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Determination and interpretation of earthquake magnitude spectra

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Magnitude spectra are determined for three major earthquakes of the year 1985: the Xianjiang earthquake of 23 August, and the Mexico earthquake of 19 September with its largest aftershock of 21 September. Broad-band recordings obtained at the Central Seismological Observatory of the Federal Republic of Germany (GRF) are used for the analysis. Pass-band seismograms are obtained by way of filtering the broad-band seismogram. The magnitude spectrum of an earthquake is determined from the velocity amplitude density spectrum of the signal by calculating the spectral magnitude for each Fourier component. The magnitude spectrum represents the velocity amplitude density spectrum at the earthquake source scaled in magnitude units. A comparison of the magnitude spectra shows significant differences between the focal parameters of the earthquakes, even if their conventional magnitudes (m_b, M_S) are similar.

Određivanje i interpretacija spektara magnitude potresa

Određeni su spektri magnituda za tri velika potresa koji su se dogodili 1985. godine: 23. kolovoza (Xianjiang), 19. rujna (Meksiko) i najveći naknadni potres od 21. rujna (Meksiko). Analizirani su širokopojasni zapisi zabilježeni na Središnjem seizmološkom opservatoriju Savezne Republike Njemačke (GRF). Postupkom filtriranja izračunati su uskopojasni seizmogrami. Spektar magnituda potresa određen je iz spektra gustoće amplituda brzine signala proračunom spektralne magnitude za svaku Fourierovu komponentu. Spektar magnituda potresa predstavlja spektar gustoće amplituda brzine u žarištu potresa izražen jedinicama magnitude. Usporedbom spektara magnitude uočavaju se značajne razlike među hipocentralnim parametrima potresa, iako su im konvencionalne magnitude (m_b, M_S) slične.

1. Introduction

The earthquake is being understood as a radiation process of mechanical energy. The process is caused by a tectonic movement inside the Earth. The mechanical energy is radiated primarily in the form of seismic body waves.

The radiation from the earthquake focus is characterised by a spectrum of the *P*-wave and the *S*-wave signal. During the propagation through the Earth the spectrum is altered in accordance with the physical properties of the Earth material. A second alteration is taking place in the seismograph system. The seismograms thus reflect the spectral properties of the radiated body waves only remotely.

The body wave magnitude as introduced by Gutenberg and Richter (1956), can be considered as the first attempt to restore the level of the radiated spectrum, with the intention to arrive at a single quantity characterizing the strength of the earthquake.

The attempt appears today crude, in view of the progress which has been made in the meantime in seismological instrumentation, in the control of the physical properties of the Earth material, and in the knowledge about the source rupture process.

New calibration functions for *P*- and *S*-waves have been presented by Nortmann and Duda (1983). They depend not only on the epicentral distance and focal depth, as it is the case with the calibration function of Gutenberg and Richter, but also on the period of the spectral component of the wave, as well as on the bandwidth of the seismometer system employed. Based on these calibration functions magnitude spectra can be determined from broad-band seismograms for a given earthquake. The magnitude spectrum reflects the velocity amplitude density spectrum of the respective seismic phase radiated from the earthquake focus.

In the present paper the method is presented to determine magnitude spectra and to derive from them other focal parameters. The presentation is made on the basis of three recent earthquakes, for which the focal parameters are determined and subsequently interpreted as to their tectonophysical significance.

2. Data

Three major earthquakes of the year 1985 are analysed in this study: the Xinjiang, China, earthquake of 23 August 1985 and the Mexico earthquake of 19 September 1985, with its largest aftershock of 21 September 1985. Basic information on these earthquakes is given in Table 1.

