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Low-Sensitivity, Single-Amplifier, Active-*RC* Allpole Filters Using Tables

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A single-amplifier, active-*RC* filters design procedure for common filter types, such as Butterworth and Chebyshev, using tables with normalized filter component values, is presented. The considered filters consist of *RC* ladder network in operational amplifier's positive feedback path. Tables with normalized component values having equal capacitors and equal resistors were presented by some authors [1, 2]. In this paper, we presented new tables for designing filters with optimized sensitivity to passive circuit components. A considerable improvement in sensitivity is achieved using the design technique called »impedance tapering«. The presented filters are up to the 6$^{\text{th}}$-order. The sensitivity problem generally limits the use of higher than 6$^{\text{th}}$-order single-amplifier filters. Low-pass and high-pass filters design is presented.

Key words: active-*RC* filters, minimal-*RC* filters, low power, low sensitivity, normalized components

1 INTRODUCTION

Passive filters are usually designed using handbooks with filter tables [3–5]. They are typically applied for the design of passive ladder-*RLC* filters. Besides filter transfer function parameters for many common filter types, handbooks usually contain tables with normalized filter component values, as well [3, 4].

Active filters are commonly designed using closed-form design equations and/or filter-design programs [6, 7]. In a majority of applications they are realized as cascade structures of second-order (»biquads«) and/or third-order (»bitriplets«) sections filters, and explicit formulas for calculation of filter components, as well as, step by step design procedures are available.

However, the high-order, single-amplifier, non-cascade filter designs have very often complicated design procedures and since there exists no explicit formulas, numerical calculations have to be performed. Those filters are referred to as *canonical* or *minimal-RC* because they have minimum number of components. In this paper we present filter tables with components for designing single-amplifier, active-*RC* filters up to the 6$^{\text{th}}$-order, for Butterworth and Chebyshev approximation types. Based on the preliminary results from previous paper [8], extended (final) results including practical design examples as guidance for engineers are presented. The need for design tables with normalized filter component values in the form of a handbook is approved. Compared to the filter design tables for some common active-*RC* filters given in [1, 2], the tables in this paper are produced using an optimization procedure for the low sensitivity to passive component tolerances.

In active-*RC* filter design, the most important parameters for evaluation of a filter's section quality are: simple realizability, repeatability, a possibility of straightforward procedure of parameter calculation, small number of components, low power consumption, low noise performance and the most often low filter's magnitude sensitivities to passive component tolerances and/or active gain variations. The filters, designed in this paper, posses minimum passive sensitivities for a given topology. The sensitivity is reduced using the design technique, called »impedance tapering« [9]. All calculations are performed numerically, including the calculation of filters components, as well as, optimizations for filters low sensitivity. Schoeffler's sensitivity measure is used as a basis for comparison of sensitivity to component tolerances of the various filters. Monte Carlo runs are performed as a double check to the Butterworth and Chebyshev filter examples.

2 SINGLE-AMPLIFIER, MINIMAL-*RC*, ACTIVE-*RC* FILTER

Consider an n^{th}-order, allpole, single-amplifier, low-pass filter circuit with positive feedback presented in Fig. 1. The filter belongs to the type of class-4 or Sallen and Key [2, 10]. The voltage transfer function $T(s)=V_2(s)/V_1(s)$ of the filter in Fig. 1 is given by

$$T(s)=\frac{N(s)}{D(s)}=\\ =\frac{Ka_0}{s^n+a_{n-1}s^{n-1}+\ldots+a_i s^i+\ldots+a_i s+a_0} \tag{1}$$

where the pass-band gain K is given by

$$K=\beta=1+\frac{R_F}{R_G} \tag{2}$$

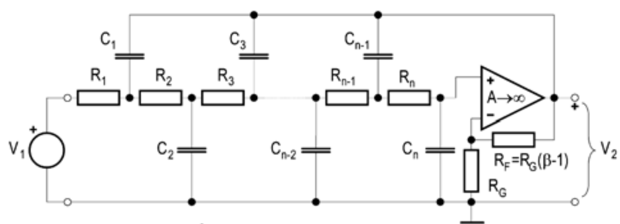


Fig. 1 General n^{th}-order single-amplifier low-pass filter

The coefficients a_i $(i=0, , n-1)$ as functions of components R_k and C_k $(k=1,\ldots, n)$ are presented in Appendix I for the canonical filters up to the 6^{th}-order. There are many different ways how those coefficients can be calculated. In this paper, in Appendix I, we present one recursive way which is easily programmed in any program that can, besides numeric, perform symbolic calculations (e.g. Mathematica, Matlab or MathCad).

Knowing coefficients a_i in the transfer function's (1) denominator polynomial, we can develop a set of nonlinear equations by equating each of the coefficients in the polynomial to the coefficient values of the appropriate Butterworth or Chebyshev polynomials. Butterworth and Chebyshev polynomials are readily obtained from tables [3, 6] or can be calculated using closed-form equations. We can, finally, numerically solve a set of nonlinear equations and calculate passive (normalized) components R_k, C_k $(k=1,\ldots, n)$ and β to build the filter circuit in Fig. 1.

The results of such computer calculation for equal capacitors and equal resistors cases are presented in [1] (and repeated in [2] p. 252). Tables presented there, are calculated for some typical amplifier gains ($\beta=2.0$ and 2.2) and are not optimized for sensitivity.

2.1 Optimization of Sensitivity

In this paper we calculate component values of the filters that are optimized for minimum sensitivity to component tolerances. Component values are obtained numerically and shown in Tables 1–2. The low sensitivity performance is achieved by »impedance tapering« design method (more precisely by capacitive tapering), first introduced in [9], and applied to the LP filter structure in Fig. 1. At the same time a value of the so-called »design frequency« $\omega_0=(R_1C_1)^{-1}$ [9] is optimized iteratively to converge to the value which provides minimum sensitivity filter. Therefore, in Tables 1–2 we obtain various optimal values for R_1 and the corresponding values of the gain β. (Suppose we have chosen $C_1=1$, thus for optimal ω_0, an optimal value of $R_1=(\omega_0C_1)^{-1}$ readily follows.) Optimal values for R_1 are calculated using the procedure shown in Fig. 2, which is implemented with program Mathematica [11].

It should be stressed that the capacitive-tapering factor ρ_C for higher-order filters is smaller, than that for filters of lower-order n. For example, if we try to find solution of the 6^{th}-order Butterworth filter with larger ρ_C (for example $\rho_C\geq3$), it is not possible, because the Newton's method does not converge. Furthermore, larger capacitive tapering factor ρ_C is not permitted since the last capacitor C_n (e.g. $C_6=C_1/\rho_C{}^5$) becomes too small and comparable to the parasitic capacitance of the circuit. Luckily, in high-order filters, even the small tapering factor satisfies enough our needs in degree of desensitization (e.g. $\rho_C=2.0$ is good enough for the filter order $n=6$).

In the sequel, we briefly explain the block diagram shown in Fig. 2. The procedure in Fig. 2 uses Newton's iterative method to calculate the components of the high-order (i.e. 4^{th}-, 5^{th}- and 6^{th}-order) allpole low-pass filters. In addition, the filters are capacitively tapered and numerically optimized for low sensitivity. Optimization of 2^{nd}- and 3^{rd}-order filters follows the same steps in the block diagram shown in Fig. 2, but theirs procedures are simpler in that the calculation of filter components can be performed analytically.

