Traffic Flow: An approach towards modeling the right lane rule

James Knox, Alexander Ogle, Lauren DeVore

(Advisor: **Michael Hitchman), Department of Mathematics, Linfield College, McMinnville, OR**

BACKGROUND

In this paper we attempted to model and analyze the effect of the right hand rule. In order to analyze the right hand rule we started with Greenshield's macroscopic approach and modified it to simulate the effects. By analyzing the resulting changes in the speed, density, and flow of the system we determined the performance of the rule in varying traffic scenarios. Next we looked at the performance by modeling traffic flow when the rule is strictly adhered to, as compared to an intermediate, where the rule is followed until the critical density is reached.

Comparing freeway flow when the right lane rule is in effect to when it is not, our model showed that the right lane rule is better when the density is below Kc. We then created a piecewise function which acts as an intermediate rule. In this intermediate, velocity is described by the function for the right hand rule when density is less than K_{cr}. For densities equal to or greater than K_{cr}, we take the average velocity of the system to be described by the function of the no rule line. We also assert that this behavior will naturally occur in the right hand model as people enter the left lane in order to pass, however, they will no longer be able to re-enter the right lane to increase their velocity, thus the density of the system will now be the entire road as the right lane rule breaks down.

Our model requires two parameters to analyze a real life situation. These are free flow and jam density. Free flow is namely the speed at which most drivers are willing to drive given no restrictions due to other drivers. Jam density is determined by the length of the road being analyzed and the assumed length, or length distribution, of all cars

Abstract

We attempted to model and analyze the effect of the right hand rule for the 2014 COMAP Math Modeling Competition. In order to analyze the right hand rule we started with Greenshield's macroscopic approach and modified it to simulate the effects of the right hand rule. By analyzing the resulting changes in the flow and density of the system we determined the performance of the rule in varying traffic densities. Next we looked at the performance by modeling traffic flow when the rule is strictly adhered to, as compared to an intermediate, where the rule is followed until the critical density is reached. In the intermediate model we show how traffic flow can be maximized if people no longer follow the right hand rule after the critical density.

Assumptions

- Linear Relationship between Velocity and Density (As assumed by Greenshield)
- In the no rule scenario $V_f = V_{fS}$ because the speeding cars can no longer pass
- Cars cannot be created nor destroyed.

Designing a Model

Macroscopic Model

To build our own model we found it manageable to consider the traffic flow on the freeway as flow through a pipe. To design the right hand rule (RHR) model we considered the effect it has on traffic flow and modeled it as a change on the flow (q), density (k), and speed (v) through a pipe. First we consider that in this pipe there are fast particles and slow particles moving through it, based upon Daganzo's behavioral theory which assumes the presence of "rabbits" and "slugs" [2]. In the right hand rule, the fast drivers (rabbits) are in the same lane as the slow drivers (slugs), however, they have the ability to pass the slower drivers. This is why we set the free flow for the RHR model higher. Because the drivers are constricted to a single lane unless passing, there is an increase in pipe density, since the rabbits must only use the lane when passing. This implies the overall speed of the system will be greater at minimal densities for which the rabbits are able to pass. In a system with no rule, there is a reduced density in the pipe as both lanes are used, however the slow drivers prevent the fast drivers from passing, thus reducing the free flow speed of the particles through the pipe. In order to account for these changes we use Greenshield's equations [1, 4] and modified them as such:

