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A MODIFICATION OF ABDELKARIM-OHNO MODEL FOR RATCHETING SIMULATIONS

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Original scientific paper

When materials are subjected to cyclic plastic loading with nonzero mean stress, strain usually accumulates in the direction of mean stress with an increasing number of cycles. This kind of strain accumulation is called cyclic creep or ratcheting. The phenomenon strongly depends on many factors and its accurate numerical modelling is problematic, especially under non-proportional loading. Two modifications of AbdelKarim-Ohno cyclic plasticity model were proposed for better ratcheting simulations. The developed model was implemented into the commercial FE software ANSYS using a radial return method. In this paper, the model capability is verified on a set of axial/torsional loading paths taken from Chen et al. The model predicts ratcheting strain reasonably well for almost all test cases.

Keywords: *finite element method, simulation, cyclic plasticity, ratcheting*

Modifikacija AbdelKarim-Ohno modela za simulacije akumulacije plastične deformacije

Izvorni znanstveni članak

Kada su materijali podvrgnuti cikličnom plastičnom opterećenju sa ne-nultim srednjim naprezanjem, deformacija se obično akumulira u smjeru srednjeg naprezanja sa sve većim brojem ciklusa. Ova vrsta akumulacije deformacije zove se ciklično puzanje ili akumulacija plastične deformacije (ratcheting). Ovaj fenomen ovisi o mnogim čimbenicima i teško je napraviti njegovo točno numeričko modeliranje, posebno u uvjetima neproporcionalnog opterećenja. U svrhu postizanja boljih simulacija akumulacije plastične deformacije predložene su dvije modifikacije AbdelKarim-Ohno modela cikličke plastičnosti. Dobiveni model je implementiran u komercijalni FE software ANSYS korištenjem metode radijalnog povratka. U ovom radu su potvrđene mogućnosti modela pomoću niza aksijalnih/torzijskih opterećenja (Chen i ostali). Model prilično dobro predviđa akumulaciju plastične deformacije za skoro sve probe.

Ključne riječi: *metoda konačnih elemenata, simulacija, ciklična plastičnost, akumulacija plastične deformacije*

1 Introduction
Uvod

Ratcheting is an important factor in the design of structural components. It has been observed by many authors that the presence of ratcheting during cyclic plastic loading may reduce the crack initiation life and thus the fatigue life of components. The fatigue of materials includes a lot of individual problems such as cyclic plasticity in low-cycle fatigue [1]. For example, manufacturing processes correspond to surface final quality [2] and so indirectly to residual stresses, which influences fatigue life of specimens or real structures [11].

The effect of ratcheting occurs for example in piping components [3] and rail/wheel system under higher traction forces [10]. Since ratcheting is the progressive deformation accumulating cycle by cycle, it is not easy to predict the development of ratcheting accurately. It is suitable to use the Finite Element Method for sufficiently accurate prediction of the number of cycles until the fatigue crack initialization under multi-axial loading. On the other hand, the plasticity models included in commercial FEM software are not able to describe well multi-axial ratcheting [4]. Hence, implementation of a more accurate model into the FE code is necessary.

2 Time-independent plasticity
Vremenski neovisna plastičnost

Constitutive equations for the mechanical behaviour of materials developed with internal variable concept are the most frequently used technique in the last two decades. In this concept, the present state of the material depends on the present values only of both observable variables and a set of internal state variables [5]. When time or strain rate influence the inelastic behaviour can be neglected, time-independent plasticity is considered. In this study viscoplasticity effects were neglected and the isothermal condition was assumed.

To solve nonlinear engineering problems by means of the constitutive equations, it is necessary to perform incremental computations of the structure. The cyclic plasticity constitutive models employed for ratcheting analysis with the assumption of rate-independent material's behaviour consist of von Mises yield criterion

$$f = \sqrt{\frac{3}{2}(\boldsymbol{s}-\boldsymbol{a}):(\boldsymbol{s}-\boldsymbol{a})} - \sigma_Y - R = 0, \tag{1}$$

the plastic flow rule

$$\mathrm{d}\boldsymbol{\varepsilon}_p = \mathrm{d}\lambda \frac{\partial f}{\partial \boldsymbol{\sigma}} \tag{2}$$

