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## Modeling a Circuit that Represents an Electrolytic Solution at Different Concentrations

Daniel C. Miller II  
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# Modeling a Circuit that Represents an Electrolytic Solution at Different Concentrations

Daniel C. Miller II

*Linfield College*

May 18th, 2016



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Modeling a Circuit that Represents an Electrolytic Solution at Different Concentrations

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**Thesis Acceptance**

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# Abstract

Electrodes are often placed within bio samples in order to measure the electrical changes that occur within a system. In order to understand the information provided through the microelectrodes, this paper will describe an experiment to measure the capacitance and the resistance between two micro electrodes as an electrolyte becomes more concentrated. This resulted a capacitance and a resistance that increased and decreased exponentially, respectively, as an electrolyte concentration increased logarithmically.

# Dedication

I would like to dedicate this to my father, Daniel C. Miller, and my mother, Dorothy G. Miller, for greatly supporting me through my four years at Linfield College. If it were not for them, I would not be here studying this research today. I also would like to mention my younger brother, Michael A. Miller, who, at this time, has recently started at a university and I wish him the best of luck to pursue his passion.

# I. Introduction

The main purpose of this experiment is to find the basic characteristics of an electrolyte solution. These results can be compared to many different scenarios. For example, biosensors is a small tool that measures a wide variety of living organisms. We can model these organisms as an electrolyte because the way it behaves can be controlled directly. For example, lactate levels found in blood can be measured using a biosensor<sup>[1]</sup>. Blood must be drawn from a person, which can then be tested to measure the levels of lactate by microelectrodes with an AC signal flowing through the system. Blood is known to be an electrolyte, but one that has fixed values that differ from person-to-person and is difficult to obtain. Instead, we can substitute blood with water that contains different levels of a salt concentration. This can then be constructed into a model (Fig. 1) that can be translated into a circuit that consists of resistors and capacitors.

A resistor is a basic circuit element that reduces an electrical current that flows through a system. A capacitor, in contrast to a resistor, stores energy that depends on the charge and the voltage applied. These two components can be used to create an infinite number of functional circuits that can be used in many different ways. With this knowledge we can model a circuit that represents an electrolyte any information that it may hold. The physical circuit can be controlled by the user and can also be used to modify and match other biological systems that may be used for future experiments.

## The Model and the Used Components

Figure 1 is a basic representation of how an electrolyte will be tested.

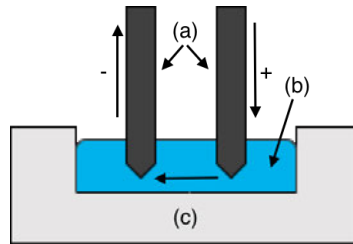


Figure 1: Basic diagram of the device, (a) tungsten probes, (b) an electrolyte, (c) tefflon chamber

From Fig. 1, a device has been constructed to represent an electrolyte through a network of resistors and capacitors<sup>[2]</sup> (Fig. 2).

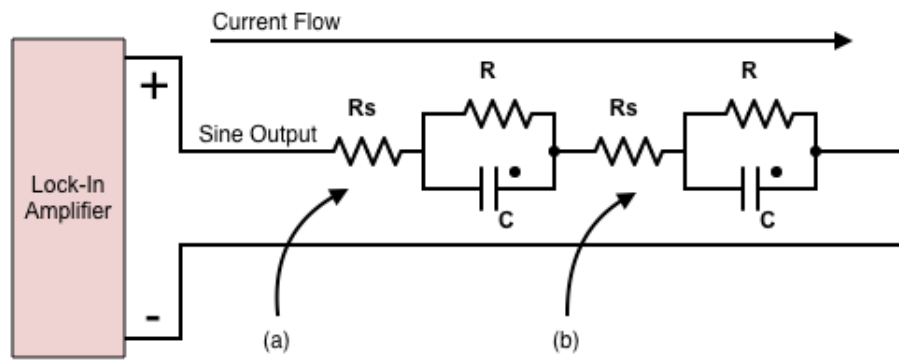


Figure 2: Complete circuit diagram based off Fig. 1, (a) a tungsten probe connected to a sine signal, (b) a grounded tungsten probe connected to a lock-in amplifier for data measurement.

Figure 2 shows the overall circuit diagram of how the system will work. There will be a brief description of what needs to be known in this figure. To get results, an AC signal from a lock-in amplifier (Fig. 7) will be applied to the system with a certain frequency,  $f$ .

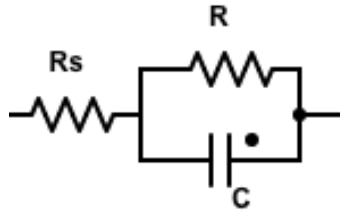


Figure 3: Electric double layer circuit in series with an external resistance,  $R_s$ .

A double layer circuit (Fig. 3) has been created to represent the probes in contact with an electrolyte. A double layer circuit consists of a resistor,  $R$ , and capacitor,  $C$ , in parallel that is in series with an external resistance,  $R_s$ . The external resistance is known to be the overall resistance of the electrolyte itself.

The first component within the double layer circuit is a resistor (Fig. 3). This can be modeled from a rare reaction called a redox reaction that takes place inside an electrolyte and the metal probe. Electrons reside inside of the metal when an electric current flows through. Ions is what makes up current inside an electrolyte. In order to complete the circuit, the electron must convert itself into an ion, which is why this reaction is best represented as a resistor<sup>[3]</sup>, which then leads us to the next component.

The second component inside of an double layer circuit is a capacitor. In between the probe and an electrolyte houses a separation of charges (Fig. 4) and within it are considered to be two parallel plates<sup>[4]</sup>.

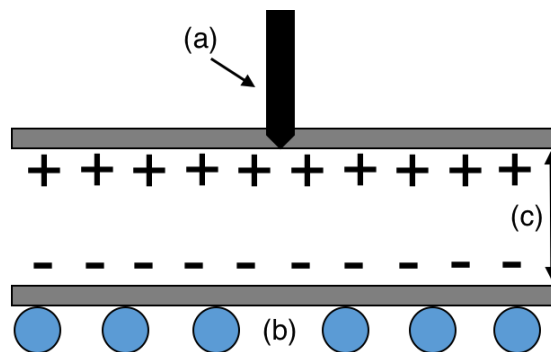


Figure 4: Parallel-plate capacitor, (a) tungsten probe, (b) ions in electrolyte, (c) parallel plates

The charge of a capacitor,  $Q$ , undergoes a change in current signal,  $I$ , and phase shift,  $\Theta$ , as it travels through the plates. This can be written as,

$$Q = \int I dt = I_0 \int \sin(\omega t + \theta) dt = -\frac{I_0}{\omega} \cos(\omega t). \quad (1)$$

where  $I_0$  is the amplitude of the current and where  $\omega$  is equal to,

$$\omega = 2\pi f. \quad (2)$$

The charge traveling through the plates is stored, therefore, resulting in,

$$C = \frac{Q}{V}. \quad (3)$$

where  $V$  is the set voltage through the system.

Since we are working with an electrolyte, we will be taking the capacitance into another direction. Since there are ions present inside an electrolyte, due to a dissolved salt, this is where the screening length comes in.

## Screening Length of a Capacitor

The screening length is the process of an electrical charge from an electron in the metal that travels in an electrolyte only for the electric field to be screened or blocked by an ion that occupies a set amount of space. This is where a redox reaction takes place.

