno stanja sustava "0", "2" ili "3". Vjerojatnost u tom slučaju iznosi:

On the other hand, analysis of the state space (Table 1) shows that successful system operation requires the realization of at least one of the events, that is, the states of the system "0", "2" or "3". The probability in that case amounts to:

$$A_S(\infty) = P_0(\infty) + P_2(\infty) + P_3(\infty) = A_1A_2A_3 + A_1N_2A_3 + A_1A_2N_3 =$$
$$= \frac{\mu_1\mu_2\mu_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)} + \frac{\mu_1\lambda_2\mu_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)} + \quad (61)$$
$$\frac{\mu_1\mu_2\lambda_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)} = \frac{\mu_1\mu_2\mu_3 + \mu_1\lambda_2\mu_3 + \mu_1\mu_2\lambda_3}{(\lambda_1+\mu_1)(\lambda_2+\mu_2)(\lambda_3+\mu_3)}$$

Naravno, izrazi (60) i (61) identični su.

3.2 Model sustava s dvije proizvodne jedinice s uključenim uvjetima okoline

Na slici 6 prikazan je model sustava koji čine dvije neovisne jedinice s uključenim uvjetima okoline, ili bolje rečeno prostor stanja sustava dviju jedinica u različitim uvjetima stanja okoline. Osnovna mu je namjena razrada modela dvostrukih kvarova.

Model čine četiri niža stanja ("0", "1", "2" i "3"), koja predstavljaju stanja sustava u uvjetima normalnog, tj. mirnog vremena, i četiri viša stanja ("4", "5", "6" i "7") koja predstavljaju stanja sustava u uvjetima olujnog vremena. Općenito se pretpostavlja da su izgledi za kvarove, pogotovo dvostruke i višestruke, puno veći tijekom olujnog vremena nego tijekom mirnog vremena. Kada se kvarovi pojave tijekom olujnog vremena, traju do pojave mirnog vremena, jer se popravci poduzimaju samo tijekom trajanja mirnog vremena. Učestalost mirnog vremena puno je veća od učestalosti kvara jedinice, a učestalost olujnog vremena puno je manja od učestalosti popravka jedinice. Sve to znači da se zbog izuzetno male vjerojatnosti ostvarenja neki prelasci među stanjima mogu zanemariti, a da pritom model ne izgubi puno na točnosti. S druge strane dobiva se mogućnost da se problem uopće može riješiti. Nakon pojednostavljenja može se uzeti da se model sustava s dvije neovisne jedinice ponavlja, jednom za uvjete mirnog vremena, a drugi put za uvjete olujnog vremena, pri čemu su ti uvjeti neovisni o kvarovima i popravcima jedinica. Pojednostavljenja i pretpostavke koji se uvode u model imaju sljedeće oblike:

Of course, the expressions (60) and (61) are identical.

3.2 Model of a system with two generating units with environmental conditions included

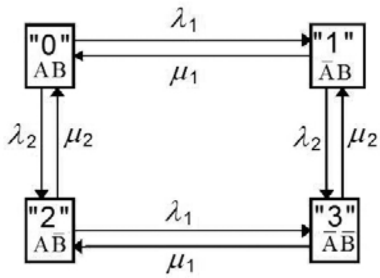
Figure 6 shows the model of the system composed of two independent units with environmental conditions included, or, better said, the space of the states of the two-unit system in different environmental state conditions. Its basic intention is to elaborate the model of double failures.

The model consists of four lower states ("0", "1", "2" and "3") which represent states of the system in the conditions of normal, that is, calm weather, and four higher states ("4", "5", "6" and "7") which represent states of the system in the conditions of stormy weather. It is generally assumed that the prospects for failures, especially double and multiple, are much greater during stormy weather than during calm weather. When failures occur during stormy weather, they last until occurrence of calm weather because the repairs are undertaken only for the duration of calm weather. The rate of calm weather is much greater than the rate of unit failure, and the rate of stormy weather is much smaller than the rate of unit repair. All of this means that because of extremely small probability of realization, certain transitions between states can be disregarded without the model losing much of its accuracy in the process. On the other hand, the possibility arises for the problem to be completely unsolvable. After the simplification, it can be assumed that the model of the system with two independent units repeats, once for calm weather conditions, and the second time for the stormy weather conditions, whereat those conditions are independent of unit failures and repairs. Simplifications and assumptions introduced into the model have the following forms:

$$\lambda_1 = \lambda_2 \approx 0 \ ,$$
$$\mu_1' = \mu_2' \approx 0 \ ,$$
$$m >> \lambda_1', \lambda_2' \ , \quad (62)$$
$$o << \mu_1, \mu_2$$
$$P_0 + P_4 \approx 1 \ .$$

Sustav linearnih diferencijalnih jednadžbi Markovljeva procesa prema slici 6 glasi:

The system of linear differential equations of the Markov process according to Figure 6 reads as follows:



Slika 6 — Model sustava koji čine dvije proizvodne jedinice s uključenim uvjetima okoline
Figure 6 — Model of a system with two generating units with environmental conditions included

Oznake:

A – prva jedinica,
B – druga jedinica,
m – učestalost pojave mirnog vremena,
o – učestalost pojave olujnog vremena,

– stanja jedinica i učestalosti tijekom mirnog vremena:
"0" – jedinice A i B spremne za pogon,
"1" – jedinica A spremna za pogon, a B u kvaru,
"2" – jedinice A i B u kvaru,
"3" – jedinica A u kvaru, a B spremna za pogon,
μ_A – učestalost popravka jedinice A,
μ_B – učestalost popravka jedinice B,

– stanja jedinica i učestalosti tijekom olujnog vremena:
"4" – jedinice A i B spremne za pogon,
"5" – jedinica A spremna za pogon, a B u kvaru,
"6" – jedinice A i B u kvaru,
"7" – jedinica A u kvaru, a B spremna za pogon,
λ'_A – učestalost kvara jedinice A,
λ'_B – učestalost kvara jedinice B.

Symbols:

A – first unit
B – second unit
m – rate of occurrence of calm weather
o – rate of occurrence of stormy weather

– states of the units and rates during calm weather:
"0" – units A and B ready for operation,
"1" – unit A ready for operation, failure of unit B,
"2" – units A and B in state of failure,
"3" – failure of unit A, unit B ready for operation,
μ_A – rate of repair of unit A,
μ_B – rate of repair of unit B,

– states of the units and rates during stormy weather:
"4" – units A and B ready for operation,
"5" – unit A ready for operation, failure of unit B,
"6" – failure of units A and B,
"7" – failure of unit A, unit B ready for operation,
λ'_A – rate of failure of unit A,
λ'_B – rate of failure of unit B.

$$
\begin{aligned}
\dot{P}_0(t) &= -oP_0(t) + \mu_2 P_1(t) + \mu_1 P_3(t) + mP_4(t)\ ,\\
\dot{P}_1(t) &= -(\mu_2 + o)P_1(t) + \mu_1 P_2(t) + mP_5(t)\ ,\\
\dot{P}_2(t) &= -(\mu_1 + \mu_2 + o)P_2(t) + mP_6(t)\ ,\\
\dot{P}_3(t) &= \mu_2 P_2(t) - (\mu_1 + o)P_3(t) + mP_7(t)\ ,\\
\dot{P}_4(t) &= oP_0(t) - (m + \lambda_1' + \lambda_2')P_4(t)\ ,\\
\dot{P}_5(t) &= oP_1(t) + \lambda_2' P_4(t) - (m + \lambda_1')P_5(t)\ ,\\
\dot{P}_6(t) &= mP_2(t) + \lambda_1' P_5(t) - mP_6(t) + \lambda_2' P_7(t)\ ,\\
\dot{P}_7(t) &= oP_3(t) + \lambda_1' P_4(t) - (\lambda_2' + m)P_7(t)\ .
\end{aligned}
\tag{63}
$$

Početne uvjete određuju pretpostavke da su u trenutku $t = 0$ obje jedinice u stanju pogonske spremnosti i da ih u okolini okružuju uvjeti mirnog vremena (stanje "0"). Iz toga slijedi:

Initial conditions are determined by the assumption that at the moment of $t = 0$ both units are in operation condition and that they are surrounded by calm weather conditions ("0" state). From this it follows:

$$\begin{aligned} &P_0(0) = 1, \\ &P_1(0) = 0, \\ &P_2(0) = 0, \\ &P_3(0) = 0, \\ &P_4(0) = 0, \\ &P_5(0) = 0, \\ &P_6(0) = 0, \\ &P_7(0) = 0. \end{aligned} \tag{64}$$

Traži se stacionarno rješenje sustava (63), dakle uzima se da vrijedi.