and the kinematic hardening rule

$$\mathrm{d}\boldsymbol{a} = g\left(\boldsymbol{\sigma}, \boldsymbol{a}, \boldsymbol{\varepsilon}_p, \mathrm{d}\boldsymbol{\sigma}, \mathrm{d}\boldsymbol{\varepsilon}_p, \text{etc.}\right) \tag{3}$$

where, $\boldsymbol{s}$ is the deviatoric part of stress tensor σ, $\boldsymbol{a}$ is the deviatoric part of back-stress α, σ_y is the initial size of the yield surface, R is the isotropic variable, ε_p is the plastic strain tensor and $\mathrm{d}\lambda$ is the plastic multiplier, which is equal to the equivalent plastic strain increment in the case of associated plasticity

$$\mathrm{d}p = \sqrt{\frac{2}{3}\mathrm{d}\boldsymbol{\varepsilon}_p : \mathrm{d}\boldsymbol{\varepsilon}_p} \tag{3}$$

where the symbol $:$ denotes contraction, i.e. using Einstein's summing rule $\boldsymbol{d} = \boldsymbol{b} : \boldsymbol{c} = b_{ij}c_{ij}$.

3 Description of the proposed model
Opis predloženog modela

As it is already known, for modeling of cyclic behaviur of ductile materials, and for catching of Bauschinger effect and simultaneously for good ratcheting description, it is necessary to use the proper kinematic hardening rule. The isotropic hardening is not assumed in this study. For example, Chaboche [5] introduced "decomposed" nonlinear kinematic hardening rule in the form

$$\boldsymbol{a} = \sum_{i=1}^{M} \boldsymbol{a}_i ,\ \mathrm{d}\,\boldsymbol{a}_i = \frac{2}{3} C_i \mathrm{d}\,\boldsymbol{\varepsilon}_p - \gamma_i\, \boldsymbol{a}_i\, \mathrm{d}p, \tag{5}$$

where C_i and γ_i are material constants. Indeed, it was found by many researchers, that the Chaboche model overpredicts biaxial ratcheting responses. On that account, AbdelKarim and Ohno [6] have proposed this modification of Chaboche nonlinear kinematic hardening rule

$$\boldsymbol{a} = \sum_{i=1}^{M} \boldsymbol{a}_i ,\ \mathrm{d}\boldsymbol{a}_i = \frac{2}{3} C_i \mathrm{d}\,\boldsymbol{\varepsilon}_p - \mu_i \gamma_i \boldsymbol{a}_i\, \mathrm{d}p - \gamma_i H(f_i) \langle \mathrm{d}\lambda_i \rangle \boldsymbol{a}_i , \tag{6}$$

where

$$f_i = \frac{3}{2} \boldsymbol{a}_i : \boldsymbol{a}_i - \left(\frac{C_i}{\gamma_i} \right)^2 , \tag{7}$$

and

$$\mathrm{d}\lambda_i = \mathrm{d}\boldsymbol{\varepsilon}_p : \frac{\boldsymbol{a}_i}{\frac{C_i}{\gamma_i}} - \mu_i \mathrm{d}p. \tag{8}$$

In equation (6), the symbol $\langle \cdot \rangle$ indicates the MacCauley bracket, i.e. $\langle x \rangle = (|x| + x)/2$. The parameters μ_i set a ratcheting rate. The original AbdelKarim-Ohno model could simulate well only ratcheting with steady-state [6]. Because of simplification the only parameter $\mu = \mu_i$ for all i is usually used in the AbdelKarim-Ohno model. Influence of the parameter μ on ratcheting is evident from Fig. 1. If $\mu = 0$ the AbdelKarim-Ohno model corresponds to the Ohno-Wang I model, which always predicts zero uniaxial ratcheting [7], if other sources of ratcheting are not present. On the contrary, if $\mu = 1$ the equation (6) reduces to the Chaboche model, as it could be easily proven. It is clear that the apropriate choice of parameter μ gives the desired ratcheting rate.

