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Pricing Behaviour of the Monopolistic and Duopolistic Firms in the Long Run with Heterogeneous Products

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Abstract: Scholars have compared the pricing behaviour where a monopolist in the short run produces heterogeneous products 1 and 2, and a duopolist i produces goods i (i = 1, 2), where there are exogenous shocks to marginal cost and/or industry demand. This pricing behaviour is short run in that no entry is considered. However, this paper considers whether the existence of a potential entrant producing heterogeneous goods affects the pricing behaviour of the established monopolist under the same random shocks.

JEL Classification: D43

Key words: Cournot profits, heterogeneous products, Nash equilibrium, probability, pricing behaviour

Since the monopolist produces two heterogeneous goods 1 and 2, it is a situation of double entry. Two cases of double entry are possible. One is that one potential entrant, if it enters, produces two heterogeneous goods 1 and 2. The other is that two potential entrants 1 and 2 try to enter, at the same time, to compete with the monopolist producing two goods, 1 and 2. The two cases do not make any difference for the

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analysis, but the former case is used for convenience. It is also assumed that the monopolist's good 1 (good 2) is substitutable for the entrant's good 1 (good 2) with a certain degree, *d*, but that the entrant's good 1 (good 2) is not related with the monopolist's good 2 (good 1). In other words, the entrant's good 1 (good 2) is substitutable for the monopolist's good 2 (good 1) with a zero degree.

These assumptions can be justified by imagining the following situations. Suppose that there are four kinds of chewing gum, which are different in the quantity of sugar contained. The first chewing gum contains 20 percent sugar, the second 40 percent, the third 60 per-cent, and the fourth 80 percent. If a chewing gum can be substitutable only for the next one, the chewing gum containing 40 percent sugar can be a substitute for the one containing 20 percent or 60 percent sugar, but not for the one containing 80 percent sugar. If the monopolist is producing the second and third chewing gums, and the entrant produces the first and fourth chewing gums, our assumption is justified. As another example, suppose that an established monopolist is producing BMWs and Pontiacs. The potential entrant considers entering the automobile market by producing Rolls Royces (highest quality car) and Yugos (lowest quality car). Rolls Royce (Yugo) is substitutable for BMW (Pontiac) with the degree of *d*, but Rolls Royce (Yugo) is not substitutable for Pontiac (BMW).

The monopolist m $(m = 1, 2)$ in period 2 has limited information on the potential entrant e's $(e = 1, 2)$ characteristics, and the potential entrant e also has only the incomplete information of the monopolist m's characteristics. Under this uncertainty, the entrant e attempts to enter for a positive payoff, while the monopolist m has a incentive to limit its price for deterrence of the entry. The monopolist's limit pricing of the imperfect information will be different from its pricing of the perfect information in Kamerschen and Park (1992a,b).

One of our tasks is to compare the long-run pricing behaviour of the monopolist m and the duopolist i. So, it is appropriate to analyze the pricing behaviour of the duopolist i that faces potential entry. However, its analytical complications would add only marginally to the duopolist i's pricing behaviour. To the duopolist i, the established rival is an imminent problem, and the pressure it feels from its potential entry is very weak, compared with what the monopolist m does. Also, since the duopolists do not experience any new shocks during the production periods 2 and 3, their pricing behaviour that is compared with the monopolist's limit pricing behaviour is assumed the same as in Kamerschen and Park (1992b).

Limit Pricing Strategy of the Monopolist Facing Heterogeneous Potential Entrants with Incomplete Information

Our analysis begins by writing the industry demand, marginal cost and marginal revenue curves of the monopolist m (m = 1, 2) as:

$$p^m = \frac{a}{b} - \frac{2}{b}q^m; \quad MC = c; \quad MR^m = \frac{a}{b} - \frac{4}{b}q^m$$

where a ~ $N(a_1, \sigma_a)$ and c ~ $N(c_1, \sigma_c)$. All the notations follow the usages of Kamerschen and Park (1992a,b,and 1994), but should be clear as stated in this paper. The monopolist m's period 2 equilibrium prices and profits are, respectively,

$$p_i(a,c) = \frac{a+bc}{2b} \text{ and } \pi_i(a,c) = \frac{(a-bc)^2}{8b}, \tag{1}$$

where the subscript *i* takes, in general, any real number in ascending order.

Suppose that the monopolist faces potential entry. The potential entrant entered the market is assumed to have the same industry demand equation as the monopolist, but a different marginal cost curve such as $MC = w$, where $w \sim N(w_1, \sigma_w)$.

In production period 3, the demand equations for the monopolist *m (m = 1, 2)* and the entrant *e (e = 1, 2)* are, respectively,

$$\begin{aligned} p^m &= \frac{a}{(b+4d)} - \frac{4}{(b+4d)}q^m + \frac{4d}{(b+4d)}p^e \\ p^e &= \frac{a}{(b+4d)} - \frac{4}{(b+4d)}q^e + \frac{4d}{(b+4d)}p^m \end{aligned} \tag{2}$$

Then, they each would maximise their respective Cournot profits:

$$\begin{aligned} c\pi^m &= \left[\frac{a}{(b+4d)} - \frac{4}{(b+4d)}q^m + \frac{4d}{(b+4d}p^e - c\right]q^m \\ c\pi^e &= \left[\frac{a}{(b+4d)} - \frac{4}{(b+4d)}q^e + \frac{4d}{(b+4d)}p^m - w\right]q^e \end{aligned} \tag{3}$$

The first order conditions of the two maximisation problems are

$$\frac{\delta c\pi^m}{\delta q^m} = \frac{a}{(b+4d)} - \frac{8}{(b+4d)}q^m + \frac{4d}{(b+4d)}p^e - c = 0$$

$$\frac{\delta c\pi}{\delta q^e} = \frac{a}{(b+4d)} - \frac{8}{(b+4d)}q^e + \frac{4d}{(b+4d)}p^m - w = 0$$

We can rewrite above two first order conditions for q^m and q^e as

$$q^m = \frac{a + 4dp^e - (b + 4d)c}{8}$$

$$q^e = \frac{a + 4dp^m - (b + 4d)w}{8}$$

Putting the above two values back into the demand equations (2) gives

$$p^m = \frac{a + 4dp^e + (b + 4d)c}{2(b + 4d)}$$

$$p^e = \frac{a + 4dp^m + (b + 4d)w}{2(b + 4d)}$$

Solving the above two equations gives Nash-equilibrium prices as

$$\begin{aligned} p^m(a,c,w,d) &= \frac{(b+6d)a + 2d(b+4d)w + (b+4d)^2 c}{2(b+6d)(b+2d)} \\ p^e(a,w,c,d) &= \frac{(b+6d)a + 2d(b+4d)c + (b+4d)^2 w}{2(b+6d)(b+2d)} \end{aligned} \tag{4}$$

Putting (4) into the first order conditions yields Nash-equilibrium quantities as

$$\begin{aligned} q^m(a,c,w,d) &= \frac{(b+4d)\left[(b+6d)a + 2d(b+4d)w - (b^2 + 8bd + 8d^2)c\right]}{8(b+6d)(b+2d)} \\ q^e(a,w,c,d) &= \frac{(b+4d)\left[(b+6d)a + 2d(b+4d)c - (b^2 + 8bd + 8d^2)w\right]}{8(b+6d)(b+2d)} \end{aligned} \tag{5}$$

Putting (4) and (5) into (3), the Cournot equilibrium profits for the monopolist *m* and the entrant *e*, respectively, are

$$\begin{aligned} c\pi^m(a,c,w,d) &= \frac{(b+4d)\left[(b+6d)a + 2d(b+4d)w - (b^2 + 8bd + 8d^2)c\right]^2}{16(b+6d)^2(b+2d)^2} \\ c\pi^e(a,w,x,d) &= \frac{(b+4d)\left[(b+6d)a + 2d(b+4d)c - (b^2 + 8bd + 8d^2)w\right]^2}{16(b+6d)^2(b+2d)^2} \end{aligned} \tag{6}$$

So the reward to the monopolist *m* (*m* = *1, 2)* from deterring entry is equal to the excess of its period 2 monopoly profit over its period 3 Cournot equilibrium profit:

$$R(a,c,w,d)=(a,c)-c\pi^{m}(a,c,w,d)$$

$$=\frac{(a-bc)^2}{8b}-\frac{(b+4d)\left[(b+6d)a+2d(b+4d)w-(b^2+8bd+8d^2)c\right]^2}{16(b+6d)^2(b+2d)^2} \quad (7)$$

For simplicity, the post-entry profits of the monopolist *m* are normalised to be zero [$c\pi^m(a, c, w, d) = 0$] if entry occurs. In this event, it receives only its second period profit as its payoff:

$$\pi(p_i,a,c)=(p_i-c)(a-bp_i)/2 \quad (8)$$

where $p_i(a, c)$ is the monopolist *m*'s equilibrium price in period 2. If entry does not occur, however, its payoff is its second period profit plus the discounted value of the reward to deterring entry:

$$p(p_i, a, c) + \delta^m R(a, c, w, d), \quad (9)$$

whered δ^m is the present value to the monopolist m $(m = 1, 2)$ of \$1 accruing after entry.

The payoff to the entrant *e* $(e = 1, 2)$ if no entry occurs is zero, but his payoff if entry occurs in period 3 is

$$\delta^e\frac{(b+4d)\left[(b+6d)a+2d(b+4d)c-(b^2+8bd+8d^2)w\right]^2}{16(b+6d)^2(b+2d)^2}-K \quad (10)$$

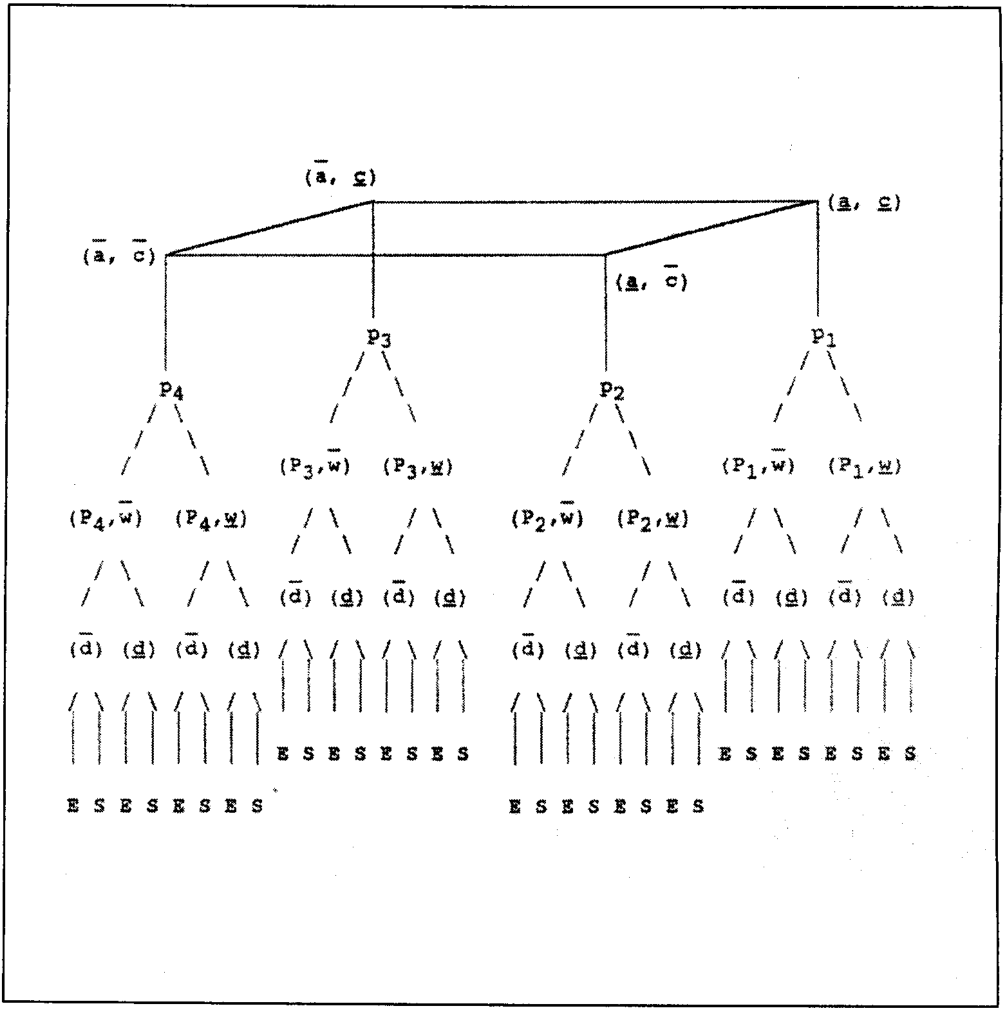
where δ^e is the present value to the entrant *e* of \$1 accruing after entry, and *K* is the entry cost.