Table 1. List of earthquakes. Data published by the National Earthquake Information Service (NEIS)

No.	Region	Date and origin time	Epicenter	Depth (km)	m_b	*Ms*
1	Southern Xinjiang, China	23.08.85 12:41:56	39.43N 75.22E	7	6.4	7.3
2	Michoacan Mexico	19.09.85 13:17:47	18.19N 102.53W	28	6.8	8.1
3	Near Coast of Guerrero, Mexico	21.09.85 01:37:13	17.80N 101.65W	31	6.3	7.6

Broad-band seismograms recorded at the Central Seismological Observatory of the Federal Republic of Germany (GRF) are employed in analysis. Figure 1 shows the magnification of the seismographs. The actual digital broad-band recordings were subjected to inverse filtering, so as to simulate broad-band recordings corresponding to an instrument with a velocity response flat in the period range 0.2 sec - 200 sec. The corresponding broad-band record of the vertical component of the *P*-wave is shown in Figures 2a, b, c (top line), for the three earthquakes. Ten band-pass seismograms are obtained from each of the broad-band seismograms by way of filtering the seismogram with a system of 10 nonoverlapping filters, each having the width of one octave. The system of filters is shown in Figure 1 on the background of the magnification curves of the broad-band seismographs. The band-pass seismograms are shown in Figures 2a, b, c. Each of the seismograms, as well as the broad-band seismogram, has the length of 3 min 24.8 sec (4096 sampling points).

For the Southern Xinjiang earthquake, the maximum ground velocity at the station is seen to occur at a period of 2 sec (Fig. 2a). In contrast, we find dominant periods of 16 and 32 sec for the main shock in Mexico (Fig. 2b) and of 16 sec for the aftershock (Fig. 2c). A comparison of the amplitudes in the respective period ranges indicates a similarity between the two Mexican earthquakes, although the high frequency content (at a period of about 2 sec) is higher for the main shock.

The analysis of pass-band seismograms offers the possibility to study the energy content in different period bands radiated from the earthquake focus.

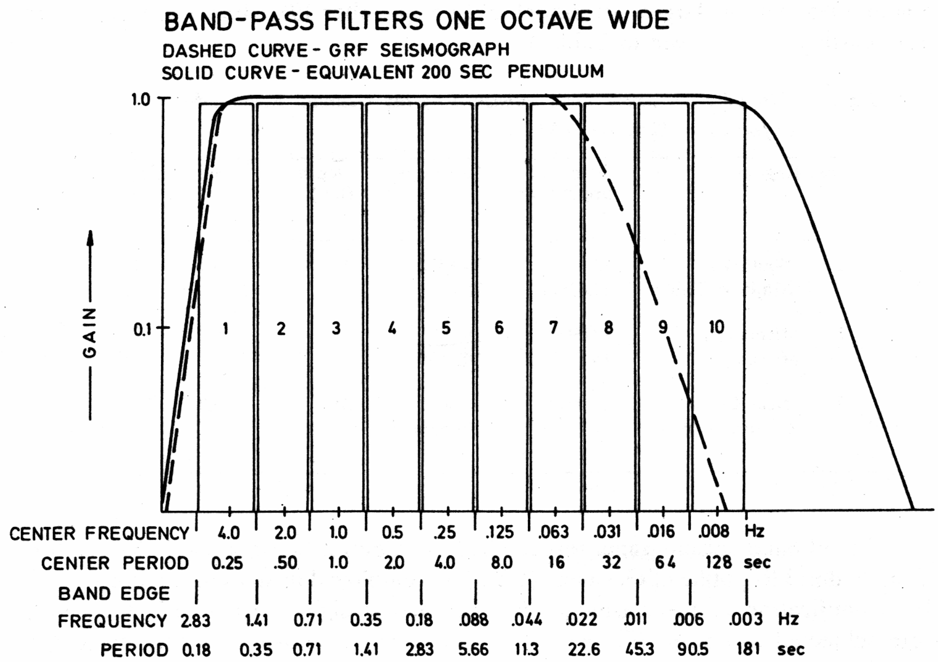

Figure 1. Velocity response of seismographs at Central Seismological Observatory of the Federal Republic of Germany at Erlangen (GRF) (dashed curve). The solid curve is the velocity response of a hypothetical seismograph with free period of oscillation of 200 s. Superimposed are 10 one-octave band-pass filtres used to obtain band-pass seismograms (see Fig. 2)

