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# Modelling of Propulsion Shaft Line and Basic Procedure of Shafting Alignment Calculation

Original scientific paper

The main propulsion shafting is exposed to various operating conditions throughout the entire lifetime of a modern ship. The necessary condition for the shafting to withstand and survive all possible situations is its proper dimensioning and manufacture, as well as its assembly and testing onboard. Its alignment is of utmost importance during the assembly process itself.

The aim of this paper is to present the shafting alignment calculation procedure in order to help the designer to understand the whole alignment process. Calculation presumptions, modelling of shafting parts, material properties and loading are given in detail. The advantages of the transfer matrix methods over the finite element methods in this particular case have been described. The important part is to establish the designed shafting elastic line onboard the ship, during the outfitting in the shipyard. It is proposed in the conclusion that the presented matter be included into a future edition of the CRS Technical Rules.

**Keywords:** *propulsion system, propulsion shafting, shaft alignment, transfer matrix method*

## Modeliranje i osnove proračuna centracije brodskog porivnog vratilnog voda

Izvorni znanstveni rad

Tijekom životnog vijeka suvremenog broda, porivni vratilni vod izložen je vrlo promjenjivim radnim stanjima. Osnovni su uvjeti da vratilni vod ispuni svoju funkciju u svim mogućim radnim uvjetima pravilno dimenzioniranje i izrada, kao i montaža i ispitivanje na brodu. U provođenju montaže posebnu važnost ima postupak centracije.

Cilj je ovoga rada prikazati metodologiju proračuna centracije vratilnog voda, sa svrhom da se projektantima omogući lakše razumijevanje cjelovitoga postupka centracije. Potanko su prikazane proračunske pretpostavke, modeliranje dijelova vratila, kao i značajke materijala i opterećenja. Opisana je prednost primjene metode početnih parametara u matričnom prikazu (tzv. metode prijenosnih matrica) u odnosu na metodu konačnih elemenata u ovom specifičnom slučaju. Naglašena je važnost postizanja projektne elastične linije vratilnog voda, tijekom opremanja broda. U zaključku se predlaže da se prikazani pristup uključi u buduća izdanja Tehničkih pravila *HRB*-a.

**Ključne riječi:** *porivni sustav, porivni vratilni vod, centracija vratilnog voda, metoda prijenosnih matrica*

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## 1 Introduction

The purpose of the propulsion machinery (main engine, gearbox, propulsion shaft line, propeller and pertinent auxiliary systems) is to propel the ship and to control manoeuvring, thus enabling the navigator to be in control of the ship's speed and course. The main propulsion shaft line is the essential part of a modern ship propulsion system, exposed to various conditions throughout the ship's lifetime. It has to function properly under all possible operating conditions. Consequently, the shaft line preliminary and final design, its static and dynamic behaviour shall be carefully considered by the designer and by the classification society.

Shafting alignment procedure considers static and pseudo-static loading of the shafting in order to determine its static response. This procedure consists of three phases: calculation, assembly and validation of the assembled shaft line onboard the ship. The main goal of this procedure is to determine and ensure onboard achievement of the bearings designed positions in athwart direction in order to comply with the loading criteria for propulsion system and shafting parts. For this purpose the shaft line is usually modelled as a continuous multi-span beam on several supports. They may be modelled as absolutely stiff or linearly elastic (in the case of static and pseudo-static response), or even as real radial journal bearings (in the case of dynamic response).

The goal of this paper is to provide designers with the basic information how to model real shafting systems in order to perform shaft alignment calculations. The paper aims to present the conventional shafting alignment calculation procedure and its presumptions. Modelling of shafting parts, material properties and loading is given in detail, in order to help the designer understand the whole calculation process. The advantage of the transfer matrix method over the finite element method in this particular case is briefly described. The important part is to establish the designed shafting elastic line onboard the ship, during the outfitting phase in the shipyard. The results of a real life calculation example are presented in the end.

