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# Foot Morphometric Phenomena

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**ABSTRACT**

*Knowledge of the foot morphometry is important for proper foot structure and function. Foot structure as a vital part of human body is important for many reasons. The foot anthropometric and morphology phenomena are analyzed together with hidden biomechanical descriptors in order to fully characterize foot functionality. For Croatian student population the scatter data of the individual foot variables were interpolated by multivariate statistics. Foot morphometric descriptors are influenced by many factors, such as life style, climate, and things of great importance in human society. Dominant descriptors related to fit and comfort are determined by the use 3D foot shape and advanced foot biomechanics. Some practical recommendatios and conclusions for medical, sportswear and footwear practice are highlighted.*

***Key words:*** *foot morphology, foot biomechanics, comfort*

## Introduction

The human foot is a flexible structure, playing an important role in human everyday life. It is a part of the body that acts on external surface, providing support and balance during stance and gait. The foot structure description, beside anthropometric and morphology descriptors needs biomechanical factors such as muscle deformation, tissue stiffness, stress and strain distribution[1]. The full morphology description of the foot for more than 26 descriptive measures is desired. Foot dynamic morphology has a vital role in medical rehabilitation, sport science, and footwear design among others[2]. For example, some studies indicate that foot morphology and deformity affect peak plantar pressure, which is related to ulcer development. Several techniques have been developed to study the morphology, architecture and kinematics of the foot[3]. Integrated experimental technique is able to measure simultaneously both the kinematics and dynamic structural behaviours of the foot during gait. A biomechanical model of the foot for specified external load gives a stress and strain distribution under foot structure (Figure 1).

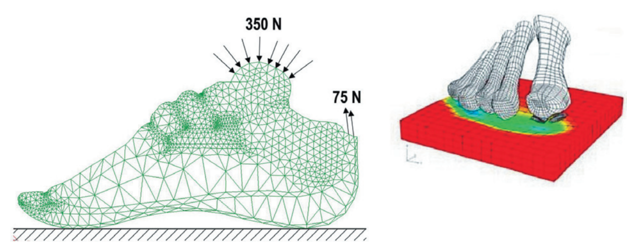


*Fig. 1. 2D biomechanical model of the foot.*

The objective of this work is to understand foot shape, morphology and foot biomechanics and derive usefulness regression relationship for Croatian student population. Scatter data of the individual foot variables are interpreted by multivariate statistics. This is multiple regression analysis, in which various combinations of these descriptors were regressed against each other with physical explanation all relevant phenomena.

## Materials and Methods

### *Foot shape modelling*

Shape and size have important influence when designing product and equipment. The distribution of foot sizes is related to production planning: how many and what size of shoes should be produced to satisfy the consumers needs. Anthropometric variables such as foot length, joint girth, bottom width, are stochastic variables in geometric description of the foot[4]. Probability distribution of these variables is determined by the measure-

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ment for target population in determined geographic region in specified time interval. Rapid advancements in morphometrics and multivariate statistics of the shape objects encourage the replacement of shoemaking craftsmen with computer designer[4]. In the footwear practice, manufacturers assume that most foot dimensions follow multivariate $n$ – dimensional normal distribution

$$f(x)=\frac{1}{(2\pi)^{n/2}|\Sigma|^{1/2}}\exp\left[-\frac{1}{2}\{x-\mu\}^T\left[\Sigma^{-1}\right]\{x-\mu\}\right] \quad (1)$$

Here $\{x\}^T = \{x_1, x_2, ... x_n\}$ is an n–dimensional vector, $\{\mu\}^T = \{\mu_1, \mu_2, ... \mu_n\}$ is a mean vector, and $[\Sigma]$ is $n \times n$ – covariance matrix. Traditional shoe size system has been constructed using bivariate normal distribution with adequate classes and intervals for both foot length and joint girth[5]. These few measures are not sufficient for proper foot fit and comfort under shoe; hence a good description of the 3D foot shape is necessary. A foot in CAD (Computer Aided Design) is represented as a set of digitized point clouds on the surface that more closely approximate an average shaped foot. The 3D foot surface can be represented using Bezier or B spline, as the closest interpolating surface across digitizing points. According to the shape representation theory, there are a few 3D shape representation categories. Global feature-based methods use global properties of the 3D model such as moments, spherical harmonics and Fourier coefficients in order to represent shape[6].