3. Analysis

A spectral magnitude can be determined basically from each of the band-pass seismograms. Up to 10 spectral *P*-wave magnitudes can thus be given for a given earthquake. The procedure is described by Nortmann and Duda (1983). The spectral magnitudes are a measure of the energy spectral density in the period range corresponding to the given filter (1-10) of the signal radiated from the earthquake focus in the form of the respective body wave. The spectral magnitude has to be given thereby together with the period at which it is determined. Conventional body-wave magnitudes, as published e.g. by the National Earthquake Information Service (NEIS), Boulder, Colorado, U.S.A., also correspond to a filter whose frequency characteristic is determined by that of the seismometer and the galvanometer.

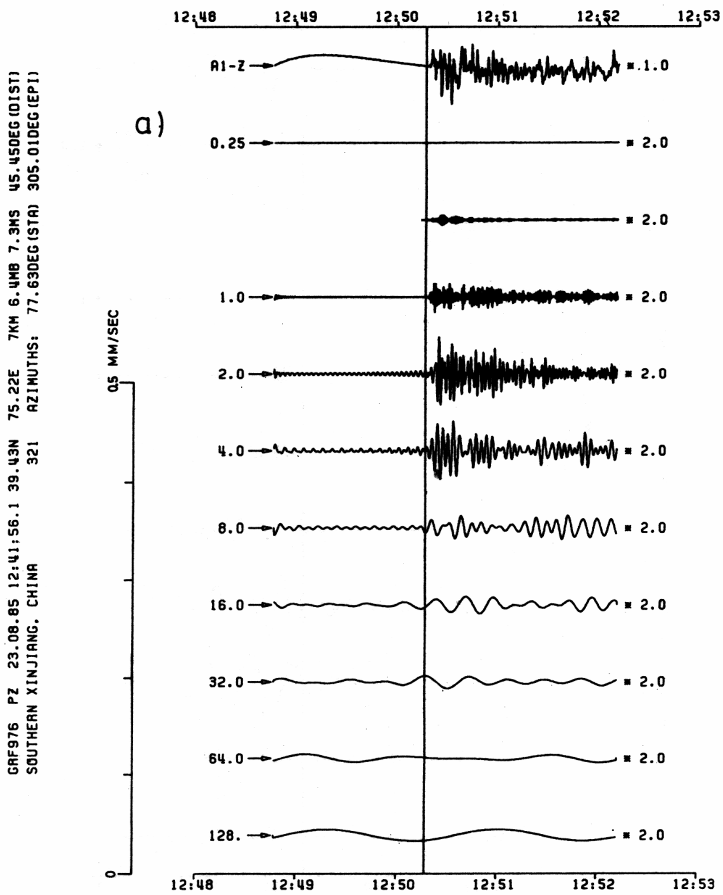

Figure 2a. Vertical component of the broad-band seismogram and ten band-pass seismograms. The center period of the one-octave band-pass filter is given on the left. The magnification relative to that of the broad-band seismogram is given on the right side of each trace. The vertical line marks the theoretical *P*-wave arrival time. Southern Xinjianng, China, 23 August, 1985