The input data are the values of Butterworth or Chebyshev coefficients a_i $(i=0,\ldots, n–1)$, chosen values of ρ_C, and C_1. Because of capacitive tapering, the procedure calculates $C_k=C_1/\rho_C{}^{k-1}$ $(k=2,\ldots,$

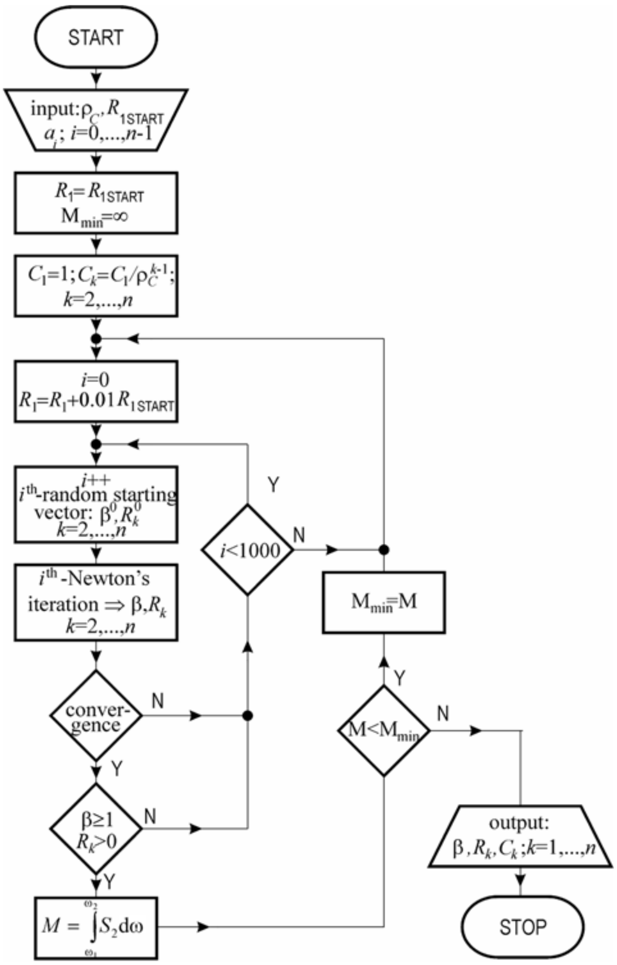


Fig. 2 Block-diagram for solving capacitively-tapered 4th-, 5th- and 6th-order filters and optimising design frequency ω_0 for minimum sensitivity

n). In the first cycle the initial value of the resistor R_1 has to be defined. The resistor R_1 is the design parameter to be adjusted. We choose C_1=1, then by varying the value of R_1 we indeed vary the value of the design frequency $\omega_0=(R_1C_1)^{-1}$. With the value of R_1 we solve the system of non-linear equations for the vector $R_2,\ldots, R_n$ and β. To achieve a solution, we start with vector of random values for $R_2^0,\ldots, R_n^0$ and β^0. Random initial resistors' values R_k^0 are within the interval <0.2, 20> and gain β has a value in the range <1.0, 5.0>. If the proper starting vector is chosen, the Newton's method will, with prescribed accuracy, converge in several steps to the solution, i.e. to the vector $R_2,\ldots, R_n$ and β. If the method fails to converge, we must try another random starting vector. If the convergence is achieved but we have solution with negative resistor values or gain β less than unity, then we, again, choose another random starting vector. We perform random starting vectors for maximum 1000 times. This process of finding solution is known as *random search*. If we choose starting vectors for Newton's iterative solving method by applying some rule, which tries to find all possible solutions, we have *exhaustive search* (brute force algorithm). In solving some problems, if there exist several local minimums, *exhaustive search* tries to find global minimum, but only for a particular type of problems.

If we do not find all real and positive component values $R_2,\ldots, R_n$ and gain $\beta \geq 1$, we proceed with another value of resistor R_1. Finally, when we find realizable elements we proceed to the sensitivity optimization. To accomplish it we calculate multi-parametrical statistical measure M, which is defined in [12], with

$$M = \int_{\omega_1}^{\omega_2} S_2(\omega)\,\mathrm{d}\omega \tag{3}$$

where $S_2(\omega)$ is Schoeffler's sensitivity function defined by

$$S_2(\omega)\,\mathrm{d}\omega = \sum_{i=1}^{N}\left(S_{x_i}^F\right)^2 \tag{4}$$

Function $S_{x_i}^F$ in (4) represents relative sensitivity of the function F (it is the transfer function magnitude, i.e. $F=|T(j\omega)|$ to the parameter x_i (there are $N=2n+2$ passive filter components x_i in an n^{th}-order filter). $S_2(\omega)$ in (3) is a function of frequency, while M is a number instead of a function, and can be used as a goal of optimizing process. M represents area under the function $S_2(\omega)$, with borders of integration from ω_1 to ω_2. Disadvantage is the dependence of number M on the selected borders of integration $\omega_1$1 and ω_2. Therefore, during whole optimization process, we choose the same pair of ω_1 and ω_2. The whole above procedure is repeated with new value for R_1, until minimum value of M is found.

For the 2$^{\text{nd}}$-order filters we do not perform numerical optimization. Instead we apply capacitive tapering factors ρ_C in Tables 1–2 that must obey the following constraint

$$\rho_C \leq \rho_{\text{maks}} = q_p^2 \frac{\left(1+\dfrac{R_2}{R_1}\right)^2}{\left(\dfrac{R_2}{R_1}\right)} \tag{5}$$

The value $\rho_{\max}$ in (5) represents an upper bound of resistive tapering factor ρ_C. For equal resistors

Table 1 **Normalized components of min. sensitivity LP filters using capacitive tapering: Butterworth approximation**

n	ρ_C	C_1	C_2	C_3	C_4	C_5	C_6	R_1	R_2	R_3	R_4	R_5	R_6	β
2	2	1	0.5					1.41421	1.41421					1.0
3	3	1	0.3333	0.1111				1.09	6.01255	4.11983				1.14231
4	3	1	0.3333	0.1111	0.0370			0.7	7.07694	18.1004	8.13008			1.34647
5	2.5	1	0.4	0.16	0.064	0.0256		2.29	2.26474	8.21287	26.8796	8.32969		1.5333
6	2	1	0.5	0.25	0.125	0.0625	0.03125	0.675	8.63542	5.19291	8.97413	23.3141	5.17416	1.74047

Table 2 **Normalized components of min. sensitivity LP filters using capacitive tapering: 0.5 dB Chebyshev approximation**

n	ρ_C	C_1	C_2	C_3	C_4	C_5	C_6	R_1	R_2	R_3	R_4	R_5	R_6	β
2	2.9841	1	0.3351					1.40289	1.40289					1.0
3	3	1	0.3333	0.1111				1.71	6.5827	3.35148				1.31082
4	3	1	0.3333	0.1111	0.0370			1.31	9.61917	19.9327	7.65695			1.4989
5	2.5	1	0.4	0.16	0.064	0.0256		3.96	4.86466	11.4959	29.8471	8.06382		1.66703
6	2	1	0.5	0.25	0.125	0.0625	0.03125	1.8	14.2659	9.77697	11.7128	23.8853	5.17416	1.84611

(i.e. $R_1=R_2$), we have minimum sensitivity of the 2nd-order filters and another form of the constraint (5) given by

$$\rho_C \le \rho_{\max}\Big|_{R_1=R_2} = 4q_p^2 \qquad (6)$$

If we use maximum desensitization with $\rho_C=4q_p^2$ in (6) we obtain in Tables 1–2 »low-Q« realization of 2nd-order filters (having unity gain, i.e. $\beta=1$) [6]. Furthermore, it is apparent from tables that for the 3rd-order filter we have the values of R_2 and R_3 close to each other. This corresponds to the conclusions for the 3rd-order low-pass circuits with minimum sensitivity derived in [9]. For higher-than-third order filters no such rules exist for the minimum sensitivity design; instead the numerical optimization should be carried out.

2.2 Sensitivity Analysis

In this section we compare sensitivities of newly designed capacitively-tapered filters to the filters having equal capacitors and equal resistors.

A sensitivity analysis was performed to the logarithmic gain function $\alpha(\omega)=20\log|T(j\omega)|$/dB, assuming the relative changes of the resistors and capacitors to be uncorrelated random variables, with a zero-mean Gaussian distribution and 1% standard deviation. The standard deviation $\sigma_\alpha(\omega)$/dB (which is related to the Schoeffler's sensitivities) of the variation of the logarithmic gain $\Delta\alpha=8.68588\,\Delta|T(j\omega)|/|T(j\omega)|$/dB, with respect to the passive elements, is calculated for the optimized element values for Butterworth filters given in Table 1, as well as for the equal capacitors and equal resistors filters designed using tables in [1, 2]. For all those filters the standard deviations curves $\sigma_\alpha(\omega)$ are presented in Fig. 3. Observing Fig. 3 we conclude that the capacitively impedance tapered filters have minimum sensitivities to component tolerances of the circuits for all filter orders. Very close results are achieved with filter circuits, having equal resistors. One possible explanation is in the slightly tapered capacitor values, which can be seen in Table 1. The worst sensitivity performances have filters with equal capacitors. Although it is usually not practical to mass produce discrete component active-*RC* filters having unequal capacitors, in the case of IC design various (tapered) capacitor values are acceptable. The sensitivity curves in Fig. 3 are repeated in Fig. 4, sorted by different designs.

Same investigations performed for Chebyshev filter approximations lead to the same conclusions, but they are not presented here.