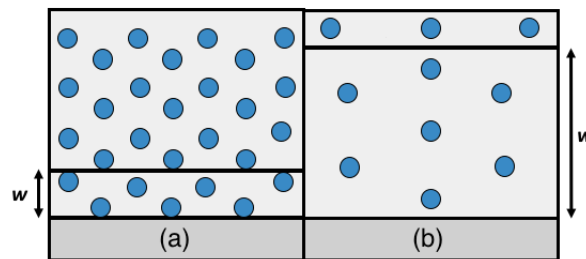


Figure 5: Screening length of an electrolyte solution, (a) a highly-concentrated electrolyte, (b) a weakly-concentrated electrolyte.

Figure 5 shows us two different electrolytes each with a different concentration represented as small circles. The gap width,  $w$ , shown in Fig. 5 represents a space that houses the same number of ions. In Fig. 5a the gap is noticeably smaller compared to Fig. 5b. As an electric charge flows through the electrolyte, the charge is screened out inside the gap width, leaving excess space for the remaining charge to flow through. Think of it as a bridge. The wider the bridge the faster one can travel over it. If there are many people trying to cross a single narrow one, it is going to take much longer to move across. To translate the screening length of a capacitor into an equation, we get,

$$C = \frac{\epsilon A}{w} \quad (4)$$

where  $\epsilon$  is the permittivity of free space and  $A$  is the measured area of the electrolyte in a given space. [5].

## II. Experimental Methods

We will be explaining the hardware used throughout this experiment. Figure 2 represents a circuit diagram of what the entire system consists of.

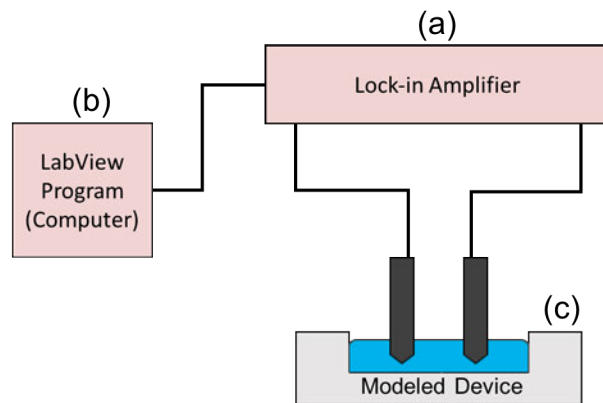


Figure 6: Schematic of the experiment, (a) lock-in amplifier, (b) LabView program, (c) the device holding an electrolyte (Fig. 1).

Figure 6 is a schematic of the experimental setup. Figure 6a is a lock-in amplifier, Fig. 6b is the LabView computer program for recording the data from a lock-in amplifier, and Fig. 6c is a drawing of the device that will be shown in Fig. 9. Each of these will be described throughout the experimental methods section.



## The Lock-In Amplifier

This first device that will be described is a lock-in amplifier (Fig. 6a).

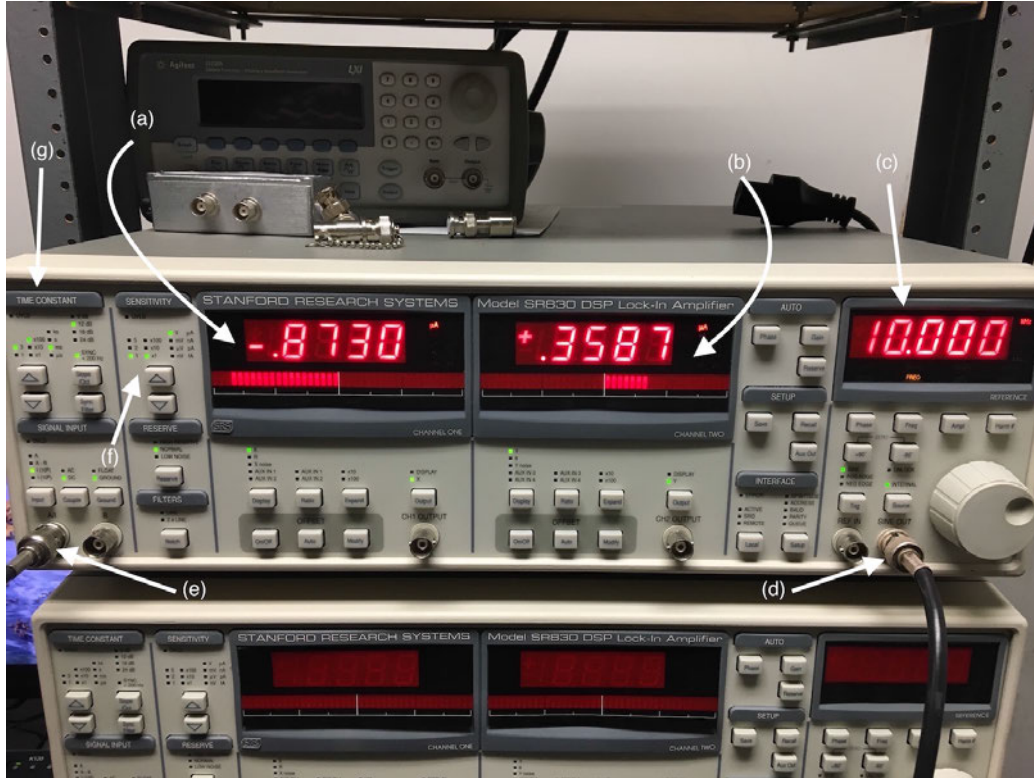


Figure 7: A lock-in amplifier displays of (a) the real current, (b) the imaginary current, (c) the frequency being used with amplitude of 0.02 V, (d) the lock-in output to the constructed device, (e) the AC signal measured after being in contact with the model, (f) the sensitivity, (g) the time constant.

A lock-in amplifier (Fig. 7) is used to detect and measure small AC signals<sup>[6]</sup>. What makes this device so special is that it can block out any unnecessary signals, such as noise, that may obscure the data being recorded.

In order to reduce external noise, a lock-in amplifier uses a process called phase-sensitive detection (PSD). Suppose that there is some AC signal output with some square wave reference signal that may be synchronized with the output in a function generator that is provided (Fig. 7e),

$$v_r(t) = V_{sig} \sin(\omega_r t + \theta_{sig}) \quad (5)$$

where  $v_r(t)$  is the reference amplitude over time,  $V_{sig}$  is the initial amplitude signal,  $\omega_r$  is the reference frequency, and  $\Theta_{sig}$  is the signal phase shift.

The lock-in is then able to generate its own signal in the form of a sine wave, this can be written out as (Fig. 7d),

$$v_L(t) = V_L \sin(\omega_L t + \theta_{ref}) \quad (6)$$

where  $v_L(t)$  is the final lock-in amplitude over time,  $V_L$  is the initial lock-in signal amplitude,  $\omega_L$  is the lock-in frequency, and  $\Theta_{ref}$  is the lock-in reference.

The lock-in will then take the output signal from Eq. 5 and the reference signal from Eq. 6 and will multiply the two together resulting in a function while making the assumption that an experiment is occupied by a specific frequency,

$$V_{PSD} = \frac{1}{2} V_{sig} V_L \cos([\omega_r - \omega_L]t + \theta_{sig} - \theta_{ref}) - \frac{1}{2} V_{sig} V_L \cos([\omega_r + \omega_L]t + \theta_{sig} + \theta_{ref}) \quad (7)$$

There are two outputs of the PSD. One that includes the sum between the frequencies and the other that includes the difference between the frequencies. These frequencies go through a low-pass filter which will ultimately remove these signals. The result is that there is nothing left, but if  $\omega_r$  is equal to  $\omega_L$ , a DC signal will be left which will leave us with,

$$V_{PSD} = \frac{1}{2} V_{sig} V_L \cos(\theta_{sig} - \theta_{ref}) \quad (8)$$

This is the signal that is recorded with minimal noise distortion. To acquire these data from a lock-in amplifier, we will be using an application on a computer called LabView. It is a program that one can use manually to record data efficiently and accurately.