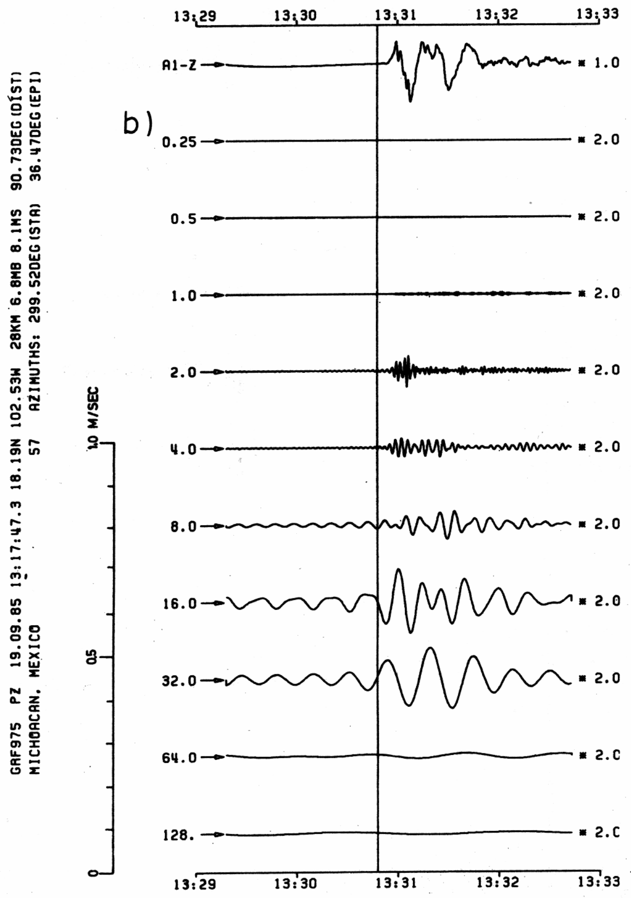

Figure 2b. See the description of Figure 2a. Michoacan, Mexico, 19 September, 1985.

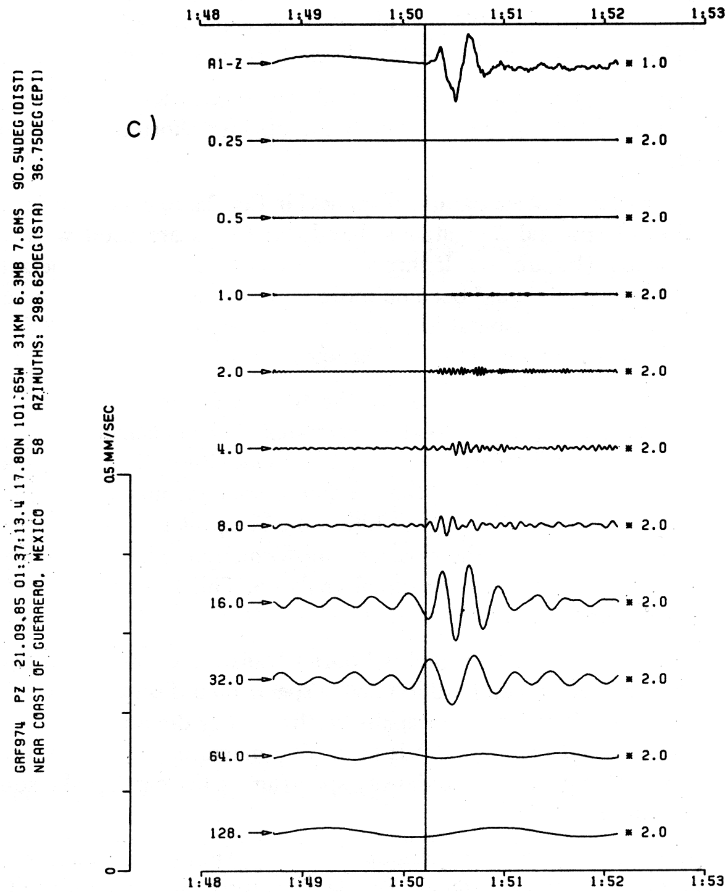

Figure 2c. See the description of Figure 2a. Near Coast of Guerrero, Mexico, 21 September, 1985.

Unfortunately, the width of this filter system is not taken into account in the determination of the conventional body-wave magnitude, and is usually unknown. Also, the period is not being published due to expectation that the same magnitude figure can be determined at all periods. Consequently, the conventional body-wave magnitudes fluctuate for a given earthquake and are lacking a clear physical meaning.

The spectral magnitude as defined above is intended to put the magnitude concept on a solid footing and to remedy the obvious shortcomings of the conventional magnitude.

For the bandpass seismograms displayed in Fig. 2a, b, c, and for the subsequent determination of spectral magnitudes, bandpass filters are used with an arbitrary width of 1 octave (Figure 1). If larger widths were used (e.g. 2 octaves), the filtered body-wave signal would become more complex and could not be equally easily approximated by a sinusoidal line, as it is necessary for the amplitude and period measurement. Also, the resolution of the spectral magnitudes would be decreased.

