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Buckling Response of Thick Functionally Graded Plates

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ABSTRACT

In this paper, the buckling of a functionally graded plate is studied by using first order shear deformation theory (FSDT). The material properties of the plate are assumed to be graded continuously in the direction of thickness. The variation of the material properties follows a simple power-law distribution in terms of the volume fractions of constituents. The von Karman strains are used to construct the equilibrium equations of the plates subjected to two types of thermal loading, linear temperature rise and gradient through the thickness are considered. The governing equations are reduced to linear differential equation with boundary conditions yielding a simple solution procedure. In addition, the effects of temperature field, volume fraction distributions, and system geometric parameters are investigated. The results are compared with the results of the no shear deformation theory (classic plate theory, CPT).

1 Introduction

The idea of the construction of functionally graded materials (FGMs) was first introduced in 1984 by a group of Japanese materials scientists [1, 2]. During the past two decades, FGMs have experienced a noteworthy increase in terms of research and development programs. World wide distribution and dissemination of the results through publications, international meetings and exchange programs testifies to this increasing growth. They have many gained applications in rocket engine components, space plan body, nuclear reactor components, first wall of fusion reactor, engine components, turbine blades, hip implant and other engineering and technological applications. A detailed discussion on their design, processing and applications can be found in [3]. FGMs are also promising candidates for future intelligent composites [4]. They are multifunctional composite materials, mechanical properties of which vary smoothly and continuously from one side to the other. This is achieved by a continuous change in composition of the constituent materials.

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The most well-known FGM is compositionally graded from a ceramic to a metal to incorporate such diverse properties as heat, wear and oxidation resistance of ceramics with the toughness, strength, machinability and bending capability of metals.

Buckling and post-buckling characteristics are one of the major design criteria for plates/panels for their optimal usage. Hence, it is, therefore, important to study the buckling and post-buckling characteristics of FGM plates under mechanical, thermal or thermo-mechanical loading for accurate and reliable design. The buckling of rectangular plates has been the subject of study for many investigators during the past. However, investigations on the post-buckling behavior of FGM plates are rather limited in number. Praveen and Reddy [5] investigated the response of functionally graded ceramic-metal plate, using finite element procedure. Reddy [6] presented the theoretical and finite element formulations for linear and nonlinear thermomechanical response of FGM plates employing higher order shear deformation theory. Javaheri and Eslami [7,8] obtained the buckling of the FGM plate for uniform in-plane compressive loading and thermal loading using variational approach, based on classical plate theory. They employed equilibrium and stability relations to study the buckling behavior of functionally graded plates with all edges simply supported. Vel and Batra [9] found an exact solution for the thermoelastic deformation of functionally graded thick rectangular simply supported plates. Yang and Shen [10] presented a semi-numerical approach for nonlinear bending analysis of shear deformable functionally graded plates subjected to thermo-mechanical loads, incorporating Reddy's higher order shear deformation theory.

Employing Galerkin-differential quadrature iteration based scheme and incorporating Reddy's higher order shear deformation theory, Liew et al. [11] investigated the postbuckling behavior of the piezoelectric FGM plate. Using Fourier series, Woo et al. [12] presented an analytical solution based on mixed Fourier series for post-buckling of functional graded material plates and shallow cylindrical shells under thermo-mechanical loading. Ma and Wang [13] presented the relationships between axisymmetric bending and buckling of FGM circular plate. Bouazza et al [14] obtained the closed form solution for the thermal buckling of sigmoid functionally graded rectangular simply supported plates subjected to three types of temperature fields; uniform temperature rise; linear temperature rise and sinusoidal temperature rise, employing the first-order shear deformation theory. Shen [15] obtained the post-buckling response of axially loaded functionally graded cylindrical panels subjected to thermal loading. Based on Reddy's higher order shear deformation theory and von-Karman–Donnell type nonlinearity, Shen and Leung [16] presented the post-buckling analysis of functionally graded cylindrical panel under lateral pressure and temperature. Najafizadeh and Eslami [17] presented the buckling analysis of solid circular clamped and simply supported FGM plate. Based on classical plate theory, Yang and Shen [18] presented a semi-analytical approach for the large deflection and post-buckling response of a functionally graded rectangular plate with two opposite edges clamped and two simply supported and subjected to transverse and in-plane loads. Ma and Wang [19] investigated the axisymmetric thermal post-buckling behavior of a functionally graded circular plate.

