Krunoslav HORVAT¹
Vladimir KOROMAN²
Ognjen KULJAČA³

Nonlinear Resonant Jump in Rudder Positioning Electrohydraulic Systems

Original scientific paper

The rudder positioning electrohydraulic system, as many other complex systems, is a nonlinear system. In nonlinear systems, a nonlinear resonant jump can occur. The paper deals with the analysis of the rudder positioning electrohydraulic system regarding the nonlinear resonant jump caused by saturation of the electrohydraulic proportional valve. The meaning of the term "resonant jump" is explained in the introduction. Then, the basics needs for occurrence of nonlinear resonant jump phenomenon are given. The rudder positioning electrohydraulic system is also described. Two methods of analyzing system with respect to resonant jumps are described: the analytical method and the simulation method. When performing the analytical analysis based on the describing function approach, the analysis of the variations of important parameters, including saturation, is performed. Then, the confirmation of the results of analytical analysis is illustrated by the simulation method. Also, the simulation procedures, like creating an excitation signal and accelerating the simulation by using vectors in Simulink/Matlab simulation software package, are explained in detail.

Keywords: *electrohydraulics, nonlinear resonant jump, rudder positioning system*

Nelinearni rezonantni skok u elektrohidrauličkim kormilarskim sustavima

Izvorni znanstveni rad

Elektrohidraulički sustav pozicioniranja kormilarskog uređaja, kao i mnogi ostali složeni sustavi, je nelinearni sustav. U nelinearnim sustavima može se pojaviti nelinearni skok. Članak se bavi analizom elektrohidrauličkog sustava pozicioniranja kormilarskim uređajem u odnosu na rezonantni skok uzrokovan zasićenjem u elektrohidrauličkom proporcionalnom ventilu. U uvodu je objašnjeno značenje termina "rezonantni skok". Nakon toga dane su osnove fenomena nelinearnog rezonantnog skoka. Također je opisan elektrohidraulični sustav pozicioniranja kormilarskim uređajem. Opisane su dvije metode za analizu sustava u odnosu na rezonantni skok: analitička metoda i simulacijska metoda. Korištenjem analitičke metode, zasnovane na načelu opisne funkcije, izvodi se analiza varijacija značajnih parametara, uključujući zasićenje. Nakon toga, potvrda rezultata analitičke analize dana je simulacijskom metodom i posebno su objašnjeni simulacijski postupci kao kreiranje pobudnog signala i ubrzanje simulacije korištenjem vektora u simulacijskom paketu Simulink/Matlab.

Ključne riječi: *elektrohidraulika, nelinearni rezonantni skok, kormilarski sustav*

¹ *Brodarski Institute*, *Control Systems Department,* Ave. V. Holjevca 20, Zagreb, Croatia
Phone: +385 1 6504 324
E-mail: krunoslav.horvat@hrbi.hr
² *Brodarski Institute*, Ave. V. Holjevca 20, Zagreb, Croatia
Phone: +385 1 6504 111
E-mail: vladimir.koroman@hrbi.hr
³ *Alcorn State University, Systems Research Institute, School of Agriculture and Applied Sciences, Department of Advanced Technology, Simmons Technology Building*, Room 113, 1000 ASU Drive 360, Alcorn State, MS 39096-7500
Phone: +1 601 877 2344
E-mail: okuljaca@alcorn.edu

1 Introduction

Nonlinear resonant jump is a phenomenon that occurs only in nonlinear systems. The term "resonant jump" is used in the case of a sudden jump of amplitude and/or frequency and/or phase of a periodic output signal of a nonlinear system. This is clearly not a beneficial for a system performance and the frequencies at which this phenomenon can occur should be avoided when designing the control system. Resonance jump was investigated more in the theory of oscillation of mechanical systems ([7], [11]) than in the theory of control systems ([1], [4], [6]). The resonant jump can occur in nonlinear systems and is often not desirable state in the system. The resonant jump cannot occur if excitation is such that the response of the system is transient and cannot be defined by solving nonlinear differential equations. It is also not recommended to use experimental tests on the plant during operation in order to determine if the system can have tendency to the occurrence of this phenomenon. For that purpose the best way of analysis is to use analytical and simulation methods. However, analytical approach can be cumbersome and sometime even impossible in the case of more complex systems. The paper shows that simulation approach to resonant jump phenomenon that can occur in rudder positioning electrohydraulic systems is an efficient way of analysis.