On the other hand, a decrease of the filter width increases the resolution of the spectral magnitudes, understood as estimates of the spectrum in the given passband. However, it also increases the number of magnitude figures, which eventually become difficult to handle. Here, the *maximum* spectral magnitude with its period gains special significance. For digital filters the smallest feasible bandwidth equals the elementary frequency of the Fourier transformation. The elementary frequency is determined by the length of the signal analysed. This leads to the concept of the magnitude spectrum.

The *P*-wave (or *S*-wave) signal is Fourier transformed and from each Fourier-component of the velocity amplitude density spectrum a discrete spectral magnitude is calculated. For this calculation, again the frequency-dependent calibration function of Nortmann and Duda (1983) is used. The magnitude spectrum thus obtained represents the velocity amplitude density spectrum at the earthquake source, scaled in magnitude units.

In order to estimate the resolution, four time windows are chosen for Fourier-transformation of the *P*-wave signal (Fig. 3a, b, c, top), each beginning at 0.0-the expected arrival time of the phase. The shortest window (1) has a length of 6.4 sec (128 sampling points) and the longest window (4) - of 51.2 sec (1024 sampling points). The magnitude spectra for time windows 1 to 4 are shown in Figure 3a, b, c. As can be seen, the length of the time window has some effects on the magnitude spectrum, especially at the long periods.

No general rule can be given for the required window length as requirements antagonize with each other. On one side the window should include the total *P*-wave (or *S*-wave) signal, but at same time the window should exclude other phases (*pP, sP, pS, sS,* etc). Above all, however, the window should be long enough for a sufficient resolution of the magnitude spectrum at long periods.

In Figure 3a the time window 3 (25.6 sec) appears to be long enough, as the doubling of the window length (51.2 sec, window 4) virtually does not alter the magnitude spectrum 4, if compared with spectrum 3. In particular, the maxima of both spectra $(mf)_{max}$, occur at practically the same period. Disregarding instantaneous peaks in the spectrum, we find $(mf)_{max}$ = 7.9 and T_o = 2.4 sec, for the Southern Xinjiang earthquake.

In Figures 3b and 3c the time window 3 (25.6 sec) is evidently too short. For the two Mexican earthquakes we consider the time window 4 (51.2 sec), and the corresponding spectra as representative. We find $(mf)_{max}$ = 8.9 and 8.7, and T_o = = 18 and 14 s for the two Mexican earthquakes (see also Table 2).

The conventional body wave magnitude, as published by NEIS is being determined from *P*-waves with periods around 1 sec. Also, the period-independent calibrating function of Gutenberg and Richter (1956) is applied.

At the period of 1 sec from the magnitude spectra (Figure 3a, b, c) spectral magnitudes are found, amounting to mf (1 sec) = 7.3, 7.3, and 6.6 for the three earthquakes, respectively (compare Table 2). These spectral magnitudes are numerically larger than the conventional body-wave magnitudes published by NEIS (6.4, 6.8, 6.3). This is due to the fact that in the former case the anelastic attenuation, particularily high for waves at 1 sec (and shorter), is taken into account, while conventional body wave magnitudes neglect this in employing the period-ndependent calibrating functions of Gutenberg and Richter (1956).

Nortmann and Duda (1983) find at the period of 1 sec an average difference of 0.4 units between spectral magnitudes and NEIS-magnitudes.

For the earthquakes studied here, the difference amounts to 0.9, 0.5, and 0.3 respectively, in, at least, qualitative agreement with the expectation.

At 1 sec, the Xinjiang earthquake and one of the Mexican earthquakes (main shock) feature the same spectral *P*-wave magnitude (mf (1 sec) = 7.3).

