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Some properties of hydrodynamic instability in a quasi-geostrophic atmosphere

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The baroclinic hydrodynamic instability of zonal flow has been studied by a two-level model which takes into account the influence of a barotropic, nonlinear shear in a quasigeostrophic atmosphere. There are regions where a horizontal wind shear supports the baroclinic amplification of an unstable wave. It can be found near the inflexional part of the zonal flow meridional profile.

Neke osobine hidrodinamičke nestabilnosti u kvazigeostrofičkoj atmosferi

Dvoslojnim modelom razmatrana je baroklina hidrodinamička nestabilnost zonalne struje uzimajući u obzir i utjecaj barotropnog, nelinearnog smicanja u kvazigeostrofičkoj atmosferi. Postoje područja u kojima horizontalno smicanje pomaže baroklinu amplifikaciju poremećenja. Ono se nalazi u blizini infleksionog dijela meridionalnog profila zonalne struje.

1. Introduction

Hydrodynamic instability (HI) is caused by nonlinear space variations of the air flow in the statically stable atmosphere above the planetary boundary layer. It comprises a complex influence of vertical (baroclinic HI) and horizontal (barotropic HI) zonal flow variations. Charney and Stern (1962), as well as Pedlosky (1964), derived a criterion for the onset of the entire HI which includes barotropic and baroclinic effects. Based on the potential vorticity, q, and conservation in a quasigeostrophic and hydrostatic flow, they found that in the case of hydrodynamic instability the integral value of the meridional variation of q through the whole atmosphere must equal zero. The values of $\partial q/\partial y$ have to be determined on as many as possible levels in the atmosphere — and the problem can be solved numerically. Still, Pedlosky (1964) demonstrates HI appearance in a quasigeostrophic model with only two levels.

The modelling of hydrodynamic instability usually assumes various theoretical forms of the zonal flow velocity, U, meridional and vertical profiles (Brown, 1968 a, b; James, 1987). This fact probably gives vise to some inconsistency of the results obtained by various models. According to James (1987), meridional variations of zonal flow always disturb a baroclinic development of perturbations, while Brown (1968 a, b) finds that a baroclinic HI development appears to be supported in regions of barotropic hydrodynamic instability ($\partial^2 U/\partial y^2 \approx \beta$). Simmons and Hoskins (1980) discuss the energetics of total HI and point out a perturbations reduced ability to grow in case of the zonal flow strenghtening (regardless of its direction), i.e. in case of a growing barotropic shear.

Phillips (1954) investigates the baroclinic HI by means of a two-level quasigeostrophic model and finds that for macroperturbations with a high wave number a neglect of the horizontal shear makes the air flow stable, regardless of the amount of vertical shear. Pedlosky (1964) comes to the same conclusion making use of the integral value of $\partial\bar{q}/\partial y = 0$.

An application of spectral analysis (Šinik, 1986) can help to determine the length of a perturbation which is supposed to amplify first.

Our results offer some new information on the relationship between baroclinic and barotropic instabilities in the real atmosphere. We use the quasigeostrophic model with two levels and without upper boundary and lateral restrictions (Charney, 1947). Unlike Phillips (1954), we introduce a nonlienar horizontal shear of zonal flow into the model. That way, besides $\partial\bar{q}/\partial y = 0$ integrally, a new possibility to estimate barotropic effects in a baroclinic, hydrodynamically unstable zonal flow has been obtained. These effects have been examind by the model application to real cases of hydrodynamically unstable flow in the free atmosphere over the region of Yugoslavia during the period of 6 days in January 1987. The results indicate that a barotropic, horizontal flow shear may support — as well as disturb — the baroclinic HI dependent on the magnitude of the zonal flow monlinear meridional shear in the β-plane.

2. Derivation of HI criterion

The two-layer quasigeostrophic atmosphere (Fig. 1) can be well described by the mean and thermal vorticity equations (Haltiner and Martin, 1957):

$$\frac{\partial}{\partial t}\nabla^2\phi + J\left(\phi, \frac{1}{f_0}\nabla^2\phi + f\right) + \frac{1}{4}J\left(\pi, \frac{1}{f_0}\nabla^2\pi\right) = 0 \qquad (1)$$

$$(\nabla^2 - M^2)\frac{\partial\pi}{\partial t} + J\left(\phi, \frac{1}{f_0}\nabla^2\pi\right) + J\left(\pi, \frac{1}{f_0}\nabla^2\phi + f\right) - \frac{M^2}{f_0}J(\phi, \pi) = 0 \qquad (2)$$

where

$\phi = \frac{1}{2}(\phi_1 + \phi_2)$ the mean geopotential

$\pi = \phi_1 = \phi_3$ relative topography

f_0 a constant value of Coriolis parameter

$M^2 = \frac{8f_0^2}{\rho_0^2 \sigma}$ with a static stability $\sigma = -\frac{\alpha}{\Theta}\frac{\partial \Theta}{\partial p}$ and $p_0 = 1000\,\mathrm{hPa}$ (Θ is potential temperature and α specific volume).