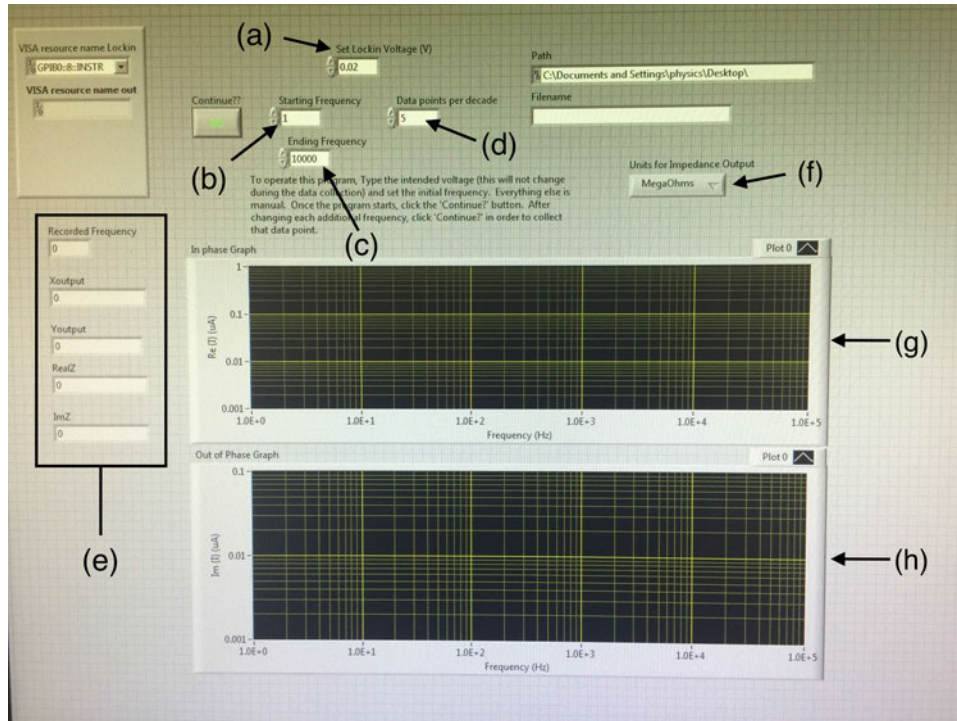


Figure 8: LabView computer program used to record data, (a) lock-in amplitude, (b) initial frequency, (c) ending frequency, (d) number of data points per decade of frequency, (e) data information, (f) units for impedance, (g) graph of real impedance, (h) graph of imaginary impedance.

In LabView, one can input an initial low frequency up to a certain maximum. The lowest and highest frequencies a lock-in amplifier can record is 0.01 Hz and 100 kHz, respectively. However, this application is not a necessity, it only makes the data collection process easier and more efficient.

## The Constructed Device

The lock-in amplifier is then connected to a physical device which consists of two tungsten probes connected to a copper wire placed vertically into a teflon chamber where an electrolyte of some concentration is held.

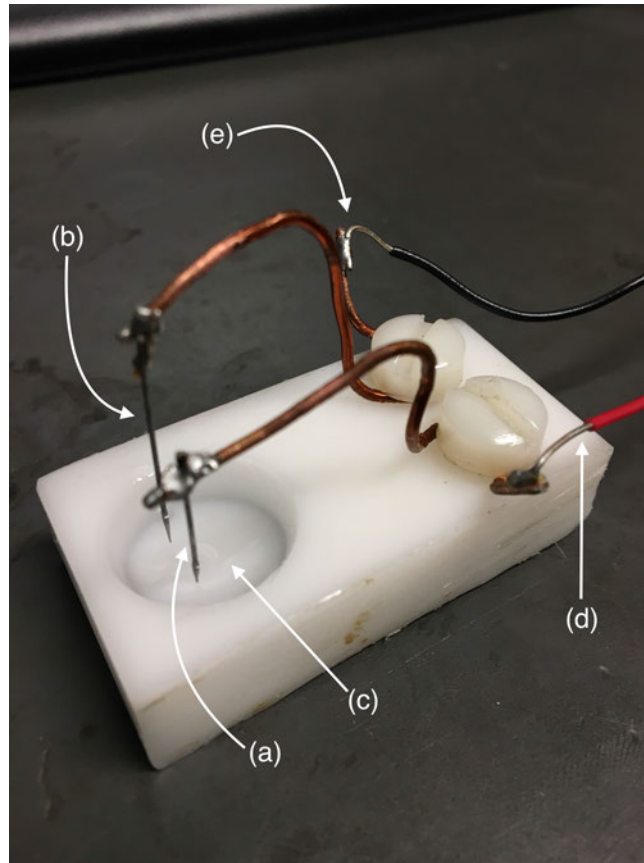


Figure 9: A close-up of the constructed model, (a) is the first tungsten probe, (b) is the second tungsten probe, (c) is the chamber where an electrolyte is held, (d) and (e) represent the lead from end to the lock-in, respectively.

The output signal from the lock-in flows through one of the probes, into an electrolyte, then back through the grounded probe which flows back to the lock-in amplifier for it to take measurements through LabView (Fig. 8).

A signal at amplitude of 20.0 mV is applied throughout the device with a range of frequencies starting at 0.01 Hz that is manually cycled through each decade up until 100 kHz. For each data point in between decades, a complex current will be recorded through the lock-in amplifier, that is composed of real and imaginary components of the current. The lock-in amplifier is connected to a computer where a program in LabView is used to record both components of the current where it is then calculated in terms of impedance, both real and imaginary.

When taking data, it is important to take note of different factors. First, ensure that the lock-in amplifier is on the correct sensitivity. If one is not on the correct value of sensitivity, it can alter the data by some degree. Second, make sure that the time constant is correct. For example, let's say the frequency is a 0.01 Hz, the time constant should at least be at a time constant of 300 seconds. The time constant is the number of seconds needed to complete one period. However, in order to record an accurate set of data, the signal needs to average itself out. To do that, add, at least, another power of ten to the time constant to make sure that the signal has time to do so. When reaching the decade of frequency in the kilohertz range, the data collection should be much faster.

## The Electrolytes and their Concentrations

Throughout the experiment, we will be using a range of electrolytes.