## 2 Shafting alignment calculation procedure

The shafting alignment calculation comprises evaluation of the shafting elastic line and the reaction forces of supports for the pre-determined offsets of supports. In case of propulsion systems with gearboxes (mainly small, medium and high-speed four-stroke diesel engines) the scope of the analysis is restricted to the shaft line from the propeller to the output shaft of the gearbox, together with its bearings and the bull gear. The remaining shaft line parts (clutches, input shaft, as well as the engine itself) need not be taken into account. A typical shaft line layout of this kind is schematically shown in Figure 1.

In the case of directly coupled engines (mainly large slow-speed two-stroke diesel engines) the shafting alignment analysis takes into account the model and the static behaviour of the engine crankshaft. The complete crankshaft need not be modelled in detail, as almost every slow-speed diesel engine manufacturer provides drawings describing this model as a girder system available to the shaft line designers.

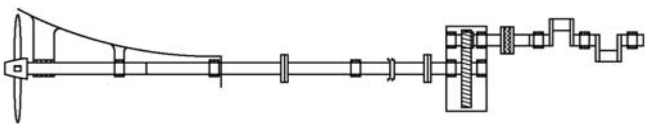

Figure 1 **Schematic of a typical marine shaft line including a gearbox [1]**
Slika 1 **Shematski prikaz tipičnog brodskog vratilnog voda s reduktorom [1]**

### 2.1 Input data and modelling of the system

The data describing dimensions, material and loading of the shafts, together with the data describing the bearings concept (slide or roller), bearing clearances and lubrication means are to be available for shafting alignment calculations. This real system is modelled as a statically indeterminate system of variable section beams with multiple supports. The shaft line elements are modelled by means of circular section model elements, and the shaft line bearings are modelled by means of absolutely stiff or linearly elastic supports. In general, the cross-section varies from one beam to another.

In general, model elements are of conical shape. A special case of conical element is the element of cylindrical shape, as a cone with equal diameters on both ends.

Elements are made of homogenous material, of specific density $\rho$, submerged (completely, partially, or not at all) into sea-water of specific density $\rho_w$. The shaft material elastic properties are described by means of Young modulus of elasticity $E$ and shear modulus $G$.

As the calculation presumes the ship afloat, after assembling all the parts of shaft line, loading of elements consists of:

- self-weight of the element;
- buoyancy in sea water (for submerged elements);
- external concentrated force $F$ in the centre of the cross section of the left element end, [N];
- external concentrated moment $T$ in the centre of the cross section of the left element end, [Nm];
- external uniformly distributed load $q$ along the element (owing to other possible forces, additional to the shaft self weight and buoyancy), [N/m].

A general element model, together with the support at its right end is shown in Figure 2.

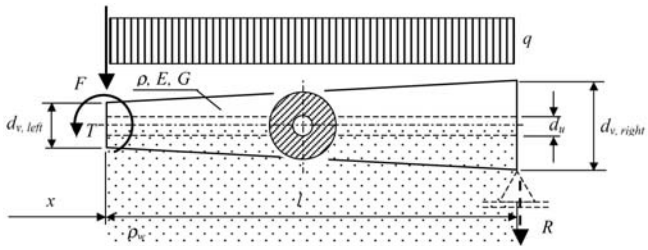


Figure 2 **General model of shaft line element [3]**
Slika 2 **Općeniti model elementa vratilnog voda [3]**

All the calculations are to be performed for the vertical plane, where the influence of self-weight and buoyancy shall be taken into consideration within the loading of the model. In the case of propulsion systems with gearboxes, where gearing forces in horizontal direction have a significant influence, the separate calculations for the horizontal plane are also needed.

### 2.2 Calculation presumptions

The calculations are based upon the real element dimensions, and the following presumptions:

- Propeller is completely or semi-submerged into water;
- Volumetric forces (self-weights and buoyancy) are uniformly distributed along each element;
- All the bearings may be modelled by means of absolutely rigid or linearly elastic supports;
- The influence of shear forces and deformations is to be taken into account;
- The axial position of each reaction force is on the half way of the bearing length.

If necessary, the inclination of shafting with respect to the ship waterline (horizontal plane) may be taken into account by calculating of components (for concentrated forces) and correction of gravity constant (for volumetric forces).