The object of this investigation is to present an analytical solution for buckling of P-FGM plates subjected to linear temperature rise or non-linear temperature rise across the thickness. The material properties are assumed to be graded in the thickness direction according to a simple power-law distribution in terms of the volume fractions of the constituents. The FSDT and the von Karman-type linear strains are used to construct the problem governing equations. The effects of volume fraction index on the critical temperature buckling behavior are studied.

2 Theoretical Formulation

The functionally graded material (FGM) can be produced by continuously varying the constituents of multi-phase materials in a predetermined profile. The most distinct features of an FGM are the non-uniform microstructures with continuously graded properties. A FGM can be defined by the variation in the volume fractions. Most researchers use the power-law function, to describe the volume fractions.

2.1 P-FGM structures

In order to analyze P-FGM structures as shown in Fig. 1, the simple power-law distribution (Praveen and Reddy [5, 6]) can be employed in this study. The volume fraction using power-law functions to ensure smooth distribution of stresses is defined.

$$V_f(z) = (z/h + 1/2)^k \quad (1)$$

where h is the thickness of the plate and k is the material parameter that dictates the material variation profile through the thickness.

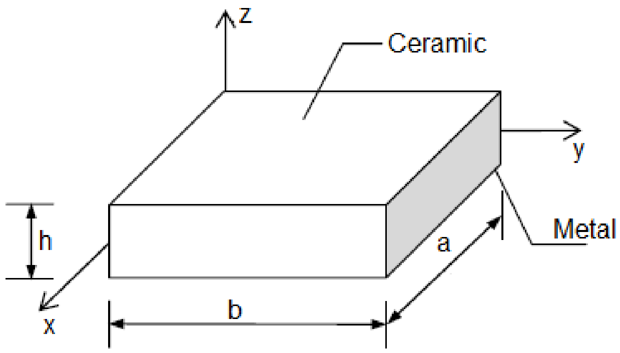

Fig. 1. Typical FGM square plate.

By using the rule of mixture, the material properties of the P-FGM can be calculated by [5, 8, 14]

$$\begin{aligned} E(z) &= E_m + E_{cm} V_f(z) & E_{cm} &= E_c - E_m \\ \alpha(z) &= \alpha_m + \alpha_{cm} V_f(z) & \alpha_{cm} &= \alpha_c - \alpha_m \\ \nu(z) &= \nu_0 \end{aligned} \quad (2)$$

where $E(z)$ denotes a generic material property such as modulus, E_c and E_m indicate the property of the top and bottom faces of the structure, respectively.

2.2 Stability equations

Assume that u, v, w denote the displacements of the neutral plane of the plate in x, y, z directions respectively; ϕ_x, ϕ_y denote the rotations of the normals to the plate midplane. According to the first order shear deformation theory, the strains of the plate can be expressed [20-22]

$$\begin{aligned} \varepsilon_x &= u_{,x} + z\phi_{x,x} \qquad \varepsilon_y = \upsilon_{,y} + z\phi_{y,y} \\ \gamma_{xy} &= u_{,y} + \upsilon_{,x} + z(\phi_{x,y} + \phi_{y,x}) \\ \gamma_{xz} &= \phi_x + w_{,x} \qquad \gamma_{zy} = \phi_y + w_{,y} \end{aligned} \quad (3)$$

The forces and moments per unit length of the plate expressed in terms of the stress components through the thickness are

$$N_{ij} = \int_{-h/2}^{h/2} \sigma_{ij}\, dz \;;\; M_{ij} = \int_{-h/2}^{h/2} \sigma_{ij} z\, dz \;;\; Q_{ij} = \int_{-h/2}^{h/2} \tau_{ij}\, dz \quad (4)$$

The nonlinear equations of equilibrium according to Von Karman's theory are given by:

$$\begin{aligned} & N_{x,xx} + 2N_{xy,xy} + N_{y,yy} = 0 \\ & M_{x,xx} + 2M_{xy,xy} - Q_{x,x} - Q_{y,y} = 0 \\ & Q_{x,x} + Q_{y,y} + q + N_x w_{,xx} + N_y w_{,yy} + 2N_{xy} w_{,xy} = 0 \end{aligned} \quad (5)$$