The information structure in this paper is similar to that in Kamerschen and Park (1994) and the difference arises by adding an additional parameter, *d.* But the parameter of substitutable degree, *d*, is different from other random parameters. The random parameters, *a*, *c* and *w,* are not controllable by the firms, but determined solely by nature, and they were assumed to be distributed normally. The parameter of *d* is determined by the entrant *e*, not by nature, and its probability is assumed to have a uniform density function. We assume that each of the four random variables, *a, c, w*, and d, takes only two values, high and low, which are denoted by the upper bar and lower bar, respectively, in those random variables. The monopolist's period 2 equilibrium price, $p_i(a, c)$, takes only four values from p_1 to p_4.

The monopolist *m*'s $(m = 1, 2)$ information sets are realised values of industry demand, *a*, and its marginal cost, *c*, while the entrant *e*'s $(e = 1, 2)$ information sets are its cost *w*, a measure of substitutable degree *d*, and the monopolist *m*'s choice of $p_i(a,$

c). The entrant *e* has a belief about the demand shock, *a*, and the monopolist *m*'s cost shock, *c*, from observing $p_i(a, c)$. The monopolist *m* has a belief about the entrant *e*'s cost *w* from experiences, but its belief about the value of *d* does not matter, and is indifferent to all circumstances. This monopolist *m*'s indifferent belief about *d* is represented by the uniformly distributed density function.

Figure 1.: Game Tree



Therefore, the monopolist *m*'s strategy is a map s from R^2 of each possible combination (a, c) into its decision $\{p_i\}$ *for* $i = 1, ..., 4$, where p_1 is the lowest price and

p_4 is the highest price. The entrant e's strategy is a map t from R^3 of each possible combination (p_i, w, d) into its decision $\{E, S\}$, where E is "enter" and S is "stay out." This information framework is summarised in a game tree of Figure 1.

Then, an equilibrium consists of a pair of strategies (s^*, t^*) and a pair of conjectures $(s°, t°)$ such that:

(i) For any $a \in [\underline{a}, \bar{a}]$, $c \in [\underline{c}, \bar{c}]$, and any $s: [a,c] \to \{P_i\}$

$$\pi(s^*(a,c),a,c) + \delta^m \int_{(\underline{w},\underline{d})}^{(\bar{w},\bar{d})} R(a,c,w,d)\left[1 - t°(s^*(a,c),w,d)\right] dH^e(w,d)$$

$$\geq \pi(s(a,c),a,c) + \delta^m \int_{(\underline{w},\underline{d})}^{(\bar{w},\bar{d})} R(a,c,w,d)\left[1 - t°(s(a,c),w,d)\right] dH^e(w,d)$$

where $[(\underline{w}, \underline{d}), (\bar{w}, \bar{d})]$ is the range of possible values of (w, d) and H^e is the probability distribution function for (w, d) [monopolist m's beliefs about (w, d)]. That is, the monopolist m's pricing policy s^* is a best response to its conjecture $t°$ about the entrant e's entry rule.

(ii) For any $P \in [P_1, P_4]$, $w \in [\underline{w}, \bar{w}]$, $d \in [\underline{d}, \bar{d}]$, and any t :

$$[P_i, w, d] \to \{E, S\}$$

$$\int_{(\underline{a},\underline{c})}^{(\bar{a},\bar{c})} \left[\delta^e c\pi^e(a,c,w,d) - K\right] t^*(s°(a,c),w,d) dH^m(a,c)$$

$$\geq \int_{(\underline{a},\underline{c})}^{(\bar{a},\bar{c})} \left[\delta^e c\pi^e(a,c,w,d) - K\right] t(s°(a,c),w,d) dH^m(a,c)$$

where $[(\underline{a}, \underline{c}), (\bar{a}, \bar{c})]$ is the range of possible values of (a, c) and H^m is the probability distribution function for (a, c) [entrant e's beliefs about (a, c)]. That is, the strategy t^* is a best response for the entrant e to its conjecture $s°$.

(iii) $(s^*, t^*) = (s°, t°)$.

The actual and conjectured strategies coincide.

Numerical Examples

In the numerical examples of Kamerschen and Park (1992b, in Tables 2 and 3), where the random parameter values were $a_2 = 10 \pm 2$, $b = 1$, $c_2 = 4 \pm 0.8$ and $d = 0.1/0.35$ in time 2, the monopolist m's $(m = 1, 2)$ equilibrium price and profit were

$$
\begin{aligned}
&P_4(\bar{a},\bar{c})=8.4, \quad P_3(\bar{a},\underline{c})=7.6\\
&P_2(\underline{a},\bar{c})=6.4, \quad P_1(\underline{a},\underline{c})=5.6\\
&\pi_4(\bar{a},\bar{c})=6.48, \quad \pi_3(\bar{a},\underline{c})=9.68\\
&\pi_2(\underline{a},\bar{c})=1.28, \quad \pi_1(\underline{a},\underline{c})=2.88
\end{aligned}
$$

These examples are used in this paper, with additional parameters of $w_2 = 4.5 \pm 0.9$, $K = 1.5$, and $\delta^m = \delta^e = 1$, so that the pricing behaviour of the monopolist m facing the potential entrant e ($e = 1, 2$) can be compared with that of the duopolist i ($i = 1, 2$) in Kamerschen and Park (1992b). The random parameter value for w satisfies the assumed proportionality. The monopolist m's period 3 Cournot equilibrium profits are computed for each combination of (a, c, w, d), by putting above parameter values into (6):

$$
\begin{aligned}
&c\pi^m(\bar{a},\bar{c},\bar{w},\bar{d})=2.94 \quad c\pi^m(\bar{a},\bar{c},\bar{w},\underline{d})=3.24\\
&c\pi^m(\bar{a},\bar{c},\underline{w},\bar{d})=2.23 \quad c\pi^m(\bar{a},\bar{c},\underline{w},\underline{d})=2.97\\
&c\pi^m(\bar{a},\underline{c},\bar{w},\bar{d})=5.18 \quad c\pi^m(\bar{a},\underline{c},\bar{w},\underline{d})=5.13\\
&c\pi^m(\bar{a},\underline{c},\underline{w},\bar{d})=4.22 \quad c\pi^m(\bar{a},\underline{c},\underline{w},\underline{d})=4.78\\
&c\pi^m(\underline{a},\bar{c},\bar{w},\bar{d})=0.64 \quad c\pi^m(\underline{a},\bar{c},\bar{w},\underline{d})=0.66\\
&c\pi^m(\underline{a},\bar{c},\underline{w},\bar{d})=0.34 \quad c\pi^m(\underline{a},\bar{c},\underline{w},\underline{d})=0.54\\
&c\pi^m(\underline{a},\underline{c},\bar{w},\bar{d})=1.86 \quad c\pi^m(\underline{a},\underline{c},\bar{w},\underline{d})=1.63\\
&c\pi^m(\underline{a},\underline{c},\underline{w},\bar{d})=1.31 \quad c\pi^m(\underline{a},\underline{c},\underline{w},\underline{d})=1.44
\end{aligned}
$$

Taking the differences between the period 2 monopoly profits and period 3 Cournot profits of the monopolist m for each corresponding combination of random variables, the monopolist m's period 3 rewards from deterring entry are

$$
\begin{aligned}
&R(\bar{a},\bar{c},\bar{w},\bar{d})=3.54 \quad R(\bar{a},\bar{c},\bar{w},\underline{d})=3.24\\
&R(\bar{a},\bar{c},\underline{w},\bar{d})=4.25 \quad R(\bar{a},\bar{c},\underline{w},\underline{d})=3.51\\
&R(\bar{a},\underline{c},\bar{w},\bar{d})=4.50 \quad R(\bar{a},\underline{c},\bar{w},\underline{d})=4.55\\
&R(\bar{a},\underline{c},\underline{w},\bar{d})=5.46 \quad R(\bar{a},\underline{c},\underline{w},\underline{d})=4.90\\
&R(\underline{a},\bar{c},\bar{w},\bar{d})=0.64 \quad R(\underline{a},\bar{c},\bar{w},\underline{d})=0.62\\
&R(\underline{a},\bar{c},\underline{w},\bar{d})=0.94 \quad R(\underline{a},\bar{c},\bar{w},\underline{d})=0.74\\
&R(\underline{a},\underline{c},\bar{w},\bar{d})=1.02 \quad R(\underline{a},\underline{c},\bar{w},\underline{d})=1.25\\
&R(\underline{a},\underline{c},\underline{w},\bar{d})=1.57 \quad R(\underline{a},\underline{c},\underline{w},\underline{d})=1.44
\end{aligned}
\tag{11}
$$

According to (10), the entrant e's ($e = 1, 2$) third period payoffs if entry occurs, for each combination of different parameter values (a, c, w, d), are

$$
\begin{aligned}
&c\pi^e(\bar{a},\bar{c},\bar{w},\bar{d})-1.5 = 0.54 && c\pi^e(\bar{a},\bar{c},\bar{w},\underline{d})-1.5 = 1.06\\
&c\pi^e(\bar{a},\bar{c},\underline{w},\bar{d})-1.5 = 2.75 && c\pi^e(\bar{a},\bar{c},\underline{w},\underline{d})-1.5 = 3\\
&c\pi^e(\bar{a},\underline{c},\bar{w},\bar{d})-1.5 = 0.02 && c\pi^e(\bar{a},\underline{c},\bar{w},\underline{d})-1.5 = 0.85\\
&c\pi^e(\bar{a},\underline{c},\underline{w},\bar{d})-1.5 = 1.98 && c\pi^e(\bar{a},\underline{c},\underline{w},\underline{d})-1.5 = 2.72\\
&c\pi^e(\underline{a},\bar{c},\bar{w},\bar{d})-1.5 = -1.23 && c\pi^e(\underline{a},\bar{c},\bar{w},\underline{d})-1.5 = -1.12\\
&c\pi^e(\underline{a},\bar{c},\bar{w},\bar{d})-1.5 = -0.18 && c\pi^e(\underline{a},\bar{c},\bar{w},\underline{d})-1.5 = -0.21\\
&c\pi^e(\underline{a},\underline{c},\bar{w},\bar{d})-1.5 = -1.4 && c\pi^e(\underline{a},\underline{c},\bar{w},\underline{d})-1.5 = -1.2\\
&c\pi^e(\underline{a},\underline{c},\underline{w},\bar{d})-1.5 = 0.59 && c\pi^e(\underline{a},\underline{c},\underline{w},\underline{d})-1.5 = -0.36
\end{aligned}
\qquad (12)
$$

We can specify the probability distribution for numerical examples. For both $H^m(a, c)$ and $H^e(w, d)$ of the plane distribution, all the random parameters are assumed to be independently distributed with $H^m(a = a) = x = 1 - H^m(a = \underline{a})$, $H^m(c = c) = y = 1 - H^m(c = \underline{c})$, $H^e(w = \bar{w}) = z = 1 - H^e(w = \underline{w})$, *and* $H^e(d = d) = 1/2 = 1 - H^e(d = \underline{d})$.