The paper is organized as follows. In Section 2 the basics of the nonlinear jump phenomenon are explained. The short descrip-

tion of rudder position electrohydraulic system is given in Section 3. Section 4 details the analytical approach to the problem while Section 5 explains the simulation method. Finally, conclusions are given in Section 6.

2 Basics of the nonlinear resonant jump phe- nomenon

The resonant jump can be observed through the analysis of the frequency characteristics of a nonlinear system given in Figure 1.
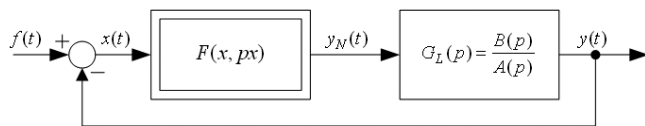

Figure 1 **Block diagram of the forced system**
Slika 1 **Blok dijagram prinudnog sustava**

The quantities from Figure 1 are:

$f(t) = F_v \sin(\omega_v t)$ - harmonic input to the system

$x(t) = X_m \sin(\omega_v t + \varphi_x)$ - input to the nonlinear part of the system

$F(x, px)$ - nonlinear part of the system

$G_L(p) = \dfrac{B(p)}{A(p)}$ - linear part of the system.

Input excitation signal $f(t)$ that is described in more detail in [1], [3], [5] and [9] is of the harmonic form ([1], [6]):

$$f(t) = F_v \sin(\omega_v t) \tag{1}$$

Frequency ω_v is being changed from some $\omega_{v\,min}$ to $\omega_{v\,max}$ and afterwards from $\omega_{v\,max}$ to $\omega_{v\,min}$. There are some issues of impor- tance for the successful simulation and analysis of the nonlinear resonant jump. It is important that change of the frequency of the excitation signal may be executed only when the input signal in the nonlinear part is stabilized. In other words, the input signal in the nonlinear part must have constant values of amplitude X_m and frequency ω_v during one simulation step.

It is very important that the curve is smooth with respect to frequency of the input excitation signal.

3 Description of the system

The main parts of the analyzed rudder positioning electro- hydraulic system are electro-hydraulic proportional valve and hy- draulic cylinder. A position sensor is built in within the hydraulic cylinder. In the real system analyzed here the position controller was implemented using PLC (Programmable Logical Control- ler). Apart from the saturation inherent in hydraulic cylinder, the controller is also designed with the output saturation in order to

insure that the maximal permitted velocity of the piston of the hydraulic cylinder will not be reached, Figure 2.
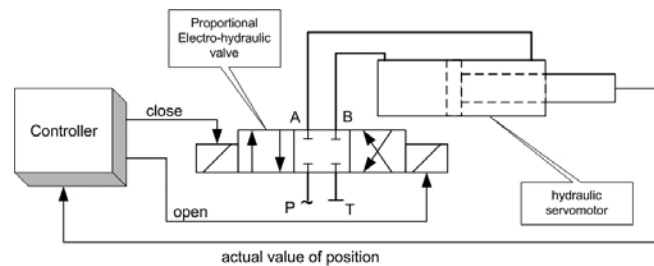

Figure 2 **Block diagram of the system**
Slika 2 **Blok dijagram sustava**

4 Analytical method

The most suitable method to determine resonant jump of a nonlinear system is the describing function method ([1], [5], [6] and [7]). The process of determining nonlinear resonant jump is based on equation (2) [1]:

$$X_m \frac{A(j\omega_v) + B(j\omega_v)\big[P(X_m, \omega_v) + jQ(X_m, \omega_v)\big]}{A(j\omega_v)} =$$
$$= F_v e^{-j\varphi_x} \tag{2}$$

which can be rewritten as:

$$X_m \big[1 + G_L(j\omega_v)G_N(X_m, \omega_v)\big] = F_v e^{-j\varphi_x} \tag{3}$$

where $G_L(j\omega_v) = B(j\omega_v)/A(j\omega_v)$ is the linear part of the system, $P(X_m, \omega_v)$ is the real part of the describing function of nonlinear part of the system, $Q(X_m, \omega_v)$ is imaginary part of the describing function of the nonlinear part of the system, $F_v e^{-j\varphi}$ is harmonic input to the system (excitation) and X_m represents the magnitude of the input of the nonlinear part of the system.