Using Eqs.(2), (3) and (4), and assuming that the temperature variation is non-uniform with respect to x- and y-directions, the equilibrium Eq. (5) may be reduced to a set of one equation as

$$\nabla^4 w + \frac{2(1+\nu)}{E_1}\nabla^2 (N_x w_{,xx} + N_y w_{,yy} + 2N_{xy} w_{,xy} + q) - \frac{E_1(1-\nu^2)}{E_1E_3 - E_2^2}(N_x w_{,xx} + N_y w_{,yy} + 2N_{xy} w_{,xy} + q) = 0 \tag{6}$$

where

$$(E_1, E_2, E_3) = \int_{-h/2}^{h/2} (1, z, z^2)\, E(z)\, dz \tag{7}$$

$$(\Phi, \Theta) = \int_{-h/2}^{h/2} (1, z)\; E(z)\alpha(z)T(x, y, z)\; dz \tag{8}$$

To establish the stability equations, the critical equilibrium method is used. Assuming that the state of stable equilibrium of a general plate under thermal load may be designated by w_0. The displacement of the neighboring state is $w_0 + w_1$, where w_1 is an arbitrarily small increment of displacement. Substituting $w_0 + w_1$ into Eq. (6) and subtracting the original equation, results in the following stability equation

$$\nabla^4 w_1 + \frac{2(1+\nu)}{E_1}\nabla^2 (N_x^0 w_{1,xx} + N_y^0 w_{1,yy} + 2N_{xy}^0 w_{1,xy}) - \frac{E_1(1-\nu^2)}{E_1E_3 - E_2^2}(N_x^0 w_{1,xx} + N_y^0 w_{1,yy} + 2N_{xy}^0 w_{1,xy}) = 0 \tag{9}$$

where, N_x^0, N_y^0 *and* N_{xy}^0 refer to the pre-buckling force resultants

To determine the buckling temperature difference ΔT_{cr}, the pre-buckling thermal forces should be found firstly. Solving the membrane form of equilibrium equations, gives the pre-buckling force resultants:

$$N_x^0 = -\frac{\Phi}{1-\nu}, \quad N_y^0 = -\frac{\Phi}{1-\nu}, \quad N_{xy}^0 = 0 \tag{10}$$

Substituting Eq(9) into Eq. (8), one obtains

$$\nabla^4 w_1 - \frac{2(1+\nu)}{E_1}\frac{\Phi}{1-\nu}\nabla^4 w_1 + \frac{E_1(1-\nu^2)}{E_1E_3 - E_2^2}\frac{\Phi}{1-\nu}\nabla^2 w_1 = 0 \tag{11}$$

The simply supported boundary condition is defined as

$$\begin{aligned} w_1 = 0, \; M_{x1} = 0, \; \phi_{y1} = 0 \;\; on \;\; x = 0, a \\ w_1 = 0, \; M_{y1} = 0, \; \phi_{x1} = 0 \;\; on \;\; y = 0, b \end{aligned} \tag{12}$$

The following approximate solution is seen to satisfy both the governing equation and the boundary conditions

$$w_1 = c\, \sin(m\pi\, x/a)\sin(n\,\pi\, y/b) \tag{13}$$

where m, n are number of half waves in the x and y directions, respectively, and c is a constant coefficient.

3 Buckling Analysis

In this section, the thermal buckling behaviors of simply supported square metal-ceramic plates under thermal environment are analyzed. The thermal load is assumed to be linear temperature rise and nonlinear temperature change through the thickness direction. The reference temperature is assumed to be 5°C. The effects of volume fraction index and geometric parameter a/h are investigated in each case.

3.1 Linear temperature rise

The temperature field under linear temperature rise through the thickness is assumed as

$$T(z)=\frac{\Delta T}{h}(z+h/2)+T_m \tag{14}$$

where z is the coordinate variable in the thickness direction which measured from the middle plane of the plate.