### 2.3 Selection of calculation method (FEM vs. transfer-matrix method)

The most appropriate modelling and calculation procedures in this case are the method of initial parameters in its matrix form (the so called: transfer-matrix method) and finite element method (FEM). Practically equivalent results may be obtained by means of either of these two methods, except in the case of trapezoidal loading along the element itself.

However, the transfer matrix method is chosen and preferred, as it requires linear systems of significantly smaller ranges to be solved. Particularly, FEM requires solving of $2m$ equations (where $m$ is the number of shaft line elements). On the other hand, the transfer-matrix method requires solving only of $z$+2 equations (where $z$ is the total number of stiff supports between the system ends). In addition to this, the transfer matrix method is purely analytical, implementing the solutions to differential equations for beams in bending and shear.

There is an additional advantage of the transfer-matrix method over FEM in this case. FEM calculation results (solutions) are valid in the nodes only, whereas the transfer-matrix method allows the user to obtain deflections, slopes, bending moments and shear forces along the element itself (i.e. between the nodes) on the basis of calculated results in the nodes.

Consequently, the calculation model based on transfer matrices in a single (e.g. vertical) plane is chosen and described further on.

## 2.4 Element transfer matrices and selection of initial parameters

For calculation purposes the whole shaft line is modelled as a system of multi-span beams, supported in rigid (absolute stiff) or linearly elastic supports. Each beam has a uniform circular cross-section (solid or hollow). Conical shafting elements are modelled as cylindrical with mid-section diameters, for the evaluation of stiffness and loading by volumetric forces.

The basic goal of the transfer matrix method is to determine the state vectors $\mathbf{v}_i$ in each section of the whole system. It is necessary to determine these vectors at each end section of each element:

$$\mathbf{v}_i = \begin{bmatrix} -w_i & \beta_i & M_i & Q_i & 1 \end{bmatrix}^T \qquad (1)$$

Considering the system element ($i$), the state vector ($\mathbf{v}_{i+1}$) at the right section of the element right end is related with the state vector ($\mathbf{v}_i$) at the right section of the element left end as follows:

$$\mathbf{v}_{i+1}^{(right)} = \mathbf{L}_{i,support} \cdot \mathbf{v}_{i+1}^{(left)} = \mathbf{L}_{i,support} \cdot \mathbf{L}_{i,elem} \cdot \mathbf{v}_i^{(left)} = \mathbf{L}_i \cdot \mathbf{v}_i^{(left)} \qquad (2)$$

In the equation (2) $\mathbf{L}_i = \mathbf{L}_{i,support} \cdot \mathbf{L}_{i,elem}$ denotes the total transfer matrix of the element $i$ (including the support at its right end). It may also be written in the expanded form (3):

$$\mathbf{L}_i = \begin{bmatrix} 1 & \ell_i & \dfrac{\ell_i^2}{2EI_i} & \dfrac{\ell_i^3}{6EI_i} - \dfrac{\kappa_i \ell_i}{GA_i} & -\dfrac{\ell_i^4}{EI_i} \cdot \left( \dfrac{T_i}{2} + \dfrac{F_i \ell_i}{6} + \dfrac{q_i \ell_i^2}{24} \right) + \dfrac{\kappa_i \ell_i}{GA_i} \cdot \left( F_i + \dfrac{q_i \ell_i}{2} \right) \\ 0 & 1 & \dfrac{\ell_i}{EI_i} & \dfrac{\ell_i^2}{2EI_i} & -\dfrac{\ell_i}{EI_i} \cdot \left( T_i + \dfrac{F_i \ell_i}{2} + \dfrac{q_i \ell_i^2}{6} \right) \\ 0 & 0 & 1 & \ell_i & -\left( T_i + F_i \cdot \ell_i + \dfrac{q_i \ell_i^2}{2} \right) \\ 0 & 0 & 0 & 1 & -\left( F_i + q_i \ell_i + R_{i+1} \right) \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

The quantities in the equations (1) to (3) have the following meaning:

$l_i$ – element length, [mm]
$EI_i$ – element bending stiffness, [Nm$^2$]
$GA_i$ – element shear stiffness, [N]
$\kappa_i = f(d_u/d_v)$ – shear form factor for the circular (solid or hollow) section, $\kappa_i$=1,11 … 1,45
$q_i$ – uniform distributed external loading along the element (see Figure 2), [N/m]
$F_i$ – concentrated force at the element left end (see Figure 2), [N]
$T_i$ – concentrated moment at the element left end (see Figure 2), [Nm]
$R_{i+1}$ –reaction of the support (if any), at the element right end, positive downwards, [N].
$w$, $\beta$ – displacement components (deflection, [m] and slope, [m/m]),
$M$, $Q$ – internal forces (bending moment, [Nm] and shear force, [N]).