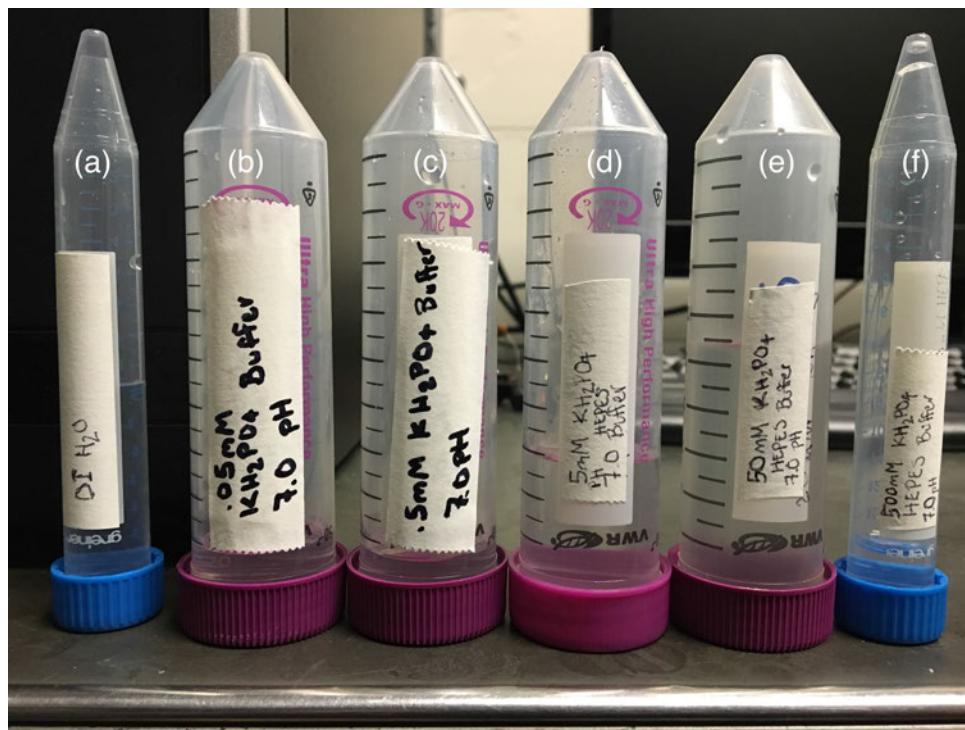


Figure 10: The electrolyte concentrations, (a) distilled water, (b) 0.05 mM, (c) 0.50 mM, (d) 5.00 mM, (e) 50.0 mM, (f) 500 mM. All of which consist of potassium phosphate ( $\text{KH}_2\text{PO}_4$ ).

These are the solutions that were used throughout the experiment. Keep in mind that sodium chloride ( $\text{NaCl}$ ), table salt, works the same way as potassium phosphate. In order to properly obtain these concentrations, basic chemistry has been applied<sup>[7]</sup>,

$$n = \frac{M}{V}. \quad (9)$$

The variable of  $M$  denotes the moles of the solute,  $V$  is the volume of the solution. When dividing the two together, the molarity,  $n$ , is found. Note that one should create their highest concentrated electrolyte first so that it can easily be diluted with distilled water.

Diluting the solution to obtain the weaker concentrations is rather simple by using solution stoichiometry<sup>[8]</sup>,

$$n_1V_1 = n_2V_2. \quad (10)$$

The molarity of the initial electrolyte,  $n_1$ , should be the strongest concentration that has been created. The goal is to find the final molarity,  $n_2$ , that is needed with a reasonable final volume  $V_2$ . In order to do this, one must choose a reasonable volume of the initial electrolyte  $V_1$ ,

$$V_1 = \frac{n_2V_2}{n_1} \quad (11)$$

From this, the amount of distilled water needed to create the diluted solution comes out to be the difference between the initial and final solution volumes,

$$V_{water} = V_2 - V_1. \quad (12)$$

In general, the reason why an electrolyte is used is because the signal from the lock-in amplifier can easily travel through an electrolyte due to the ions that allow an electrical current to flow through. This will then be applied to the circuit constructed from the model, which will potentially give us resistor and capacitor values at different concentrations of electrolytes.

# III. Theory

## Impedance of the Model through the Lock-In Amplifier

The lock-in amplifier will be used to record data for the first half of the experiment, we must find an equation that represents the measured data from the lock-in itself. With the equations, the main priority is to find the complex impedance,  $Z$ , with respect to the measured current,  $I$ , over the device with a voltage,  $V$ .

$$Z = \frac{V}{I} \quad (13)$$

From the experimental methods section, we know that the lock-in amplifier has an AC signal output,  $v(t)$ , of a simple sine wave from Eq. 6 that consists of<sup>[9]</sup>,

$$v(t) = V_L \sin(\omega t) \quad (14)$$

where  $V_L$  is the amplitude of the signal produced from the lock-in amplifier.

Since the current changes over time,  $I(t)$ , due to this wave, we can break up the current into its real,  $I_x$ , and imaginary components,  $I_y$ ,

$$I(t) = I_x \cos(\omega t) + I_y \sin(\omega t) \quad (15)$$

Phasors is what makes the current complex<sup>[10]</sup>. This current is shown through a sinusoidal function. By observing the wave functions of sine and cosine, cosine waves correspond to the real current while the sine wave correspond to the imaginary. This means that cosine waves are in-phase while the sine waves are out-of-phase. A simple trigonometric identity explains a cosine wave of  $\cos(\omega t)$  is equal to a sine wave with a phase shift that consists of  $\sin(\omega t + \Theta)$  where the value for the phase shift, is  $\pi/2$ .



By plugging in the alternating current from Eq. 15 into the impedance from Eq. 13,

$$Z = \frac{V}{(I_x + jI_y)}. \quad (16)$$

where  $j$  represents an imaginary number,

$$j = \sqrt{-1}. \quad (17)$$

This shows that the impedance is dependent on a real and imaginary current that is within the signal. Since there is a real and imaginary current, there has to be a real and imaginary impedance. We now simplify Eq. 16 into two separate equations that will be able to justify the real and imaginary impedance that is dependent on the current. We get this by multiplying Eq. 16 by its complex conjugate,

$$Z = \frac{V(I_x - jI_y)}{(I_x^2 + I_y^2)} \quad (18)$$

Now that there is no imaginary number that resides in the denominator, we can now separate the equation into terms of its real and imaginary values. By doing so,

$$Z_{total} = \frac{V(I_x)}{(I_x^2 + I_y^2)} - \frac{V(jI_y)}{(I_x^2 + I_y^2)}. \quad (19)$$

Note that the sum of the real,  $Z_x$ , and imaginary,  $Z_y$ , impedances is equal to the total,  $Z_{total}$ ,

$$Z_{total} = Z_x + jZ_y \quad (20)$$

Eq. 19 is the measured impedance from the lock-in amplifier that uses the real and imaginary current.

## Measuring the Impedance of the Electric Double Layer Circuit

We will now directly observe the double layer circuit (Fig. 3) and as mentioned in the introduction, the double layer circuit consists of a resistor and capacitor in parallel in series with an external resistance. Looking back at the lock-in amplifier scenario for finding impedance, we can assume that there will be two impedance components and from there a complex impedance will be distinguished into each component within the double electric circuit.

The resistance is modeled from a redox reaction because of electron conversion into an ion in between the probe and an electrolyte. Since the resistance is considered a real value, according to a brief explanation of phasors early in the theory section, the resistance is in-phase with the current and the voltage which can be written as<sup>[11]</sup>,

$$Z_x = Z_r = R. \quad (21)$$

where  $Z_x$  is the real impedance and  $Z_r$  is the impedance of the resistor.

The second component within this system is the capacitor. The capacitance is dependent on frequency which will ultimately change the capacitance when measuring at different frequencies. If the current and the voltage are implemented in a sinusoidal wave, we can determine that,

$$Z_y = Z_c = \frac{1}{j\omega C}. \quad (22)$$

where  $Z_y$  is the imaginary impedance,  $Z_c$  is the impedance of the capacitor.