The independence assumption on the distribution of random variables does not contradict the assumption of proportionality.

At equilibrium, the only values of $P_i(a, c)$ to be observed are $s^*(\bar{a},\bar{c})$, $s^*(\bar{a},\underline{c})$, $s^*(\underline{a},\bar{c})$, and $s^*(\underline{a}, \underline{c})$. If all the possible random pairs of the above four observable prices are categorised, there are 24 possible combinations as the separating case, and 232 combinations as the pooling case.

Among 24 separating cases, the only sensible equilibrium set is $s^*(\bar{a},\bar{c})=8.4$, $s^*(\bar{a},\underline{c})=7.6$, $s^*(\underline{a},\bar{c})=6.4$, and $s^*(\underline{a},c)=5.6$.

Thus, the observation of $P_i(a, c)$ in the above separating equilibrium allows the values of a and c to be exactly inferred by the entrant e.

According to the entrant e's payoff list, (12), entry occurs if $s^*(\bar{a})$ is observed and will not if $s^*(\underline{a})$ is observed. Therefore, the probability of entry is simply x, which is $Pr\left[a=\bar{a}\right]$.

In pooling equilibrium, observing $P_i(a, c)$ gives no information about the actual values of a and c. In this situation, the entrant e enters only if its expected payoff is positive: that is,

$$
\begin{aligned}
&xyc\pi^e(\bar{a},\bar{c},w,d)+x(1-y)c\pi^e(\bar{a},\underline{c},w,d)+\\
&(1-x)yc\pi^e(\underline{a},\bar{c},w,d)+(1-x)(1-y)c\pi^e(\underline{a},\underline{c},w,d)-K\geq 0
\end{aligned}
$$

We can determine the specific range of probabilities, x and y, for which the entrant e can make a positive expected payoff, by looking at the following four possibilities.

First, consider where $(w,d)=(\bar{w},\bar{d})$. For the entrant *e (e = 1, 2)* to enter, it must be that:

$$
\begin{aligned}
&0.54xy+0.02x(1-y)-1.23(1-x)y-1.4(1-x)(1-y)\geq 1.5\\
&\rightarrow 0.35xy+1.42x+0.17y\geq 2.9\\
&\rightarrow (x+0.49)(y+4.06)\geq 10.28\\
&y\geq\frac{10.28}{(x+0.49)}-4.06
\end{aligned}
$$

$$
\begin{array}{lll}
\textit{if } x=2.04, y\geq 0, & \textit{or if } y=0, & x\geq 2.04;\\
\textit{if } x=1.54, y\geq 1, & \textit{or if } y=1, & x\geq 1.54;\\
\textit{if } x=1, \quad y\geq 1, & \textit{or if } y=2.84, & x\geq 1;\\
\textit{if } x=0.5, \ y\geq 1, & \textit{or if } y=6.32, & x\geq 0.5;\\
\textit{if } x=0, \quad y\geq 1, & \textit{or if } y=18.3, & x\geq 0;
\end{array}
$$

Therefore, when $(w,d)=(\bar{w},\bar{d})$, there does not exist any combination of x and y that allows the entrant e to make a positive payoff. Second, when $(w,d)=(\bar{w},\underline{d})$, for the entrant *e (e = 1, 2)* to enter, it must be that:

$$
\begin{aligned}
&1.06xy+0.85x(1-y)-1.12(1-x)y-1.2(1-x)(1-y)\geq 1.5;\\
&\rightarrow 0.13xy+2.05x+0.8y\geq 2.7\\
&\rightarrow (x+0.62)(y+15.77)\geq 30.55\\
&y=\frac{30.55}{(x+0.62}-15.77
\end{aligned}
$$

$$
\begin{array}{lll}
\textit{if } x=1, \quad y\geq 3.09 & \textit{or if } y=3.09, & x\geq 1;\\
\textit{if } x=1.32, \ y\geq 1 & \textit{or if } y=0, & x\geq 1.32;\\
\textit{if } x=1.26, \ y\geq 0.5 & \textit{or if } y=0.5, & x\geq 1.26;\\
\textit{if } x=1.2, \ y\geq 1 & \textit{or if } y=1, & x\geq 1.2;\\
\textit{if } x=0, \quad y\geq 33.5 & \textit{or if } y=33.5, x\geq 0; &
\end{array}
$$

Therefore, when $(w,d)=(\bar{w},\underline{d})$, there does not exist any combinationof x and y that allows the entrant e to make a positive payoff. Thus, if $w=\bar{w}$, there is no chance for the entrant e to enter, regardless of whether d is high or low. Third, when $(w,d)=(\underline{w},\bar{d})$, for the entrant *e (e = 1, 2)* to enter, it must be that:

$2.75xy + 1.98x(1 - y) - 0.18(1 - x)y - 0.59(1 - x)(1 - y) \geq 1.5$
$\rightarrow 0.36xy + 2.57x + 0.41y \geq 2.09$
$\rightarrow (x + 1.14)(y + 7.14) \geq 13.95$

$$y \geq \frac{41.55}{(x+1.15)} - 23.69$$

$if\, x = 1,\ y \geq -0.62,\quad or\ if\quad y = -0.62,\ x \geq 1;$
$if\, x = 0.81,\ y \geq 0,\quad or\ if\quad y = 0,\quad x \geq 0.81;$
$if\, x = 0.69,\ y \geq 0.5,\ or\ if\quad y = 0.5,\quad x \geq 0.69;$
$if\, x = 0.57,\ y \geq 1,\quad or\ if\quad y = 1,\quad x \geq 0.57;$
$if\, x = 0,\ y \geq 5.1,\quad or\ if\quad y = 5.1,\quad x \geq 0.$

Therefore, when $(w,d) = (\underline{w},\bar{d})$, if the value x is smaller than or equal to 0.57, even the low cost entrant e has no chance to make a positive payoff for $y \in [0, 1]$. If the value of x or $\Pr\left[a = \bar{a}\right]$ is larger than 0.57, there are chances for the entrant e of $(\underline{w},\bar{d})$ to enter by making a positive payoff. That is, if, $0.81 < x \leq 1$, the low cost entrant e would enter for all y. For $0.57 < x \leq 0.81$, if the value x is higher, the low cost entrant e enters even against the low cost monopolist m, and if the value x is lower, the entrant e enters only against the high cost monopolist m.

Finally, when $(w,d) = (\underline{w},\underline{d})$, for the entrant e $(e = 1, 2)$ to enter, it must be that

$3xy + 2.72x(1 - y) - 0.21(1 - x)y - 0.36(1 - x)(1 - y)\ \ 1.5$
$\rightarrow 0.13xy + 3.08x + 0.15y\ \ 1.86$
$\rightarrow (x + 1.15)(y + 23.69)\ \ 41.55$

$$y \geq \frac{41.55}{(x+1.15)} - 23.69$$

$if\, x = 1,\quad y \geq -4.36,\ or\ if\, y = -4.36,\ x \geq 1;$
$if\, x = 0.61,\ y \geq 0,\quad or\ if\, y = 0,\quad x \geq 0.61;$
$if\, x = 0.57,\ y \geq 0.5,\quad or\ if\, y = 0.5,\quad x \geq 0.57;$
$if\, x = 0.53,\ y \geq 1,\quad or\ if\, y = 1,\quad x \geq 0.53;$
$if\, x = 0,\quad y \geq 12.44,\ or\ if\, y = 12.44,\ x \geq 0.$

Therefore, if $x \leq 0.53$, the entrant e of $(\underline{w}, \underline{d})$ has no chance to enter for all y, while if $x > 0.53$, there are chances for the entrant e of $(\underline{w}, \underline{d})$ to enter by making a positive payoff. More specifically for the latter case, if $x > 0.61$, the low cost entrant e enters for all y. If $0.53 < x \leq 0.61$, the low cost entrant e with the higher value of x would enter even against the low cost monopolist m, but the low cost entrant e with the lower value of x would enter only against the high cost monopolist m.

Inference and Equilibrium

When the potential entrant *e* $(e = 1, 2)$ is not directly knowledgeable about the values of a and c, it attempts to make inferences about them from the observed monopolist's price. If the entrant can infer the actual values of a and c, there would be no point to limit pricing, and the monopolist *m* $(m = 1, 2)$ would simply set its price at the short-run profit-maximising levels. When the entrant cannot exactly infer the true values of a and c, it calculates its expected payoff. If it is positive, it enters, but it stays out, otherwise. Therefore, the monopolist's pricing strategies can possibly influence the entrant's decision. With nonexact inference, two possibilities are discussed: first, the entrant cannot infer any values of a and c; second, it can infer only partially.

A. Exact inference case

1. Separating equilibrium.

We can construct a separating equilibrium condition which is consistent with the results of the numerical examples. That is, the monopolist *m*'s $(m = 1, 2)$ pricing strategy (s^*) and the entrant *e*'s
$(e = 1, 2)$ response (t^*) to it are, respectively, given as:

$$s^*(\bar{a},\bar{c})=8.4, \quad s^*(\bar{a},\underline{c})=7.6, \quad s^*(\underline{a},\bar{c})=6.4, \quad s^*(\underline{a},\underline{c})=5.6$$

$$t^*(\underline{a})=S, \quad t^*(\bar{a})=E$$

$t^*(a)$ is clearly a best response to $s^*(a, c)$. We can show that $s^*(a, c)$ is optimal, given $t^*(a)$.

(1) When $(a,c)=(\bar{a},\bar{c})$, any type of firm *e* with any pair of (w, d) would enter. Therefore, the monopolist *m* would earn second period profits only, regardless of what the established price levels are. That is, $p_i(a, c)$ yields, by (8), the monopolist *m*'s payoffs as:

$$\pi(9,\bar{a},\bar{c})=6.3; \quad \pi(8.4,\bar{a},\bar{c})=6.48; \quad \pi(7.6,\bar{a},\bar{c})=6.16$$

$$\pi(6.4,\bar{a},\bar{c})=4.48; \quad \pi(5.6,\bar{a},\bar{c})=2.56.$$

Since $\pi(7.6,\bar{a},\underline{c})$ is the largest payoff, $s^*(\bar{a},\bar{c})=8.4$ is globally optimal to the monopolist *m*.