It is well known that Butterworth filter compared to a Chebyshev filter of equal order has lower pole Qs. Since the sensitivities are related to the magnitudes of filter Q-factors, from the sensitivity point, a Butterworth filter is always preferable to a Chebyshev filter and a low-ripple Chebyshev filter is always preferable to a Chebyshev filter with higher ripple. Preferable filters with regard to low sensitivity have lowest possible pole Qs. Furthermore, filters with high order n have both larger pole Qs and larger number of components than low-order filters and therefore much higher sensitivities. The reason for high sensitivities to component variations of high-order canonical filters is obvious from filter coefficients, presented as functions of components, in Appendix I. Qualitatively this can be explained by the fact that in a canonic

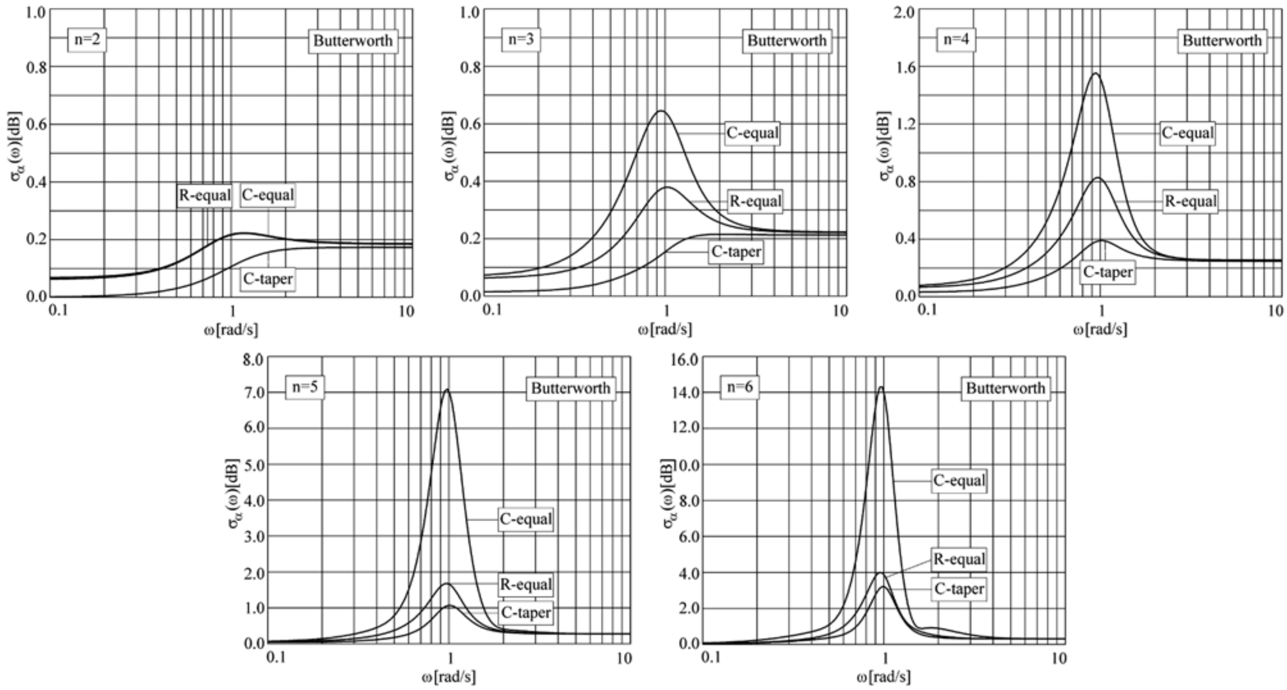


Fig. 3 Schoeffler's sensitivity of normalized Butterworth low-pass filter circuits (up to 6^{th}-order), with components given in Table 1 and in tables presented in [1] *and* [2] *p. 252*

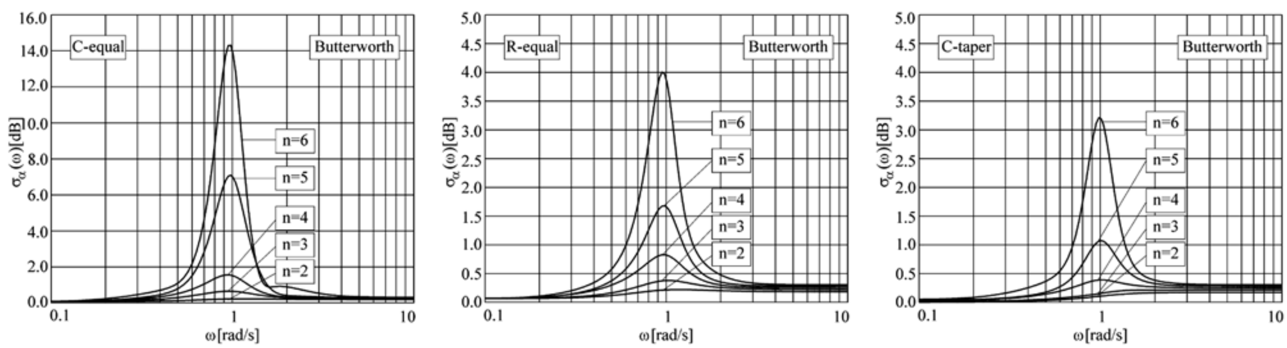


Fig. 4 Schoeffler's sensitivity of normalized Butterworth LP filter circuits in Fig. 3, sorted by type of impedance tapering

realization all the components interact with each other, and a change in any component will be magnified through its repeated occurrence in all coefficients. The larger order n of the filter, the more complicated and larger are the filter coefficients and therefore the larger sensitivities. In cascade realization, in contrast, the change in a component will affect only a localized segment, i.e. only one biquad in a cascade. Therefore, the cascade design has lower sensitivities. But it has more amplifiers (consumes more power) and passive components (resistors) than single-amplifier filters.

As stated above, in the design of active-*RC* filters, very important quality of the filter is its low power consumption. Thus, for low-pass filters of reasonably low order (for example $n \leq 6$), the use of single-amplifier filters, as in Fig. 1 is advantageous over the cascaded 2^{nd}- and 3^{rd}-order Sallen and Key sections [10], although the latter have smaller sensitivities to component tolerances of the circuit. Another advantage of canonical, single-amplifier filters, which are presented here, is that they have only n capacitors and n resistors as well, and their sensitivities are improved applying »impedance tapering« technique. Finally, advantages of single-amplifier filters lie also in the fact that there is no need for signal level optimization procedure which must be performed in multiple amplifier configurations.

3 TRANSFORMATIONS

3.1 LP-LP Transformation

Low-pass to low-pass (LP-LP) frequency transformation (i.e. denormalization) defined by

$$s \to \frac{s}{\omega_0} \tag{7}$$

can be performed by new elements calculations. From normalized element values in Tables 1–2 we calculate denormalized element values by applying

$$R_{LP} = R_0 \cdot R_{nLP}, \quad C_{LP} = \frac{C_{nLP}}{\omega_0 R_0} \tag{8}$$

where index n denotes normalized components values in tables. It is an advantage of normalized component values in that they can be used as a template to directly construct the filter with any cut--off frequency. We do not consider inductances because we deal with active-*RC* filters. Denormalization constants ω_0 is in [rad/s] and R_0 in Ω.

3.2 LP-HP Transformation

Low-pass to high-pass (LP-HP) frequency transformation defined by

$$s \to \frac{\omega_0}{s} \tag{9}$$

can be performed by new elements calculations. From normalized LP filter components in Tables 1–2, HP filter components can be readily calculated using (*RC*-*CR* transformation)

$$R_{HP} = \frac{R_0}{C_{nLP}}, \quad C_{HP} = \left(R_{nLP}\,\omega_0\,R_0\right)^{-1} \tag{10}$$

Note, that while applying LP-HP transformation (9) [i.e. while calculating HP filter components using (10)]; *capacitive tapering* by a factor ρ_C at LP filters, transforms itself into *resistive tapering* factor at HP filters. The value of gain β remains the same as in the LP prototypes.

4 DESIGN OF LOWPASS FILTERS

Suppose we build mixed-signal-processing circuitry on the chip for the hearing aids. It consists of continuous-time to discrete-time converter (C/D) which prepares the signal for the digital signal processing circuitry. Before C/D converter we have to prepare voice signal using an integrated low-pass filter to suppress high frequency components and eliminate aliasing. On the other hand the low-pass filter has to be as simple as possible (therefore we realize it using an active-*RC* filter), must have low power consumption (it has a single amplifier), and must be selective (it is a high order filter).