The area in which resonant jumps of magnitude $X_m(\omega_v)$ can occur is determined by the envelope of a family of circles drawn on the characteristics, $-1/G_N(X_m, \omega_v)$ as explained in [1].

To determine the frequency area where the occurrence of resonant jump is possible, the parametric equation (4) [1] of the envelope is determined by:

$$U(\omega_v) = -\frac{P\left(2\dfrac{dP}{dX_m} + X_m \dfrac{d^2 P}{dX_m^2}\right) + \dfrac{dP}{dX_m}\left(P + X_m \dfrac{dP}{dX_m}\right)}{P^2\left(2\dfrac{dP}{dX_m} + X_m \dfrac{d^2 P}{dX_m^2}\right) + \dfrac{dP}{dX_m}\left(P + X_m \dfrac{dP}{dX_m}\right)^2}$$

$$V(\omega_v) = \pm\frac{-X_m \dfrac{dP}{dX_m}\sqrt{\dfrac{dP}{dX_m}\left(2\dfrac{dP}{dX_m} + X_m \dfrac{d^2 P}{dX_m^2}\right)}}{P^2\left(2\dfrac{dP}{dX_m} + X_m \dfrac{d^2 P}{dX_m^2}\right) + \dfrac{dP}{dX_m}\left(P + X_m \dfrac{dP}{dX_m}\right)^2} \tag{4}$$
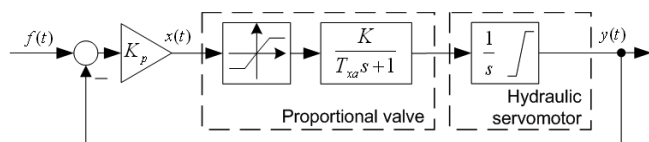
Consider our system, Figure 3:

Figure 3 **Block structure of the system for the analytical method**
Slika 3　**Blok struktura sustava za analitičku metodu**

Linear part of the system is:

$$G_L(s) = \frac{K}{s(sT_{xa}+1)} \tag{5}$$

or:

$$
\begin{aligned}
G_L(j\omega_v) &= \frac{K}{j\omega_v(j\omega_v T_{xa}+1)} = \\
&= -\frac{KT_{xa}}{1+\omega_v^2 T_{xa}^2} - j\frac{K}{\omega_v\left(1+\omega_v^2 T_{xa}^2\right)} = \\
&= U(\omega_v) + jV(\omega_v)
\end{aligned} \tag{6}
$$

The describing function of nonlinear part of the system is:

$$G(X_m,b) = P(X_m,b) = \frac{2}{\pi}\left(\arcsin\frac{b}{X_m} + \frac{b}{X_m}\sqrt{1-\frac{b^2}{X_m^2}}\right) \tag{7}$$
$$; X_m \geq b$$

where b is upper and lower limit saturation.

From (4) it follows:

$$\frac{dP}{dX_m} = \frac{4b}{\pi}\frac{b^2-X_m^2}{X_m^3\sqrt{X_m^2-b^2}} \tag{8}$$

$$\frac{d^2P}{dX_m^2} = \frac{4b}{\pi}\frac{2X_m^2-3b^2}{X_m^4\sqrt{X_m^2-b^2}} \tag{9}$$

From (7), (8) and (9), for $b = 0.3$ we obtain:

$$P(X_m) = \frac{2}{\pi}\left(\arcsin\frac{0.3}{X_m} + \frac{0.3}{X_m}\sqrt{1-\frac{0.3^2}{X_m^2}}\right); \tag{10}$$
$$X_m \geq 0.3$$

$$\frac{dP}{dX_m} = \frac{1.2}{\pi}\frac{0.09-X_m^2}{X_m^3\sqrt{X_m^2-0.09}} \tag{11}$$

$$\frac{d^2P}{dX_m^2} = \frac{1.2}{\pi}\frac{2X_m^2-0.27}{X_m^4\sqrt{X_m^2-0.09}} \tag{12}$$