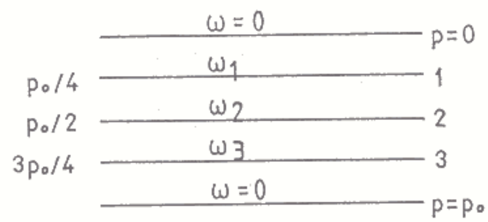

Figure 1. A scheme of the two-level atmospheric model.

Next, we linearize eqs. (1) and (2) (see e.g. Labović, 1963) and introduce the possibility of nonlinear meridional variations of zonal flow i.e. $\frac{\partial^2 U}{\partial y^2} \neq 0$. The mean fields ϕ and π vary only meridionally and the perturbed ones $\phi' = \phi_0 e^{ik(x-ct)}$ and $\pi' = \pi_0 e^{ik(x-ct)}$ with ϕ_0 and π_0 being amplitudes, c the phase velocity and k zonal wave number. The condition that the set of two equations for ϕ_0 and π_0 has nontrivial solutions finally leads to an expression for the phase velocity

$$C = U - \frac{1}{2}\frac{M^2 + 2k^2}{k^2(M^2+k^2)}\left(\beta - \frac{\partial^2 |U|}{\partial^2 y}\right) \pm$$

$$\left\{\left[\frac{(M^2+2k^2)^2}{4k^4(M^2+k^2)^2} - \frac{1}{k^2(M^2+k^2)}\right]\cdot\left(\beta - \frac{\partial^2|U|}{\partial y^2}\right) - \frac{U_T^2(M^2 - k^2)}{4(M^2+k^2)}\right\}^{1/2} \tag{3}$$

where U_T is the thermal wind. In the two layer model $U_T = U_1 - U_3 = \mathrm{const}$. Obviously, c may become a complex number, i.e. $c = c_r + ic_i$ if the discriminant

$$D = \left[\frac{(M^2+2k^2)^2}{4k^2(M^2+k^2)^2} - \frac{1}{k^2(M^2+k^2)}\right]\left(\beta - \frac{\partial^2|U|}{\partial y^2}\right) - \frac{U_T^2(M^2 - k^2)}{4(M^2+k^2)} < 0 \tag{4}$$

The absolute value $|U|$ in our model indicates — in accordance with the findings of Simmons and Hoskins (1980) — that barotropic nonlinear influences are independent of the zonal flow direction (west or east).

Equations (3) and (4) clearly illustrate a dependence of hydrodynamic instability HI upon the zonal flow vertical shear (thermal wind, U_T), and also upon its nonlinear meridional variations $\left(\frac{\partial^2|U|}{\partial y^2}\right)$.

Making use of the thermal wind equation, we introduce a meridional change od the zonal mean temperature ($\frac{\partial \overline{T}}{\partial y}$, $\overline{T}$ — zonal mean temperature, relevant for the whole layer of the width δp between the upper and lower levels) and get

$$D=\left[\frac{(M^2+2k^2)^2}{4k^4\,(M^2+k^2)^2}-\frac{1}{k^2\,(M^2+k^2)}\right]\left(\beta-\frac{\partial^2|U|}{\partial y^2}\right)-\left(\frac{R\delta p}{f_0 p}\right)^2\left(\frac{\partial \overline{T}}{\partial y}\right)^2\left(\frac{(M^2-k^2)}{4\,(M^2+k^2)}\right) \tag{5}$$

The case of $D=0$ finally gives a criterion for the onset of total hydrodynamic instability. The total HI depends upon a sum of baroclinic $\left(\frac{\partial \overline{T}}{\partial y}\right)$ and barotropic $\left(\frac{\partial^2|U|}{\partial y^2}\right)$ effects. They are included in the eq. (5) in a similar way as they appear in the integral meridional change of potential vorticity (Charney and Stern, 1962).