### *Principal component analysis*

The recent development of 3D laser scanner has provided another efficient way for surface registration and analysis. The scattered grid point data are fitted by small polynomial surface patches. At each point it is possible to calculate the principal curvatures, mean curvature, Koenderink shape index[6], which are shape representing descriptors. The initial foot shape are collection 3D surfaces patches $S_i$ each represented by a set of n surface points

$$S_i=\begin{bmatrix} x_{i1} & y_{i1} & z_{i1} \\ x_{i2} & y_{i2} & z_{i2} \\ . & . & . \\ . & . & . \\ x_{in} & y_{in} & z_{in} \end{bmatrix} \qquad i=1,2...\mathrm{M} \quad (2)$$

where $M$ represents the number of surfaces. Principal component analysis (PCA) is applied to reduce large number parameters in modelling of the foot shape. Each array, $S_i$ was converted to a column vector $U_i^T=\{x_{i1},y_{i1},z_{i1},x_{i2},y_{i2},z_{i2},...x_{in},y_{in},z_{in}\}$ of length $3n$, and its magnitude was normalised to unity such that $\hat{U}_i^T \cdot \hat{U}_i = 1$. The $(3n \times 3n)$ variance-covariance matrix $\Xi$ for the $M$ surface vectors can be determined as follows

$$\Xi=\sum_{i=1}^{M}\hat{U}_i \cdot \hat{U}_i^T \quad (3)$$

The principal components $\lambda_J$ $j=1, 2, ... 3n$ of $\Xi$ were then determined by eigenvector decomposition; and their magnitudes were normalised. The eigenvalue represents the variance of the data with respect to the direction of its principal component, while eigenvector defined a principal component direction. Thus, for the i-th surface vector, each co-ordinate (or weight) $v_{ij}$ on the j-th principal component is

$$v_{ij}=U_i^T \cdot \lambda_j \qquad j=1, 2, ... 3n \quad (4)$$

This vector can be transformed back to the original co-ordinate system by following transformation procedure

$$U_i=\sum_{j=1}^{3n}v_{ij}\lambda_j \quad (5)$$

The first few eigenvectors $\lambda_k$, k=1,...m, m≤3n (with greatest eigenvalues) can explain most of the variance in the data. This means that the 3n dimensional space is approximated by m dimensional space. The m is the smallest number of the modes such that the sum of their variances explains a sufficiently large proportion of all the variables. Now any shape $\hat{S}_i$ in the data can be obtained by writing

$$\hat{U}_i=\sum_{j=1}^{m}v_{ij}\lambda_j \quad (7)$$

The procedure has a three steps; extraction of its principal components, projection of each surface onto the principal components, and reconstruction of each surface based on a small subset of the principal components. In a geometric sense, the direction of the first principal component lies along a line in n-dimensional space about which the variance of the n-dimensional data is maximised. The variance is defined as the sum of the squared distances from each data point to the line. The direction of the second principal component is orthogonal to that of the first principal component, such that the variance with respect to the line is similarly maximised. The case for the third principal component is similar, and so forth on to the n-th principal component. A virtual shape at the edge of a distribution is useful for finding the individual differences within a group. It is quite obvious to see how shapes change along the axis, which was obtained by multi-dimensional scaling, by making a movie by lining up the virtual shapes on the axis. It is also possible to calculate the virtual shapes on probability ellipsoids in which 95% of the samples will fall. Virtual shape in principal component space is illustrated by Figure 2. The shape distribution maps visualize the distance relationship between individuals calculated on the basis of the homologous shape model. According to the shape theory, the distance metric could be established by using multi-dimensional scaling. After that, we can categorize a foot into sizes or types for target population.