(2) When $(a,c)=(\bar{a},\underline{c})$, as in the previous situation, any type of firm *e* would enter. Therefore, $p_i(a, c)$ yields the monopolist *m*'s payoffs as:

$\pi(8.4,\bar{a},\underline{c}) = 9.36; \quad \pi(7.6,\bar{a},\underline{c}) = 9.68;$

$\pi(6.4,\bar{a},\underline{c}) = 8.96; \quad \pi(5.6,\bar{a},\underline{c}) = 7.68.$

Since $\pi(7.6,\bar{a},\underline{c})$ is the largest payoff, $s^*(\bar{a},\underline{c}) = 7.6$ is globally optimal to the monopolist *m*.

(3) When $(a,c) = (\underline{a},\bar{c})$, any type of firm *e* with any pair of *(w, d)* would not enter. Therefore, the monopolist *m* would earn second period profits plus the discounted value of *a* reward to deterring entry, and $p_i(a, c)$ yields, by (9), the monopolist *m*'s expected payoffs as:

$\pi(7.6,\underline{a},\bar{c}) + 0.63z + 0.84(1-z) = 1.4 - 0.21z;$

$\pi(6.4,\underline{a},\bar{c}) + 0.63z + 0.84(1-z) = 2.12 - 0.21z; \mathit{and}$

$\pi(5.6,\underline{a},\bar{c}) + 0.63z + 0.84(1-z) = 1.8 - 0.21z;$

Since the expected payoff, *(2.12 - 0.21z)*, is the largest one, $s^*(\underline{a},\bar{c}) = 6.4$ is globally optimal for all *z*.

(4) When $(a, c) = (\underline{a}, \underline{c})$, any type of firm *e* would not enter. Therefore, $p_i(a, c)$ yields the monopolist *m*'s expected payoffs as:

$\pi(6.4, \underline{a}, \underline{c}) + 1.14z + 1.51(1 - z) = 4.07 - 0.37z$; and
$\pi(5.6, \underline{a}, \underline{c}) + 1.14z + 1.51(1 - z) = 4.39 - 0.37z.$

Since the expected payoff, *(4.39 - 0.37z)*, is the largest one, $s^*(\underline{a}, \underline{c}) = 5.6$ is globally optimal for all *z*.

Therefore, $s^*(a, c)$ is globally optimal for all *z*. Since $s^*(a, c) = P_i(a, c)$ with exact inference $s^*(a, c)$ is not a limit pricing strategy. The separating equilibrium set, $s^*(a, c)$, holds for $x \in [0, 1]$ and $y \in [0, 1]$.

This separating equilibrium can be compared with the one with exact inference in Kamerschen and Park (1994) when the entrant could infer the values of *a* and *c*, and the monopolist could also infer the value of *w*. When the monopolist could not infer the true value of *w*, it could simply set its price at the short-run profit-maximising levels (instead of the one separating and two pooling equilibria), as if it knew the value of *w*. However, the separating equilibrium is a sensible Nash-equilibrium, regardless of the monopolist's knowledge of *w*. This difference is due to the different payoff list of the entrant: that is, in this paper, the entrant's decision to enter does not depend on *w* in the entrant's payoff list (12); whereas in Kamerschen and Park (1994) it depends on w for the random movements $(\bar{a},\underline{c})$ and $(\underline{a},\bar{c})$, in the entrant's payoff list (14).

Therefore, if the potential entrant e $(e = 1, 2)$ infers the true values of a and c, the monopolist m $(e = 1, 2)$ would not limit its price. For the entrant e's decision to enter is not influenced by the monopolist m's pricing strategies. This is true regardless of whether the monopolist can infer the actual value of w. If the entrant e cannot infer the monopolist m's characteristics, a and c, it would enter if and only if its expected payoff is positive. Therefore, the entrant e's decision is affected by the monopolist m's pricing strategy.

To look at specific equilibria, we review the entrant e's strategy with incomplete information. If $w = \bar{w}$, there is no chance for the entrant e to enter, but if $w = \underline{w}$, it has a chance to enter. That is,

i) if $(w,d)=(\underline{w},\bar{d})$ and $0.57 < x \leq 0.81$, or $(w, d) = (\underline{w}, \underline{d})$ and $0.53 < x \leq 0.61$, the low cost entrant e with higher value x enters even against the low cost monopolist m, but the low cost entrant e with lower value x enters only against the high cost monopolist $\bar{m}$:

ii) if $(w,d)=(\underline{w},\bar{d})$ and $0 \leq x \leq 0.57$, or $(w,d)=(\underline{w}, \underline{d})$ and $0 \leq x \leq 0.53$, even the low cost entrant e has no chance to enter;

iii) if $(w,d)=(\underline{w},\bar{d})$ and $0.81< x \leq 1$, or $(w, d) = (\underline{w}, \underline{d})$ and $0.61< x \leq 1$, the low cost entrant e always has chances to enter.

The first one involves zero inference, and the second and third together involves partial inference. However, both zero and partial inference give the same result in these specific numerical examples. They both are completely complementary for the same equilibrium set. We investigate partial inference for the two different degrees of substitutability: one is high substitutable degree $d = \bar{d}$; the other is low substitutable degree $(d = \underline{d})$.

B. Partial inference case of $(w,d)=(\underline{w},\bar{d})$ *and* $0.81 < x \leq 1$ *or* $0 \leq x \leq 0.57$

2. Separating equilibrium

The monopolist m's $(m = 1, 2)$ pricing strategies (s^*) and the entrant e's $(e = 1, 2)$ responses (t^*) to it are, respectively, given as

$$s^*(\bar{a},\bar{c})=8.4 \quad s^*(\bar{a},\underline{c})=7.6,$$

$$t^*(\bar{a},\bar{w})=S \quad t^*(\bar{a},\underline{w},\bar{d})=\begin{bmatrix} E \text{ if } p_i(a,c)>6.4 \\ S \text{ otherwise} \end{bmatrix}$$

and

$s^*(\underline{a},\bar{c})=6.4, \quad s^*(\underline{a},\underline{c})=5.6$

$t^*(\underline{a})=S.$

Clearly, $t^*(a, w, d)$ are the best responses to $s^*(a, c)$ for x ∈ [0.81, 1] and x ∈ [0, 0.57]. But it should be shown that $s^*(a, c)$ are optimal, given $t^*(a, w, d)$.

(1) When $(a,c)=(\bar{a},\bar{c})$ the entrant e with the pair of $(\underline{w},\bar{d})$ would enter, as long as $p_i(a, c)$ is greater than 6.4. But if $p_i \leq 6.4$, even the lower cost firm e would not. Therefore, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(9,\bar{a},\bar{c})+1.77z=63+1.77z$$

$$\pi(8.4,\bar{a},\bar{c})+1.77z=6.48+1.77z$$

$$\pi(7.6,\bar{a},\bar{c})+1.77z=6.16+1.77z$$

$$\pi(6.4,\bar{a},\bar{c})+1.77z+2.13(1-z)=6.61-036z$$

Since (6.48 + 1.77z) > (6.61 - 0.36z) for z > 0.06, $s^*(\bar{a},\bar{c})$ is globally optimal for z ∈ [0.06, 1].

(2) When $(a,c)=(\bar{a},\underline{c})$, as in the previous results the firm e of $(\underline{w},\bar{d})$ only enters if $p_i(a, c) > 6.4$. Thus, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(8.4,\bar{a},\underline{c})+225z=936+225z$$

$$\pi(7.64,\bar{a},\underline{c})+225z=9.68+225z$$

$$\pi(8.4,\bar{a},\underline{c})+225z+2.73(1-z)=11.69-0.48z$$

Since *(9.68 + 2.25z) > (11.69 - 0.48z)* for *0.74<* z, $s^*(\bar{a},\underline{c})$ is globally optimal for z ∈ [0.74, 1].

(3) When $(a,c)=(\bar{a},\underline{c})$, any type of entrant e would not enter. Thus, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(7.6,\underline{a},\bar{c})+032z+0.47(1-z)=1.03-0.15z;$$

$$\pi(6.4,\underline{a},\bar{c})+032z+0.47(1-z)=1.75-0.15z; \textit{and}$$

$$\pi(5.6,\underline{a},\bar{c})+032z+0.47(1-z)=1.43-0.15z.$$

Since the expected payoff, *(1.75 - 0.15z)*, is the largest one, $s^*(\underline{a},\bar{c})=6.4$ is globally optimal for all z.

(4) When $(a, c) = (\underline{a}, \underline{c})$, as in the preceding situation, any type of firm e would not enter. Therefore, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(6.4, \underline{a}, \underline{c}) + 0.51z + 0.79(1 - z) = 3.35 - 0.28z; \text{ and}$$
$$\pi(5.6, \underline{a}, \underline{c}) + 0.51z + 0.79(1 - z) = 3.67 - 0.28z.$$

Since the expected payoff, $(3.67 - 0.28z)$, is the largest one, $s^*(\underline{a}, \underline{c}) = 5.6$ is globally optimal for all z.

Therefore, $s^*(a, c)$ are globally optimal for $z \in [0.74, 1]$. Since $s^*(a, c) = p_i(a, c)$, $s^*(a, c)$ is not a limit pricing strategy.

3. One-pair pooling equilibrium

The monopolist m's $(m = 1, 2)$ pricing strategies (s^*) and the entrant e's $(e = 1, 2)$ responses (t^*) to it are, respectively, given as

$$s^*(\underline{a}, \underline{c}) = 8.4 \quad s^*(\bar{a}, \underline{c}),$$
$$t^*(\bar{a}, \bar{w}) = S, \quad t^*(\bar{a}, \underline{w}, \bar{d}) = \begin{bmatrix} E \text{ if } p_i(a,c) > 6.4 \\ S \quad otherwise \end{bmatrix}$$

and

$$s^*(\underline{a}, \bar{c}) = 6.4, \quad s^*(\underline{a}, \underline{c}) = 5.6$$
$$t^*(\underline{a}) = S.$$

Clearly, $t^*(a, w, d)$ are the best responses to $s^*(a, c)$ for $x \in [0.81, 1]$ and $x \in [0, 0.57]$. We can show that $s^*(a, c)$ are optimal, given $t^*(a, w, d)$.

(1) When $(a,c) = (\bar{a}, \bar{c})$, the low cost firm e of $(\underline{w}, \bar{d})$ would enter, as long as $p_i(a, c) > 6.4$. So, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(8.4, \bar{a}, \bar{c}) + 1.77z = 6.48 + 1.77z;$$
$$\pi(7.6, \bar{a}, \bar{c}) + 1.77z = 6.16 + 1.77z;$$
$$\pi(6.4, \bar{a}, \bar{c}) + 1.77z + 2.13(1 - z) = 6.61 - 0.36z.$$

Since $(6.48 + 1.77z) > (6.61 - 0.36z)$ for $0.06 < z$, $s^*(\bar{a}, \bar{c})$ is globally optimal for $z \in [0.06, 1]$.