In addition, assuming the only parameter for ratcheting

$$\mathrm{d}\eta = \omega (\eta_\infty - \eta) \mathrm{d}p, \tag{9}$$

transient effect in initial cycles could be introduced [8]. Indeed, for a large transient effect, it is more suitable to compose increment dη from two parts

$$\mathrm{d}\eta = \mathrm{d}\eta_1 + \mathrm{d}\eta_2 , \tag{10}$$

where

$$\mathrm{d}\eta_1 = \omega_1 (\eta_{\infty 1} - \eta_1) \mathrm{d}p,\ \mathrm{d}\eta_2 = \omega_2 (\eta_{\infty 2} - \eta_2) \mathrm{d}p, \tag{11}$$

Then, the six material constants $\eta_{01}, \eta_{02}, \eta_{\infty 1}, \eta_{\infty 2}, \omega_1, \omega_2$ should be determined by fitting an uniaxial ratcheting test. In this procedure it is useful to get the values of the ratcheting rate $\mathrm{d}\varepsilon_p/\mathrm{d}N$ (for each cycle) and corresponding values of the accumulated equivalent plastic strain p from experimental data. The original AbdelKarim-Ohno model shows near linear dependence of the ratcheting rate $\mathrm{d}\varepsilon_p/\mathrm{d}N$ on the ratcheting parameter $\mu(p)$, i.e. $\mathrm{d}\varepsilon_p/\mathrm{d}N = k\,\mu(p)$. After determination of the parameter k by a FE computation with the appropriate choice of ratcheting parameter (for example μ=0,1) the corresponding values $\mu(p)$ can be calculated. The last step of the procedure is fitting by the function

$$\left. \begin{aligned} \mu(p) = \eta = \eta_{\infty 1} + (\eta_{01} - \eta_{\infty 1}) e^{(-\omega_1 p)} + \\ + \eta_{\infty 2} + (\eta_{02} - \eta_{\infty 2}) e^{(-\omega_2 p)} \end{aligned} \right| , \tag{12}$$

which can be obtained by integration of equations (10) and (11). The Levenberg-Marquardt nonlinear least square method was applied for that purpose in this study.

Furthermore, it is difficult to simulate simultaneously the uniaxial and the multiaxial ratcheting responses with the AbdelKarim–Ohno model as was found by Chen et al. [8]. This problem can be solved by introduction of the nonproportional term into the ratcheting parameter

$$\mu_i = \eta \left\langle \frac{\partial f}{\partial \boldsymbol{\sigma}} : \frac{\boldsymbol{a}_i}{\bar{a}_i} \right\rangle^{\chi} ,\ \bar{a}_i = \sqrt{\frac{3}{2} \boldsymbol{a}_i : \boldsymbol{a}_i} , \tag{13}$$

where material constant χ should be determined from a multiaxial ratcheting test. In contrast to previous work [3] the MacCauley bracket is used instead of the absolute value as was originally proposed by Chen for a modification of Ohno-Wang II model [8]. For uniaxial loading, terms (13) become a unity and the multiaxial parameter χ is ineffective. For multiaxial loading, terms (13) are less than a unity and the multiaxial parameter χ will influence the ratcheting response. A proper value of χ should also be found from experimental data.

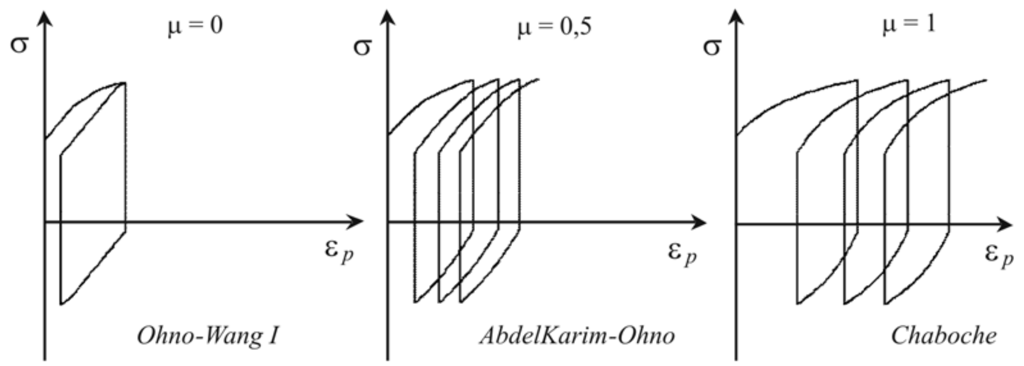

Figure 1 Influence of parameter μ on ratcheting in AbdelKarim-Ohno model for uniaxial loading
Slika 1 Utjecaj parametra μ na akumulaciju plastične deformacije u AbdelKarim-Ohnovom modelu za jednoosno opterećenje