The impedance of a capacitor is purely an imaginary number from Eq. 17<sup>[11]</sup>,

Now that we were able to distinguish impedance equations from both the resistance and the capacitance within the double layer circuit, everything can be put together. First factor that is noticed is that there is a resistor and capacitor in series with an external resistance. From this we can generate an equation<sup>[12]</sup>,

$$Z_{total} = R_s + \frac{R}{1 + j\omega RC}. \quad (23)$$

Looking back at Eq. 19, the impedance will need to be separated into terms of its real and imaginary components, by performing a derivation of Eq. 23 and plugging into Eq. 20,

$$Z_{total} = R_s + \frac{R}{(1 + \omega^2 R^2 C^2)} - \frac{j\omega R^2 C}{(1 + \omega^2 R^2 C^2)} \quad (24)$$

Now that we have a set of equations we can say that Eq. 19 will be used for measuring the complex current through the lock-in to find impedance, whereas Eq. 24 will be used to calculate the impedance from the values of the resistance and capacitance within the double layer circuit. By using the data from the lock-in amplifier and device, (Fig. 7 and 9), we can calculate specific values for the resistance and capacitance within the double layer circuit (Fig. 3) and compare the two together.

# IV. Results and Analysis

## The Controlled System

This first section will be focused on a controlled system which will consist of a physical resistor and capacitor in parallel. Since the test circuit only consists of these two components we may consider the external resistance negligible only for this case. However, this is subject to change once we begin analyzing the data.

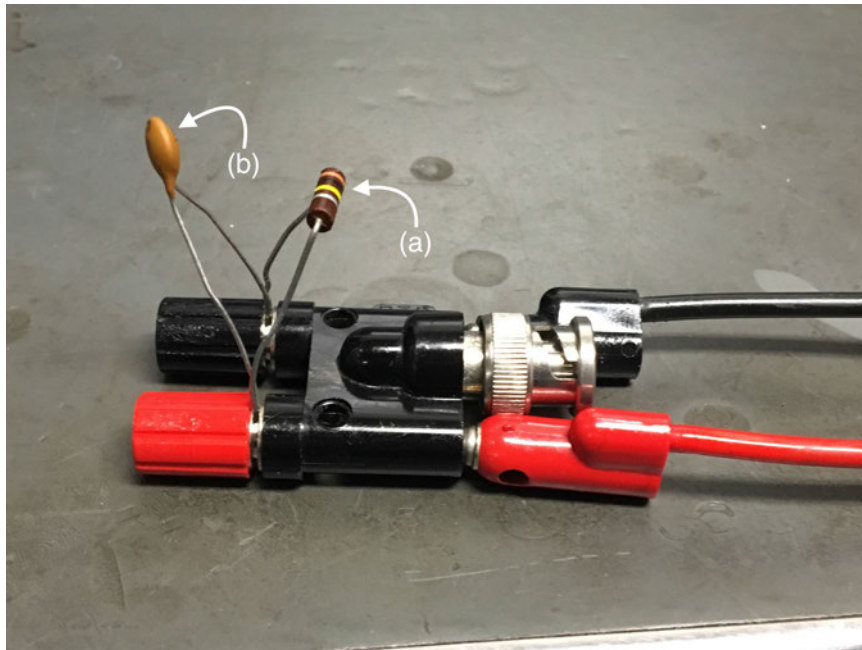


Figure 11: A resistor and capacitor in parallel that represents the double layer circuit, (a)  $R = 108 \text{ k}\Omega$ , (b)  $= 300 \text{ pF}$

This will be the backbone of the entire series of data collection. We will find the specific values of resistance and capacitance for the controlled system that will match that of the future data at different concentrations.

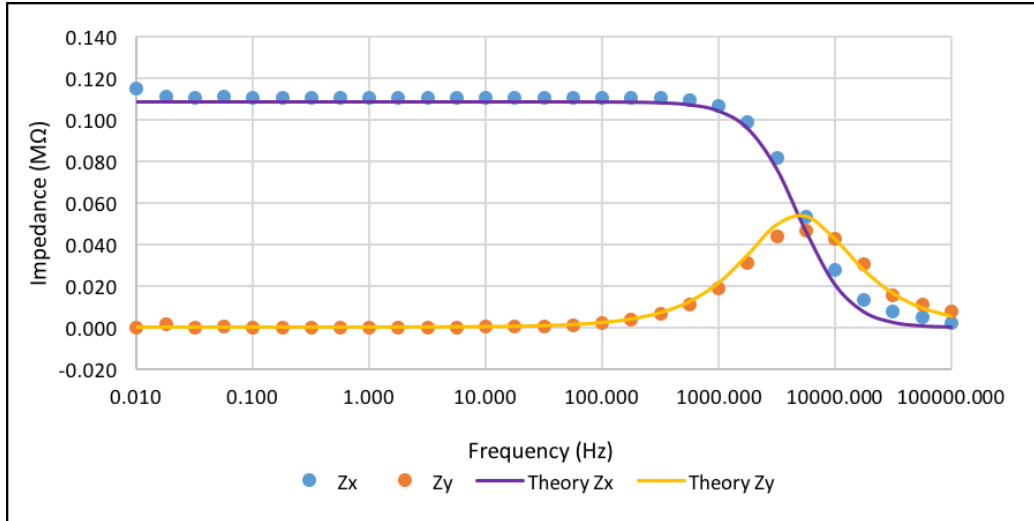


Figure 12: Plot of  $Z_x$  and  $Z_y$  versus frequency for a test circuit with component values of  $R = 108 \text{ k}\Omega$  and  $C = 300 \text{ pF}$

Here is the controlled data with a fixed value of  $108 \text{ k}\Omega$  for the resistance and  $300 \text{ pF}$  for the capacitance. The real impedance is represented through a blue-scattered plot while the imaginary is represented through an orange-scattered plot. The best-fit solid lines is used to find values for the resistance and capacitance. Since the two plots match fairly well together, the values of the components are accurate representations. In this scenario, the frequency range peaked up to  $100 \text{ kHz}$  because there was not enough points to justify the data. When observing an electrolyte, the maximum recorded frequency will be  $10.0 \text{ kHz}$ . The best-fit line is what will be focused on the most.

Keep in mind that the best-fit line resembles Eq. 24 and the scattered plot resembles Eq. 19.

## Electrolyte Concentration of Distilled Water

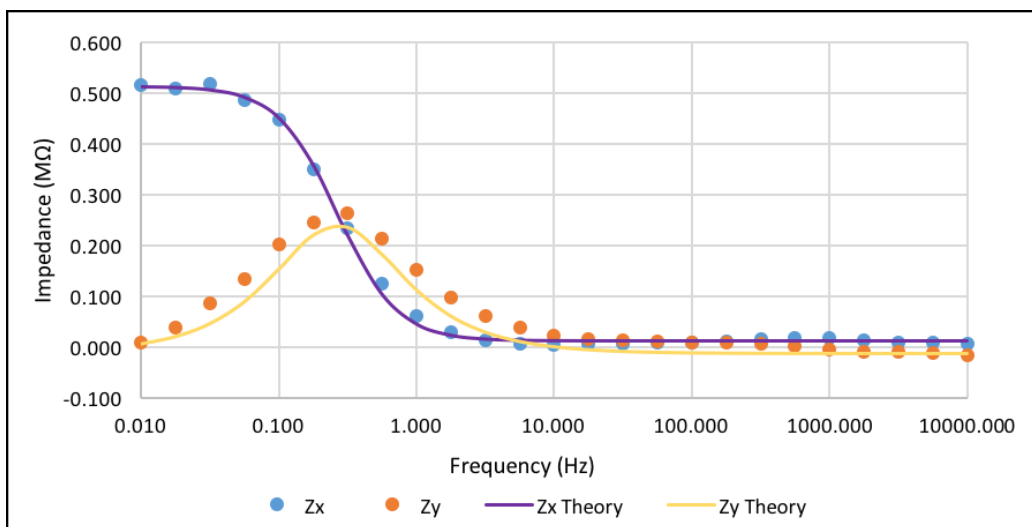


Figure 13: Plot of impedance versus frequency for distilled water. Solid line is best-fit with  $R = 500 \text{ k}\Omega$ ,  $C = 1.20 \text{ }\mu\text{F}$ , and  $R_s = 12.0 \text{ k}\Omega$