A steady solution of the system (63) is being searched for and therefore the following is taken to apply:

$$\begin{aligned} &\dot{P}_n(t) = 0, \\ &n = 0,\ 1,\ 2,\ 3,\ 4,\ 5,\ 6,\ 7. \end{aligned} \tag{65}$$

Uz uvjet (65) sustav (63) poprima oblik:

With the condition (65), the system (63) assumes the form:

$$\begin{aligned}
0 &= -oP_0(t) + \mu_2 P_1(t) + \mu_1 P_3(t) + mP_4(t)\ , \\
0 &= -(\mu_2 + o)P_1(t) + \mu_1 P_2(t) + mP_5(t)\ , \\
0 &= -(\mu_1 + \mu_2 + o)P_2(t) + mP_6(t)\ , \\
0 &= \mu_2 P_2(t) - (\mu_1 + o)P_3(t) + mP_7(t)\ , \\
0 &= oP_0(t) - (m + \lambda_1' + \lambda_2')P_4(t)\ , \\
0 &= oP_1(t) + \lambda_2' P_4(t) - (m + \lambda_1')P_5(t)\ , \\
0 &= mP_2(t) + \lambda_1' P_5(t) - mP_6(t) + \lambda_2' P_7(t)\ , \\
0 &= oP_3(t) + \lambda_1' P_4(t) - (\lambda_2' + m)P_7(t)\ .
\end{aligned} \tag{66}$$

Jednadžba identiteta glasi:

The identity equation reads:

$$P_0 + P_1 + P_2 + P_3 + P_4 + P_5 + P_6 + P_7 = 1\ . \tag{67}$$

Rješenje sustava (66), tj. približne stacionarne vjerojatnosti stanja sustava uz uvjete (62) jesu:

The solution of the system (66), that is, approximate steady values of the system state, with the conditions (62) are:

$$
\begin{aligned}
P_0 &\approx \frac{mmm\mu_1\mu_2(\mu_1+\mu_2)}{\Delta}\\
P_1 &\approx \frac{mmo\mu_1\lambda_2'(\mu_1+\mu_2)}{\Delta}\\
P_2 &\approx \frac{2mo\mu_1\mu_2\lambda_1'\lambda_2'}{\Delta}\\
P_3 &\approx \frac{mmo\mu_2\lambda_1'(\mu_1+\mu_2)}{\Delta}\\
P_4 &\approx \frac{mmo\mu_1\mu_2(\mu_1+\mu_2)}{\Delta}\\
P_5 &\approx \frac{mo\mu_1\mu_2\lambda_2'(\mu_1+\mu_2)}{\Delta}\\
P_6 &\approx \frac{2o\mu_1\mu_2\lambda_1'\lambda_2'(\mu_1+\mu_2)}{\Delta}\\
P_7 &\approx \frac{mo\mu_1\mu_2\lambda_1'(\mu_1+\mu_2)}{\Delta}\\
\Delta &\approx mm(m+o)\mu_1\mu_2(\mu_1+\mu_2)
\end{aligned}
\quad . \qquad (68)
$$