T_m is the metal temperature and ΔT is the temperature difference between ceramic surface and metal surface, i.e., $\Delta T = T_c - T_m$. For this loading case, the thermal parameter Φ can be expressed as

$$\Phi = P\,T_m + X\,\Delta T \tag{15}$$

where

$$X = \frac{E_m \alpha_m h}{2} + \frac{(E_m \alpha_{cm} + E_{cm}\alpha_m)h}{(k+2)} + \frac{E_{cm}\alpha_{cm} h}{(2k+2)} \tag{16}$$

From Eq.(15) one has

$$\Delta T_{cr} = \frac{\Phi - PT_m}{X} \tag{17}$$

3.2 Nonlinear temperature change

The functionally graded materials are designed in order to resist high temperature rise by ceramic, so the temperature change will be quite different at the two sides of the FGM structures. When the temperature rises differently at the inner and outer surfaces of the plate, the temperature distribution across the thickness is governed by the steady state heat conduction equation and boundary condition as follows:

$$\frac{d}{dz}\left(K(z)\frac{dT}{dz}\right)=0 \quad , \quad T(h/2)=T_c \quad , \quad T(-h/2)=T_m \tag{18}$$

where $K(z)$ is the coefficient of thermal conduction. Similar to the elasticity and thermal expansion properties, we assume that the thermal conductive coefficient is also a power form function as

$$K(z)=K_{cm}(z/h+1/2)^k + K_m \tag{19}$$

where

$$K_{cm} = K_c - K_m \tag{20}$$

The solution of Eq. (18) is obtained by means of polynomial series. Taking the first seven terms of the series, the solution for temperature distribution across the plate thickness becomes [8]

$$T(z)=T_m + \frac{\Delta T}{C}\eta \tag{21}$$

with

$$\eta = (z+1/2)\sum_{n=0}^{n=\infty}\frac{\left(-(z+1/2)^k K_{cm}/K_m\right)^n}{nk+1} \tag{22}$$

$$C = \sum_{n=0}^{n=\infty}\frac{\left(-K_{cm}/K_m\right)^n}{nk+1} \tag{23}$$

where $\Delta T = T_c - T_m$ is defined as the temperature difference between ceramic-rich and metal-rich surfaces of the plate. Substituting Eq. (21) into the thermal parameter equation (8), yields

$$\Phi = PT_m + H\Delta T \tag{24}$$

where

$$H = \frac{h\sum_{n=0}^{n=\infty}\frac{\left(-K_{cm}/K_m\right)^n}{nk+1}\left[\frac{E_m\alpha_m}{nk+2}+\frac{E_m\alpha_{cm}+E_{cm}\alpha_m}{(n+1)k+2}+\frac{E_{cm}\alpha_{cm}}{(n+2)k+2}\right]}{\sum_{n=0}^{n=\infty}\frac{\left(-K_{cm}/K_m\right)^n}{nk+1}} \tag{25}$$

From Eq. (24) one has

$$\Delta T_{cr} = \frac{\Phi - PT_m}{H} \tag{26}$$

The critical temperature difference is obtained for the values of m=n=1.

4 Numerical Results and Discussion

Based on the derived formulation, a computer program is developed to study the behavior of P-FGM plates in thermal buckling. The analysis is performed for pure materials and different values of volume fraction exponent, k, for aluminum–alumina FGM. The Young’s modulus and Poisson’s ratio for aluminum are: 70 GPa and 0.3 and for alumina: 380GPa and 0.3, respectively. Note that the Poisson’s ratio is chosen to be 0.3 for simplicity. Fig. 2 depicts the variation of the Young's modulus through the thickness with different values of volume fraction exponent k.

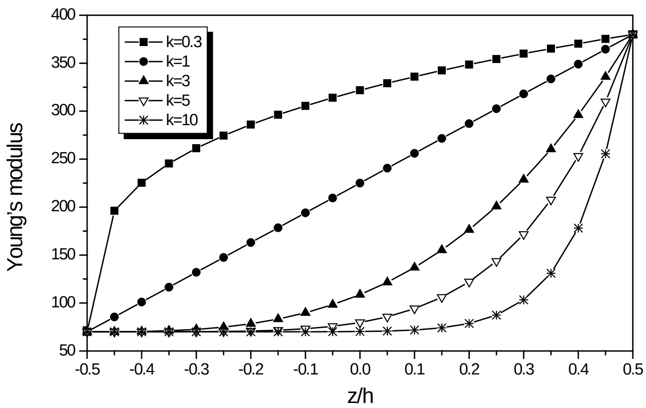

Fig 2. Young’s modulus variation associated with different exponent indexes for a P-FGM plate.