Because of relatively high filter order it must have acceptably small sensitivity to component tolerance to be realizable. For those reasons we decided to use the n^{th}-order allpole low-pass filter circuit presented in Fig. 1. Because, this filter is rather complicated to calculate (especially for the orders higher than 3$^{\text{rd}}$, where there are no closed form equations) the Tables 1–2 with component values shows to be of enormous help. In the design process we will first decide which type of filter (Butterworth or Chebyshev) best suites our needs.

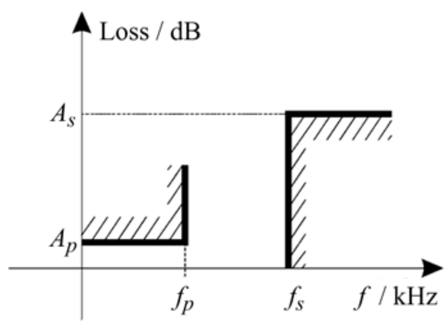


Fig. 5 The attenuation specifications for the LP filter

An anti-aliasing low-pass filter has to satisfy the specifications imposed by the tolerance scheme shown in Fig. 5. The requirements are the maximum pass-band attenuation of A_p = 0.5 dB for the frequencies up to the f_p = 20 kHz, and the minimum stop-band attenuation of A_s = 10 dB for the frequencies above f_s = 32 kHz. The filter should have a unity gain in the pass band (K = 1).

The specifications in Fig. 5 can readily be converted to the normalized LP specifications having normalized cut-off frequency $F_s = f_s/f_p$ (or Ω_s= $=\omega_s/\omega_p$, where $\Omega_s = 2\ \pi F_s$). In that case we write 1 instead f_p and we write F_s instead f_s in Fig. 5. In all subsequent design equations the ratio f_s/f_p is then replaced by the single value F_s.

Butterworth transfer function magnitude has the following form

$$\frac{A_{Loss}(\omega)}{\mathrm{dB}} = 20\log\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^{2n}} \tag{11}$$

The filter order n for the Butterworth filter can be readily calculated

$$n \geq \frac{1}{2}\cdot\frac{\log\left[\dfrac{\left(10^{\frac{A_s}{10}}-1\right)}{\left(10^{\frac{A_p}{10}}-1\right)}\right]}{\log\left(\dfrac{f_s}{f_p}\right)} \tag{12}$$

We choose next larger integer value for n. The cut-off frequency ω_0 is given by

$$\omega_0 = \frac{\omega_p}{\sqrt[2n]{10^{\frac{A_p}{10}}-1}} \tag{13}$$

where the radian frequency ω_p=2 πf_p. The cut-off frequency ω_0 is the frequency on which the filter's magnitude has the 3 dB attenuation. Note that for the calculation of the Butterworth cut-off frequency ω_0, we use the lower frequency ω_p, i.e. the pass-band edge in specifications, because the suppressing of the high frequency components in the anti-aliasing filter is then guaranteed. The Table 1 provides a set of necessary component values for the designing required Butterworth filter.

Chebyshev transfer function magnitude has the following form

$$A_{Loss}(\omega)[\mathrm{dB}] = 20\log\sqrt{1+\varepsilon^2 T_n^2\left(\frac{\omega}{\omega_0}\right)} \tag{14}$$

where the constant $0<\varepsilon\leq 1$ describes the pass-band ripple and $T_n(\omega)$ an n^{th}-order Chebyshev polynomial. The constant ε readily follows from minimum pass-band attenuation (i.e. pass-band ripple) A_p given in dB

$$\varepsilon = \sqrt{e^{\frac{A_p}{10}}-1} \tag{15}$$

The filter order n for the Chebyshev filter can be calculated using

$$n \geq \frac{\mathrm{Arch}\sqrt{\dfrac{10^{\frac{A_s}{10}}-1}{10^{\frac{A_p}{10}}-1}}}{\mathrm{Arch}\,\dfrac{f_s}{f_p}} \tag{16}$$

The cut-off frequency ω_0 in (14) is the highest frequency on which the filter's attenuation reaches the maximum value in the pass-band (i.e. A_p). In the Chebyshev case we choose the cut-off frequency to be

$$\omega_0 = \omega_p \tag{17}$$

where the radian frequency ω_p=2 πf_p. Note the similarity between (16) and the Butterworth formula (12). The Table 2 provides a set of necessary component values for the designing required Chebyshev filter.

If the cut-off frequency in the Chebyshev filters design is required to be the frequency at which the magnitude reaches –3dB value (not the pass-band ripple value A_p) then we have to calculate the value of ω_0 in (14) another way using

$$\omega_0 = \frac{\omega_p}{\omega_{-3\mathrm{dB}}} \tag{18}$$

where

$$\omega_{-3\mathrm{dB}} = \mathrm{Cosh}\left[\left(\frac{1}{\mathrm{n}}\right)\cdot \mathrm{ArCosh}\left(\frac{1}{\varepsilon}\right)\right] \tag{19}$$

The ε value is given by (15). In this case we also use normalized component values of the LP low-sensitivity Chebyshev filter in Table 2. This case will not be considered in this paper, instead we will design Chebyshev filters having cut-off frequency at the pass-band ripple value A_p /dB.

4.1 Example

Using equations (11) to (19) we can readily calculate the filter order n and the cut-off frequency ω_0 for the design of the Butterworth or Chebyshev filters. Indeed those calculations are the only that we need. The filter design then proceeds using tables and is straightforward.

As descriptive examples we present the designs of filters that satisfy our specifications in Fig. 5: i) Butterworth, ii) Chebyshev with pass-band ripple R_p=0.2 dB and iii) Chebyshev with R_p=0.5 dB.

Using (12) and (16) we calculate required filter order n. Thus we have for the three possible realizations the following filter orders and cut-off frequencies: i) Butterworth $n=5$, $\omega_0=155084$ rad/s; ii) Chebyshev $n=4$, $\omega_0=125664$ rad/s; and iii) Chebyshev $n=3$, $\omega_0=125664$ rad/s. Note that the order n of the Chebyshev filters is smaller than the order of the Butterworth filter. Also note that the Chebyshev filter with higher ripple requires lower filter order. Consequently, in what follows we realize the Butterworth filter and the Chebyshev filter with 0.5 dB ripple.

Using normalized predefined component values from above design tables, we proceed with the denormalization procedure. To calculate filter components we need to choose denormalizing resistor R_0 value and the denormalization frequency ω_0. Generally speaking ω_0 and R_0 are free constants. But, in our case, the frequency ω_0 is already known and follow from filter specifications (e.g. it is $\omega_0=155084$ rad/s for the Butterworth filter example). The resistor value R_0 could be chosen to obtain, for example, desired total capacitance value C_{TOT}. The latter constraint arises because we intend to realize the filter circuit on the chip, thus we are limited with maximum allowable capacitance still realizable on the chip.

Suppose that we want to obtain a total capacitance $C_{TOT}=300$ pF, we have to calculate denormalization resistance R_0. For the filters with tapered capacitors by a factor ρ_C of the order n the total capacitance equals

$$C_{TOT}=\sum_{i=1}^{n}C_i=C_1\cdot\sum_{i=0}^{n-1}\rho_C^{-i}=C_1\cdot\frac{\rho_C^{-n}-1}{\rho_C^{-1}-1} \quad (20)$$

For the 5th-order Butterworth filter from Table 1 and using (20) we obtain normalized value $C_{TOT}=C_1\cdot 1.6496$ and therefore maximum value for capacitor C_1 should be C_1=300 pF/1.6496= =181.862 pF. Starting from this we calculate minimum denormalization resistance R_0

$$R_{0\,\min}=\frac{1}{\omega_0 C_1}=\frac{1}{155084\cdot 181.86\,\text{pF}}= =35.4561\,\text{k}\Omega \quad (21)$$