Note: In case there is no support at the element right end, the transfer matrix $\mathbf{L}_{i,elem} = \mathbf{L}_i$ for the sole element is obtained from (3), taking $R_{i+1}$=0.

The initial parameters to be selected are the two additional unknowns at the whole system left end. They are finally determined from the two known parameters at the system right end, together with the reactions in all rigid supports. The system ends may either be free, propped, or fixed. Any case may be chosen, however, the most common situation is that both of the ends are free. In the case of free left end of the system the unknown initial parameters are:

$w_1$ – deflection of the system left section;
$\beta_1$ – slope of the system left section.

These parameters, together with all the reaction forces in rigid supports ($R_1, R_2, \ldots, R_z$) are determined from the known boundary conditions at the right end of the system, i.e.

$M_{m+1}$=0; $Q_{m+1}$=0 – in case of free right end;
$w_{m+1}$=0; $M_{m+1}$=0 – in case of propped right end;
$w_{m+1}$=0; $\beta_{m+1}$=0 – in case of fixed right end.

The total number of equations to be solved is thus $z$+2 only.

## 2.5 Calculation of influence coefficients, initial reactions of supports and system response

The whole elastic system is described by means of the system matrix **A**, and the system vector **b**. Both of them are assembled on the basis of the boundary conditions at each fixed support and at the rightmost end of the system, by means of span transfer matrices. For each span these span transfer matrices are simply matrix products of transfer matrices that relate the state vector in the section of one stiff support (or system leftmost end) to the next one:

$$\mathbf{v}_{j+1}^{(R)} = \mathbf{L}_{span,j} \cdot \mathbf{v}_j^{(L)} = \mathbf{L}_k \cdot \mathbf{L}_{k-1} \cdot \ldots \cdot \mathbf{L}_1 \cdot \mathbf{v}_j^{(L)} \qquad (4)$$

where:

$k$ – number of elements in the present span.

In case of both the left and right ends free, the vector of unknowns **k** consists of the two initial parameters ($w_0$ and $\beta_0$) and of the reaction forces in stiff supports ($R_j$), as follows:

$$\mathbf{k} = \begin{bmatrix} -w_0 & \beta_0 & R_1 & R_2 & \ldots & R_z \end{bmatrix}^T \qquad (5)$$

In this particular case, the boundary conditions used to assemble matrix **A** and vector **b** are:

- The zero displacements of the fixed supports (forming the first $z$ equations). This condition may also be expressed by the null-vector $\mathbf{p}_0$ of initial offsets of supports ($\mathbf{p}_0$=**0**).

- $M_{m+1}$=0 and $Q_{m+1}$=0, used to form the remaining 2 equations (i.e. the 2 rows of **A** and the 2 components of **b**).

The best practice is to calculate the influence coefficients prior to the calculation of components of **k**. That is the essential part of the complete analysis. The influence coefficient $h_{ij}$ quantitatively expresses the change of reaction force (in N) in the movement direction of the support *i*, when the support *j* moves for 1 mm in that direction.

The matrix of influence coefficients **H**, which is independent of the actual support offsets, is determined as:

$$\mathbf{H} = \mathbf{A}^{-1} \tag{6}$$

The vector of unknowns $\mathbf{k}_0$, containing the initial reactions in the stiff supports (i.e. those for zero support offsets) then becomes:

$$\mathbf{k}_0 = \mathbf{H} \cdot \mathbf{b}_0 \tag{7}$$

Once the components of the vector $\mathbf{k}_0$ are known, the state vectors in each section of the system may be easily found by a simple matrix multiplication, beginning from the known state vector at the leftmost end of the system. This is the system bending and shear response in terms of deflection, slope, bending moment and shear force at both ends of each element.