However, a comparison of the magnitude spectra (Fig. 3a and 3b) reveals a considerable difference in the spectral contents, signifying that the comparison of earthquakes on the basis of a single magnitude may lead to erroneous conclusions. As indicated above, the maximum spectral magnitude for the Xinjiang earthquake, $(mf)_{max}$ = 7.9, occurs at a period of 2.4 sec, while for the Mexican earthquake the maximum magnitude $(mf)_{max}$ = 8.9 occurs at a period of 18 sec. Thus, the maximum radiation, expressed by way of the maximum velocity amplitude spectral density, is found to take place at widely differing periods. The magnitude spectra demonstrate thus significant differences in the physical parameters of the rupture process in different earthquakes.

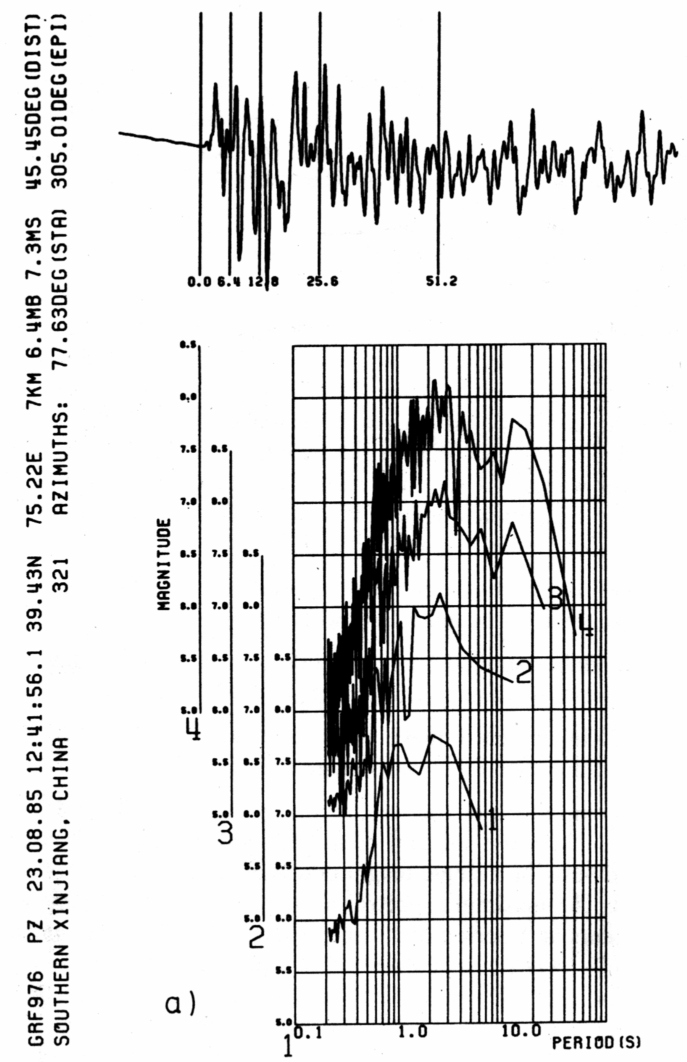

Figure 3a. Magnitude spectra determined from the broad-band seismogram of the *P*-wave as in Figure 2. Vertical lines mark the portion of the seismogram that is windowed and Fourier-transformed. The time windows begin at 0.0 s - the expected arrival time - and end at 6.4 s, 12.8 s, 25.6 s, 51.2 s, respectively. The magnitude spectra 1, 2, 3, 4 correspond to the above windows.