(2) When $(a,c) = (\bar{a}, \underline{c})$, as in the preceding scenario, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(8.4,\bar{a},\underline{c}) + 2.25z = 9.36 + 2.25z;$$

$$\pi(7.6,\bar{a},\underline{c}) + 2.25z = 9.68 + 2.25z;$$

$$\pi(6.4,\bar{a},\underline{c}) + 2.25z + 2.73(1-z) = 11.69 + 0.48z;$$

Since *(11.69 - 0.48z) > (9.68 + 2.25z)* for $z < 0.74$, $s^*(\bar{a},\underline{c})$ is globally optimal for $z \in [0, 0.74]$. The situations of $(a,c) = (\underline{a},\bar{c})$ and $(a, c) = (\underline{a}, \underline{c})$ in this one-pair pooling equilibrium are exactly the same as those in the separating equilibrium of partial inference. That is $s^*(\underline{a},\bar{c})$, $s^*(\underline{a}, \underline{c}) = 5.6$ are globally optimal for all z. Therefore, $s^*(a, c)$ are globally optimal for $z \in [0.06, 0.74]$. Since $s^*(\underline{a},\bar{c}) \le p_i(\bar{a},\underline{c})$, $s^*(a,c)$ is a limit pricing strategy.

4. Triple pooling equilibrium

The monopolist m's $(m = 1, 2)$ pricing strategies (s^*) and the entrant e's $(e = 1, 2)$ responses (t^*) to it are, respectively, given as

$$s^*(\bar{a},\bar{c}) = s^*(\bar{a},\underline{c}) = 6.4,$$

$$t^*(\bar{a},\bar{w}) = S, \qquad t^*(\bar{a},\underline{w},\bar{d}) = \begin{bmatrix} E & \text{if } p_i(a,c) > 6.4 \\ S & \text{otherwise} \end{bmatrix}$$

and

$$s^*(\bar{a},\underline{c}) = 6.4, \quad s^*(\underline{a},\underline{c}) = 5.6$$

$$t^*(\underline{a}) = S$$

Clearly, $t^*(a, w, d)$ are the best responses to $s^*(a, c)$ for $x \in [0.81, 1]$ and $x \in [0, 0.57]$. We can demonstrate that $s^*(a, c)$ are optimal, given $t^*(a, w, d)$.

(1) When $(a,c) = (\bar{a},\bar{c})$, the firm e of $(\underline{w},\bar{d})$ enters, as long as $p_i(a, c) > 6.4$. Therefore, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(8.4,\bar{a},\bar{c}) + 1.77z = 6.48 + 1.77z;$$

$$\pi(7.6,\bar{a},\bar{c}) + 1.77z = 6.16 + 1.77z;$$

$$\pi(6.4,\bar{a},\bar{c}) + 1.77z + 2.13(1-z) = 6.61 - 0.36z;$$

Since *(6.61 - 0.36z) > (6.48 + 1.77z)* for $z < 0.06$, $s^*(\bar{a},\bar{c})$ is globally optimal for $z \in [0, 0.06]$.

(2) When $(a,c)=(\bar{a},\underline{c})$, as in the above case, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$\pi(8.4,\bar{a},\underline{c})+225z=936+225z;$

$\pi(7.6,\bar{a},\underline{c})+225z=9.68+225z;$

$\pi(8.4,\bar{a},\underline{c})+225z+2.73(1-z)=11.69-0.48z;$

Since $(11.69 - 0.48z) > (9.68 + 2.25z)$ for $z<0.74$, $s^*(\bar{a},\underline{c})$ is globally optimal for $z \in [0, 0.74]$. The situations of $(a,c)=(\underline{a},\bar{c})$ and $(a, c) = (\underline{a}, \underline{c})$ in this triple pooling equilibrium are the same as the of partial inference. That is, $s^* = (\underline{a},\bar{c}) = 6.4$ and $s^*(\underline{a}, \underline{c}) = 5.6$ are globally optimal for all z. Therefore, $s^*(a, c)$ are globally optimal for $z \in [0, 0.06]$. Since $s^*(\bar{a},\bar{c})=s^*(\bar{a},\underline{c})< p_i(\bar{a},\underline{c})$, $s^*(a,c)$ is a limit pricing strategy. We can now search for equilibria with zero inference of $(w,d)=(\underline{w},\bar{d})$ and $0.57< x \leq 0.81$. But the equilibrium sets are the same as those with partial inference. Even if the potential entrant e cannot infer the value of a, it would enter if $P_i(a, c) > 6.4$, and this entrant e's behaviour is exactly the same as if it is able to infer the true value of a. This is because the entrant e's third period payoffs, (12), are symmetrically structured into the division of positive and negative payoff by the criterion of whether the value of a is high or low. With this symmetric structure of the entrant e's payoffs, the inference of the value of a only is not useful at all. Since the three equilibrium sets with partial inference, one separating and two pooling, hold true with zero inference, they are the equilibrium price sets of $(w,d)=(\underline{w},\bar{d})$ for all x.

C. Partial inference case of $(w, d) = (\underline{w}, \underline{d})$, and $0.61< x \leq 1$ or $0 \leq x \leq 0.53$

5. Separating equilibrium

The monopolist m's $(m = 1, 2)$ pricing strategies (s^*) and the entrant e's $(e = 1, 2)$ responses (t^*) to it are, respectively, given as

$$s^*(\bar{a},\bar{c})=8.4 \quad s^*(\bar{a},\underline{c})=7.6,$$

$$t^*(\bar{a},\bar{w})=s, \quad t^*(\bar{a},\underline{w},\underline{d})=\begin{bmatrix} E & \text{if } p_i(a,c)>6.4 \\ S & \text{otherwise} \end{bmatrix}$$

and

$$s^*(\underline{a},\bar{c})=6.4 \quad s^*(\underline{a},\underline{c})=5.6$$

$$t^*(\underline{a}) = S.$$

Obviously, $t^*(a, w, d)$ are the best responses to $s^*(a, c)$ for $x \in [0.61, 1]$ and $x \in [0, 0.53]$. We can prove that $s^*(a, c)$ are optimal, given $t^*(a, w, d)$.

(1) When $(a,c)=(\underline{a},\underline{c})$, the firm e with the pair of $(\underline{w}, \underline{d})$ would enter, if $p_i(a, c) > 6.4$. Therefore, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(9,\bar{a},\bar{c})+1.62z=63+1.62z$$

$$\pi(8.4,\bar{a},\bar{c})+1.62z=6.48+1.62z$$

$$\pi(7.6,\bar{a},\bar{c})+1.62z=6.16+1.62z$$

$$\pi(6.4,\bar{a},\bar{c})+1.62z+1.76(1-z)=624+0.14z$$

Since $(6.48 + 1.62z) > (6.24 - 0.14z)$ for $z \geq 0$, $s^*(\bar{a},\bar{c})$ is globally optimal for all z.

(2) When $(a,c)=(\underline{a},\bar{c})$, as in the preceding situation, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(8.4,\bar{a},\underline{c})+228z=936+228z;$$

$$\pi(7.6,\bar{a},\underline{c})+228z=9.68+228z;$$

$$\pi(6.4,\bar{a},\underline{c})+228z+2.45(1-z)=11.41-0.17z;$$

Since $(9.68 + 2.28z) > (11.41 - 0.17z)$ for $z > 0.71$, $s^*(\bar{a},\underline{c})$ is globally optimal for $z \in [0.71, 1]$.

(3) When $(a,c)=(\underline{a},\bar{c})$, any type of firm e would not enter. Thus, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(7.6,\underline{a},\bar{c})+0.31z+037(1-z)=0.93-0.06z;$$

$$\pi(6.4,\underline{a},\bar{c})+0.31z+037(1-z)=1.65-0.06z;\ and$$

$$\pi(5.6,\underline{a},\bar{c})+0.31z+037(1-z)=133-0.06z;$$

Since the expected payoff, $(1.65 - 0.06z)$, is the largest one, $s^*(\underline{a},\bar{c})=6.4$ is globally optimal for all z.

(4) When $(a, c) = (\underline{a}, \underline{c})$, as in the previous scenario, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(6.4,\underline{a},\underline{c})+0.63z+0.72(1-z)=328-0.09z;\ and$$

$$\pi(5.6,\underline{a},\underline{c})+0.63z+0.72(1-z)=3.6-0.09z$$

Since the expected payoff, $(3.6 - 0.09z)$, is the largest one, $s^*(\underline{a}, \underline{c}) = 5.6$ is globally optimal for all z.

Therefore, $s^*(a, c)$ are globally optimal for $z \in [0.71, 1]$. Since $s^*(a, c) = p_i(a, c)$, $s^*(a, c)$ is not a limit pricing strategy.

5. One-pair pooling equilibrium

The monopolist m's $(m = 1, 2)$ pricing strategies (s^*) and the entrant e's $(e = 1, 2)$ responses (t^*) to it are, respectively, given as

$$s^*(\bar{a},\bar{c}) = 8.4, \quad s^*(\bar{a},\underline{c}) = 6.4$$

$$t^*(\bar{a},\bar{w}) = S, \quad t^*(\bar{a},\underline{w},\underline{d}) = \begin{bmatrix} E & \textit{if } p_i(a,c) > 6.4 \\ S & \textit{otherwise} \end{bmatrix}$$

and

$$s^*(\underline{a},\bar{c}) = 6.4, \quad s^*(\underline{a},\underline{c}) = 5.6$$

$$t^*(\underline{a}) = S.$$

It is obvious that $t^*(a, w, d)$ are the best responses to $s^*(a, c)$ for $x \in [0.61, 1]$ and $x \in [0, 0.53]$. We show that $s^*(a, c)$ are optimal, given $t^*(a, w, d)$.

(1) When $(a,c) = (\bar{a},\bar{c})$, the firm e of $(\underline{w}, \underline{d})$ would enter, as long as $p_i(a, c) > 6.4$. Therefore, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(8.4,\bar{a},\bar{c}) + 1.62z = 6.48 + 1.62z;$$

$$\pi(7.6,\bar{a},\bar{c}) + 1.62z = 6.16 + 1.62z;$$

$$\pi(6.4,\bar{a},\bar{c}) + 1.62z + 1.76(1-z) = 6.24 - 0.14z.$$

Since $(6.48 + 1.62z) > (6.24 - 0.14z)$ for $z \geq 0$, $s^*(\bar{a},\bar{c})$ is globally optimal for all z.

(2) When $(a,c) = (\bar{a},\underline{c})$, as in the previous case, $p_i(a, c)$ yields the monopolist m's expected payoffs as:

$$\pi(8.4,\bar{a},\underline{c}) + 2.28z = 9.36 + 2.28z;$$

$$\pi(7.6,\bar{a},\underline{c}) + 2.28z = 9.68 + 2.28z;$$

$$\pi(6.4,\bar{a},\underline{c}) + 2.28z + 2.45(1-z) = 11.41 - 0.17z.$$

Since $(11.41 - 0.17z) > (9.68 + 2.28z)$ for $z < 0.71$, $s^*(\bar{a},\underline{c})$ is globally optimal for $z \in [0, 0.71]$.