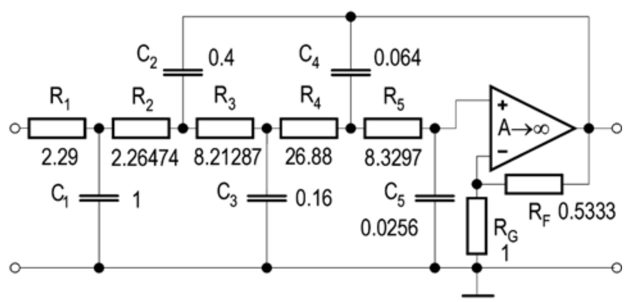
We can choose for example reference resistor $R_0=36$ kΩ thus we have $R_0\geq R_{0\min}$. We calculate reference capacitor value $C_0=1/(\omega_0 R_0)$=179.114 pF and according to (8) the circuit capacitor is then $C=C_0\cdot C_{nLP}$ and the circuit resistor is $R=R_0\cdot R_{nLP}$, where C_{nLP} and R_{nLP} represent normalized values from Tables 1–2. The fifth capacitor C_5 has a value of $C_5=C_0\cdot C_{5nLP}$=4.5853 pF. This is very small value and it is dangerously near to the order of the parasitic capacitance in the circuit. We choose $R_G=10$ kΩ and calculate

$$R_F=R_G(\beta-1)=5.333\,\text{k}\Omega \quad (22)$$

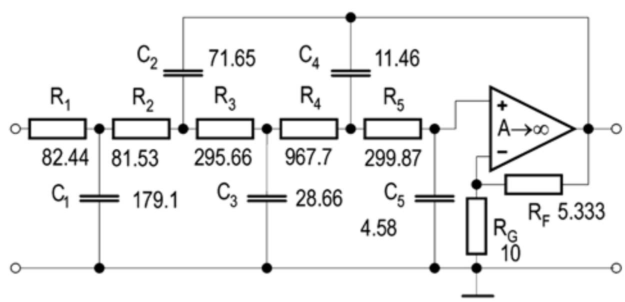
Figure 6a shows the normalized network with the reference values $\omega_0=155084$ rad/s and $R_0=36$ kΩ (all normalized values follow directly from the 5th row in Table 1). Figure 6b shows the denormalized network with resistor values in kΩ and capacitors in pF. A simple first-order check for the correctness of these results is to verify that $a_0=(R_1R_2R_3R_4R_5C_1C_2C_3C_4C_5)^{-1}$.

Now we can continue with the next example of the 3rd-order Cheby-0.5 dB filter. Using the same design steps as above we choose $R_0=39$ kΩ. This provides that the total capacitance is less than 300 pF. The reference capacitor value is $C_0=1/(\omega_0 R_0)=$ $=204.044$ pF.

We choose $R_G=10$ kΩ and calculate $R_F=3.1082$ kΩ. Note that the third capacitor C_3 has a value of $C_3=22.67$ pF, which is, in this case, quite larger than the parasitic capacitance in the integrated circuit.



a)



b)

Fig. 6 Butterworth LP filters (n=5). a) Normalized elements. b) Denormalized elements

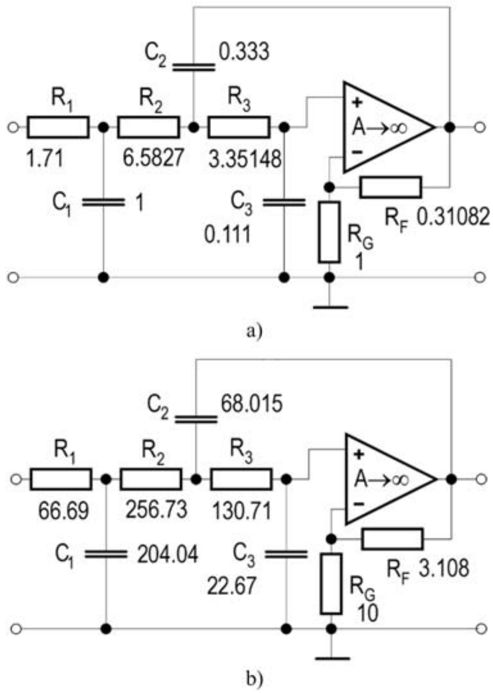


Fig. 7 Chebyshev-05 dB LP filters (n=3). a) Normalized elements. b) Denormalized elements

Figure 7a shows the normalized network with the reference values ω_0=125664 rad/s and R_0=39 kΩ, whereas Figure 7b shows the denormalized network.

In what follows for both denormalized filters shown in Fig. 6b and Fig. 7b, magnitudes are shown in Fig. 8 to demonstrate their functionality and the satisfaction of filter specifications. Notice that magnitudes in Fig. 8 have unity pass-band gain

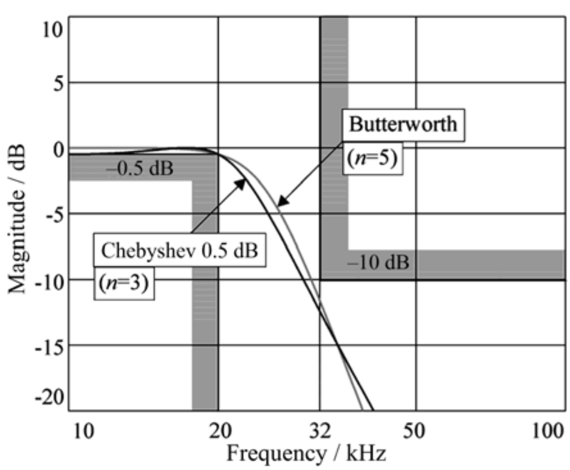


Fig. 8 Butterworth and Chebyshev LP filters transfer function magnitudes

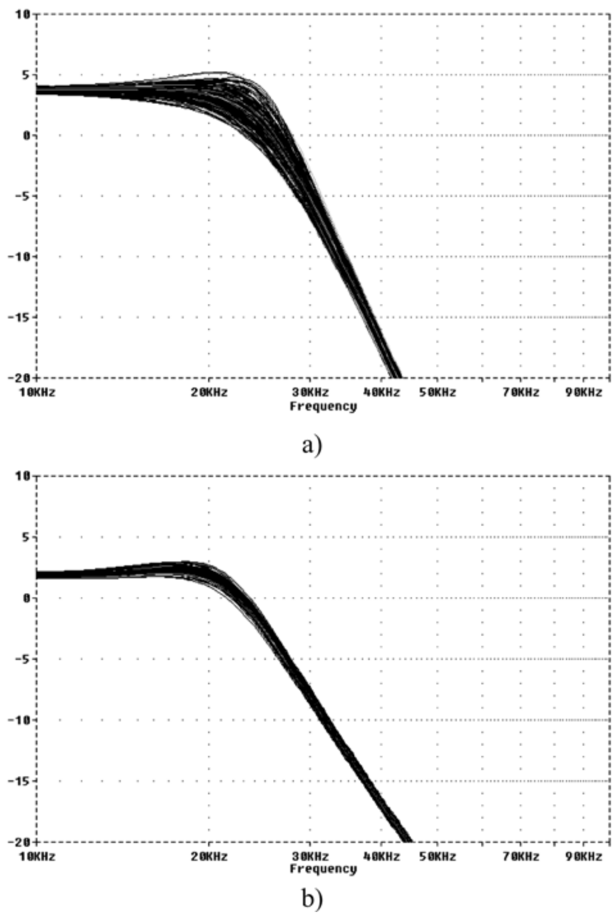


Fig. 9 MC runs (a) Butterworth (n=5) and (b) Chebyshev 0.5 dB (n=3) filters

(K=1, i.e. 0 dB). The realization of the desired pass-band gain value K will be presented in the section 4.2, below. As a double check of filter sensitivities, Monte Carlo runs (using PSPICE simulation) are performed and shown in Fig. 9.

Note that Chebyshev filter although has 0.5 dB ripple in the pass-band and larger pole Q factors, shows lower sensitivity than the other realization, a higher order Butterworth filter. This is because lower filter order has substantially lower sensitivity, especially when filters are realized using single opamp circuit as in Fig. 1. Therefore our choice will be the Chebyshev filter realization.

Recall that the filters presented in design tables above are capacitively tapered and optimized for low sensitivity.

4.2 The Gain Factor *K*

The DC forward gain K of the filter transfer function will generally not coincide with the amplifier gain β, required to obtain filter transfer function parameters. The gain factor K may be specified by the filter designer, but the amplifier gain is determined by the values for β given in design tables. Recall that all values in above tables including β are obtained using optimization procedure for low sensitivity filter. Thus, the value of β cannot be freely chosen; it depends on the design equations for the filter, whereas the overall DC filter gain K may very likely be required to have a different value. Fortunately, there are various schemes for the decoupling of K and β [9], one of which will be presented in what follows. In terms of our filter, this implies that

$$\alpha = \frac{K}{\beta} \tag{23}$$

If the desired value of α is less than unity, i.e. $\beta > K$ (and because $\beta \geq 1$), then a resistive voltage divider can be inserted at the input of the network, as shown in Fig. 10. The gain decoupling was applied to the 3rd-order Chebyshev filter with 0.5 dB shown in Fig. 7b. In this case input resistor R_1 was substituted by voltage divider (by factor α) consisting of resistors R'_1 and R''_1. We have

$$\alpha = \frac{R''_1}{R'_1 + R''_1} \quad \text{and} \quad R_1 = \frac{R'_1 R''_1}{R'_1 + R''_1} \tag{24}$$

i.e. $R'_1 = R_1/\alpha$ and $R''_1 = R_1/(1-\alpha)$. Since α is, in this case, less than unity, R''_1 is always positive. In our example to realize $K=1$ we have $\alpha = K/\beta = 0.7629$, and $R'_1 = R_1/\alpha = 87.419$ kΩ; $R''_1 = 281.251$ kΩ.