Računanje s prosječnim učestalostima kvara i popravka kada su uključeni uvjeti okoline ograničeno je samo na jednostruke kvarove. Vjerojatnost dvostrukih i višestrukih kvarova, odnosno kvarova dviju ili više jedinica kada su uključeni uvjeti okoline, mogu se izračunati s prosječnim učestalostima samo ako se uvede pretpostavka da su jedinice podvrgnute vremenskim uvjetima koji su neovisni jedan o drugome. Naravno, ovdje su upravo interesantni uvjeti kada su dvije i više jedinica podvrgnuti istim uvjetima okoline, pogotovo stanja i vjerojatnosti stanja kada su obje jedinice u kvaru, prvenstveno onom tijekom trajanja olujnog vremena, dakle u stanju 6. Do vjerojatnosti stanja 6 može se doći i na drugi način. Naime, vjerojatnost jednostrukog kvara tijekom olujnog vremena određena je izrazom (22). Uvažavajući činjenicu da se može pokvariti bilo koja od dviju istovrsnih jedinica znatno pojednostavljen izraz može se napisati u obliku:

Calculation with average failure and repair rates, when environmental conditions are included, is limited to single failures. The probability of double and multiple failures, that is, failures of two or more units, when environmental conditions are included, can be calculated with average rates only if the assumption is introduced that the units are subjected to weather conditions which are mutually independent. Of course, those conditions when two or more units are subjected to the same environmental conditions is exactly what is interesting here, especially as regards the states and probabilities of the state when both units are in the state of failure, primarily failure during stormy weather, that is, state 6. The probability of state 6 can also be reached in a different way. Particularly, probability of single failure during stormy weather is defined by expression (22). Taking into consideration the fact that any of the two units of the same type can break down, a significantly simplified expression can be written in the form:

$$
P_{5,7} \approx 2\frac{o}{m(m+o)}\lambda' = 2O\frac{O}{M+O}\lambda' \qquad (69)
$$

gdje je:

- $\frac{o}{m+o} = \frac{O}{M+O}$ – vjerojatnost pojave olujnog vremena,
- λ_o – učestalost kvara jedinice tijekom trajanja olujnog vremena,
- $\frac{1}{m} = O$ – srednje trajanje olujnog vremena.

where it is as follows:

- $\frac{o}{m+o} = \frac{O}{M+O}$ – probability of occurrence of stormy weather
- λ_o – rate of unit failure during stormy weather,
- $\frac{1}{m} = O$ – average duration of stormy weather.

Kada se već pojavio jedan kvar, drugi kvar tijekom olujnog vremena može se pojaviti s učestalošću λ',

When failure has already occurred, the other failure during stormy weather can occur at the rate λ', tako da će frekvencija tog događaja biti:

so that the frequency of that event will be:

$$f_6 = P_{5,7}\lambda' \approx \frac{2(\lambda' O)^2}{M+O} \quad . \tag{70}$$

Vjerojatnost stanja kvara dviju neovisnih jedinica tijekom trajanja olujnog vremena iznosi:

The probability of the state of failure of two independent units during stormy weather amounts to:

$$P_6 \approx \frac{2(\lambda' O)^2}{M+O} \; O \cdot \tag{71}$$

Kod sustava koji čini više jedinica srednje vrijeme do kvara određuje se polazeći od prostora stanja sustava, odnosno odgovarajuće matrice učestalosti prijelaza između stanja, ispuštajući stanja kvara kao apsorbirajuća stanja. Stanja kvara određuju se prema modelu pouzdanosti sustava o kojem se radi.

Za sustav s dvije identične jedinice od kojih obje moraju biti ispravne da bi sustav bio ispravan, čime se podrazumijeva serijski model pouzdanosti, srednje vrijeme do kvara iznosi:

In systems composed of several units, average time to failure is determined starting from the system state space, that is, the relevant state transition rate matrix, excluding states of failure as absorbing states. States of failure are determined according to the relevant system reliability model.