When $(a,c) = (\underline{a},\bar{c})$ and $(a, c) = (\underline{a}, \underline{c})$ are exactly the same as those in the separating equilibrium of partial inference. That is, $s^*(\underline{a},\bar{c})$ and $s^*(\underline{a}, \underline{c}) = 5.6$ are globally optimal for all z. Therefore, $s^*(a, c)$ are globally optimal for $z \in [0, 0.71]$. Since $s^*(\bar{a},\underline{c}) <$, $s^*(a, c)$ is a limit pricing strategy.

The equilibrium price sets with zero inference of $(w, d) = (\underline{w}, \underline{d})$ and $0.53 < x \leq 0.61$ are the same as with partial inference with $(w, d) = (\underline{w}, \underline{d})$ and $0 \leq x \leq 0.53$ or $0.61 < x \leq 1$. This is because, with the symmetric structure of period 3 payoffs, (12), the entrant e's inferability of whether the industry demand is high or low doesn't provide a useful information to itself. Therefore, these two equilibrium sets with partial inference, one separating and one pooling, hold true for all x.

D. Comparison and implications

With exact inference, where the entrant e $(e = 1,2)$ can infer the monopolist m's $(m = 1, 2)$ characteristics, one separating equilibrium exists for all x, y and z. With non-exact inference, where the low cost entrant e of both high and low d cannot infer or partially infer the true values of a and c, there exist both separating and pooling equilibria, each of which holds for all x and y. The reason that all the optimal pricing strategies of the monopolist m hold for all x in non-exact inference is because of the symmetrical structure of the entrant e's payoffs by the criterion of a. Once the monopolist m's optimal pricing is satisfied for all x, it should also hold true for all y. This is because the monopolist m's price is dominated by the demand shock, rather than cost shock.

To be more specific with non-exact inference, if the firm e of $(\underline{w}, \bar{d})$ cannot exactly infer the firm m's characteristics, a and c (including the possibility that it infers the value of a only), one separating equilibrium exists for $z \in [0.74, 1]$; one-pair pooling equilibrium exists for $z \in [0.06, 0.74]$; and triple pooling equilibrium exists for $z \in [0, 0.06]$. On the other hand, if the firm e of $(\underline{w}, \underline{d})$ cannot exactly infer the actual values of a and c (including the case that it infers the value of a only), one separating equilibrium exists for $z \in [0.71, 1]$ and one-pair pooling equilibrium exists for $z \in [0, 0.71]$.

All the three pooling equilibria involve the limit pricing in different degrees, whereas all the three separating equilibria including the one with exact inference do not. With exact inference, the probability of entry is x, and with non-exact inference of both high and low d is $x(1 - z)/2$, satisfying the respective probability restriction for each equilibrium price set. The probability of entry in the former is greater than that in the latter two cases.[1] That is, the probability of entry in limit pricing equilibrium is lower than that with complete information. This result differs from Milgrom and Roberts' (1982) conclusion that the probability that entry actually occurs in limit pricing equilibrium can be lower, the same, or even higher than in a regime of complete information. Therefore, the tradeoff for society between lower prices and deterred entry exists with heterogeneous products, and the monopolist can rationalise its behaviour of limit pricing.

Table 1. contrasts the monopolist m's *(m = 1, 2)* pricing behaviour when it does and when it does not face potential entry. When the entrant e *(e = 1, 2)* can infer the monopolist m's characteristics of a and c, the firm e's decision of whether to enter is not influenced by the firm m's limit pricing. The firm m's optimal decision of separating equilibrium is not constrained by any of the probabilities, x, y, and z (see the last column of Table 1). When the entrant e cannot exactly infer the monopolist m's characteristics, there is a incentive for the firm m to deter the entry by limiting price. The firm m's optimal pricing behaviour is constrained to a certain range of probability z, which represents the firm m's belief about the firm e's cost, w.

The restriction on the probability z for each equilibrium set tells how the monopolist m comes up with a specific equilibrium set. For example, consider non-exact inference with high d. The probability restriction, $z \in [0.06, 0.74]$ for one-pair pooling equilibrium, is the probability range required for the monopolist m to deter entry or to accept the low cost firm e only, with its price set, (8.4, 6.4, 6.4, 5.6). Each price of the equilibrium price set can be either limiting or unlimiting, depending on the pair of random variables, a and c. If the monopolist m believes that the entrant e's cost is so high as to fall in the probability range, [0.74, 1], it sets the separating equilibrium set, without worrying about whether the entrant e is able to infer. When the monopolist m believes the firm e's cost is so low that $z \in [0, 0.06]$, it commands the maximum limit pricing such as (6.4, 6.4, 6.4, 5.6). The same interpretation can be applied to non-exact inference with low d. Therefore, the higher entrant e's cost perceived by the monopolist m, the less it limits price, and *vice versa*.

The perceived demand curve can be derived for the monopolist m *(m = 1, 2)* that faces the heterogeneous entrant e *(e = 1, 2)*. It is discontinuous at a certain output level and horizontal for a certain range of output levels, and the horizontal part of the demand curve not only depends on what kinds of belief the firms have about the unknown nature moves, but also on how substitutable the two firms' goods are. With exact inference, the monopolist m would not limit its prices, and thus has the same downward-sloping curve as it had without any potential entry. With non-exact inference, however, the monopolist m has an incentive to limit price to deter entry.

Thus, its perceived demand curve is horizontal and also shows that the more substitutable goods the entrant e produces, the more the monopolist m limits price.

Consider the situation with a high degree of substitutability,

(a) in Figure 2. If $0.74 < z \leq 1$, the monopolist m's perceived demand curve is the downward-sloping curve, *dx*, for all output levels. However, if $0.06 < z \leq 0.74$ ($0 < z \leq 0.06$), its perceived demand curve is the discontinuous one, *dghkx (defkx)*, and becomes horizontal for the output range, *hk (fk)*. With a low degree of substitutability,

(b) in Figure 2, if $0.71 < z \leq 1$, the monopolist m's perceived demand curve is the downward-sloping curve, *dx*, for all output levels. If $0 < z \leq 0.71$, its perceived demand curve is the discontinuous one, *dnrsx*, and horizontal for the output range, *rs*.

Therefore, in both cases of substitutable degree, if the monopolist m believes that the entrant e's cost is high, its perceived demand curve includes no or a small flat part, while if the monopolist m believes that the entrant e has a low cost, its perceived demand curve includes a large flat part.

Table 1.: Comparison of the monopolist m's pricing behaviour with and without potential entry

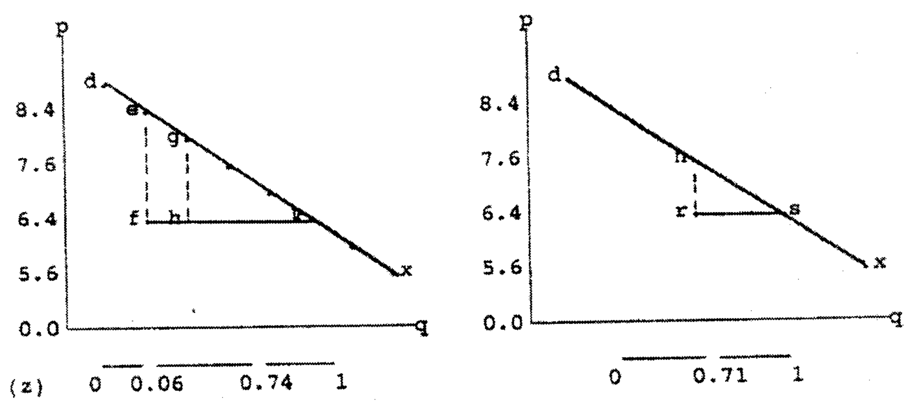
			$s^*(\bar{a},\bar{c})$	$s^*(\bar{a},\underline{c})$	$s^*(\underline{a},\bar{c})$	$s^*(\underline{a},\underline{c})$	Probability
Monopolist m's Price without Potential Entery			8.4	7.6	6.4	5.6	
Monpolist m's Price with Potential Entry	Exact Inference	Separating equili.	8.4	7.6	6.4	5.6	$0 < z \leq 1$
	Non-Exact Inference ($d = \bar{d}$)	Separating equili.	8.4	7.6	6.4	5.6	$0.74 < z \leq 1$
		One-pair pooling equili.	8.4	6.4	6.4	5.6	$0.06 < z \leq 0.74$
		Triple pooling equili.	6.4	6.4	6.4	5.6	$0 \leq z \leq 0.06$
	Non-exact Inference ($d = \underline{d}$)	Separating equili.	8.4	7.6	6.4	5.6	$0.71 < z \leq 1$
		One-pair pooling equili.	8.4	6.4	6.4	5.6	$0 < z \leq 0.711$

Note: This table assumes that the coefficients have the following values: $a_2 = 10 \pm 2$, b = 1, $c_2 = 4 \pm 0.8$, $w_2 = 4.5 \pm 0.9$, $d = 0.1/0.35$ and $K = 1.5$, which satisfy the assumed proportionality. All the equilibrium prices are satisfied for $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

It is useful to compare the limit pricing behaviours of the monopolist in the homogeneous products and with heterogeneous products. Table 2 contains the relevant information for both zero and partial inference. This comparison is possible, because we have used consistent parameter values in those numerical examples of both Kamerschen and Park (1994) and here. A general tendency, as shown in the Table 2., is that, for both zero and partial inference, the more substitutable goods the potential entrant produces to enter with, the more the monopolist limits its price. This was, in fact, shown in Figure 2.

To investigate the general tendency, we can look at the partial inference with homogeneous goods and the heterogeneous products with a of high degree of substitutability degree ($d = \bar{d}$) in Table 2. The monopolist's pricing behaviours are apparently similar, because there are three equilibria, separating, one-pair and triple, in each case. However, facing the probability restrictions of z in the last column, the monopolist i with heterogeneous goods case of $(d = \bar{d})$ sets its triple pooling equilibrium price only if it believes that the entrant's cost is extremely low ($0 \leq z \leq 0.06$), while the monopolist with homogeneous goods sets the same triple pooling price even when it believes that the entrant's cost is not very low ($0 \leq z \leq 0.55$). We can make a similar comparison for other illustrations.

Figure 2.: Unconventional demand curve with heterogeneous goods



(a) With High Degree of Substitutability $(d = \bar{d})$

(b) With Low Degree of Substitutability $(d = \underline{d})$

To investigate the general tendency, examine partial inference with homogeneous goods and with heterogeneous goods with high substitutable degree $(d = \bar{d})$ in Table 2. The monopolist's pricing behaviours in both settings are apparently similar, because there are three equilibria, separating, one-pair and triple, in each case. However, if we look at the probability restrictions of z in the last column, the monopolist m with heterogeneous products of $(d = \bar{d})$ will set its triple pooling equilibrium price only if it believes that the entrant's cost is extremely low ($0 \leq z \leq 0.06$), whereas the monopolist with homogeneous products sets the same triple pooling price even when it believes that the entrant's cost is not very low ($0 \leq z \leq 0.55$). We can make a similar comparison for other possibilities.

