Mathematic Models: Searching a lost plane

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Introduction

Malaysia plane MH370 disappeared en route from Kuala Lumpur to Beijing on 8, March 2014. Whereas the seas and the oceans are relatively large and may pose a big challenge for search teams to locate a place. There is no doubt that air transport is so far one of the best modes of communication throughout the world. With increased levels of communication and the need to carry out overseas businesses and operations, it is inevitable for countries and individuals to carry out expansion and integrate air transport as part of the growth strategy. The only challenge that remains has to do with the issue of safety in as far as air transport is concerned.

Model Overview

We develop a model including two steps: confirming the area and then the optimum searching in the area.

First of all, to confirm the area that the plane probably falls, we assume it falls along with a parabola ignoring wind resistance and horizontal lift. The Poisson Probability Distribution is adapted to calculate the probability distribution of the accident and the probability of direction of the airplane. We use this theory to model the situation of general types of airplanes.

The second layer is to propose the famous Bayesian statistics and mathematical model for search of lost planes. The Bayesian statistical model can indeed be used to help locate planes after they go missing. The Bayesian search theory and model can indeed be useful method from which the search for a plane can be enhanced.

Notation Table

Symbol	Meaning
X	Horizontal displacement
Y	Vertical displacement
m	Mass of the plane
g	Gravitational acceleration
k	Drag coefficient of the airplane
t	Time
$\vec{f}$	Wind resistance of the plane
k_1	Lift
ρ	Air density
ρ_s	Sea density
C_D	Wind resistance coefficient
v_x	The velocity of the plane on the real-axis
v_y	The velocity of the plane on the imaginary-axis
s	The area of the head of the airplane
F	The buoyancy of the black box

Methodology

- Poisson Probability
- Drag equation: Drag depends on the properties of the fluid and on the size, shape, and speed of the object.

We assume the point where airplane cast a shadow in the open ocean as origin in a Cartesian coordinate system.
Probability about declination of airplane course
Analyze crashed airplane trajectory

Confirming layer

Probability about declination of airplane course
When airplane lost powder during high-altitude flying, we could assume it was affected by different factors to crash.

Thus, we can build up an equation,

E(n)=A(x)+B(y)+C(z)

Where: A(x) average affect by wind force factor
B(y) average air current factor
C(z) average airplane speed factor
We would get E as a real positive number from reality data that powder that affect airplane after lose power.

Since this lost airplane is flying from point A to point B, we could depend on time when airplane lose contact to get point C. Then according to drag equation, we could find that resistance in the point C as :

$$f=1/2\rho v2\boldsymbol{C_D}S$$

where:

f is the drag force,
ρ is the density of the fluid
v is the speed of the object relative to the fluid
S is the cross sectional area
C_D is the drag coefficient – a dimensionless number

Since this airplane affected by four different forces in the air, we simplify that lift just affect vertical direction. We can use Newton's laws of motion to get:

$$m\frac{d^2\vec{r}}{dt^2} = m\vec{g} + \vec{f} + \vec{F}$$
$$\kappa = \frac{1}{2}\rho \boldsymbol{C_D} S \qquad (2)$$

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Using Orthogonal Decomposition Theorem to solve equation let $y(0)=y_0$, $x(0)=X_0$ then we could get:

$$m\frac{d^2x}{dt^2} = -k\left(\frac{dx}{dt}\right)^2$$
$$(3) \qquad m\frac{d^2y}{dt^2} = -mg + k\left(\frac{dx}{dt}\right)^2 - k_1 g$$

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This model is through the way that lost airplane would be to find the region that airplane fall into water. We using the above two equations can solve (x, y) which is the approximately geographical position where fallen airplane is.

Optimum layer

Optimum-the Bayesian search model

1. The first procedure in using this model would involve formulation of hypothesis with regard to the probable locations of the plane.
2. After the first procedure, the next step would involve the construction of probability density function for location X.
3. Third step would involve the construction of a function on getting the plane in location X.
4. The above data and information would then have to be brought together as part of the need to develop the density map.
5. The fifth step would then involve construction on the search path.
6. The six and the last procedure calls for the revision of the probabilities.

Strengths and Weaknesses

Strengths: Our model analyzes the problem as two layers: confirming the area and optimizing searching area. Our model has kinds of precise data and graphs to exhibit.
Weakness: In order to simplify the model, we neglect the wind resistance that would influence the direction of the rout of falling.