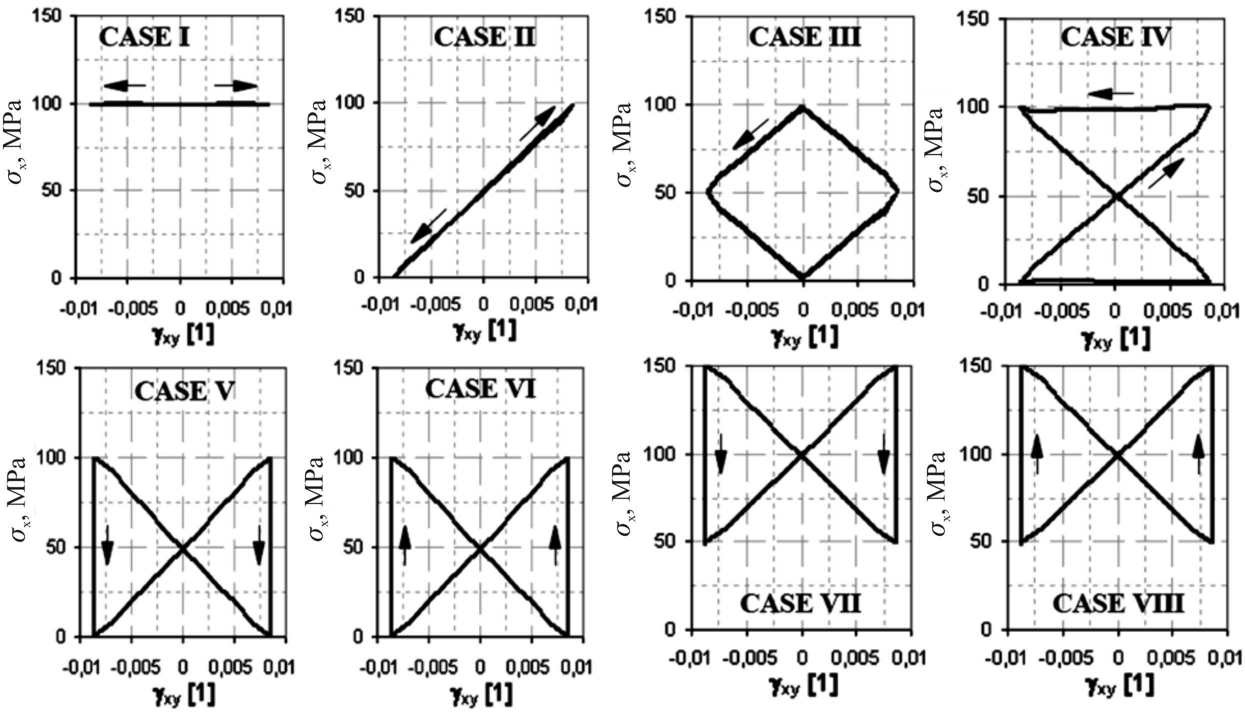

Figure 2 Loading histories – tension/torsion ratcheting tests (curves from simulations)
Slika 2 Povijest opterećenja – naprezanje/uvijanje testovi akumulacije plastične deformacije (krivulje iz simulacija)

The described cyclic plasticity model has been coded into the FE software ANSYS as a user material subrutine. The return mapping algorithm with successive substitution utilized by Kobayashi and Ohno [9] was used for implicit stress integration. The quadratic convergence of the Newton-Raphson method in solving the global equilibrium equations is ensured by use of elasto-plastic stiffness matrix consistent with integration scheme, so called consistent tangent modulus. Because of the nonlinear nonproportional term usage in the ratcheting parameter, the analytical way for the consistent tanget modulus expression is difficult. An approximate solution of the consistent tanget modulus was derived using the classical differential approach and confronted with the exact one for the original AbdelKarim-Ohno model. The approximate solution will be described in a future paper. Due to the nonlinear kinematic hardening law, the consistent tangent modulus is unsymmetrical. For the evaluation of the stiffness matrix the unsymmetrical tangent operator is symmetrized in this study.