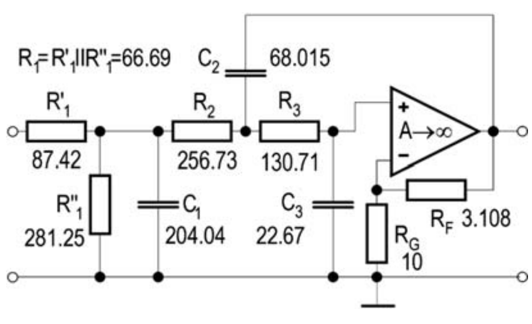


Fig. 10 Final Chebyshev LP filter circuit with unity gain in pass-bad (K=1). (Rs in kΩ and Cs in pF)

The input of the LP filter with K and β decoupled is shown in Fig. 10 when $K < \beta$. The decoupling for $K > \beta$ will not be shown in this paper and is presented in [9].

5 DESIGN OF HIGHPASS FILTERS

In this section we present the design of high--pass (HP) filters starting from given specifications and using the same tables as in the low-pass filter (LP) case presented above. The filters are designed in the same straightforward way and final filter circuits are optimal in the sense that they have minimum sensitivity to passive components variations of the circuit, and low power consumption.

Consider an example of HP filter which satisfies the specifications shown in Fig. 11. The specification requires the maximum pass-band attenuation of $A_p = 0.5$ dB for the frequencies above $f_p = 32$ kHz and the minimum stop-band attenuation of $A_s = 10$ dB for the frequencies below $f_s = 20$ kHz. The filter should have a unity gain in the pass band ($K=1$).

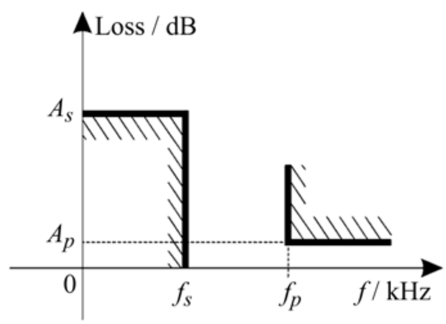


Fig. 11 The attenuation specifications for the HP filter

When we include specifications values into (12) and (16) we readily calculate required filter order n. Recall that the HP specifications can be transformed to the normalized LP specifications with the normalized cut-off frequency $F_s = f_p/f_s$. In both LP and HP filter examples we obtain the same orders n because both specifications have the same requirements, i.e. we have the same frequency F_s.

In what follows we are going to design a HP filter of the 3rd-order with Chebyshev approximation 0.5 dB ripple, which is dual to the filter circuit in Fig. 7b.

To design HP filter we should calculate capacitors values as reciprocals of resistors values in Tables 1–2 and vice versa, resistors values are reciprocals of capacitors values (*RC-CR* transformation), which is accomplished by (10). In (10) the denormalization of components values is performed, as well. The denormalization is performed to the cut-off frequency $f_p = 32$ kHz or $\omega_0 = 2\pi f_p = 201061.9$ rad/s. To properly select the denormalization resistance R_0 we must be able to realize capacitors on the chip.

Therefore we calculate (with C_{TOT}= 300 pF)

$$R_{0\min} = \frac{1}{\omega_0 C_1} = \frac{\sum C_{in}}{\omega_0 C_{TOT}} = = \frac{1.03508}{201062 \cdot 300\,\mathrm{pF}} = 17160\,\Omega \quad (25)$$

We choose reference resistor value R_0= 18 kΩ and this provides reference capacitor value C_0= = $1/(\omega_0 R_0)$=276.311 pF. The circuit capacitor is then $C = C_0 \cdot C_{nHP}$ and the circuit resistor is R= = $R_0 \cdot R_{nHP}$, where index *n* denotes normalized values. Thus we have the following values for components in HP filter

$$R_{1HP} = \frac{R_0}{C_{1nLp}} = R_0 \cdot R_{1nHP} = 18\,\mathrm{k\Omega}$$
$$R_{2HP} = 54\,\mathrm{k\Omega}; \quad R_{3HP} = 162\,\mathrm{k\Omega};$$
$$C_{1HP} = \frac{1}{R_{1nLp}\omega_0 R_0} = C_0 \cdot C_{1nHP} = 161.6\,\mathrm{pF}; \quad (26)$$
$$C_{2HP} = 41.98\,\mathrm{pF}; \quad C_{3HP} = 82.44\,\mathrm{pF}$$

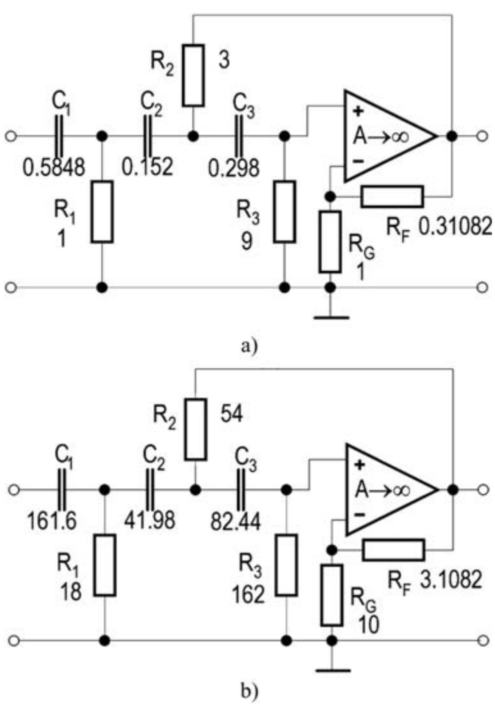


Fig. 12 Chebyshev HP filters. a) Normalized elements. b) Denormalized elements (Rs in kΩ and Cs in pF)

Note that in (26) R_{nHP}=1/C_{nLP} and C_{nHP}=1/R_{nLP}. Fig. 12a shows the normalized network with the ω_0=201061.9 rad/s and R_0=18 kΩ, whereas the denormalized network is shown in Fig. 12b. It is obvious from Fig. 12a that the circuit is resistively tapered by the factor 3 (as in the LP case, where the capacitors are scaled by the factor ρ_C=3). The resistors R_G and R_F in the amplifier feedback realize the same gain β as in the LP case.

To realize the gain K= 1 we substitute an input capacitor C_1 by voltage divider (by factor α) consisting of capacitors C'_1 and C''_1. We have

$$\alpha = \frac{C_1'}{C_1' + C_1''} \text{ and } C_1 = C_1' + C_1'' \quad (27)$$

i.e. C'_1=C_1/(1−α) and C''_1=C_1/α. In our example we have α=K/β=0.7629, thus the input capacitor C_1

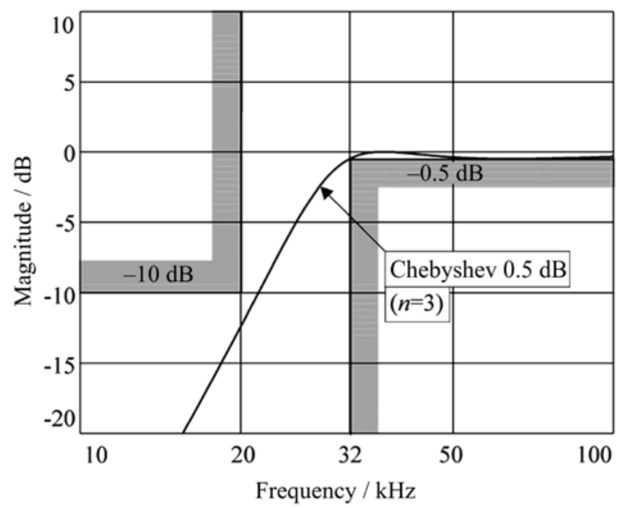


Fig. 13 Chebyshev HP filter transfer function magnitude

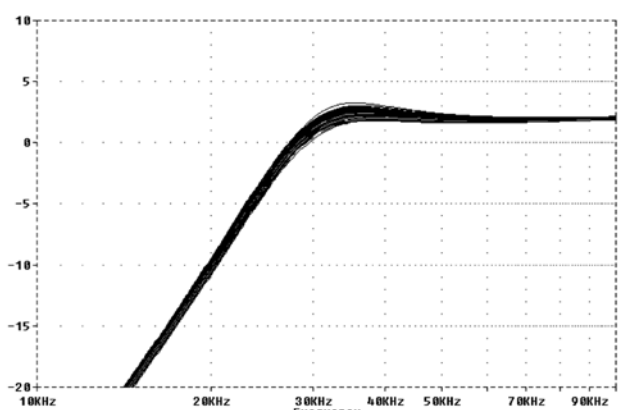


Fig. 14 MC runs of Chebyshev HP filter

is split into C'_1=681.45 pF and C''_1=211.81 pF (C'_1 is connected to the input signal generator). Corresponding magnitudes and MC runs are shown in Fig. 13 and Fig. 14, respectively. The obtained HP filter circuit in Fig. 12b satisfies the specifications in Fig. 11.