From (5) and (6) for $K = 200$ and $T_{xa} = 0.8$ we obtain:

$$G_L(s) = \frac{200}{s(0.8s+1)} \tag{13}$$

$$U = -\frac{160}{1+0.8^2\omega_v^2}; \quad V = -\frac{200}{\omega_v\left(1+0.8^2\omega_v^2\right)} \tag{14}$$
$$V^2 = -\frac{U^3}{U+KT_{xa}}; \quad U > -160$$

By substituting the obtained relations into (4), the intersection points of the envelope and the linear part characteristics are determined. The area of resonant jump is defined by the frequency interval (ω_{v1}, ω_{v2}), Figure 4, Figure 5, Figure 6.
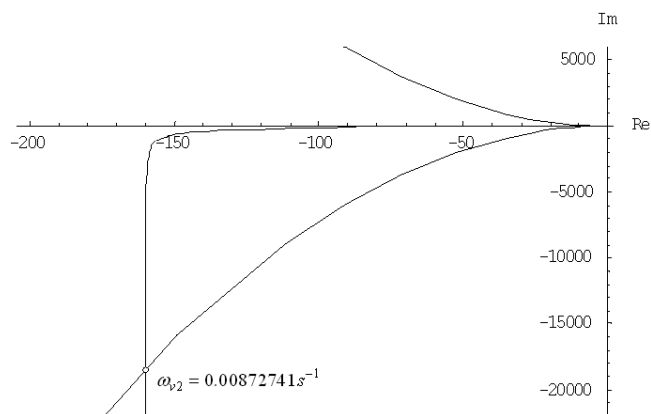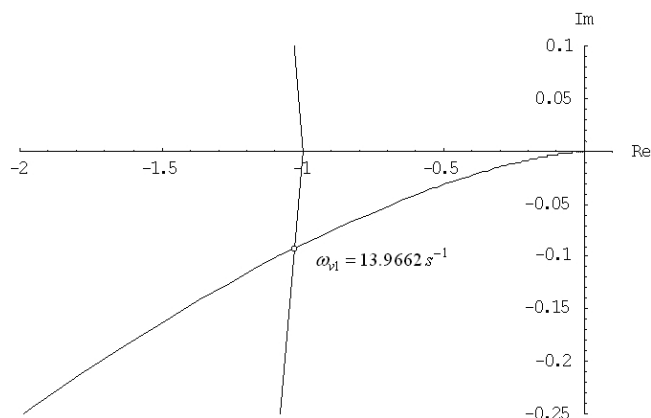

Figure 4　**Frequency characteristic of the linear part of the system $G_L(j\omega_v)$ and the envelope of the nonlinear part of the system**
Slika 4　**Frekvencijska karakteristika linearnog dijela sustava $G_L(j\omega_v)$ i ovojnice nelinearnog dijela sustava**

Figure 5　**Frequency characteristic of the linear part of the system $G_L(j\omega_v)$ and the envelope of the nonlinear part of the system - Detail**
Slika 5　**Frekvencijska karakteristika linearnog dijela sustava $G_L(j\omega_v)$ i ovojnice nelinearnog dijela sustava – detalj**
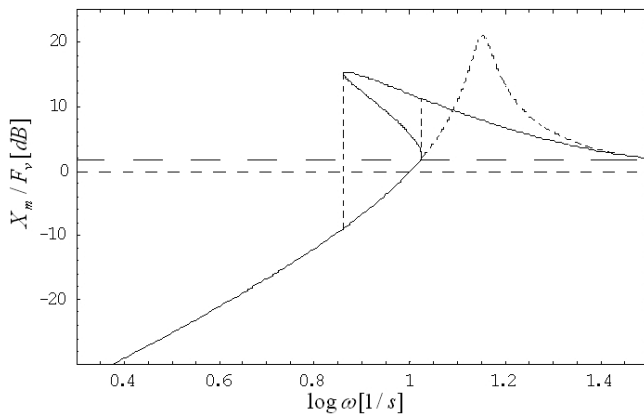
Figure 6 **Frequency characteristic $X_m(\omega_v)$ determined analytically (dashed line is linear system frequency response)**
Slika 6 **Frekvencijska karakteristika $X_m(\omega_v)$ određena analitički (isprekidana crta je frekventni odziv linearnog sustava)**

The system analyzed here is relatively simple, but the analytical approach resulted in fairly complex equations. In the case of presence of additional nonlinearities in controlled system the complexity of calculations rises to the level of insolvability for day-to-day engineering practice. One way to address this problem is to use simulation in order to access frequency range where resonant jump could occur.