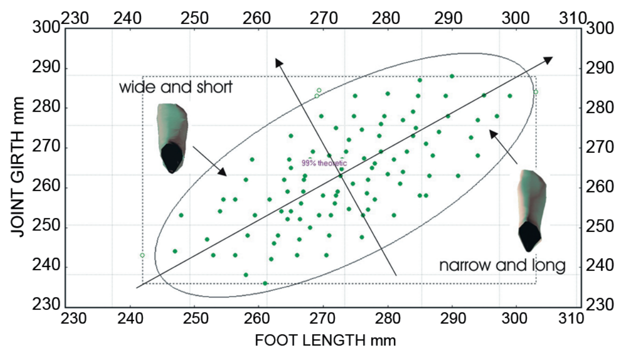


*Fig. 2. Principal component space.*

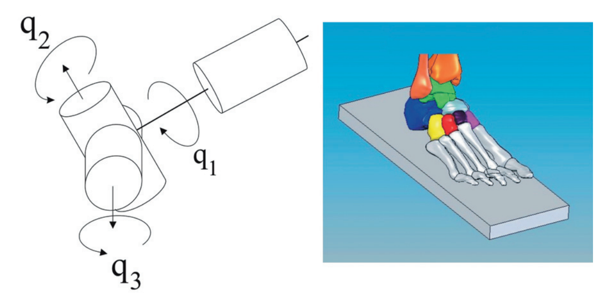


*Fig. 4. Foot with kinematics model with $q_i$ as generalized coordinates.*

## *The morphological parameters*

Foot shape and its variation is an important topic of medical engineering research. Understanding morphological changes caused by a particular disorder, ageing and growth phenomena are some of them which are need computational morphometry. The 3D reconstruction of the bone by magnet resonance (MR) is of main importance, because the reconstructured bone surface provides the morphological basics for finite element models of the foot. The 3D reconstruction of the bones is based on segmented bony parts in the respective MR images. For example, the shape of each foot bone is defined by the confidence ellipse in eigenvalue space of the inertial tensor. The separation between bone types is evident[7], (Figure 3). Some known morphological measures (shape distance metric, segmentation, clustering) applied to 3D foot shape can explain many foot morphology phenomena in the best possible way. The inertial tensor of the group bones can be useful morphological descriptor of the foot biomechanical functionality

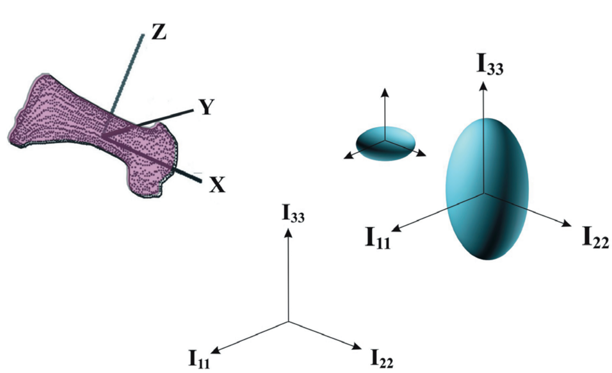


*Fig. 3. Bone inertial tensor space with $I_{11}$, $I_{22}$, $I_{33}$ as principal moments of inertia.*

The foot structure can be modelled as a kinematics system, a series of links connected by revolute joints that represent musculoskeletal joints (Figure 4). The kinetic energy K of the foot bones as scalar quantity can be expressed as follows

$$K = \frac{1}{2}\sum_{i=1}^{n}\left(\sum_{p=1}^{i}\sum_{r=1}^{i} Tr(D_{ip}\, I_i\, D_{ir}^{T})\,\dot{q}_p \cdot \dot{q}_r\right) \tag{8}$$

Where $\dot{q}_r$ is time derivative of the generalized coordinate $q_r$, $I_i$ is inertia tensor (matrix form) of the i-th bone, n is number bones in foot structure. The matrix $D_{ij}$ comes from Denavit-Hartenberg representation of bone structure as mechanisms[8]. Some degenerative forms of foot structure are evident from the structure of the matrix $D_{ij}$. The envelope all trajectories of the point of interest define workspace (Figure 5). The workspace manifolds has vital role in medical rehabilitation, modelling sports activities.