Table 2.: Comparison of limit pricing behaviours of the monopolist facing homogeneous and heterogeneous entry

	Equlibrium	$s^*(\bar{a},\bar{c})$	$s^*(\bar{a},\underline{c})$	$s^*(\underline{a},\bar{c})$	$s^*(\underline{a},\underline{c})$	Probability
Homogeneous Goods For Zero Inference	Separating	8.4	7.6	6.4	5.6	$0.7 < z \leq 1$
	One-pair pooling	8.4	7.6	5.6	5.6	$0.6 < z \leq 0.7$
	Triple pooling	8.4	5.6	5.6	5.6	$0.13 < z \leq 0.6$
	Quadruple pooling	5.6	5.6	5.6	5.6	$0 < z \leq 0.13$
Homogeneous Goods For Partial Inference	Separating	8.4	7.6	6.4	5.6	$0.86 < z \leq 1$
	One-pair pooling	8.4	7.6	5.6	5.6	$0.55 < z \leq 0.86$
	Triple pooling	8.4	5.6	5.6	5.6	$0 < z \leq 0.55$
Heterogeneous Goods $(d=\bar{d})$ For both Zero and Partial Inference	Separating	8.4	7.6	6.4	5.6	$0.74 < z \leq 1$
	One-pair pooling	8.4	6.4	6.4	5.6	$0.06 < z \leq 0.74$
	Triple poolong	6.4	6.4	6.4	5.6	$0 < z \leq 0.06$
Heterogeneous Goods $(d=\underline{d})$ For Both Zero and Partial Inference	Separating	8.4	7.6	6.4	5.6	$0.71 < z \leq 1$
	One-pair poolong	8.4	6.4	6.4	5.6	$0 < z \leq 0.71$

Comparison of the Limit Pricing behaviour of the Monopolist with the Pricing Behaviour of the Duopolistic Firm[2]

Tables 3. and 4. summarise the main results of the parameterised examples respectively, for the low degree $(d = \underline{d})$ and high degree $(d = \bar{d})$ of substitutability. Therefore, in each case of substitutable degree, the monopolist m's $(m = 1, 2)$ price changes are compared with the duopolist i's $(i = 1, 2)$ for each pair of random movements.

First, compare the first and fourth columns in both tables, to determine when the two random variables move in the same direction. For the low substitutable degree $(d = \underline{d})$ of Table 3, we see the maximum possible fluctuations of price in both firms, m and i. In all cases, the firm m's price change, (5.6 - 8.4), is greater than the firm i's (5.38 - 8.07). As the substitutable degree rises to $d = 0.35$ in Table 4, the firm i's price change, (4.98 - 7.47), get smaller than before, and the firm m also exercises a maximum limit pricing if it believes that the entrant e's cost is extremely low $(0 \leq z \leq 0.06)$. If the degree of substitutability increases further, then the firm i's price change gets much smaller than before, whereas the range of the probability z for the firm m's maximum limiting price gets widened. When the substitutable degree goes up, the firm m's greater limit pricing goes with the firm i's smaller price change, and thus their relative pricing behaviour remains the same. The overall pricing behaviour of the firm m is relatively more fluctuating than that of the firm i, when the two random variables of a and c move in the same direction.

Second, compare the second and third column of both tables to when the two random variables move in the opposite direction. For the low substitutable degree $(d = \underline{d})$ of Table 3, the firm i's price moves between 6.25 and 7.2. But the price of the firm m facing potential entry makes different movements, depending on its belief about the firm e's cost. If the firm m believes that the entrant e's cost is very high (0.71 > z), its prices move between 6.4 and 7.6, and fluctuate more than the firm i's. This is the same for exact inference. If, however, the firm m believes that the entrant e's cost is not very high ($0 \le z \le 0.71$), its price of one-pair pooling equilibrium doesn't move at all. Therefore, it is difficult to make a clear statement of which price fluctuates more, because it depends on whether the potential entrant e is exactly informed of the actual values of a and c, and on what kind of belief the firm m has about the firm e's cost. With a high substitutable degree $(d = \bar{d})$ of Table 4, the price comparison of the firms, i and m, is similar to that in the case of $d = \underline{d}$, because as the substitutable degree rises, the firm i's price change, (5.98 - 6.46), becomes smaller than before, and the firm m exercises more limit pricing ($0 \le z \le 0.74$).

Table 3.: Comparison of the pricing behaviours of the monopolist m and the duopolist i with a substitutable degree $d = 0.1$

			$s^*(\bar{a},\bar{c})$	$s^*(\bar{a},\underline{c})$	$s^*(\underline{a},\bar{c})$	$s^*(\underline{a},\underline{c})$	Probability
Duopoly i's $(i=1,2)$			6.73				
Duopoly i's Price in Period 2			8.07	7.2	6.25	5.38	
Monopolist m's $(m=1,2)$ Price in Period 1			7				
Monopolist m's Price without Potential Entry			8.4	7.6	6.4	5.6	
Monopolist m Price with Potential Entry	Exact Inference	Separating equili.	8.4	7.6	6.4	5.6	$0 \le z \le 1$
	Non-Exact Inference $(d = \underline{d})$	Separating equili.	8.4	7.6	6.4	5.6	$0.71 \le z \le 1$
		One-pair pooling equili.	8.4	7.6	6.4	5.6	$0 \le z \le 0.71$

Note: This table assumes that the coefficients have the following values: $a_2 = 10 \pm 2$, $b = 1$, $c_2 = 4 \pm 0.8$, $w_2 = 4.5 \pm 0.9$, $d = 0.1$ and $K = 1.5$, which satisfy the assumed proportionality. All the relevant equilibrium prices are satisfied for $0 \le x \le 1$ and $0 \le y \le 1$.

Table 4: Comparison of the pricing behaviours of the monopolist m and the duopolist i with substitutable degree $d = 0.35$

			$s^*(\bar{a},\bar{c})$	$s^*(\bar{a},\underline{c})$	$s^*(\underline{a},\bar{c})$	$s^*(\underline{a},\underline{c})$	Probability
Duopoly i's $(i=1, 2)$ Price in Period 1			6.22				
Duopoly i's Price in Period 2			7.47	6.46	5.98	4.98	
Monopolist m's $(m=1, 2)$Price in Period 1			7				
Monopolist m's Price without Potential Entry			8.4	7.6	6.4	5.6	
Monopolist m Price with Potential Entry	Exact Inference	Separating equili.	8.4	7.6	6.4	5.6	$0 \leq z \leq 1$
	Non-Exact Inference $(d = \bar{d})$	Separating equili.	8.4	7.6	6.4	5.6	$0.74 \leq z \leq 1$
		One-pair pooling equili.	8.4	6.4	6.4	5.6	$0.06 \leq z \leq 0.74$
		Triple pooling equili.	8.4	6.4	6.4	5.6	$0 \leq z \leq 0.06$

Note: This table assumes that the coefficients have the following values: $a_2 = 10 \pm 2$, $b = 1$, $c_2 = 4 \pm 0.8$, $w_2 = 4.5 \pm 0.9$, $d = 0.35$ and $K = 1.5$, which satisfy the assumed proportionality. All the relevant equilibrium prices are satisfied for $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

Third, compare the third and fourth column in both tables, to investigate when demand is low. The firm i's price in Table 3 and Table 4 moves, respectively, between 5.38 and 6.25 and between 4.98 and 5.98, depending on whether the marginal cost is low or high. The firm m's prices move between 5.6 and 6.4 with both exact and non-exact inference and for both low and high d. In this specific illustration of lower demand, therefore, the firm i's price change is greater than the firm m's for all case, and this dominance of the firm i over the firm m in the price fluctuation becomes greater, as the degree of substitutability increases.

Fourth, compare the first and second column in both tables to the high demand case. The firm i's price in Table 3 moves between 7.2 and 8.07. If the firm m believes that the firm e's cost is high ($0.71 > z$), its price moves between 7.6 and 8.4, and it is the same for exact inference. If, however, the firm m believes that the firm e's cost is not high ($0 \leq z \leq 0.71$), its price moves between 6.4 and 8.4. Therefore, which price fluctuates more depends on what kinds of belief the firm m has about the firm e's cost. As the substitutable degree increases to $d = 0.35$ in Table 4, the firm i's price change, (6.46 - 7.47), gets larger than before, but at the same time, the firm m's price that moves between 6.4 and 8.4, shifts up to a more plausible probability range, ($0.06 < z \leq 0.74$). But the firm m's price is stuck at 6.4 if it believes that the entrant e's cost is extremely low ($0 \leq z \leq 0.06$). Therefore, the firm *m*'s pricing behaviour becomes erratic when the substitutable degree increases, even though it is still not clear which price change of the firms *i* and *m* is greater.

Fifth, compare the second and fourth column in both tables to study low cost, which is analytically quite similar to the fourth case. The firm *i*'s price in Table 3 moves between 5.38 and 7.2. If the firm *m* believes that the firm *e*'s cost is high ($z > 0.71$), its price moves between 5.6 and 7.6, and it is the same for exact inference. If, however, the firm *m* believes that the firm *e*'s cost is not very high ($0 \leq z \leq 0.71$), its price moves between 5.6 and 6.4. Therefore, which price fluctuates more depends on what kinds of belief the firm *m* has about the firm *e*'s cost. As the substitutable degree increases, as shown in Table 4, the firm *i*'s price change, (4.98 - 6.46) gets smaller than before, but at the same time, the firm *m*'s price, that moves between 5.6 and 6.4, has a greater probability range, $0 \leq z \leq 0.74$. Even if the degree of substitutability increases, the firm *m*'s overall pricing behaviour, relative to the firm *i*'s, doesn't change, and it is still not clear which price change of the firms *i* and *m* is greater.

Finally, compare the first and third columns in both tables for the high cost scenario. The firm *i*'s price in Table 3 moves between 6.25 and 8.07. The firm *m*'s price moves between 6.4 and 8.4 in all cases, and fluctuates more than the firm *i*'s. When the substitutable degree goes up, as shown in Table 4, the firm *m*'s price is stuck at 6.4 if it believes that the firm *e*'s cost is extremely low ($0 \leq z \leq 0.06$), but at the same time, the firm *i*'s price change becomes smaller (6.46 and 7.47) than before. As the substitutable degree increases further, the probability range of *z*, in which the firm *m*'s price is stuck at 6.4, expands, and also the firm *i*'s price change becomes even smaller. Therefore, the greater price change of the firm *m* over the firm *i* holds true for any degree of substitutability.

The previous analyses of the six random pair changes are, in order, summarised in Table 5. The price changes from one price to the other (e.g., 5.6 - 8.4) are caused by the random pair changes of the variables, *a* and *c*, such as from $(\underline{a}, \underline{c})$ to $(\bar{a}, \bar{c})$. For a given random pair changes of *a* and *c*, the monopolist *m*'s ($m = 1, 2$) price changes are different, depending on the potential entrant *e*'s ($e = 1, 2$) inferability (exact or non-exact inference) and also on the substitutable degree of the two goods. Even for a given inferability and substitutable degree of the entrant *e* with the given random pair change (indicating a cell in the table), the monopolist's price changes are different, depending on its beliefs about *w*.