The variation of the critical temperature change ΔT_{cr} of alumina-aluminium FGM plates under linear temperature rise for two different geometric parameters and volume fraction index are plotted in Fig. 3. The isotropic alumina and aluminium cases correspond to fully ceramic plates and fully metallic plates, respectively. While the other cases, $k = 0.3, 1, 5$, are for the graded plates with two constituent materials.

In Fig. 3, it is found that the critical temperature change of FGM plates is higher than that of the fully metal plates but lower than that of the fully ceramic plates. In addition, the critical temperature change decreases as volume fraction index k is increased. This is because for FGMs, as the volume fraction index is increased, the contained quantity of metal increases. In all material cases, the critical temperature change decreases, when the geometric parameter a/h is increased. The responses are very similar comparing to of sigmoid functionally graded plates (Bouazza et al. [14]).

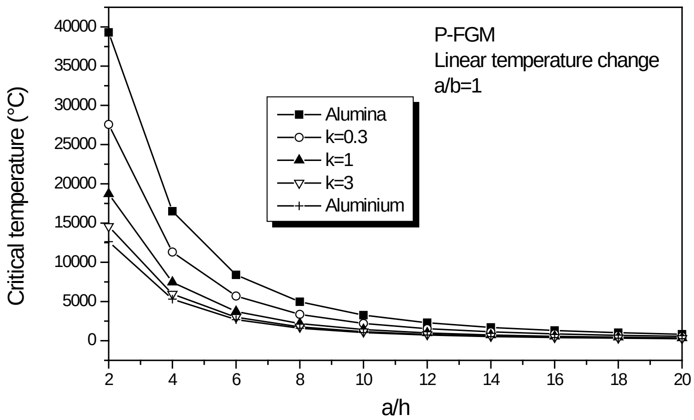

Fig.3. Critical temperature gradient as a function of the side-to-thickness ratio (a/h) of a P-FGM square plate, under linear temperature rise.

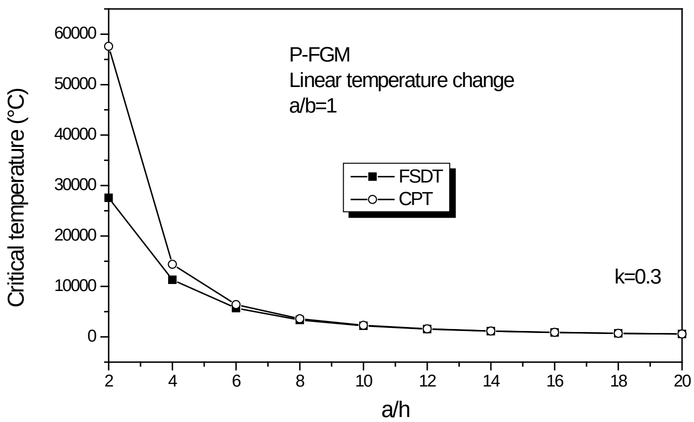

Fig.4. Comparison between temperature graphs vs. ratio Ah (a/h) based on first order shear deformation theory, classic plate theory in the case of linear temperature rise with square plate, k=0.3.

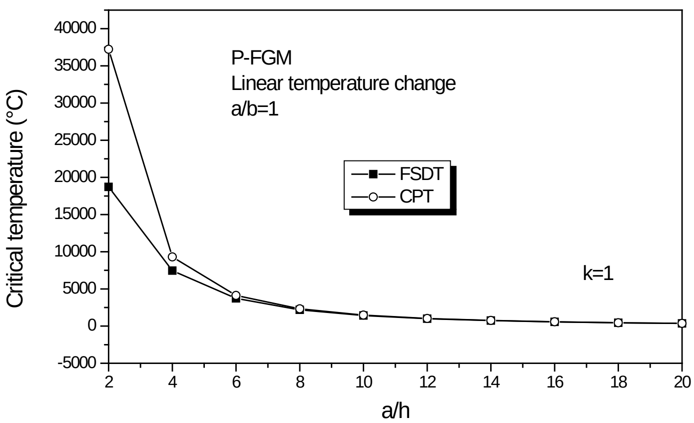

Fig.5. Comparison between temperature graphs vs. ratio Ah (a/h) based on first order shear deformation theory, classic plate theory in the case of linear temperature rise with square plate, k=1.