$$K_R = \frac{(L-1)}{L} * K, for\ L \in \mathbb{Z}, for\ L \geq 2 \qquad [1]$$

$$V_{fR} = \frac{V_{fs}+V_{ff}}{2} \qquad [2]$$

$$V_{fN} = V_{fS} \qquad [3]$$

RESULTS

Speed and Density

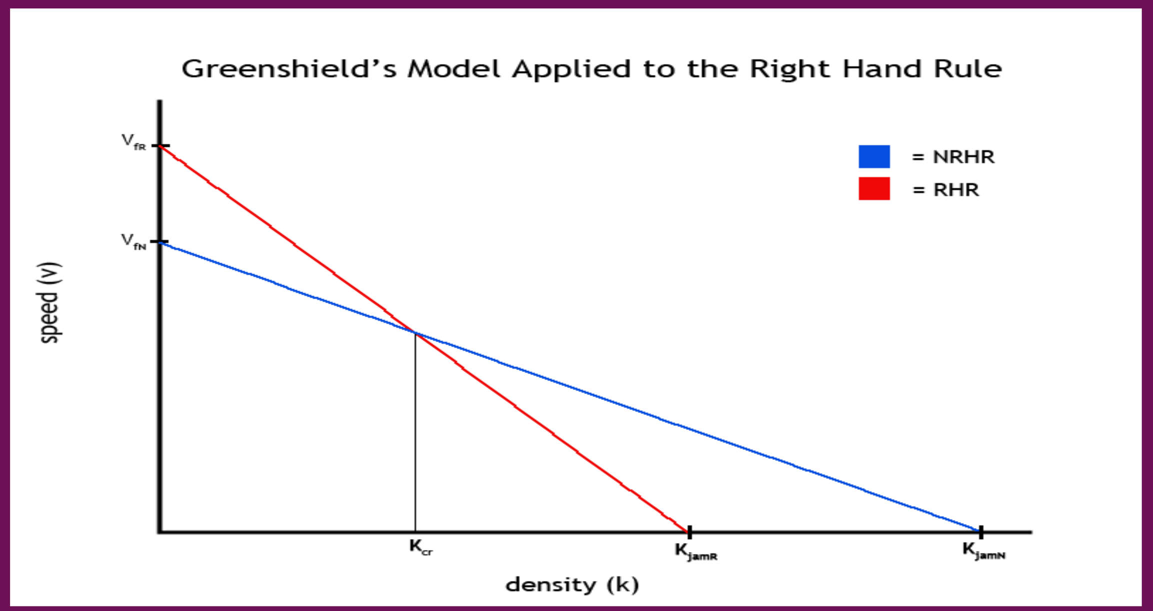

Figure 1. The end behavior when the right hand rule is in effect has a higher free flow speed, and a lower jam density, as a result of the assumptions made. Where the lines cross is the critical density, and corresponds to the point when the right hand rule is no longer beneficial. After this point if people strictly adhered to the right hand rule we see that their overall speed at the given densities is reduced, making the right hand rule superior up until this critical density. However, it is at this point where we assume that both the fast and slow cars will attempt to utilize the left lane for passing, however they will no longer be able to travel at their desired speeds. Therefore at this density we assume people will begin utilizing the left lane.

$$V = \begin{cases} V_R(k), & K < K_{cr} \\ V_N(k), & K > K_{cr} \end{cases} \qquad [4]$$

Flow and Density

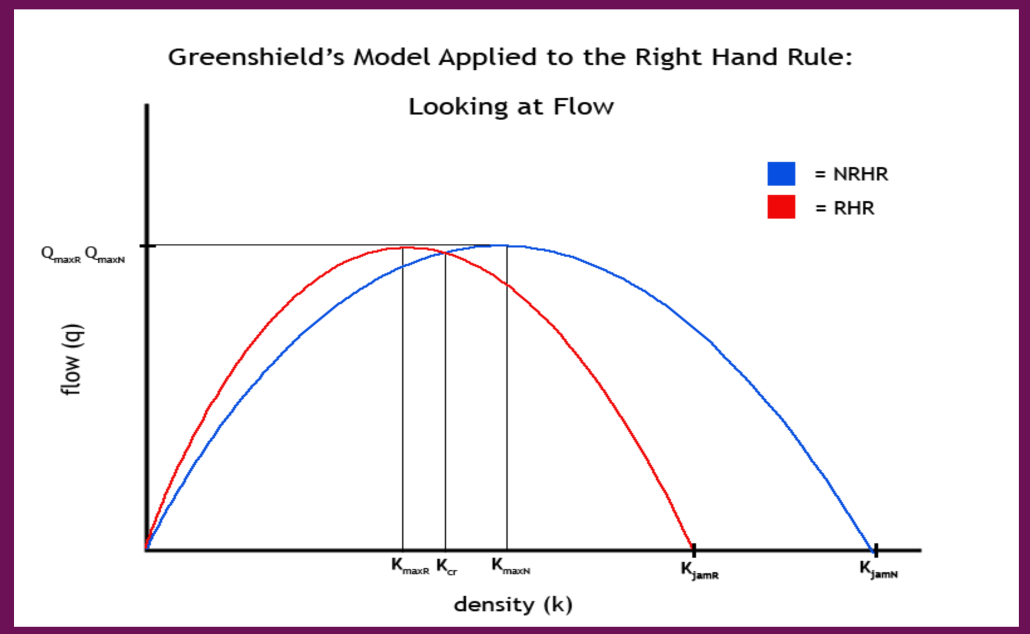

Figure 2. The relationship of flow and density with and without the RHR

This shows that optimal flow can be maintained by following the piecewise function as stated in the following equation:

$$Q = \begin{cases} Q_R(k), & K < K_{cr} \\ Q_N(k), & K > K_{cr} \end{cases} \qquad [5]$$

CONCLUSIONS

We modeled the right hand rule using Greenshield's macroscopic model with adjusted free flow speed and critical densities and compared it to the unmodified Greenshield's equation to evaluate the differences. From these comparisons we were able to visualize the effect of the right hand rule on traffic flow, density, and speed. If the right hand rule were followed exactly we have showed that it will result in better flow in low densities, until the critical density is reached, after which it is where the system with no rules results in better flow.

To optimize the flow through the entire model we designed a piecewise function which allows for the increased flow of the right hand rule in low traffic as well as the increased flow of no rules in the high density traffic. We also assert that this behavior will naturally occur in the right hand model as people enter the left lane in order to pass, however, they will no longer be able to reenter the right lane to increase their velocity, and thus the density of the system will now be the entire road as the right lane rule breaks down.