4 Comparison of simulations with experimental data
Usporedba simulacija s eksperimentalnim podacima

In the previous study [3], a similar cyclic plasticity model has been evaluated for a set of uniaxial and biaxial ratcheting responses of a carbon steel 1026. Now, the verification of the proposed model will be done on experiments with different loading path shape in axial/torsional tests taken from paper [8]. The material used in experiments was a medium carbon steel S45C. The specimen has a tubular geometry with outer and inner diameters of 12,5 mm and 10 mm respectively. Completely reversed, uniaxial and torsional tests at several strain amplitudes, a uniaxial ratcheting test and eight tests under tension/torsion combined loading with different loading histories were realised by Chen et. al [8]. The last mentioned tests were conducted under force control for axial loading and under strain control for torsional loading (at room temperature), hence the ratcheting occurs in axial direction. The loading paths in axial stress – shear strain plane are ilustrated in Fig. 2.

The benefits of the proposed model will be clear from a comparative analysis performed by the Chaboche model with three decomposed hardening rules, M=3 in eq. (5). The material parameters C_i , γ_i of both models have been estimated by the engineering approach published in Bari and Hassan [4] from the largest uniaxial hysteresis loop (the parameters of the modified AbdelKarim-Ohno model can be determined using the same relations as for OhnoWang model). Table 1 includes all material parameters of the proposed model and the Chaboche model, except for elastic constants (Young modulus E=206000 MPa and Poissons ratio ν=0,3).

Tab. 1 Parameters of used cyclic plasticity models
Tablica 1 Parametri korištenih modela ciklične plastičnosti

Model	Parameters
Chaboche (CHAB)	σ_Y = 220 MPa, C_{1-3} = 300000, 90000, 7500 MPa γ_{1-3} = 3000, 600, 1
Proposed (MAKO)	σ_Y = 220 MPa, C_{1-6} = 434968, 74544, 39359, 15263, 6439,8607 MPa γ_{1-6} = 6077, 1551, 735, 345, 194, 10 $\eta_{01} = -0{,}2; \eta_{\infty 1} = 0{,}1; \omega_1 = 1{,}5; \eta_{02} = 0{,}27; \eta_{\infty 2} = 0; \omega_2 = 0{,}7; \chi = 0{,}5$