## Electrolyte Concentration of 0.05 mM

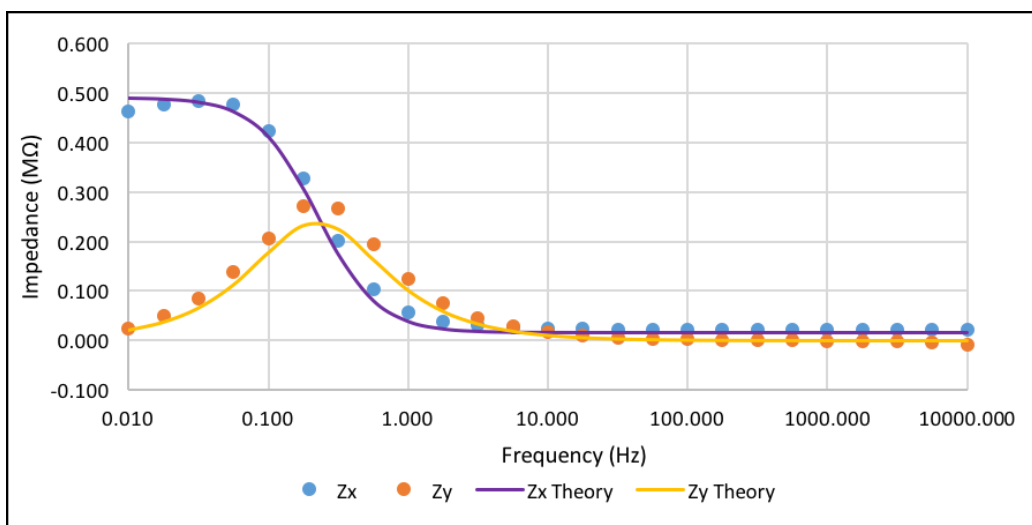


Figure 14: Plot of impedance versus frequency for a 0.05 mM concentration. Solid line is best-fit with  $R = 475 \text{ k}\Omega$ ,  $C = 1.50 \text{ }\mu\text{F}$ , and  $R_s = 15.0 \text{ k}\Omega$

## Electrolyte Concentration of 0.50 mM

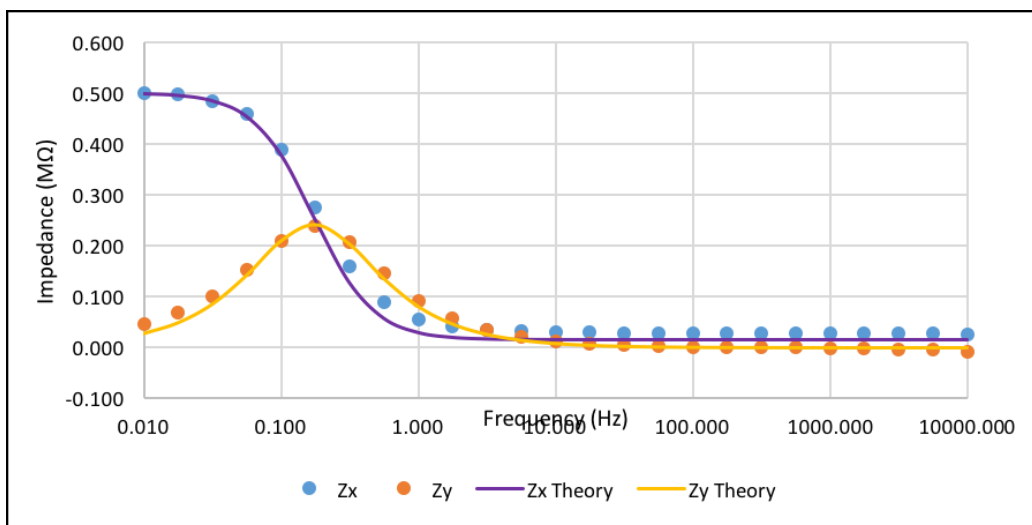


Figure 15: Plot of impedance versus frequency for a 0.50 mM concentration. Solid line is best-fit with  $R = 485 \text{ k}\Omega$ ,  $C = 1.90 \mu\text{F}$ , and  $R_s = 15.0 \text{ k}\Omega$

## Electrolyte Concentration of 5.00 mM

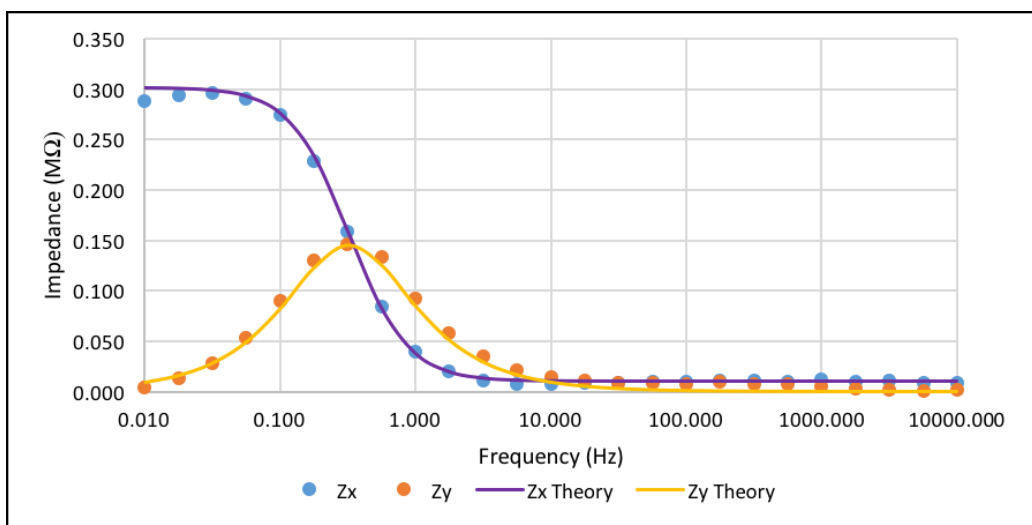


Figure 16: Plot of impedance versus frequency for a 5.00 mM concentration. Solid line is best-fit with  $R = 285 \text{ k}\Omega$ ,  $C = 1.70 \mu\text{F}$ , and  $R_s = 11.0 \text{ k}\Omega$

## Electrolyte Concentration of 50.0 mM

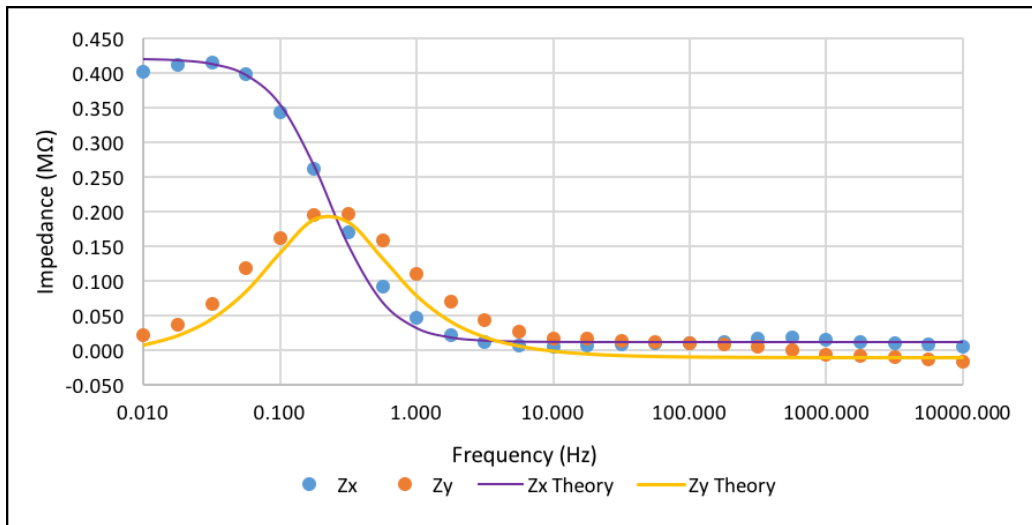


Figure 17: Plot of impedance versus frequency for a 50.0 mM concentration. Solid line is best-fit with  $R = 410 \text{ k}\Omega$ ,  $C = 1.75 \text{ }\mu\text{F}$ , and  $R_s = 11.0 \text{ k}\Omega$