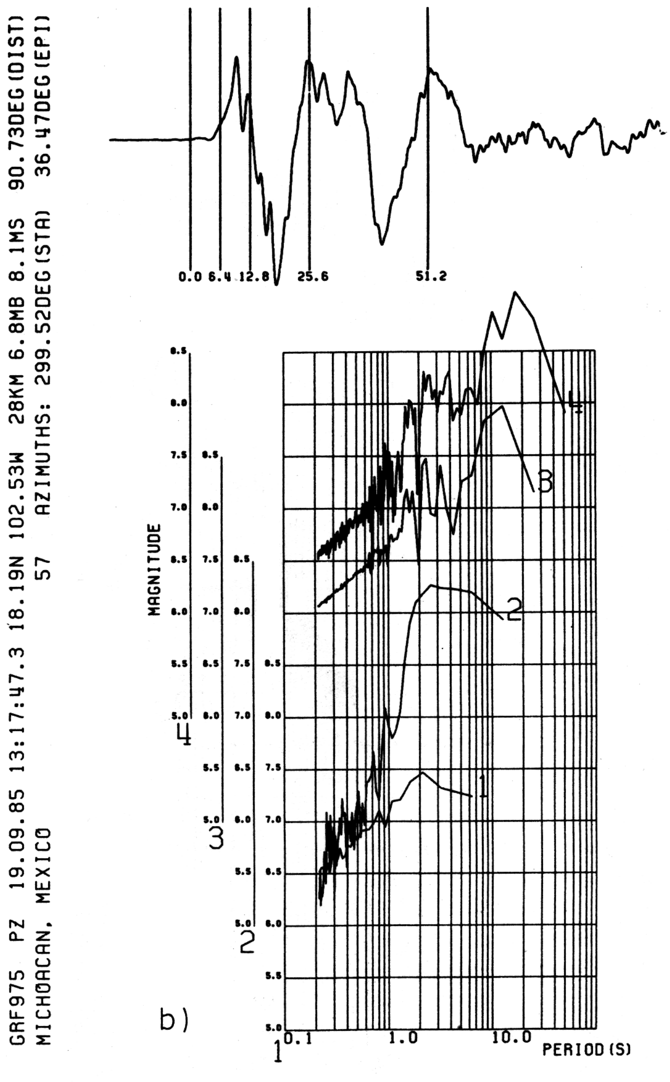

Figure 3b. See description of Figure 3a.

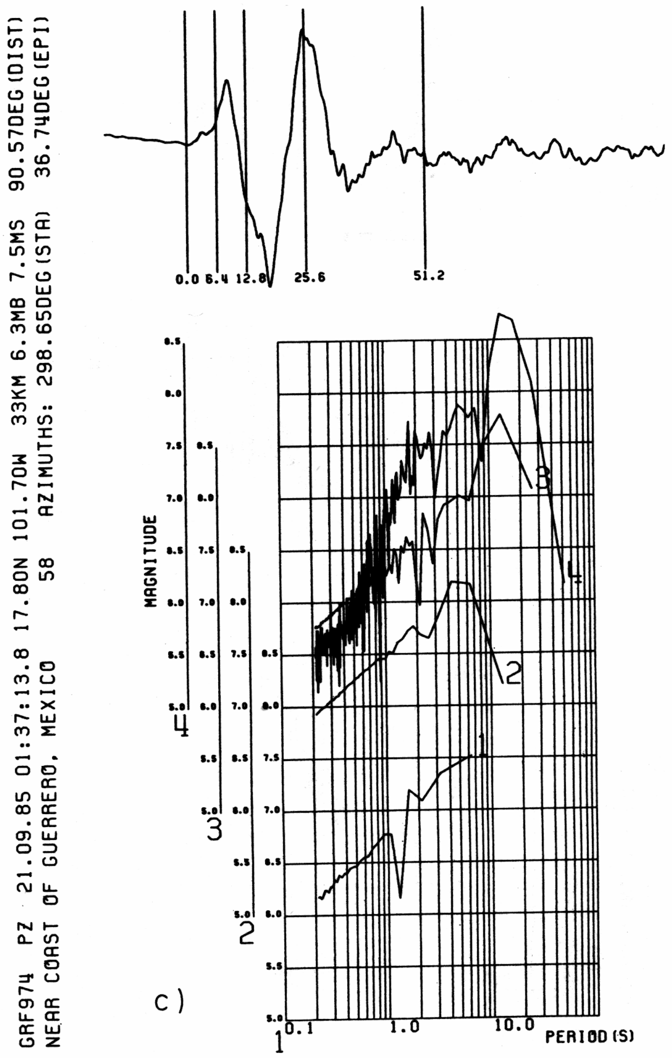

Figure 3c. See description of Figure 3a.