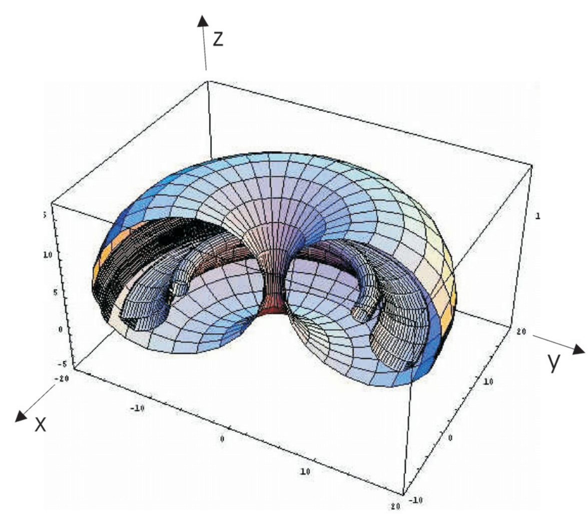


*Fig. 5. Workspace manifolds of the foot.*

Topology is typically represented in the form of a relational data structure such as graphs and trees. The shape can be represented by boundary representation, spectral, Reeb or skeletal graph. One of the shape description methods is medial representation of the foot. The m-rep parameters are the elements of the Lie groups, and therefore all statistical calculations must be performed in tangent space[9]. As one of the morphological characteristics of human foot, flexion angles of medial axis of foot outline have been proposed in order to improve shoe comfort.

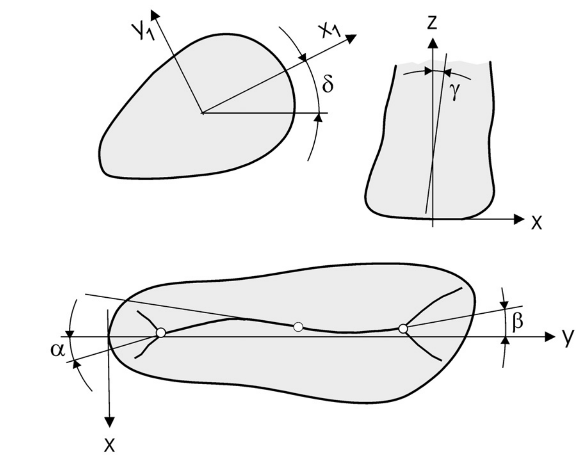


*Fig. 6. Medial axis of the foot with α – posterior flexion angle, β – anterior flexion angle, γ – heel inclination, δ – direction of principal axes $x_1$ and $y_1$.*

### *A footwear fit*

Fit is one of the most important functional aspects in footwear comfort. Proper fit is more than fit length and width; it requires a good understanding of the total 3D foot shape among others. The starting point is the geometric similarity between two feet and between foot and last. The basic idea is to compare the lasts which were used to manufacture the shoes and the scanned feet of the person. The foot geometric similarity can be described by two-step procedures: the pose estimation and object comparison[10]. The appropriate pose estimation, which consists of computing the scaling, translation and rotation of the objects, so that their surfaces lay one on the other. Any allowable shape can be reached by adding a linear combination of the eigenvectors to the standard model $\bar{v}$

$$v = \bar{v} + \Gamma \cdot C \tag{9}$$

where $\Gamma = [\lambda_1 \; \lambda_2, \ldots \; \lambda_m]$ is the matrix of the first m eigenvectors $\lambda_1$, and $C^T = \{c_1, c_2, \ldots c_m\}$ is a vector of weights. The above equations allow us to generate new samples of the shapes by varying the shape parameters $c_k$. A distance map is calculated to indicate the difference between the shoes last and foot shape. The cost function had minimized the following least-squared distance metrics

$$\min_{R,T} \sum_i \left\| A_i - (R \circ B_i + T) \right\|^2 \tag{10}$$

Where $A_i$ and $B_i$ represent points on the last and foot, respectively. The goal is to find a rotation R and translation T matrix that minimize the least-squared distance metric. After the pose estimation, we determine for each object how big portion of its volume is outside of the other object. This comparison problem is possible to be resolved by discrete 3D distance field concept, which is described in[10]. The first step is to estimate the normal vector of each triangle of the foot and last surface, respectively. The next step is to calculate distance along normal vectors between the surfaces of the aligned foot and last. For every data point of the shoe last, the nearest foot data point can be found by searching along the normal direction.

