Social Dynamics of Gang Involvement: A Mathematical Approach

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Abstract

Gangs have played a signi cant role in the social and political history of the United States, and continue to impact the country today, as gang violence and participation rates continue to grow. In this paper, we explore the dynamics of gang involvement between at-risk individuals, gang members, and reformed gang members using an SIR-type model. We investigate the effect that social in uence of reformed gang members has on the "at-risk" population using a general function, which takes into account the cost of gang membership and a threat factor. Our results show that the in uence of the reformed population is highly sensitive to initial gang member population size, recidivism, and cost.

1. Introduction

- The NYGS estimates a 28% increase from 2002 to 2009 in the number of gangs throughout the country, with over 28,000 active gangs.
- Gang involvement is a product of personal choice, sociocultural setting, and social in uence; gangs commonly involve youth, are concentrated in areas of poverty, and are composed of racial and ethnic minorities.
- We reason that reformed gang members, who have experienced many of the same challenges, may serve as the most effective mentors for "at-risk" individuals.

2. Model

Our model is based primarily on the model constructed by Sánchez et al., which models the impact of nonlinear social in uence on drinking behavior(1). The key difference is that our model includes a reducing function, $f(\ ;\ ;r\ )$, on , to represent the effect that reformed gang members serving as mentors have on at-risk youth. $f(\ ;\ ;r\ )$ must be a positive, decreasing smooth function that includes a threatening factor, , which represents the potential risks to reformed gang members who work to prevent gang involvement.

Figure 1: Gang involvement grouped into s (at-risk population), g (gang members), and r (reformed gang members).

Our system of nonlinear differential equations is given by:

$$s' = \quad f(\ ;\ ;r\ )sg \quad s;$$
$$g' = \ f(\ ;\ ;r\ )sg + \ rg \quad (\ +\ )g;$$
$$r' = \ g \quad rg \quad r:$$

where $s + g + r = 1$. We work with the following example of the reducing function:

$$f(\ ;\ ;r\ ) = \frac{}{1 + \ r}:$$

Table 1: Parameter De nitions

Parameter	De nition	Values
	recruitment rate	0.009
	departure rate	0.00015
	cost of gang membership	[0,1]
	threat factor	[0,1]
	recidivism rate	0.005
	gang departure rate	0.0027

3. Mathematical Analysis

Gang-free equilibrium: $(s_0; g_0; r_0) = (1; 0; 0)$

Reproductive numbers: $R_0 = \frac{}{+}$ and $R\ = \frac{}{+}$

We study two cases of the endemic equilibria: when $= 0$ and when $0 <\quad 1$.

3.1 = 0 (Absence of Threat Factor)

Solving for the endemic equilibria when $= 0$ yields $a_2 g^2 + a_1 g + a_0$, where:

$$a_2 = \ R_0;$$
$$a_1 = (\ +\ )R_0(1 \quad R\ ) + R\ ;$$
$$a_0 = \ (1 \quad R_0):$$

Figure 2: (a) Backward bifurcation with $= 0{:}00015$, $= 0{:}009$, $= 0{:}0027$, $= 0$, $= 0{:}005$ and varied ; (b) Time series with $= 0{:}00015$, $= 0{:}009$, $= 0{:}0027$, $= 0{:}004$, $= 0{:}3$, $R_0 = 0{:}9474$, $R\ = 1{:}4035$ and initial gang populations 5% (endemic) and 1% (gang-free).

3.2 0 < 1 (Presence of Threat Factor)

Solving for the endemic equilibria when $0 <\quad 1$ yields $a_3 g^3 + a_2 g^2 + a_1 g + a_0$, where:

$$a_3 = \ ^2R_0;$$
$$a_2 = \quad (\ +\ )R_0\ 1 \quad R\ \ + \quad R_0 + R\ + \frac{}{+} \quad ;$$
$$a_1 = \quad +(\ +\ )\ R_0 + R\ \ + R\ \quad 2R_0\ ;$$
$$a_0 = \ ^2(1 \quad R_0):$$

Figure 3: Forward-backward bifurcation with parameters $= 0{:}00015$, $= 0{:}0027$, $= 0{:}009$, $= 0{:}8$, and varied . (a) $= 0{:}0044$, $R\ = 1{:}5439$ (b) $= 0{:}004$, $R\ = 1{:}4035$

Region 1: A necessary condition for no positive equilibria is $0 < R_0 < R_c < 1$ and $R\ > 1$.

Region 2: A necessary condition for two positive equilibria is $0 < R_c < R_0 < 1$ and $R\ > 1$:

Region 3: A necessary condition for three positive equilibria is $1 < R_0 < R_0$ and $R\ > 1$:

Region 4: A necessary condition for a unique positive equilibrium is $1 < R_0 < R_0$ and $R\ > 1$.

4. Numerical Analysis

We use numerical simulations to analyze the long-term gang population dynamics.

Figure 4: Gang population time series for the forward-backward bifurcation when $0 <\quad 1$. (a) Region 1; (b) Region 2; (c) Region 3; (d) Region 4.

5. Discussion

- The social in uence of reformed gang members on the at-risk population is highly sensitive to gang population size in the at-risk environment.
- Keeping recidivism rates low and encouraging reformed gang members to mentor individuals in an at-risk environment plays a major role in gang involvement. When the recidivism rate is low, a high value of can shift the forward-backward bifurcation to the point where it is possible to reach a gang-free equilibrium when $R_0 < 1$.
- A lack of opportunities could lead reformed gang members to return to gang life; therefore, programs that help gang members reintegrate into society, such as tattoo removal or job placement, could help lower recidivism rates.
- Cost of gang membership also has a signi cant impact on gang population dynamics. If cost is low, there is little that reformed gang members can do to in uence gang-involvement. Though policy cannot directly increase the cost of gang membership, programs that educate individuals about the true costs could help discourage them from joining gangs.

References

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