Table 2. Earthquake source parameters. No. refers to Table 1.

No.	m_f(1 sec)	$(m_f)_{max}$	T_o sec	M_o 10^{19}Nm	L km	$\Delta\sigma$ MPa	E_p 10^{12}J
1	7.3	7.9	2.4	1	12	23	98
2	7.3	8.9	18	75	87	4.1	1100
3	6.6	8.7	14	37	67	4.2	240

Explanations of Symbols:

$(m_f)_{max}$	maximum value of magnitude spectrum
T_o	period at which $(m_f)_{max}$ occurs (sec)
M_o	seismic moment (10^{19}Nm)
L	fault length (km)
$\Delta\sigma$	static stress drop (MPa)
E_p	seismic energy radiated from the focus as P-waves (10^{12} J)

4. Discussion and conclusions

Magnitude spectra offer the possibility to study the processes at the earthquake focus, using broad-band recordings of earthquakes at teleseismic distances. Various theoretical earthquake source models have been proposed, which predict the shape of the spectrum of P-(and S-) waves radiated from the earthquake source. By comparing these theoretical spectra with the observed spectrum and varying the model parameters until the best match is obtained, earthquake source parameters can be determined. The method for extracting source parameters from spectral magnitudes of an earthquake has been described by Sarkar and Duda (1985), and is applied in the present paper to magnitude spectra of the three 1985 earthquakes.

The period T_O at which the maximum magnitude $(mf)_{max}$ occurs, is identical with the corner period in the far-field body wave spectrum. The corner period is proportional to the fault length. The relationship given by Brune (1970, 1971) is adopted. The seismic moment is calculated using the relation given by Aki and Richards (1980). The stress drop can then be calculated, as it is related to the fault length and seismic moment (Brune, 1970). The total seismic energy radiated in the form of P-waves is determined by integrating over the energy spectral density at the earthquake focus, $E(f)$. The spectral density is related to the spectral magnitude by

$$E(f) = 10^{2m(f) - 1.4}$$

(Nortmann and Duda, 1983).

Further details about the procedure of extracting earthquake source parameters are given in Kaiser and Duda (in preparation). The source parameters for the three earthquakes under study are presented in Table 2.

From among the three earthquakes investigated, the Mexico earthquake (main shock) is by far the largest event with respect to the seismic moment as well as to the *P*-wave energy released. For the Mexico aftershock and the Xinjiang earthquake, almost identical body-wave magnitudes have been published by NEIS (m_b = 6.3 and 6.4, respectively). However, the comparison of the focal parameters reveals significant differences between these two earthquakes. The seismic moment of the Mexico aftershock is almost 40 times larger, but the stress drop more than 5 time smaller, if compared with the values for the Xinjiang earthquake. Both Mexican earthquakes are characterized by low stress drops and large seismic moments as well as large fault lengths, whereas the Xinjiang earthquake is found to be of high stress drop type.

To interpret the differences of focal parameters for Xinjiang and Mexico, we point to the difference in the tectonic setting in both regions. While the Mexican earthquakes occur in a part of the circum-Pacific seismic belt in a subduction zone, the Xinjiang earthquake occurs inside the Eurasian tectonic plate.

The focal parameters, as determined from the magnitude spectrum indicate that clear, measurable differences exist between the parameters for the intraplate and interplate earthquakes. Recently Scholz et al. (1986) observed differences in the source parameters for large intraplate and interplate earthquakes. They concluded, that the stress drops of intraplate earthquakes are systematically about 6 times greater than those of interplate earthquakes. Finally, we point to the fact that an analysis of earthquakes from the Andaman Islands region has led to the conclusion that interplate earthquakes feature lower stress drop values than other shallow-focus or deep-focus earthquakes (Sarkar and Duda, 1985). Both findings are in complete agreement with the results of the present investigation. From the magnitude spectrum it is eventually possible to recognize to which of the two classes of earthquakes a given event belongs.

Acknowledgement

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