5 Simulation method

The block diagram used for the simulation analysis is given in Figure 7.
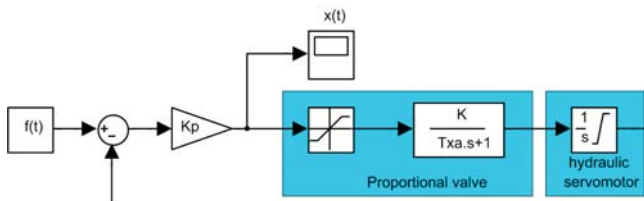

Figure 7 **Simulation block diagram of the system**
Slika 7 **Simulacijski blok dijagram sustava**

where:

$$f(t) = F_v \sin\left(\omega_v t\right) + F_{v0}; \quad F_{v0} = F_v \tag{15}$$

$$x(t) = X_m \sin\left(\omega_v t + \varphi_x\right) \tag{16}$$

In the case of simulation analysis it is important to properly set-up the simulation, otherwise the results will be insignificant and often wrong, thus leading to the wrong conclusions. The simulation analysis in this case is preformed using Matlab with Simulink. The input excitation signal $f(t)$ is generated using the Matlab script file that calls the Simulink model shown in Figure 7. It is actually a modified chirp signal, the frequency of which is changed every 100 seconds with a step of 0.01 Hz. Signal $x(t)$ is input of the nonlinear part of the system. The Integrator that represents hydraulic cylinder has the upper windup limit at 0.245 m and the lower windup limit at 0 m.

5.1 Excitation signal

The excitation signal for the block diagram that is shown in Figure 7 and described in more detail in [4] has the following harmonic form:

$$f(t) = F_v \sin(\omega_v t) \tag{17}$$

and the input to the nonlinear part is:

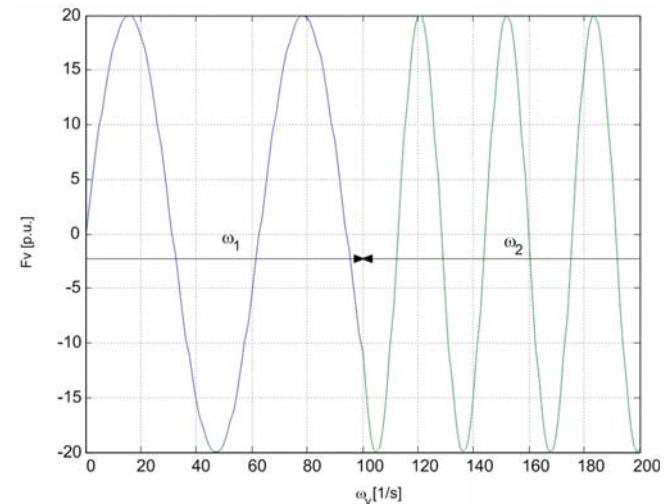$$x(t) = X_m \sin(\omega_v t + \varphi_x) \tag{18}$$

Figure 8 **Change of frequency from ω_1 to ω_2**
Slika 8 **Promjena frekvencije od ω_1 do ω_2**

The frequency ω_v is being changed from some $\omega_{v\,min}$ to $\omega_{v\,max}$ and afterwards from $\omega_{v\,max}$ to $\omega_{v\,min}$. It is important that the change of the frequency of the excitation signal may be executed only when the input signal in the nonlinear part is stabilized, Figure 8. In other words, the input signal in the nonlinear part must have constant values of magnitude X_m and frequency ω_v during one simulation step. A conventional Bode simulation will not reveal a resonant jump, so it is of utmost importance that the change of the frequency of excitation signal is performed in the way described above. Also, because resonant jump is a nonlinear phenomenon a simulation has to be performed in two ways, by rising the frequency from $\omega_{v\,min}$ to $\omega_{v\,max}$ and then by lowering the frequency from $\omega_{v\,max}$ to $\omega_{v\,min}$. Now, the facts described above for a proper simulation set-up have a profound effect on execution of simulation. Namely, the simulation tends to take a lot of data points and could be sometime a long process.