For the system with two identical units of which both need to be in sound condition in order for the system to be in sound condition, which implies a serial reliability model, average time to failure amounts to:

$$T_0 = \frac{2\lambda' + m + o}{2(2\lambda'\lambda + m\lambda + o\lambda')} \quad . \tag{72}$$

Uz pretpostavku da se promatraju samo uvjeti mirnog vremena, srednje vrijeme do kvara iznosi:

With the assumption that only calm weather conditions are being observed, average time to failure amounts to:

$$T_0 = \frac{1}{2\lambda} \quad . \tag{73}$$

Za sustav kojeg čine tri identične jedinice, pri čemu je za ispravan rad sustava potrebno da sve tri jedinice ispravno rade srednje vrijeme do kvara iznosi:

For the system composed of three identical units, whereat the proper operation of the system requires that all three units work properly, average time to failure amounts to:

$$T_0 = \frac{3\lambda' + m + o}{3(3\lambda'\lambda + m\lambda + o\lambda')} \quad . \tag{74}$$

Zanemare li se uvjeti olujnog vremena, srednje vrijeme do kvara u uvjetima mirnog vremena bit će:

If stormy weather conditions are disregarded, average time to failure in calm weather conditions will be:

$$T_0 = \frac{1}{3\lambda} \quad . \tag{75}$$

Tablica 4 prikazuje primjer utvrđenih ulaznih parametara modela stanja dvije proizvodne jedinice, s uključenim uvjetima okoline, za mirno i olujno vrijeme. Primjena modela provedena je za tri slučaja, i to:

— **slučaj 1** – dviju identičnih proizvodnih jedinica (ključni parametri modela kako su prikazani u tablici 1),
— **slučaj 2** – dviju proizvodnih jedinica od kojih je druga više izložena kvarovima za olujnog vremena, i
— **slučaj 3** – dviju proizvodnih jedinica od kojih je druga manje izložena kvarovima za olujnog vremena.

Tablica 5 prikazuje rezultat izračuna, tj. vjerojatnosti stanja u modelu stanja proizvodne jedinice za ulazne parametre iz tablice 4.

Table 4 shows the example of determined input parameters of the state model of two generating units, with environmental conditions included, for calm and stormy weather. The application of the model is implemented in three cases, and that being:

— **case 1** - in two identical generating units (key model parameters as they are shown in Table 1),
— **case 2** - in two identical productions units of which the second one is more exposed to failures during stormy weather, and
— **case 3** - in two identical productions units of which the second one is less exposed to failures during stormy weather.

Table 5 shows the calculation result, that is, the probabilities of the state in the generating unit state model for input parameters from Table 4.

Tablica 4 – Parametri modela stanja dvije proizvodne jedinice, s uključenim uvjetima okoline
Table 4 – Parameters of the state model of two generating units, with environmental conditions included

	Slučaj 1 / Case 1	Slučaj 2 / Case 2	Slučaj 3 / Case 3
	Dvije identične jedinice, jednako izložene kvarovima / Two identical units, equally exposed to failures	Druga jedinica više izložena kvarovima za olujnog vremena / Second unit more exposed to failures during stormy weather	Druga jedinica manje izložena kvarovima za olujnog vremena / Second unit less exposed to failures during stormy weather
M, h	8 493		
N_m	26		
m, 1/h	0,097 38		
o, h	267		
N_O	26		
o, 1/h	0,003 06		
c	14		
c'	8		
p	20		
p'	2		
T_{pm}	247		
T_{po}	45		
λ'_1	0,029 96	0,029 96	0,029 96
μ_1	0,080 97	0,080 97	0,080 97
λ'_2	0,029 96	0,034 46	0,025 47
μ_2	0,080 97	0,068 83	0,093 12

Tablica 5 – Vjerojatnosti stanja dvije proizvodne jedinice, s uključenim uvjetima okoline
Table 2 - Probabilities of the states of two production units, with environmental conditions included

	Slučaj 1 / Case 1	Slučaj 2 / Case 2	Slučaj 3 / Case 3
P_O	0,922 86	0,916 67	0,928 11
P_1	0,010 74	0,014 43	0,007 98
P_2	0,003 30	0,004 08	0,002 63
P_3	0,010 74	0,010 66	0,010 80
P_4	0,029 01	0,028 82	0,029 18
P_5	0,008 93	0,010 20	0,007 63
P_6	0,005 49	0,006 28	0,004 70
P_7	0,008 93	0,008 87	0,008 98
Pogreška zbog pojednostavljenja u modelu / Error due to simplification in the model	5,06 %	5,77 %	4,46 %
Obje jedinice spremne za pogon / Both units ready for operation	0,951 88	0,945 49	0,957 29
Barem jedna jedinica u kvaru / At least one unit failed	0,048 12	0,054 51	0,042 71
Barem jedna jedinica spreman za pogon / At least one unit ready for operation	0,991 20	0,989 65	0,992 68
Obje jedinice u kvaru / Both units failed	0,008 80	0,010 35	0,007 32