## Electrolyte Concentration of 500 mM

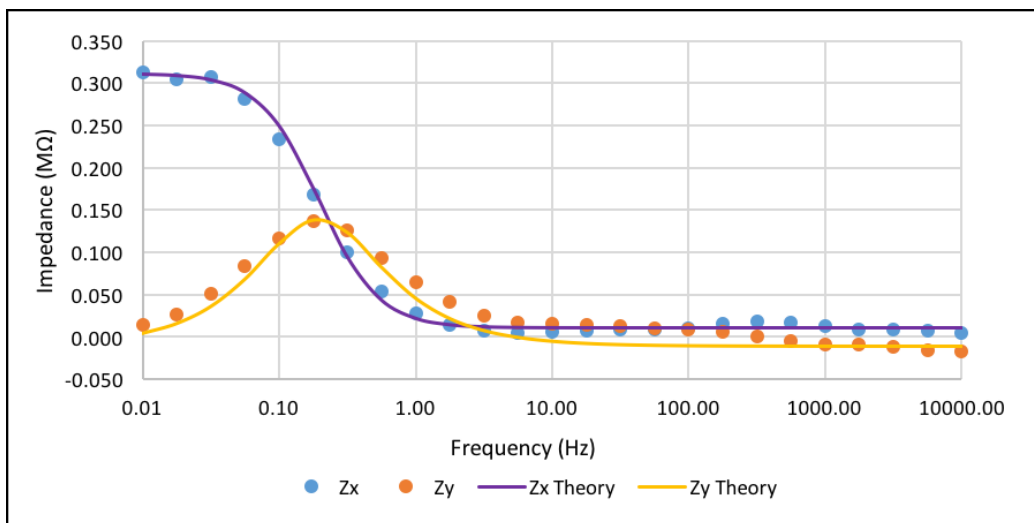


Figure 18: Plot of impedance versus frequency for a 500 mM concentration. Solid line is best-fit with  $R = 300 \text{ k}\Omega$ ,  $C = 2.70 \text{ }\mu\text{F}$ , and  $R_s = 11.0 \text{ k}\Omega$



## Data Summarization of the Electrolytes

The first set of data (Fig. 13) represents distilled water. When comparing the theory to the data, we can say that the lines seem to match up fairly well. We can now figure out specific values for the resistor and capacitor in the double layer circuit. The resistance in the first set has an approximated value of  $521\text{ k}\Omega$ , and an approximated capacitance value of  $1.20\text{ }\mu\text{F}$ . We also can make out that the external resistance has a value of  $15.0\text{ k}\Omega$ .

As explained before, the external resistance was subject to change. When comparing this data set to the controlled test circuit, there appears to be an external resistance, hence the gap between the two scattered plots in most of the graphs. We will be discussing this slightly more in-depth in the conclusion.

The second data set (Fig. 14) represents an electrolyte concentration of  $0.05\text{ mM}$ . From this graph, we were able to approximate  $500\text{ k}\Omega$  for the effective resistance and  $1.50\text{ }\mu\text{F}$  for the value of the capacitance. The external resistance has a value of  $13.0\text{ k}\Omega$ . When looking at the best-fit line, the values used give accurate representations of the component values.

The third data set (Fig. 15) is a visual representation of  $0.50\text{ mM}$  of an electrolyte. The values that were approximated resulted in  $375\text{ k}\Omega$  for the resistance and  $2.00\text{ }\mu\text{F}$  for the capacitance. The external resistance as a value of  $13.5\text{ k}\Omega$ . When comparing the other graphs to this, we can make out that the impedance is lower than those of a  $0.05\text{ mM}$  concentration and distilled water. For now, we will be observing other concentrations to see if this trend continues.

The fourth data set (Fig. 16) represents an electrolyte concentration of  $5.00\text{ mM}$ . The approximated effective resistance is  $615\text{ k}\Omega$  and the effective capacitance is  $0.90\text{ }\mu\text{F}$ . The resistor and capacitor are then in series with an external resistor that is approximated at  $10.0\text{ k}\Omega$ . From here, the theory and the data seem to be fitting nicely together showing that these values are near their exact values.

The fifth data set (Fig. 17) represents  $50.0\text{ mM}$  of an electrolyte concentration. Within this data set there is an approximation for both the resistance and capacitance which are  $400\text{ k}\Omega$  and  $1.75\text{ }\mu\text{F}$ , respectively. When comparing the theoretical data with the experimental, the trend between the two seem to be closely related to each other.

So far, within all of the concentrations, the values for the resistance and capacitance has been decently leveled. All of the theoretical data has matched up with the experiment in some way. This currently means that the theory and the experiment have some correlation to each other. The only way to confirm this is to test more concentrations.

The last set of data (Fig. 18) represents a concentration of 500 mM, which is the highest concentration is in use. From here, we approximated values of 300 k $\Omega$  for the resistance and 3.00  $\mu$ F for the approximation of the capacitance. The external resistor has a value of 7.50 k $\Omega$ . The theoretical data in contrast to the experimental data seems to be matching fairly well together when looking at the current trend in the data itself.

Now that all of the data has been stated and briefly described, we can make out a few factors. First, all the approximations for the capacitance in each of the concentrations seems to be in the same general range. In terms of consistency, the capacitance does not seem to be changing by too much. Second, the resistors within the double layer circuit do not follow a trend, but a couple of resistances vary from the others. The concentrations of 0.50 mM and 500 mM have approximated values less than 400 k $\Omega$ . However, these concentrations still follow the same trend impedance values when comparing the two to the other concentrations.

As stated before, the external resistance would be considered negligible in a test circuit. As we increase the value of the external resistance, the slope of the best-fit line will begin to even out. In other words, if the external resistance were to match to that of the resistance in the double layer, the data will output constant values for the impedance in both the real and imaginary components.