5.2 Acceleration of simulation using vectors

The simulation package tool MATLAB works with scripts as an interpreter, i.e. executed code is not compiled. The script is performed line by line or function by function. In this respect generation of vectors performed in a loop takes a lot of time and significantly slows simulation. Due to this Matlab feature, it is recommended to use generic functions from MATLAB for generating vectors and functions for operating with vectors and not working with particular elements of vectors. Namely, in this

way the generic functions that are compiled to the low level execution code have significantly reduced amount of the redundant unnecessary code or operations. Therefore, the amount of work to be performed by the interpreter is minimized. On Windows 2000 and Windows XP based systems this approach sped up the simulation for about 30%. In the case of old Windows 98 systems this was actually only approach available to avoid virtual memory overrun. Of course, the simulation time depends heavily on the processor speed and amount of available RAM. Average time for simulation results shown in this paper was about 4 hours with Windows 2000 OS, Intel Pentium IV machine and 500 GB of RAM when vector approach was used. That was about 35% improvement compared with the case when non-vector approach was used.

5.3 Acceleration of simulation by partition of the vectors and simulation processes

Since the nonlinear systems are usually very complex, the vectors have huge size and they are numerous. For these reasons the processor uses a swap memory for calculation. Swap memory is actually the place on the hard drive that the processor uses as RAM memory if there is not enough RAM memory for executing some operation. Working with hard drive, instead of RAM memory, is very slow in respect to working with RAM memory, so it takes much more time to execute the particular operation on the vector.

If we want that the processor uses only RAM memory when performs the simulation, it is necessary to split simulation process into more subprocesses. For these subprocesses it is necessary to generate corresponding vectors.

Special attention must be dedicated to the value of the state and input excitation vectors and the phase of input excitation signal when a subprocess is started. Every next subprocess must start with the vector values equal to the vector values with which the previous subprocess is finished.

When then frequency ω_v is being changed, it is necessary that the magnitude of the input excitation signal has the same value as the magnitude of the input excitation signal of the subprocess with the previous value of frequency ω_v.

5.4 Simulation parameters

The real parameters of the system are shown in Table 1.

Table 1   **The real parameters of the system**
Tablica 1 **Realni parametri sustava**

Parameter	value
upper and lower saturation limit	±0.3 [p.u.]
upper windup of hydraulic cylinder	0.245 [m]
lower windup of hydraulic cylinder	0 [m]
K - gain of the electro-hydraulic valve	200 [p.u.]
K_p - gain of proportional controller	100 [p.u.]
T_{xa} - time constant of the electrohydraulic valve	0.8 [s]
F_v - amplitude of excitation	0.1225 [p.u.]

The input excitation signal $f(t)$ that is described in more detail in [1], [5] and [9] has the following harmonic form:

$$f(t) = F_v \sin(\omega_v t) \qquad (19)$$

The frequency ω_v is being changed from some $\omega_{v\,min}$ to $\omega_{v\,max}$ and afterwards from $\omega_{v\,max}$ to $\omega_{v\,min}$. It is important that the change of the frequency of the excitation signal may be executed only when the input signal in the nonlinear part is stabilized. In other words, the input signal in the nonlinear part must have constant values of the amplitude X_m and frequency ω_v during one simulation step.

5.5 Simulation results

Simulation results of the system are shown in Figure 9 and Figure 10. The curve with the solid line represents the response on rising frequency, while the curve with the dashed line represents the response on falling frequency.

The influence of each particular parameter is described in more detail in [8]. The variations of the saturation, gain K of the electrohydraulic valve and the time constant T_{xa} of the electrohydraulic valve have big influence on resonant jump. Opposite to them, the variations of parameters of the dead zone have very small influence on resonant jump and can be eliminated from further analysis. Of course, the elimination of dead zone is not always allowed. It is allowed only if the dead zone has very small influence on the results of analysis. That can always be shown with the simulation. Thus, the simulation analysis is always a good starting point for the use of the analytical method.