### 2.6 Calculation of bearing reactions and the system response for designed support offsets

Designed support offsets are to be determined in advance so that the system response satisfies certain criteria for the final calculated case. This final case may be the static response of the assembled shaft line during the ship outfitting, or even the pseudo-static response of the shaft line in operation. If the external forces have not changed meanwhile, and transferring from the system with zero support offset to the present one, the vector of unknowns **k** will be:

$$\mathbf{k} = \mathbf{H} \cdot \left(\mathbf{p} - \mathbf{p}_0\right) + \mathbf{k}_0 \tag{8}$$

Bending and shear response of the present system with the designed support offsets is determined according to the same procedure described in 2.4 for the system with zero support offsets.

### 2.7 Design acceptance criteria and their verification

Detailed description of the design acceptance criteria would be beyond the scope of this paper, so they will be just briefly outlined here. These criteria are to be met for the pseudo-static response of the shaft line in operation, both for cold and hot working conditions, as follows [3]:

- The stresses in shafts are to be below the prescribed permissible limits. This criterion may be applied either to the normal stresses or the equivalent stresses.
- Loading of the bearings is to be within prescribed limits. In case of vertical plane calculations, bearing reactions are to be directed upwards (to avoid overloading of the neighbouring bearings) with the rule of thumb criterion for the minimal reaction value as 20% of the left and right total load of the span. Maximal reaction values shall not exceed the ones allowable by the specific pressure in the bearings, dependent upon the bearing material in question.
- Shaft line slope in the bearings is to be within allowable limits dependent upon the bearing pre-selected clearances, to avoid metal contact between the bearing and the shaft at the bearing ends. Otherwise, slope boring of the bearings will be unavoidable. The rule of thumb states that the slope may "spend" up to 50% of the bearing clearance. Some classification societies, e.g. [4], prescribe that the relative slope between the propeller shaft and the aftermost sterntube bearing is, in general, not to exceed 0.3 mm/m in the static condition.
- The shaft line shall not overload the gearbox itself, in case of propulsion systems with gearboxes. The gearbox manufacturers usually prescribe the maximal allowable load transmitted by the shaft line to the gearbox. In the absence of this data, the rule of thumb will be to limit the difference in reaction forces in the two bearings of the gearbox output shaft to maximum 20% of the weight of the bull gear.
- The shaft line is not to overload the main propulsion engine crankshaft or thrust shaft, in case of propulsion systems with directly coupled main engines. As a rule, the engine manufacturers usually prescribe the maximal allowable load transmitted by the shaft line to the engine flange in terms of shear forces and bending moments allowable range.

The stated design acceptance criteria shall be explicitly verified in the calculation phase, after all the results (system response values) have been obtained.

## 3 Calculation example

The presented calculation procedure has been implemented in the computer program *MarShAl* (Marine Shafting Alignment), coded in MS Excel/VBA, dedicated to the presented calculations. For illustration a few characteristic diagrams obtained by this computer code, which have been implemented and verified on an inland navigation ship, are presented hereafter (Figure 3). The propulsion system consists of a four-stroke engine (279 kW), connected to the shafting by a reduction gearbox.

## 4 Conclusion

Shaft line is to be properly aligned in order to ensure its reliable functioning throughout the complete ship lifetime. Careful calculation, as well as setting up of its results onboard (for the ship afloat), as well as their validation during the assembly and the testing phase is essential. This paper describes details of the calculation procedure, promoting the advantages of (somewhat forgotten) transfer matrix methods.

Details of the design acceptance criteria, a real model of radial journal bearings and the review of classification society requirements are beyond the scope of this paper. However, from the authors' long-term experience with this matter, it should be important to enhance the existing technical rules requirements of classification societies for shafting alignment to cover also the case of small size ships. A proposal of this kind is expected to be the matter of further work.