Comparing MC runs in Fig. 14 (for the HP 0.5 dB Chebyshev filter) to those in Fig. 9b for its dual counterpart LP filter (Fig. 7b), both satisfying the same LP prototype specifications, we can conclude that both of these filters have identical sensitivities. This is because they are designed starting from the same optimized component values in the second row (for $n=3$) in Table 2.

6 CONCLUSIONS

A procedure for the design of allpole low-sensitivity, low-power, active-*RC* filters using tables with predefined normalized filter component values for some common filter types (Butterworth and Chebyshev 0.5 dB) is presented. The filter uses only one operational amplifier, and a minimum number of passive components. The amplifier itself ensures realization of conjugate-complex filter poles, and low output impedance. For reasons related to the filter topology, the application of the capacitive impedance tapering has improved the sensitivity of the low-pass filters' magnitude to component tolerances [9]. The proposed design is universal and straightforward by using design tables; thus there is no need for numeric calculations. It can be extended to the design of single-amplifier, low-sensitivity high-pass filters, as well. Because, the high-pass filters are dual to the low-pass filters, resistive tapering is applied to reduce the sensitivity of the high-pass filter.

Furthermore, the reduction in power and component count achieved with the single-amplifier LP filters is obtained at a price: a cascade of impedance-tapered second-order and/or third-order sections has lower sensitivity than impedance-tapered single-amplifier filter. Thus the decision on which way to go is typically one of tradeoffs: low power and component count versus low sensitivity. In our example, of hearing aid circuit realization, we preferred former solution having low power and low component count and acceptably low sensitivity.

APPENDIX I: COEFFICIENTS OF *T*(*s*)

Consider the n^{th}-order allpole low-pass filter circuit with positive feedback presented in Fig. A.1. Note the descending notation of R_n, C_n to R_1, C_1 from the driving source to the amplifier input. This reverse order notation is convenient to develop recursive formulas for determining transfer function coefficients d_1 to d_n in eq. (A.2) of the n^{th}-order polynomial $D'_n(s)$ as functions of resistors R_i, capacitors C_i and gain β.

Recursive formulas follow from characteristics of the continuants. The continuants are used to solve ladder-networks, and they can be calculated recursively. The filter in Fig. A.1 has a ladder network in the amplifier's positive feedback loop with gain β, where gain $\beta=1+R_F/R_G$ represents the gain in the class-4 filter circuit. The transfer function of the n^{th}-order filter as presented in the Fig. A.1 has the form given by

$$T(s)=\frac{\beta}{D'_n(s)} \tag{A.1}$$

As shown in [1] and [2] p. 252, the n^{th}-order denominator polynomial in transfer function (A.1), i.e.

$$D'_n(s)=\sum_{j=1}^{n} d_j s^j+1 \tag{A.2}$$

can be calculated from polynomials of $n-1$ and $n-2$ order:

$$D'_{n-1}(s)=\sum_{j=1}^{n-1} c_j s^j+1 \tag{A.3}$$

and

$$D'_{n-2}(s)=\sum_{j=1}^{n-2} b_j s^j+1 \tag{A.4}$$

using

$$\begin{aligned} d_j=&\left(c_j-b_j\right)\frac{R_n}{R_{n-1}}+c_j+\\ &+c_{j-1}R_nC_n-\delta_{1j}\left[\frac{(-1)^n+1}{2}\right]R_nC_n\beta \end{aligned} \tag{A.5}$$

where $\delta_{1j}=0$, for $j\neq 1$; and $\delta_{1j}=1$, for $j=1$ where $1\le j\le n$. Note that $b_0=c_0=d_0=1$. Note also that, for the start of the recursive process polynomials $D'_0=1$ and $D'_1=R_1C_1s+1$.

To perform the symbolic calculations (to calculate coefficients in terms of components) we use symbolic calculation program Mathematica. At the end of recursive process we change to the ascending notation, i.e. we substitute $R_n\to R_1$, $C_n\to C_1$,

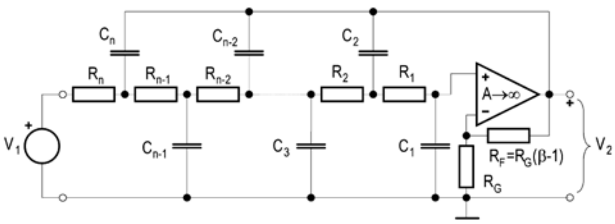


Fig. A1 General n^{th}-order single-amplifier low-pass filter with reverse notation

$R_{n-1}\to R_2$, $C_{n-1}\to C_2,\ldots$, $R_1\to R_n$, $C_1\to C_n$, resulting in the filter circuit with notation shown in Fig. 1. Consequently, we perform multiplication of nominator and denominator by the same factor

$$N(s)=\frac{N'(s)}{d_n}=\beta\cdot a_0,\quad D(s)=\frac{D'(s)}{d_n} \tag{A.6a}$$

$$a_j=\frac{d_j}{d_n},\quad 0\le j\le n \tag{A.6b}$$

We obtain the form of the transfer function of the n^{th}-order filter given by (note the unity coefficient with the highest, i.e. n^{th} power of s)

$$T(s)=\frac{N(s)}{D(s)}=\\ =\frac{\beta a_0}{s^n+a_{n-1}s^{n-1}+\cdots+a_i s^i+\cdots+a_1 s+a_0} \tag{A.7}$$

The transfer function (A.7) has the same form as the transfer function (1). In what follows we present the transfer function's coefficients up to reasonable (6^{th}) order obtained by the recursive procedure shown above. Those coefficients correspond to the filter circuit with ascending notation shown in Fig. 1, and are used in this paper for all numerical calculations carried out by Mathematica. It can be seen that increasing the filter order n, coefficients become even more complicated, and therefore the canonical high-order filter's sensitivity rapidly increases.

2^{nd} Order

a_1= (C1 R1 + C2 R1 + C2 R2 – C1 R1 β) / (C1 C2 R1 R2)

a_0= 1 / (C1 C2 R1 R2)

3^{rd} Order

a_2= (C1 C2 R1 R2 + C1 C3 R1 R2 + C1 C3 R1 R3 + C2 C3 R1 R3 + C2 C3 R2 R3 – C1 C2 R1 R2 β) / (C1 C2 C3 R1 R2 R3)

a_1= (C1 R1 + C2 R1 + C3 R1 + C2 R2 + C3 R2 + C3 R3 – C2 R1 β – C2 R2 β) / (C1 C2 C3 R1 R2 R3)

a_0= 1 / (C1 C2 C3 R1 R2 R3)

4^{th} Order

a_3= (C2 C3 C4 (R1 + R2) R3 R4 + C1 R1 (C3 C4 (R2 + R3) R4 + C2 R2 (C4 (R3 + R4) + C3 (R3 – R3 β)))) / (C1 C2 C3 C4 R1 R2 R3 R4)

a_2= (C1 C2 R1 R2 + C4 (C3 (R1 + R2 + R3) R4 + C2 (R1 + R2) (R3 + R4) + C1 R1 (R2 + R3 + R4)) – C3 (C2 (R1 + R2) R3 + C1 R1 (R2 + R3)) (–1 + β)) / (C1 C2 C3 C4 R1 R2 R3 R4)

a_1= (C1 R1 + C2 R1 + C2 R2 + C4 (R1 + R2 + R3 + R4) – C3 (R1 + R2 + R3) (–1 + β) – C1 R1 β) / (C1 C2 C3 C4 R1 R2 R3 R4)

a_0= 1 / (C1 C2 C3 C4 R1 R2 R3 R4)