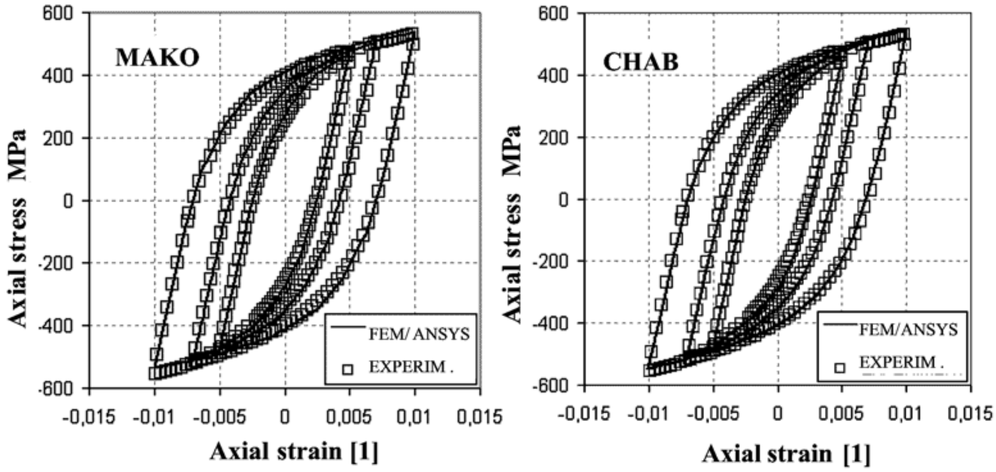

Fig. 3 Uniaxial hysteresis loops
Slika 3 Jednoosne histerezne petlje

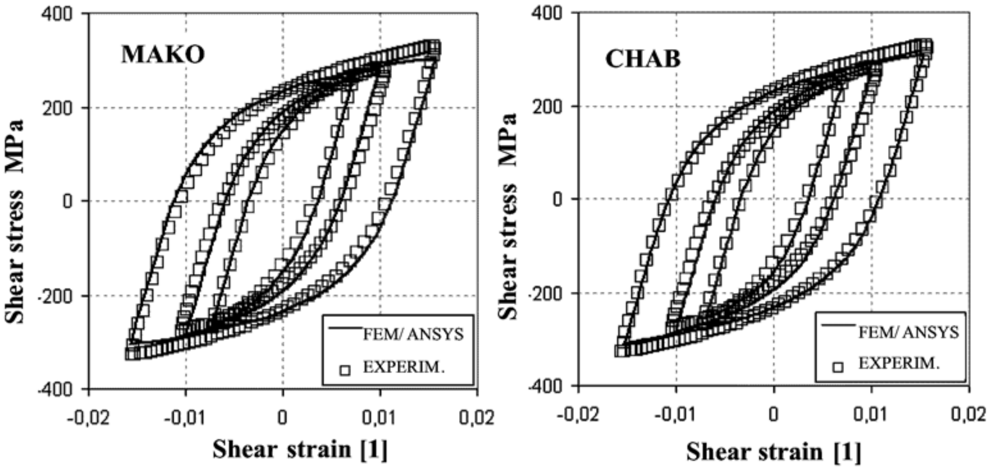

Figure 4 Torsional hysteresis loops
Slika 4 Uvojne histerezne petlje

The results from simulations of completely reversed uniaxial and torsional tests under three different strain amplitudes are shown in Fig. 3 and 4. The both models describe the cyclic stress-strain response quite well.

The next simulated case corresponds with the uniaxial ratcheting test to stress amplitude σ_a=370 MPa and mean stress σ_m =100 MPa. The simulation was also used for the estimation of the material parameter γ_3 of the Chaboche model (CHAB) and parameters $\eta_{01}, \eta_{02}, \eta_{\infty 1}, \eta_{\infty 2}, \omega_1, \omega_2$ of the proposed model (modified AbdelKarim–Ohno model,

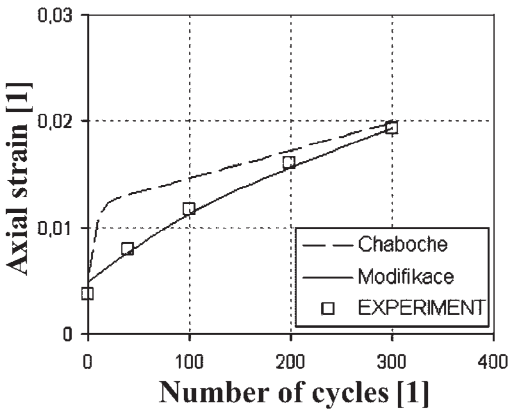

Figure 5 Uniaxial ratcheting simulations
Slika 5 Jednoosne simulacije akumulacije plastične deformacije

MAKO). The possibility of the Chaboche model calibration by appropriate non-zero estimation of parameter γ_3 was also explained in the paper [4].

The proposed model shows better results than the Chaboche model, see Fig. 5. Except for the seven described cases under proportional loading the eight tests of tension/torsion combined loading with different loading histories (Fig. 2) were simulated. The results from performed computations of the eight cases under nonproportional loading are presented graphically in Fig 6.

The second nonproportional loading case shows the best correlation with experimental results but it should be pointed out that this fact is caused by using the case for identification of nonproportional ratcheting constant χ. The MAKO model gives a very good prediction of ratcheting strain for cases I to VII but in case VIII the prediction is even worse than the Chaboche model. Tests VII and VIII differ from tests V and VI only by a higher mean axial stress value. The change of loading direction in case VI with respect to the loading direction of case V has a significant influence on ratcheting response but in cases VII and VIII it has anly minimal influence. Unfortunately, Chen [8] did not report more details about experimental results to explain the fact of poor prediction of the MAKO model in case VIII. However, it can be mentioned that the modified Ohno Wang model proposed by Chen [8] has similar behaviour as the MAKO model in simulations of cases VII and VIII.