We want to detach a barotropic influence upon the $\frac{\partial \overline{T}}{\partial y}$ and, therefore, beginning with $D=0$, derive the equation

$$\left(\frac{\partial \overline{T}}{\partial y}\right)_c=\frac{f_0 p}{R\delta p}\,\frac{1}{k^2\left[1-\left(\frac{k^2}{M^2}\right)^2\right]^{1/2}}\left(\beta-\frac{\partial^2|U|}{\partial y^2}\right) \tag{6}$$

Here the index „c" indicates the critical value of $\frac{\partial \overline{T}}{\partial y}$, denoting the onset of baroclinic instability.

The equation (6) makes the final form of the HI criterion, which we use to investigate the influence of a barotropic shear upon the baroclinic development of perturbations. It is formulated in such a way as to include the depth δp between two levels.

3. Discussion

The criterion (6) has been derived in such a way that a comparison of the theoretical $\left(\frac{\partial \overline{T}}{\partial y}\right)_c$ with an observed $\frac{\partial \overline{T}}{\partial y}$ indicates the existence and intensity of the total hydrodynamic instability. Here, the smaller value of $\left(\frac{\partial \overline{T}}{\partial y}\right)_c$ illustrates a greater possibility of a perturbation to amplify, since in that case the real $\frac{\partial \overline{T}}{\partial y}$ can sooner reach its critical value.

The critical meridional temperature gradient $\left(\frac{\partial \overline{T}}{\partial y}\right)_c$ crucially depends upon two factors: a) upon the ratio $\frac{k^2}{M_2}$ and b) upon the quantity $\frac{\partial^2|U|}{\partial y^2}$.

a) Obviously, the HI criterion (6) can be applied only to perturbations with the wave number $k \leq M$, since static stability may cause the well known "cut off" of waves which can amplify (see e.g. Kuo, 1953; Pedlosky, 1964). Allowing

σ to decrease (M increases), the model becomes able to include smaller perturbations down to localized baroclinicity (Orlansky, 1986).

b) Equation (6) enables an easy explanation of the relationship between barotropic and baroclinic parts of the hydrodynamic instability.

First, let us remember that true barotropic instability can take place only when $\frac{\partial^2 |U|}{\partial y^2} = \beta$ in a barotropic atmosphere ($U_T = 0$). All other values of $\frac{\partial^2 |U|}{\partial y^2}$ present just a barotropic shear (not instability). On the contrary, a baroclinic instability, being a much more effcient mechanism, does not require the absence of a horizontal wind shear.

A barotropic nonlinear shear of the zonal flow influences $\left(\frac{\partial \overline{T}}{\partial y}\right)_c$ in two ways:

b/1) $\frac{\partial^2 |U|}{\partial y^2} < 0$ always works destructively, i.e. increases $\left(\frac{\partial \overline{T}}{\partial y}\right)_c$ and disturbs the baroclinic instability in such a way as to grow intensively. This corresponds with the findings of James (1987) as well as of Simmons and Hoskins (1980), since the negative nonlinear horizontal shear indicates a strenghtening of the barotropic jet.

The cases with $\frac{\partial^2 |U|}{\partial y^2} = 0$ exclude barotropic influences. Such a situation occurs at linear (inflexional) parts of the zonal flow meridional profile.

b/2) Around the meridional minimum of zonal flow $\frac{\partial^2 |U|}{\partial y^2} > 0$. Here, small values of positive $\frac{\partial^2 |U|}{\partial y^2}$ (less than β) act in the sense of decreasing $\left(\frac{\partial \overline{T}}{\partial y}\right)_c$, i.e. they help a meridional temperature gradient to reach its critical value sooner. This part of zonal flow proves Brown's (1968) statement that a barotropic shear may support the baroclinic development.

b/3) When approaching the value of β, a positive $\frac{\partial^2 |U|}{\partial y^2}$ grows to the state of a barotropic atomosphere and at the same time reduces the possibility of the vertical wind shear to develop — and again the baroclinic HI is suppressed.

b/4) A positive nonlinear horizontal shear greater than β $\left(\frac{\partial^2 |U|}{\partial y^2} > \beta\right)$ indicates the kinetic energy flow from greater to smaller scales and therefore may have a dissipative meaning (James, 1987). Synoptic scale baroclinic perturbations are not supposed to deepen in such regions, but rather to move to those regions where $\frac{\partial^2 |U|}{\partial y^2}$ are postivie and small (close to zero).