Table 5: Summary of the pricing comparison

Movements of Random Variables		Duopolist i's Price Change (i=1, 2)	Monopolist m's (m=1, 2) Price Change				Duopolist i's Price Change (i=1, 2)
			Exact Inference for both d	Non-Exact Inference ($d = \underline{d}$)	Range of z	Non-Exact Inference ($d = \bar{d}$)	
(1)	$(\underline{a}, \underline{c})$ $(\bar{a}, \bar{c})$	5.38-8.07	5.6-8.4	5.6-8.4	$z > 0.06$	5.6-8.4	4.98-7.47
					$z \le 0.06$	5.6-6.4*	
(2)	$(\underline{a}, \bar{c})$ $(\bar{a}, \underline{c})$	6.25-7.2	6.4-7.6	6.4-7.6	$z > 0.74$	6.4-7.6	5.98-6.46
					$z > 0.71$		
				6.4-6.4*	$z \le 0.71$	6.4-6.4*	
(3)	$(\underline{a}, \underline{c})$ $(\underline{a}, \bar{c})$	5.38-6.25	5.6-6.4*	5.6-6.4*	$0 \le z \le 1$	5.6-6.4	4.98-5.98
(4)	$(\bar{a}, \underline{c})$ $(\bar{a}, \bar{c})$	7.2-8.7	7.6-8.4*	7.6-8.4	$z > 0.74$	7.6-8.4	6.46-7.47
					$z > 0.71$	6.4-8.4	
				6.4-8.4	$z > 0.06$		
					$z \le 0.06$	6.4-6.4*	
(5)	$(\underline{a}, \underline{c})$ $(\bar{a}, \underline{c})$	5.38-7.2	5.6-7.6	5.6-7.6	$z > 0.74$	5.6-7.6	4.98-6.46
					$z > 0.71$		
				5.6-6.4*	$z \le 0.71$	5.6-6.4*	
(6)	$(\underline{a}, \bar{c})$ $(\bar{a}, \bar{c})$	6.25-8.07	6.4-8.4	6.4-8.4	$z > 0.06$	6.4-8.4	5.98-7.47
					$z \le 0.06$	6.4-6.4*	

The monopolist m's price changes that are smaller than the duopolist i's *(i = 1, 2)* are denoted by an asterisk. Which firm's price change is greater depends on the firm e's inferability of a and c and the firm m's beliefs about w. We examine exact inference for both low and high degree of substitutability. In the two possibilities of random pair movement, (3) of low demand and (4) of high demand, the firm m's price changes are slightly dominated by the firm i's. In the other four random pair movements, however, the firm m's price changes dominate the firm i's.

Next, look at non-exact inference of low substitutable degree $(d = \underline{d})$. In the two cases of random pair movement, (1) and (6), the firm m's price changes are the same as with exact inference, and still dominate the firm i's. In (3), the firm m's price changes are also the same as in exact inference, and are still dominated by the firm i's. However, both (2) and (5), where the firm m's price changes dominate the firm i's in exact inference, include significant probability ranges ($z \le 0.71$), where the firm i's price changes dominate the firm m's. In (4), the firm m's price changes dominate the firm i's in a significant probability range ($z \le 0.71$), even though the reverse holds true with exact inference. In non-exact inference and high substitutable degree $(d = \bar{d})$, the comparison of the two firms' pricing becomes complicated. Now, (3) is the only situation in which the firm m's price change is the same as exact inference. Therefore, when the random pair moves from from $(\underline{a}, \underline{c})$ to $(\underline{a}, \bar{c})$, the firm m's price changes are dominated by the firm i's for all circumstances (both exact and non-exact inferences and both low and high degrees of the substitutability). In all the other illustrations except (4), however, the firm m's price changes are dominated by the firm i's if the firm m believes that the entrant e's cost is low, and the reverse holds if it believes that the entrant e's cost is high. More specifically, in (2) and (5) [(1) and [6)], the firm m's price changes are dominated by the firm i's for the probability range, $z \leq 0.74$ ($z \leq 0.06$), but the reverse holds otherwise. In (4), the firm m's price changes are dominated by the firm i's if it believes that the entrant e's cost is high ($z > 0.74$) and low ($z \leq 0.06$), but the reverse holds otherwise.

Therefore, the results of numerical examples in this paper are much different from Kamerschen and Park (1992b) where the monopolist did not face the potential entry. Where potential entry exists, the monopolist m's price change is not only dominated by the duopolist i's in a significant way (expressed by the probability range), but also shows internal dynamics, in that they are different for the same random pair movements of a and c, depending on the entrant e's inferability of the monopolist's characteristics and the monopolist's conjectures of the entrant e's characteristics. If the substitutable degree of two heterogeneous goods increases further, it is expected to have the result that the firm i's price changes dominate the firm m's in a more significant way. This is what Stigler (1968) showed empirically and what Rotemberg and Saloner (1987) tried to show theoretically.

Conclusions

In this paper, the monopolistic model of Kamerschen and Park (1992b) is extended with an additional assumption that the established monopolist m ($m = 1, 2$) faces potential entrant e ($e = 1, 2$) with the degree of substitutability, d. The pricing behaviour of the monopolist m facing the potential entry is compared with that of the duopolist i ($i = 1, 2$). Although it is proper to examine the pricing behaviour of the duopolist i facing potential entry, it would provide only small details. Furthermore, the established rival is an imminent problem to the firm i, rather than the potential entry.

The firm m facing the potential entry has limited information on the firm e's cost (w) with nothing about the substitutable degree (d), whereas the firm e has only the incomplete information on the firm m's cost (c) and industry demand (a). With imperfect information, the firm e attempts to enter the industry for a positive profit, whereas the firm m tries to deter the entry to maximise its long-run payoff. Therefore, the existence of potential entry influences the firm m's pricing behaviour. It is assumed that the firm *m* has a belief about the firm *e*'s characteristics, $H^e(w, d)$, whereas the firm *e* tries to infer the firm *m*'s characteristics from the observed $p_i(a, c)$, using its conjectures on the firm *m*'s pricing behaviour, $H^m(a, c)$. A Nash-equilibrium of rational expectations character is defined when the firms' actual and conjectured strategies coincide.

A parameterised family of examples is used for analysis. With exact inference, the separating equilibrium exists for all x, y and z, for the firm *e*'s entry decision would not be influenced by the firm *m*'s pricing strategy. When the firm *e* cannot infer the actual values of a and c, the firm *e* takes the rule of positive expected payoffs, and thus the firm *m*'s pricing strategy affects the firm *e*'s entry decision. In this non-exact inference circumstance, there exist two cases of zero and partial inference. However, the firm *m*'s equilibrium pricing strategies are the same in these two cases, because of the symmetric structure of the firm *e*'s payoffs by the criterion of industry demand, a. Thus, the firm *m*'s pricing behaviour is indifferent to whether the firm *e* is able to infer the value of a. This implies that, with non-exact inference, firm *m*'s equilibrium pricing strategies hold true for all x and y.

With non-exact inference, if the substitutable degree of two heterogeneous goods is low $(d = 0.1)$, one separating equilibrium exists for $z \in [0.71, 1]$, and one pooling equilibrium exists for $z \in [0, 0.71]$. If the degree of substitutability is high $(d = 0.35)$, one separating equilibrium exists for $z \in [0.74, 1]$, and two pooling equilibria exist for $z \in [0, 0.74]$. These three separating equilibria including the one with exact inference, do not involve the limit pricing, but the three pooling equilibria do in different degrees.

The probability of entry with exact inference, x, is larger than that with non-exact inference of both high and low substitutable degree, $x(1 - z)/2$. That is, the probability of entry in limit pricing equilibrium is lower than that with complete information. Therefore, the tradeoff for society between lower prices and deterred entry exists, and the monopolist *m* rationalises its behaviour of limit pricing.

The probability restrictions of z tell how the monopolist *m* comes up with a particular equilibrium, given the entrant *e*'s inferability of a and c. That is, the higher the monopolist *m* perceives the entrant *e*'s cost the less the monopolist *m* limits its price, and *vice versa*. The monopolist's equilibrium price set also depends on the substitutable degree of heterogeneous goods of the entrant. With the monopolist *m*'s limit pricing, its perceived demand curve is not necessarily downward sloping and continuous, but discontinuous at a certain output level and horizontal for a certain range of output levels. The horizontal part of its perceived demand curve depends on the monopolist's beliefs about w and the entrant's substitutable degree chosen.

Also, the limit pricing behaviour of the monopolist with homogeneous products (Kamerschen and Park, 1994) is compared with heterogeneous goods products. For both zero and partial inference, the more substitutable the goods of the potential entrant, the more the monopolist limits its price.

Using summary tables for the main results of the parameterizd examples, the price changes of the monopolist m facing the potential entry are compared with those of the duopolist i. The comparison is made for six combinations of random pair movements of *a* and *c*. For exact inference of both a low and high substitutable degree, if the cost shock only increases at either low demand [$(\underline{a}, \underline{c}) \rightarrow (\underline{a}, \bar{c})$] or high demand [$(\bar{a}, \underline{c}) \rightarrow (\bar{a}, \bar{c})$], the firm m's price changes are slightly dominated by the firm *i*'s. In the other four random pair movements, [$(\underline{a}, \underline{c}) \rightarrow (\bar{a}, \bar{c})$], [$(\underline{a}, \bar{c}) \rightarrow (\bar{a}, \underline{c})$], [$(\underline{a}, \underline{c}) \rightarrow (\bar{a}, \underline{c})$], and[$(\underline{a}, \bar{c}) \rightarrow (\bar{a}, \bar{c})$], the firm *m*'s price changes dominate the firm *i*'s.

With non-exact inference of low degree substitutability $(d = \underline{d})$, using the Table 5, the firm *m*'s price changes dominate the firm *i*'s in (1) and (6), while the reverse holds in (3). However, if (2) and (5) occur, the firm *i*'s price changes dominate the firm *m*'s in significant probability ranges ($z \leq 0.71$ for both cases). If (4) occurs, the firm *m*'s price changes dominate the firm *i*'s in a significant probability range ($z \leq 0.71$). With non-exact inference of high substitutable degree $(d = \bar{d})$, (3) is the only situation in which the firm *m*'s price change is still dominated by the firm *i*'s for all *z*. In (2) and (5) [(1) and (6)], the firm *m*'s price changes are dominated by the firm *i*'s for the probability range, $z \leq 0.74$ ($z \leq 0.06$), but the reverse holds otherwise. In (4), the firm *m*'s price changes are dominated by the firm *i*'s for high *z* (z 0.74) and low *z* ($z \leq 0.06$), but the reverse holds otherwise.

In this paper, the monopolist *m*'s price changes are generally dominated by the duopolist *i*'s, and they are also different for the same random pair movements of *a* and *c*, depending on the entrant *e*'s inferability of the monopolist's characteristics *(a, c)* and the monopolist's conjectures of the entrant *e*'s characteristics *(w, d)*. If the substitutable degree of the two heterogeneous goods rises, the firm *i*'s price changes are expected to dominate the firm *m*'s in a more significant way.

NOTES

[1] In Kamerschen and Park (1994), a comparison is made among the probability of entry with exact inference, with zero inference, and with partial inference, by using the monopolist's deterability and the entrant's enterability. However, it was not clear whether the probability of entry with exact inference was greater than and that with non-exact inference.

[2] It is appropriate to analyse the pricing behaviour of the duopolist *i* that faces potential entry. To the duopolist *i*, however, the established rival is an imminent problem, rather than the potential entry. If analysing the pricing behaviour of the duopolist *i* facing the potential entry would add only small details, its cost is too high.

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