## Resistance and Capacitance of each Electrolyte Concentration

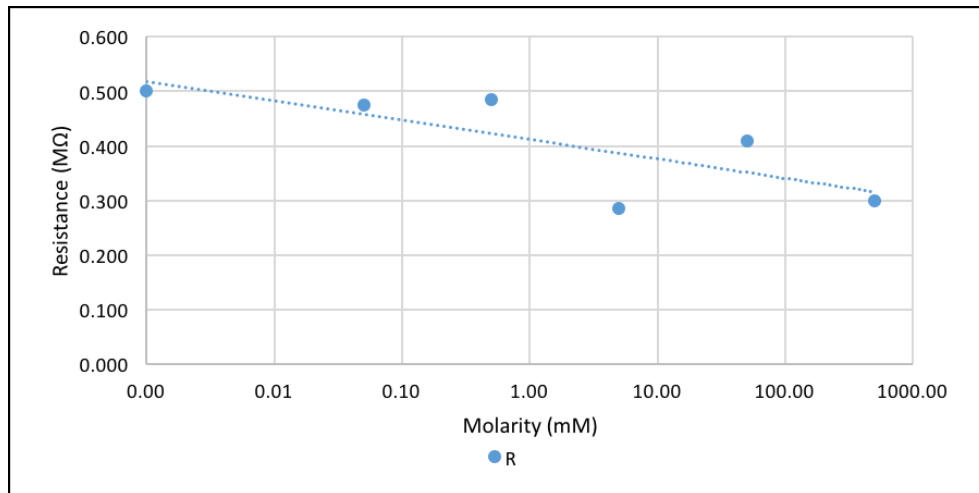


Figure 19: Resistance versus molarity concentration

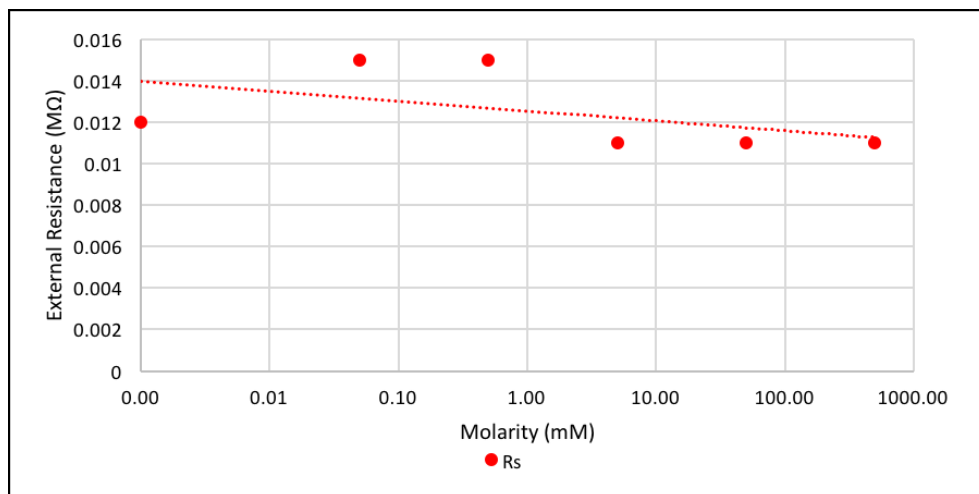


Figure 20: External resistance versus molarity concentration

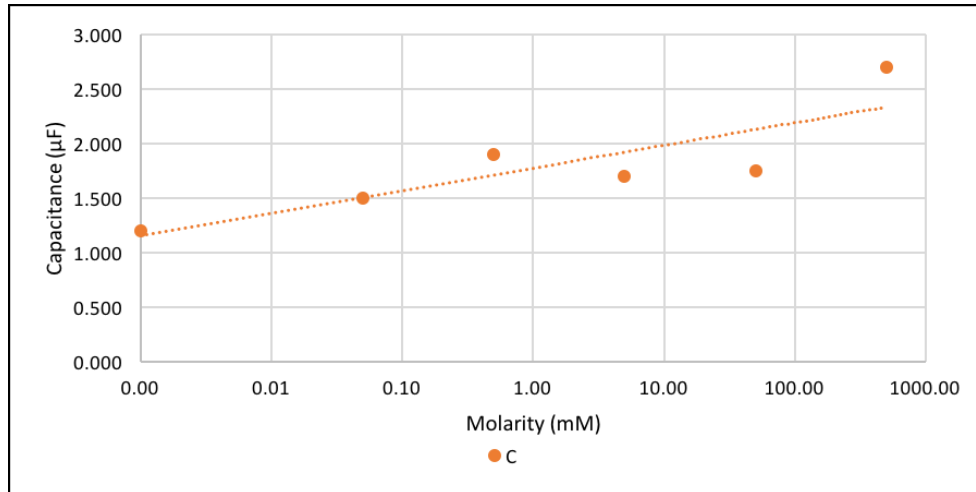


Figure 21: Capacitance versus molarity concentration

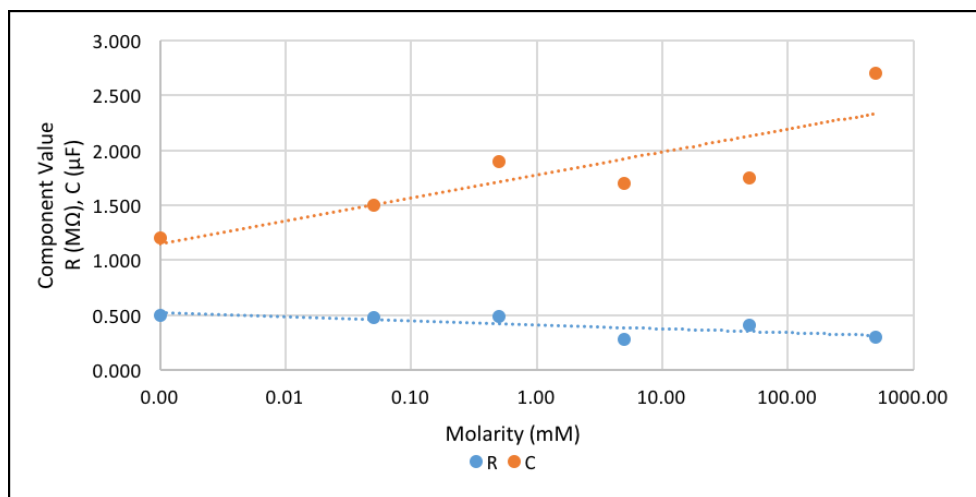


Figure 22: External resistance versus molarity concentration

The data that represents the resistance (Fig. 19) shows us that it is nearly constant throughout all of the concentrations used on a logarithmic scale, when converted into linear values, the resistance will exponentially decrease as the molarity increases. The external resistance (Fig. 20) behaves in a similar manner.

The data that represents the capacitance (Fig. 21) shows us that it will increase constantly throughout all concentrations on a logarithmic scale. When converted into linear values, the capacitance will increase exponentially as the concentration increases.

# V. Conclusion and Future Work

The goal for this experiment was to calculate values for the resistance and the capacitance of a given electrolyte solution. There were two main equations that were derived from different perspectives. One from the data of the experiment and the other from a physical circuit. By taking the two equations and working with them as one, we were able to generate the given data.

The data for the resistance involves an exponential decrease while the capacitance involves an exponential increase. The reason for this is because as the concentrations increase, the number of ions that are within the solution also increase. In other words, when there are more ions in an electrolyte, this allows the current to flow at a greater magnitude, which generates a higher charge value, which also means that this will increase the overall capacitance as the concentration increases. This is where the concept of screening length is applied (Fig. 5).

When looking back at the introduction for the definition of resistance, we know that a resistor reduces the electrical current flowing through the system, so if there is an increase in current, this could only mean that the overall resistance will decrease as the concentration increases.

Considering that an electrolyte is a basic biological organism that can be controlled, one could possibly expand this research in many ways. Since this is only involves metal-to-liquid contact, one could expand to different types of materials to see if it will alter the current findings within this research.

Another form of potential research is that one could change multiple aspects to this experiment such as: gap width between the probes, higher concentrated electrolytes, higher frequencies, and a larger volume of solution. All these factors can alter this current data in some way if these were to be used within this experiment.

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# Appendix

## Block Diagram of Lock-in Amplifier