It is to be pointed out once again that this paper presents only the basic information related to shaft alignment calculations, in

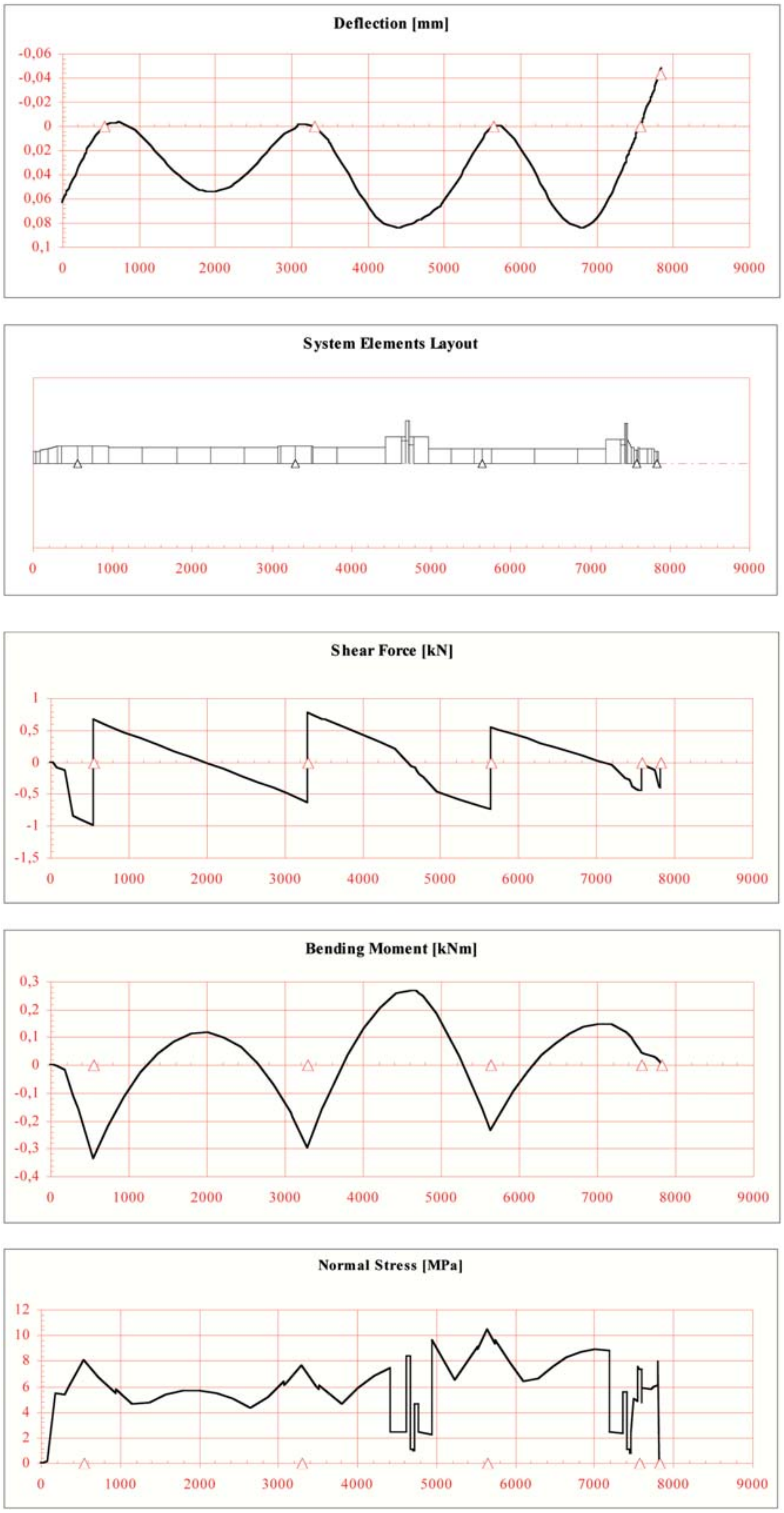


Figure 3 **Example of calculation results**
Slika 3 **Primjer dijagramskog prikaza rezultata proračuna**

order to help designers, shipbuilders, or even engineering students understand the essential calculation concepts. The authors' experience shows that it is important to make such information, presented in simple terms, widely available to the public. Comprehensive further information regarding the complex subject of shaft alignment calculation, validation and criteria may be found elsewhere in literature, e.g. [4], [5], [6] and [7], together with the detailed information about specialised and extremely powerful software.

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