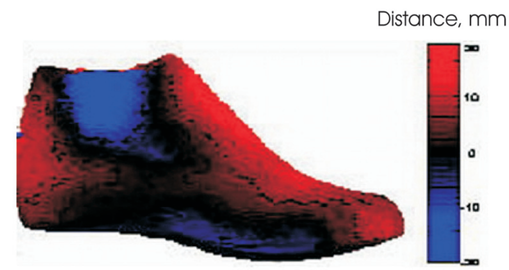


*Fig. 7. Distance map between foot and last on the last surface.*

A colour distance map is shown in Figure 7. The region in red shows the positive difference (tight), while the region in blue shows the negative difference (loose). The shape comparison between foot and particular shoe determine comfort feeling. This problem is better to analyze as contact problem between shoes as membrane shell and foot as volume preservable structure. In the simplest case, when a shoe becomes double curved membrane, the contact can be described by Laplace equations (Figure 8)

$$\frac{\sigma_1}{R_1} + \frac{\sigma_2}{R_2} = \frac{p}{h} \tag{11}$$

Where are $\sigma_1$ and $\sigma_2$ membrane stresses in merigional and circumferential direction, $R_1$ and $R_2$ are main curvature radii, p is contact pressure and h is thickness of the membrane. The above formula (11) revels that the area with a small curvature is a possible place of high stress concentration and a place of discomfort. The quality of fit in the fore-foot and mid-foot region is of the greatest importance. Subjective evaluations are the final step in determining the user-product interaction[11].

## Results and Discussion

A group of 103 normal adult males selected among student population in Croatia has participated in this study[12]. The age of participants was between 19–22

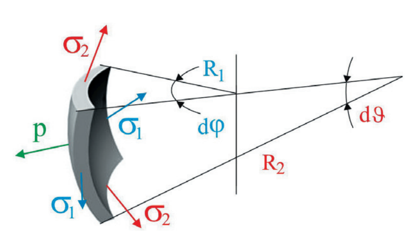


*Fig. 8. Segment of the shoe upper as membrane element with principal curvature $R_1$ and $R_2$ under membrane stress $\sigma_1$ and $\sigma_2$.*

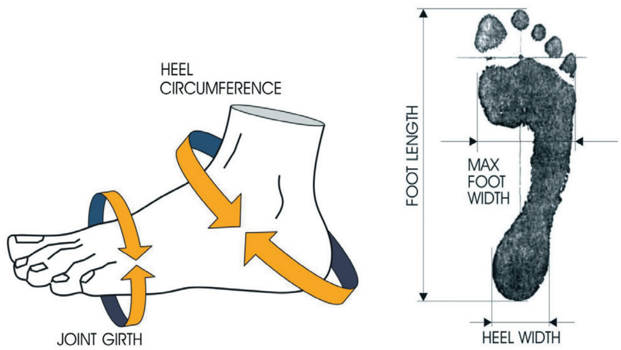


*Fig. 9. Measured foot dimensions.*

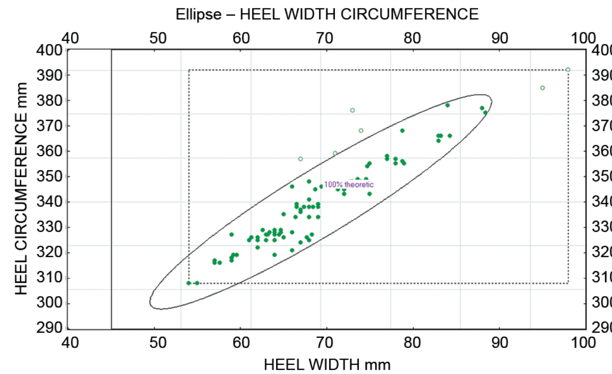


*Fig. 11. Calculated normal distribution heel width-circumference.*

years. None of the participants had any foot illnesses or deformity. Their stature height and weight were first recorded. All measurements were made under žno-load' conditions. The five dimensions (Figure 9) on the left foot have been measured for each subject (foot length, joint girth, maximum foot width, heel width and circumference).