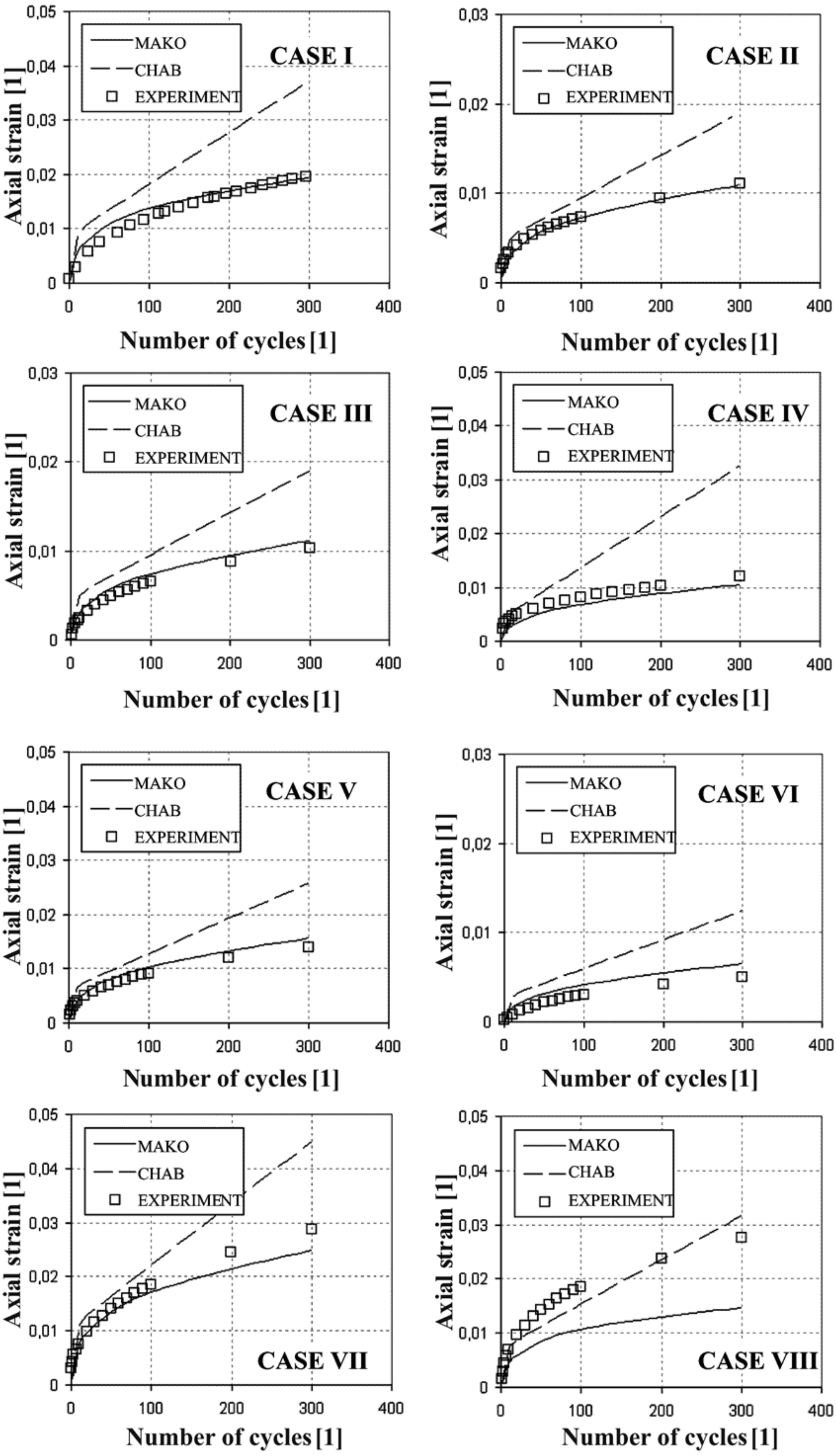

Figure 6 Results of multiaxial ratcheting simulations
Slika 6 Rezultati višeosnih simulacija akumulacije plastičnih deformacija