The above discussion is illustrated on Fig. 2.

Taken altogether — a barotropic (horizontal) wind shear suppresses the baroclinic development in almost all the cases except in the case of a small and

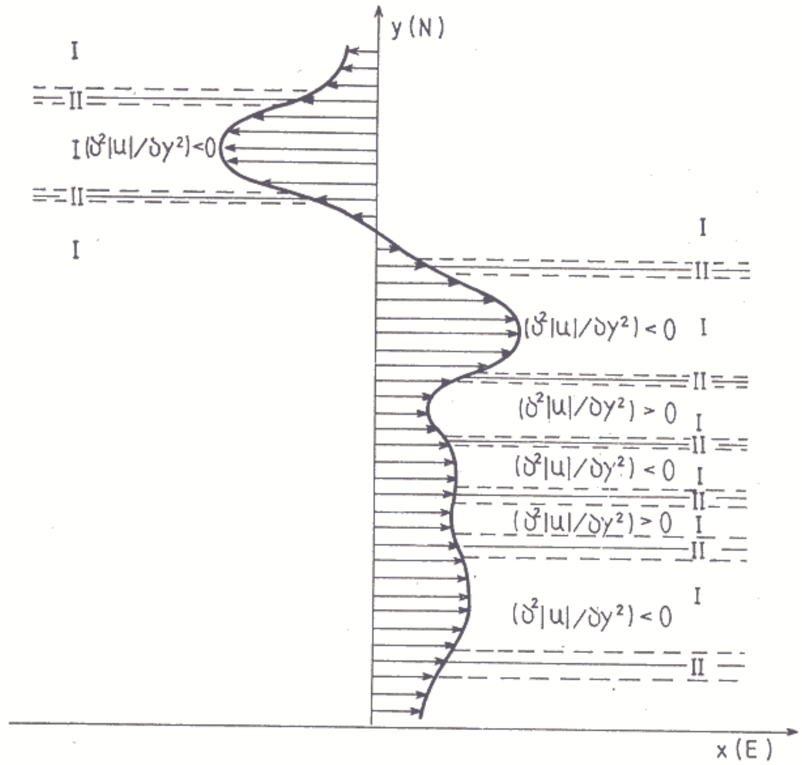

Figure 2. A general illustration of the flow meridional profile. I denotes regions of the stabilizing — and II of the labilizing influence with respect to a baroclinic development of a perturbation.

postitive $\frac{\partial^2 |U|}{\partial y^2}$. Therefore, one may conclude that a macroscale perturbation is expected to develop close to the inflexional parts of the zonal flow meridional profile (toward the side of flow minimum). Moreover, an already existing perturbation should move to those parts of the flow pattern where the barotropic shear supports its existence. In such a context, meridional profiles of zonal flow appear to be an efficient tool for macroscale diagnostic and even prognostic studies.

4. Application

Equation (6) has been used to estimate HI in the atmospheric layer between AT 850 hPa and AT 500 hPa above southern Europe during the period 14 to 19 March 1987. The whole period was characterized by a deep trough over middle and southern Europe with transient cyclones changing their position and intensity. A meridional wind shear has been determined by means of gridpoint values of U on the AT 700 hPa and the thermal wind and the static stability have been computed by means of temperature data on AT 500 hPa and AT 800 hPa (Figs. 3—8).