5^{th} Order

a_4= (C2 C3 C4 C5 (R1 + R2) R3 R4 R5 + C1 R1 (C3 C4 C5 (R2 + R3) R4 R5 + C2 R2 (C4 C5 (R3 + R4) R5 + C3 R3 (C5 (R4 + R5) + C4 (R4 – R4 β))))) / (C1 C2 C3 C4 C5 R1 R2 R3 R4 R5)

a_3= (C1 C2 C3 R1 R2 R3 + C5 (C4 (C3 (R1 + R2 + R3) R4 + C2 (R1 + R2) (R3 + R4) + C1 R1 (R2 + R3 + R4)) R5 + C3 (C2 (R1 + R2) R3 + C1 R1 (R2 + R3)) (R4 + R5) + C1 C2 R1 R2 (R3 + R4 + R5)) – C4 (C2 C3 (R1 + R2) R3 R4 + C1 R1 (C3 (R2 + R3) R4 + C2 R2 (R3 + R4))) (–1 + β)) / (C1 C2 C3 C4 C5 R1 R2 R3 R4 R5)

a_2= (C1 C2 R1 R2 + C1 C3 R1 R2 + C1 C3 R1 R3 + C2 C3 R1 R3 + C2 C3 R2 R3 + C5 (C4 (R1 + R2 + R3 + R4) R5 + C3 (R1 + R2 + R3) (R4 + R5) + C2 R2 (R3 + R4 + R5) + R1 (C1 R2 + (C1 + C2) (R3 + R4 + R5))) – C4 (C3 (R1 + R2 + R3) R4 + C2 (R1 + R2) (R3 + R4) + C1 R1 (R2 + R3 + R4)) (–1 + β) – C1 C2 R1 R2 β) / (C1 C2 C3 C4 C5 R1 R2 R3 R4 R5)

a_1= (C1 R1 + C2 R1 + C3 R1 + C2 R2 + C3 R2 + C3 R3 + C5 (R1 + R2 + R3 + R4 + R5) – C4 (R1 + R2 + R3 + R4) (–1 + β) – C2 (R1 + R2) β) / (C1 C2 C3 C4 C5 R1 R2 R3 R4 R5)

a_0= 1 / (C1 C2 C3 C4 C5 R1 R2 R3 R4 R5)

6^{th} Order

a_5= (C2 C3 C4 C5 C6 (R1 + R2) R3 R4 R5 R6 + C1 R1 (C3 C4 C5 C6 (R2 + R3) R4 R5 R6 + C2 R2 (C4 C5 C6 (R3 + R4) R5 R6 + C3 R3 (C5 C6 (R4 + R5) R6 + C4 R4 (C6 (R5 + R6) + C5 (R5 – R5 β)))))) / (C1 C2 C3 C4 C5 C6 R1 R2 R3 R4 R5 R6)

a_4= (C1 C2 C3 C4 R1 R2 R3 R4 + C6 (C5 (C4 (C3 (R1 + R2 + R3) R4 + C2 (R1 + R2) (R3 + R4) + C1 R1 (R2 + R3 + R4)) R5 + C3 (C2 (R1 + R2) R3 + C1 R1 (R2 + R3)) (R4 + R5) + C1 C2 R1 R2 (R3 + R4 + R5)) R6 + C4 (C2 C3 (R1 + R2) R3 R4 + C1 R1 (C3 (R2 + R3) R4 + C2 R2 (R3 + R4))) (R5 + R6) + C1 C2 C3 R1 R2 R3 (R4 + R5 + R6)) – C5 (C2 C3 C4 (R1 + R2) R3 R4 R5 + C1 R1 (C3 C4 (R2 + R3) R4 R5 + C2 R2 (C4 (R3 + R4) R5 + C3 R3 (R4 + R5)))) (–1 + β)) / (C1 C2 C3 C4 C5 C6 R1 R2 R3 R4 R5 R6)

a_3= (C1 C2 C3 R1 R2 R3 + C1 C2 C4 R1 R2 R3 + C1 C2 C4 R1 R2 R4 + C1 C3 C4 R1 R2 R4 + C1 C3 C4 R1 R3 R4 + C2 C3 C4 R1 R3 R4 + C2 C3 C4 R2 R3 R4 + C6 (C5 (C4 (R1 + R2 + R3 + R4) R5 + C3 (R1 + R2 + R3) (R4 + R5) + C2 R2 (R3 + R4 + R5) + R1 (C1 R2 + (C1 + C2) (R3 + R4 + R5))) R6 + C4 (C3 (R1 + R2 + R3) R4 + C2 (R1 + R2) (R3 + R4) + C1 R1 (R2 + R3 + R4)) (R5 + R6) + C2 C3 R2 R3 (R4 + R5 + R6) + R1 (C3 (C2 R3 + C1 (R2 + R3)) (R4 + R5 + R6) + C1 C2 R2 (R3 + R4 + R5 + R6))) – C5 (C4 (C3 (R1 + R2 + R3) R4 + C2 (R1 + R2) (R3 + R4) + C1 R1 (R2 + R3 + R4)) R5 + C3 (C2 (R1 + R2) R3 + C1 R1 (R2 + R3)) (R4 + R5) + C1 C2 R1 R2 (R3 + R4 + R5)) (–1 + β) – C1 C2 C3 R1 R2 R3 β) / (C1 C2 C3 C4 C5 C6 R1 R2 R3 R4 R5 R6)

a_2= (C1 C2 R1 R2 + C1 C3 R1 R2 + C1 C4 R1 R2 + C1 C3 R1 R3 + C2 C3 R1 R3 + C1 C4 R1 R3 + C2 C4 R1 R3 + C2 C3 R2 R3 + C2 C4 R2 R3 + C1 C4 R1 R4 + C2 C4 R1 R4 + C3 C4 R1 R4 + C2 C4 R2 R4 + C3 C4 R2 R4 + C3 C4 R3 R4 + C6 (C2 R2 R3 + C2 R2 R4 + C3 R2 R4 + C3 R3 R4 + C2 R2 R5 + C3 R2 R5 + C3 R3 R5 + C2 R2 R6 + C3 R2 R6 + C3 R3 R6 + C5 (R1 + R2 + R3 + R4 + R5) R6 + C4 (R1 + R2 + R3 + R4) (R5 + R6) + R1 (C1 R2 + C3 (R4 + R5 + R6) + (C1 + C2) (R3 + R4 + R5 + R6))) – C5 (C4 (R1 + R2 + R3 + R4) R5 + C3 (R1 + R2 + R3) (R4 + R5) + C2 R2 (R3 + R4 + R5) + R1 (C1 R2 + (C1 + C2) (R3 + R4 + R5))) (–1 + β) – C3 (C2 (R1 + R2) R3 + C1 R1 (R2 + R3)) β) / (C1 C2 C3 C4 C5 C6 R1 R2 R3 R4 R5 R6)

a_1= (C1 R1 + C2 R1 + C3 R1 + C4 R1 + C2 R2 + C3 R2 + C4 R2 + C3 R3 + C4 R3 + C4 R4 + C6 (R1 + R2 + R3 + R4 + R5 + R6) – C5 (R1 + R2 + R3 + R4 + R5) (–1 + β) – (C1 R1 + C3 (R1 + R2 + R3)) β) / (C1 C2 C3 C4 C5 C6 R1 R2 R3 R4 R5 R6)

a_0= 1 / (C1 C2 C3 C4 C5 C6 R1 R2 R3 R4 R5 R6)

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Projektiranje aktivnih *RC* filtara niske osjetljivosti s jednim pojačalom pomoću tablica. Prikazan je postupak projektiranja aktivnih *RC* filtara s jednim pojačalom pomoću tablica s normaliziranim filtarskim komponentama za poznate tipove aproksimacija kao što su Butterworth i Chebyshev. Filtri su građeni od *RC* ljestvičaste mreže u pozitivnoj povratnoj vezi pojačala. Tablice s normaliziranim vrijednostima elemenata s jednakim kapacitetima i s jednakim otporima su već objavljene od strane raznih autora [1, 2]. U ovom radu, prikazujemo nove tablice za projektiranje filtara koji imaju optimalnu osjetljivost na varijacije pasivnih komponenata. Značajno smanjenje osjetljivosti postiže se posebnom tehnikom projektiranja »pomoću skaliranja impedancija«. Prikazani filtri su do uključujući šestog reda. Problem osjetljivosti ne dopušta da red promatranih filtara bude veći od šestog reda. Prikazano je projektiranje niskopropusnih i visokopropusnih tipova filtara.

Ključne riječi: aktivni *RC* filtri, minimalni *RC* filtri, mala potrošnja, niska osjetljivost, normalizirane vrijednosti komponenata

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