5 McDowell's extrapolation algorithm
McDowellov ekstrapolacijski algoritam

Mainly rail steels undergo a decaying rate of strain accumulation during progressive cycling. Numerical analysis can be very time-consuming to carry out for a large number of loading cycles for a structure when such a case of ratcheting occurs. McDowell [10] developed an algorithm for extrapolation of cyclic strain accumulation to arbitrarily high cycles to solve the problem. In general, the value of any deformation quantity Δ_r in required number of cycles can be predicted using experimental or numerical data from initial cycles (practically 20-50 cycles). The decay relation was proposed

$$\frac{\mathrm{d}\Delta_r}{\mathrm{d}N} = \left(\frac{\mathrm{d}\Delta_r}{\mathrm{d}N}\right)_0 \left(\frac{N_0}{N}\right)^{1+R^*\xi}, \tag{14}$$

where

$$\left(\frac{\mathrm{d}\Delta_r}{\mathrm{d}N}\right)_0$$

is the ratcheting rate at N_0 cycles and ξ is a material constant. The parameter R^* depends on the mean stress level under uniaxial loading and it is generally defined as

$$R^* = \frac{\tau^*_{\min}}{\tau^*_{\max}}, \tag{15}$$

where τ^* is shear stress on the plane of maximum range of shear stress over the cycle. It should be noted that $\tau^*_{\max}$ corresponding to the maximum of absolute value of τ^* in the cycle (because of physically irrelevant sense of shear stress). Integration of equation (14) leads to a deformation quantity accumulation of

$$\Delta_r = \frac{-N_0}{R^*\xi}\left[\left(\frac{N_0}{N}\right)^{R^*\xi} - 1\right]\left(\frac{\mathrm{d}\Delta_r}{\mathrm{d}N}\right)_0 + \Delta_{r0}, \tag{16}$$

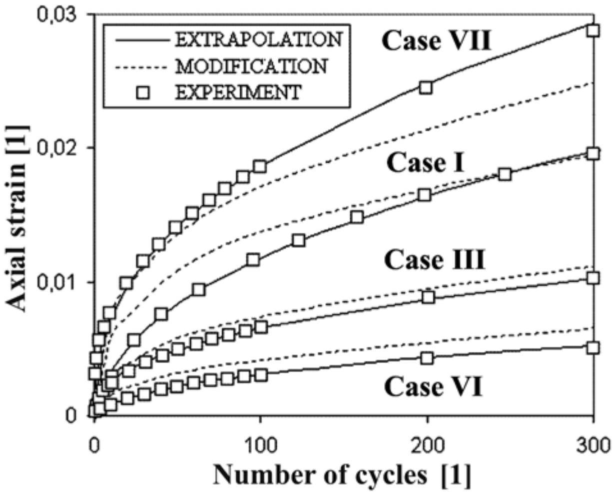

Figure 7 Results from extrapolation algorithm
Slika 7 Rezultati ekstrapolacijskih algoritama

for nonzero R^*. In equation (16) the parameter Δ_{r0} is the value of extrapolated quantity at N_0 cycles. The extrapolation method described above was used to predict the axial strain accumulation and to compare it with the results of numerical simulations. The initial number of cycles N_0=50 and R^*=-1 was assumed for all solved cases of nonproportional loading. Fig. 7 shows the correlation achieved for the chosen cases. All other cases have similar good correspondence of extrapolated results and experimental results. The value of material constant ξ was 0,41 for all calculations.

6 Conclusions
Zaključci

The paper shows possibilities of the modified cyclic plasticity model AbdelKarim-Ohno in modelling uniaxial ratcheting and multiaxial ratcheting under various load shapes in tension/torsion combined loading. For this purpose, experimental data were taken from Chen et al. [8]. The proposed model was implemented into the FEM software ANSYS and it gave better results than the Chaboche model included in the SW package for almost all solved cases. On the other hand, a certain disadvantage of the new model is the need for more parameter identification from experimental data. This trend is also clear in other recent models of cyclic plasticity, for example [8] and [11]. The smooth specimen experiments with the material S45C [8] reveal a gradual decay of ratcheting rate during progressive cycling. For such material behaviur the McDowells extrapolation method can predict strain accumulation to a higher number of cycles as the previous section describes. It is clear from the comparison of extrapolation results and simulation results that ratcheting parameters of the proposed model should be coupled rather with an additional state variable pertinent to the number of cycles than accumulated equivalent plastic strain.

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