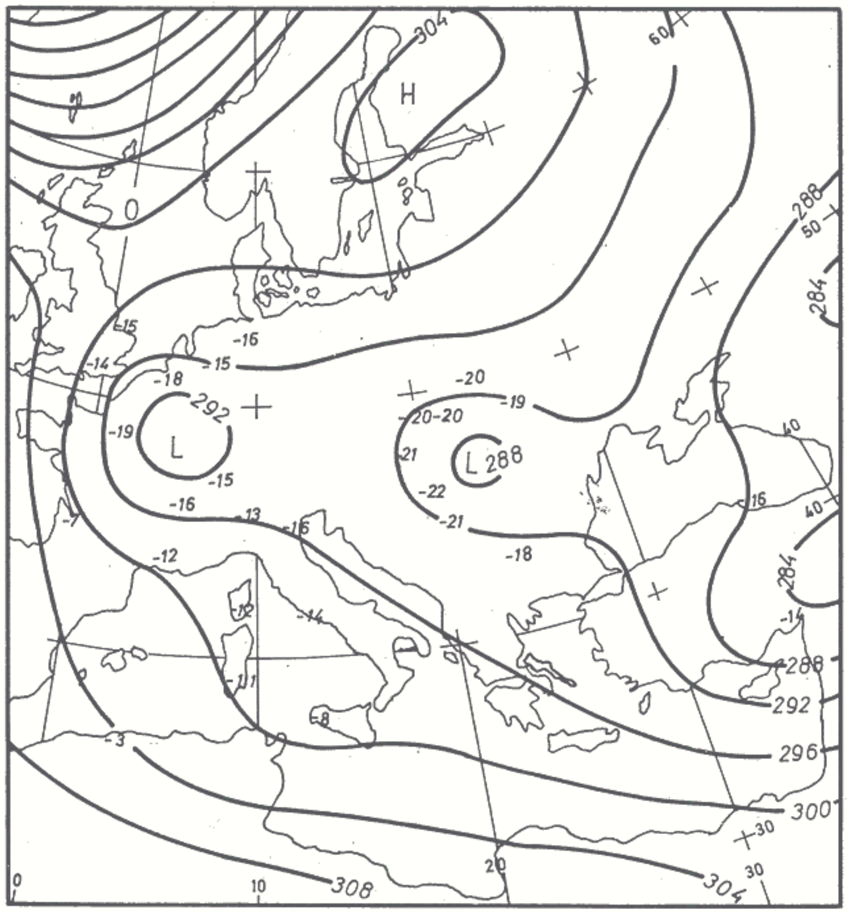

Figure 3a. AT 700 (gpdam) OO UTC, 14 March 1987. Temperature data are denoted.

The model software has been accomplished to compute $\left(\frac{\partial \overline{T}}{\partial y}\right)_c$ in a given part of the region (usually over a cyclone), as a function of 20 possible wave numbers, k. That way the model finds the wave length of the perturbation which is most likely to develop due to its hydrodynamic instability. Here our approach differs from similar investigations of Šinik (1986), who considers the energy transformations in the wave number domain.

When applying (6) one should not forget the assumptions incorporated in its derivation:

— a quasigeostrophic atmosphere; criterion (6) is to be used only on a macroscale,

— meridional temperature gradient $\frac{\partial \overline{T}}{\partial y}$, determined on AT 700 hPa, corresponds to the zonal flow vertical shear throughout the whole layer between AT 850 and AT 500,

— vertical variations of static stability are neglected.

The application of the criterion (6) will be discussed in greater detail for the first day (14 March 1987 — Fig. 3) of the period:

Temperature data enable the calculation of $\frac{\partial \overline{T}}{\partial y} = 0{,}15\,°C/°\varphi$ over the western cyclone, $\frac{\partial \overline{T}}{\partial y} = 0{,}24\,°C/°\varphi$ over the eastern one and $\frac{\partial \overline{T}}{\partial y} = 0{,}76\,°C/°\varphi$ inside the region