According to footwear practice, we assume that foot length and joint girth obey bivariate normal distribution. The scatter data are interpolated by bivariate normal distribution[12]. The calculated average foot length is $\mu_1$ = 273.29 mm (standard deviation is $\Sigma_1$ = 12.01 mm) and the average joint girth is $\mu_2$ = 263.18 mm (standard deviation is $\Sigma_2$ = 12.52 mm). We also constructed bivariate normal distribution foot length-maximum foot width (Figure 10) and heel width-heel circumference (Figure 11), respectively. Foot length and foot width show low correlation ($r^2 \approx 0.42$).

The grouping of data is evident with stronger correlation. The footprint is a used for the classification morphology based on medial longitudinal arch[12].

A bivariate normal distribution foot length-joint girth for Croatian population has been compared with some other nationality and race. We compared our data with literature data[14], for urban population in Russia and four populations in East Asia, i.e. Chinese, Japanese, Korean and Taiwanese[15]. Ethnic diversity is a significant factor and affects foot shape, too. It is possible to conclude that the Croatian population has large feet and different correlation coefficient. Therefore, establishing a national anthropometry database for the population is now inevitable[16]. Some of the subjects have been selected to test comfort and fit. The participants ask to rate the overall fit dressed shoes. The foot-shoe fit depends on many objective and subjective factors, among others. The most preferred wearing running shoes out of 10 pairs of shoes are compared. The appropriate shoe size is supplied by the last in order to make comparison procedure. The participant walked around for a few minutes until they were able to get a good feel for the fit. For overall fit, a 7-point rating scale is used (–3 = very tight, –2 = tight, –1 = acceptable, 0 = neutral, +1 = acceptable, +2 = loose, +3 = too loose). The foot's outlines were aligned with the last outlines using the heel centreline (Figure 12). The dimensional differences from each point on the foot to the last outline were computed using the shortest Euclidean distance. The measurement is done along the perimeter of the foot.

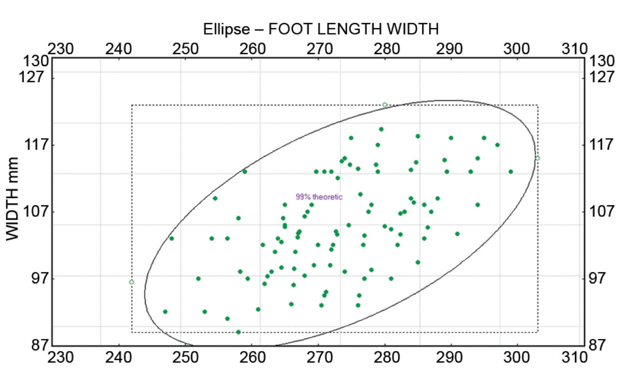


*Fig. 10. Calculated normal distribution foot length- width.*

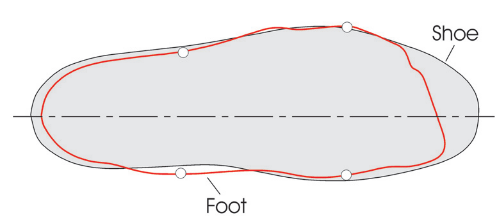


*Fig. 12. Dimensional fit between foot and shoe.*

**TABLE 1**
INTER-VARIABLE CORRELATION COEFFICIENTS

| | Mid-Foot Dim Difference | Mid-Foot Dim Difference | Fit Ratio |
|---|---|---|---|
| Mid-Foot Dim Difference | 1 | 0.78 | 0.89 |
| Fore-Foot Dim Difference | | 1 | 0.95 |
| Fit Ratio | | | 1 |