4 ZAKLJUČAK

Razvijeni su i izloženi modeli za izračun parametara i pokazatelja pouzdanosti i raspoloživosti elektroenergetskog sustava za slučajeve kada su proizvodne jedinice izložene uvjetima okoline, posebno nepovoljnim atmosferskim prilikama koji djeluju na učestalosti pojava njihova kvara ili popravka, bilo zbog nedovoljne ili nedostatne zaštite proizvodnih jedinica od takvih nepovoljnih utjecaja iz okoline, bilo zbog moguće iznimne snage takvih nepovoljnih utjecaja iz okoline. Naime, iako se proizvodne jedinice nastoje štititi od svih mogućih predvidljivih utjecaja iz okoline, pa tako i svih predvidljivih nepovoljnih atmosferskih utjecaja, ipak se dimenzioniranje svih zaštitnih mjera i sustava provodi podrazumijevajući određene vjerojatnosti pojave i težina posljedica takvih pojava i utjecaja. Dakako, niti je ekonomski opravdano niti je tehnički moguće da zaštitne mjere ili sustavi u potpunosti i za sve moguće negativne pojave i utjecaje iz okoline zaštite proizvodno postrojenje.

Dakako, ako se očekuju negativne posljedice utjecaja iz okoline, onda ih je bez obzir na složenost postupka i analize nužno uključiti u model proizvodne jedinice, odnosno u odgovarajući izračun pokazatelja pouzdanosti i raspoloživosti elektroenergetskog sustava, kako je to izloženo u ovom radu.

I obrnuto, polazeći od ekonomsko opravdane i tehničko-tehnološke prihvatljive razine zaštite proizvodne jedinice, a u svezi tražene razine raspoloživosti i pouzdanosti elektroenergetskog sustava, primjenom izloženih modela moguće odrediti mjesto i uloge nekih proizvodnih jedini-

4 CONCLUSION

The presented models for the calculation of parameters and indicators of reliability and availability of the electric power system have also been developed for the cases when generating units are exposed to environmental conditions, especially adverse weather conditions which affect the rate of occurrence of their failures or repairs, either due to insufficient or deficient protection of generating units from such adverse environmental impacts, or due to possible exceptional intensity of those adverse environmental impacts. Namely, although there is an effort to protect the generating units from all possible predictable environmental impacts, and thus also from predictable weather conditions, the dimensioning of all protective measures and systems is still being sized, presuming certain probabilities of occurrence and intensities of the consequences of such occurrences and impacts. Of course, it is neither economically justified nor technically possible for the protection measures or systems to protect the generating plant fully and from all possible negative events and environmental impacts.

Of course, if negative consequences of environmental impacts are expected, then, regardless of the complexity of the procedure or analysis, it is necessary to include them in the generating unit model, that is, in the relevant calculation of indicators of electric power system reliability and availability, as presented in this work.

And vice versa, starting from the economically justified and technically-technologically acceptable generating unit protection, and in connection with the demanded level of availability and reliability of the electric power system, by application of presented models, it is possible to determine the place and the roles of certaca u pokrivanju potreba za snagom i energijom na način da se time unaprijed djeluje na izloženost proizvodne jedinice nepovoljnim utjecajima iz okoline, ili barem njihovim posljedicama. Sljedeći korak svakako bio bi povećanje, odnosno dogradnja razine zaštitnih mjera i sustav proizvodne jedinice za postizanje tražene razine raspoloživosti i pouzdanosti elektroenergetskog sustava u cjelini.

in generating units in covering the requirements for power and energy in such a way as to impact in advance the exposure of the generating unit to adverse environmental impacts, or at least to their consequences. The next step would surely be the increase, that is, upgrade of the level of protective measures and generating unit system for the purpose of achievement of the necessary level of availability and reliability of the electric power system as a whole.

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