Figure 9   **Resonant jump for the real parameters**
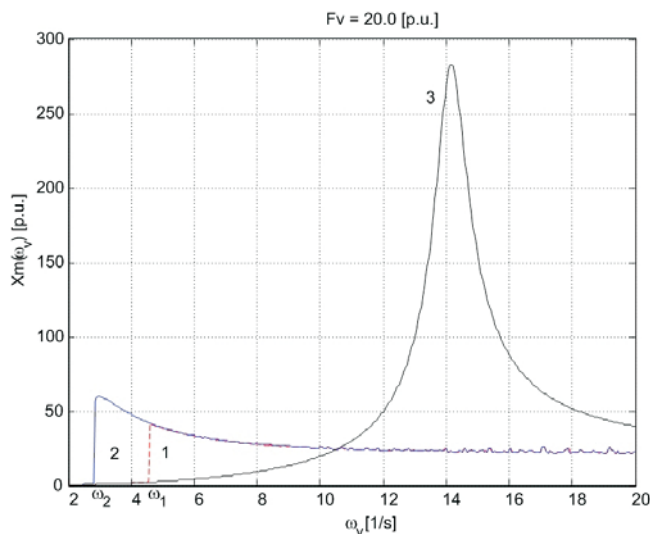Slika 9   **Rezonantni skok za realne parametre**
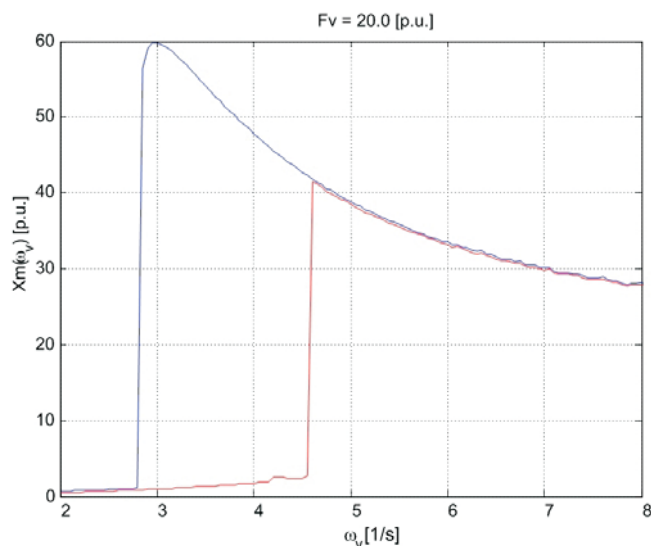
Figure 10   **Frequency characteristic $X_m(\omega_v)$ for $F_v = 20$.**
             Legend: 1 - frequency increases,
                         2 - frequency decreases,
                         3 - frequency response of the linear system
Slika 10   **Frekvencijske karakteristike $X_m(\omega_v)$ for $F_v = 20$.**
             Legenda: 1 - frekvencija raste,
                         2 - frekvencija pada,
                         3 - frekventni odziv linearnog sustava

6   Conclusion

Since the nonlinear resonant jump phenomenon can degrade dynamics of a nonlinear system, its analysis is recommended. In most cases the experimental analysis on the real system is not possible. In case of complex systems it is difficult or even impossible to apply analytical method. So, the only way to perform the analysis of complex nonlinear systems is often simulation approach. The methods for generating input excitation signal have strong influence on the quality of simulation results, while acceleration of simulation has big influence on the time that the simulation takes. The approaches presented in this paper can significantly improve design of nonlinear control systems. The results of the analysis can point out which operating regimes should be avoided so that the system does not operate in the area where the nonlinear resonant jump can occur. It is also possible to determine the elements to be added in control loop/circuit in order to modify the phase of the whole system in such way that the resonant jumps are avoided. For these reasons the nonlinear resonant jump may be difficult to avoid by using conventional controllers. It is useful to use nonconventional controllers like fuzzy [2], [10], neuro, robust, or similar controllers. The simulation presented here is performed using MATLAB development software from Mathworks. MATLAB is very much accepted as industrial standard in control systems community as far as system development and analysis are concerned. The competing tools are MATRIXx from National Instruments (commercial) and Scilab and Octave (free, under GNU). However, these packages operate in the same way as Matlab, thus, the simulation set-up described here can be applied to these development tools also. The paper details the simulation setup for resonance jump analysis in case

of rudder hydraulics systems which can be easily applied by engineers working in practice. On the other hand, the analytical method is complicated and in practice would usually require the control engineer with at least MSC degree to perform it, even in the case of just one nonlinearity in the system.

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