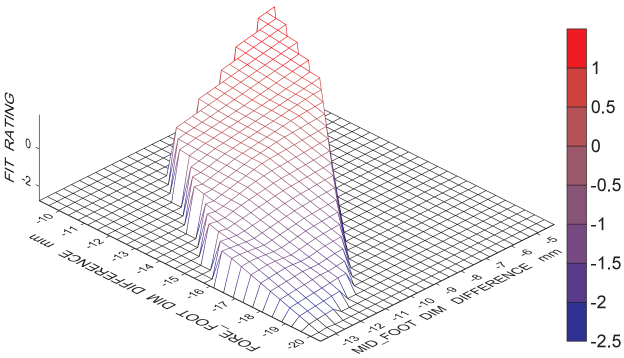


*Fig. 13. The overall fit rating dependence on mid-foot and on fore-foot dimensional difference.*

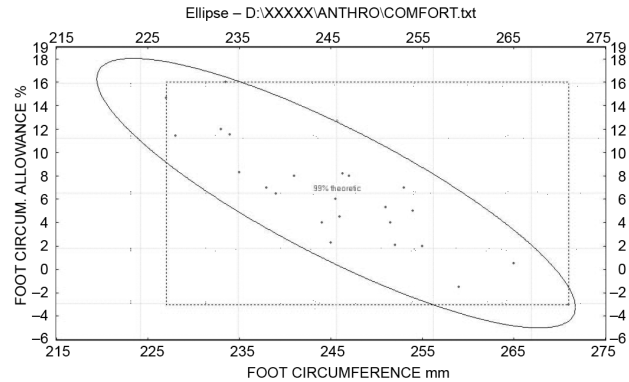


*Fig. 14. Confidence regions for foot circumference-allowance.*

Relationship between dimensional difference measured in fore-foot and mid-foot area with overall filtrating are plotted on figure 13. The negative regression coefficients indicate that higher values for both of this difference reduce the overall fit rating. We calculated inter-variable correlation coefficients for the measured data displayed in the Table 1.

Another fit indicator was circumference allowance defined as follows

$$\Theta = \frac{C_{last} - C_{foot}}{C_{foot}} \cdot 100 \quad [\%] \tag{12}$$

Where $C_{last}$ and $C_{foot}$ are the circumferences of the last and foot respectively. On Figure 14 was constructed confidence region between foot circumference and allowance. The calculated average foot circumference is is $\mu_1 = 245.57$ mm (standard deviation is is $\Sigma_1 = 10.69$ mm and the average circumference allowance is is $\mu_2 = 6.52\%$ (standard deviation is is $\Sigma_2 = 4.71\%$). It is evident negative correlation between comfort and foot circumferences is $\rho_{12} = -0.85$ (Figure 14).

According to collected data some well-known relationships for foot structure have been confirmed. The influence of the candidate's birthplace and the style of life are indicative for some foot dimensions.

## Conclusion

There are many morphology factors that influence the foot structure; therefore 3D foot shape descriptors have been supplemented by biomechanical foot model. Multivariate analysis together with foot-shoe biomechanical interaction is way how to precisely improve footwear fit and comfort. The future standardization decisions must be made to choose a few right descriptors needed for foot customization. The computational morphometry methods must be adopted in shoemaking practice. Statistical shape analysis technique based on deformable objects must include internal bone structure as multybody system dynamics. On this way most of dominant descriptors come together in interaction.

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## MORFOMETRIČKI FENOMENI STOPALA


### S A Ž E T A K

Morfometričke značajke stopala važne su za pravilnu građu i funkciju noge. Građa noge je vitalni dio ljudskog organizma iz niza razloga. Fenomeni antropometrije i morfologije razmatrani su zajedno s biomehaničkim deskriptorima s ciljem kompletne funkcionalne karakterizacije stopala. Za uzorak studentske populacije iz Hrvatske morfološki deskriptori stopala obrađeni su multivarijabilnom analizom. Morfometrički deskriptori ovise o niz faktora kao što su stil življenja, klima, osnovni čimbenici življenja u ljudske zajednice. Značajke udobnosti i komfora određene su koristeći se 3D formom stopala i biomehaničkim modelom stopala. Neke preporuke za sportsku i medicinsku praksu su istaknute.