In Figs. 4-6 the graphs of results of thermal buckling analysis for the P-FGM based on the FSDT compared to CPT are presented. These figures show that the buckling temperature increases by the decreases of the ratio a/h. In addition, based on the figures, the results obtained by first order shear deformation theory coincide with the results of classic plate theory. This is well explained by the large plate aspect ratio a/h=10, 20 or the small plate thickness h

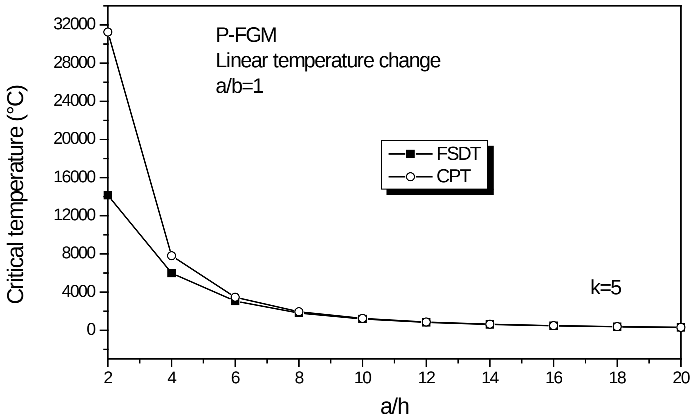

Fig.6. Comparison between temperature graphs vs. ratio Ah (a/h) based on first order shear deformation theory, classic plate theory in the case of linear temperature rise with square plate, k=5.

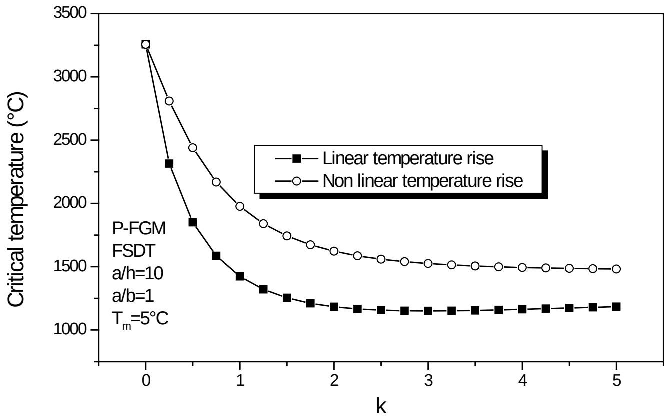

Fig. 7. Critical buckling temperature rise of a functionally graded square plate vs k (a/h=10).

Fig. 6 shows the buckling temperature vs the material gradient exponent k for a plate with a/h =10, a/b= 1. We can see that the critical buckling temperature for a homogeneous ceramic square with $k = 0$ is considerably higher than those for the functionally graded squares with $k \geq 0$. It is evident that the buckling temperature decreases as the material volume fraction exponent k increases monotonically. As the gradient index k changes from 0 to 1, the critical buckling temperature decreases significantly. When k changes from 1 to 2, it reduces very slowly, and as k becomes larger than 2, it will be a constant practically. However, the critical temperature gradient under non-linear temperature rise is higher than that linear temperature rise.

5 Conclusion

In the present paper, equilibrium and stability equations of rectangular functionally graded plates are derived. Derivations are based on the First order shear deformation theory and the P-FGM the constituent materials. The buckling analysis of such plates under linear thermal loads or nonlinear temperature change is investigated. The followings are concluded:

- Thermal buckling analysis decreases, as geometric parameter a/h is increased.

- Tcr of a functionally graded plate increases with the decreased of power law index k.

- Transverse shear deformation has considerable effect on the critical buckling temperature difference of functionally graded plate, especially for a thick plate or a plate with large aspect ratio.

- The critical temperature gradient under non-linear temperature rise is higher than that under linear temperature rise

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