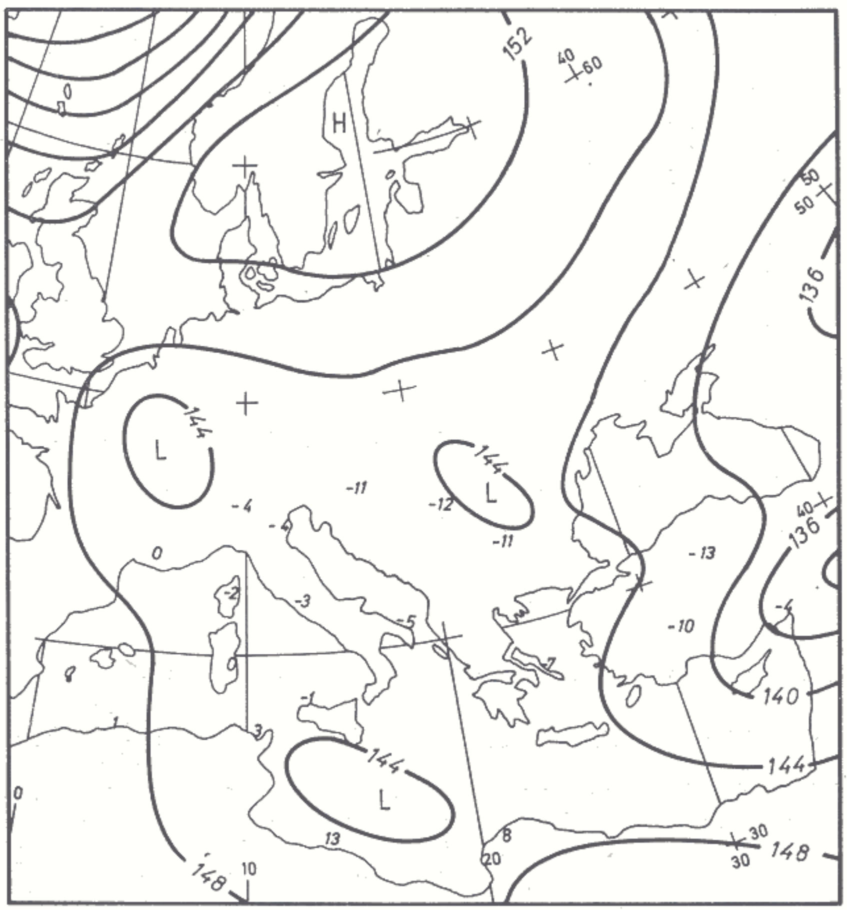

Figure 3b. AT 850 (gpdam) OO UTC, 14 March 1987. Temperature data are denoted.

of the perturbation above Sicily and Sardinia (Fig. 3a). Static stability can be estimated by the temperature values at AT 850 and AT 500 (Figs. 3b and 3c). Second derivatives of zonal flow, $\frac{\partial^2 |U|}{\partial y^2}$, have been computed at the gridpoints on Fig. 3d.

Computations of $\left(\frac{\partial \overline{T}}{\partial y}\right)_c$ and the comparisons with $\frac{\partial \overline{T}}{\partial y}$ observed indicate that

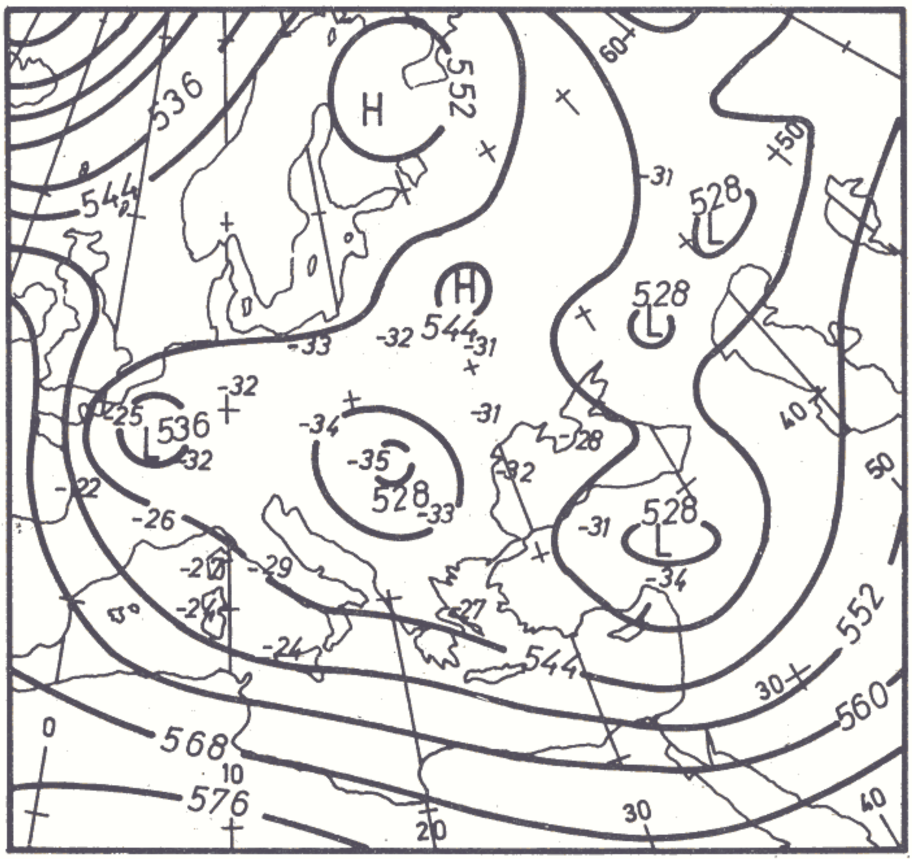

Figure 3c. AT 500 (gpdam) OO UTC, 14 March 1987. Temperature data are denoted.

both cyclones are expected to weaken. Inside the region of the perturbation an amplification is predicted for waves with lengths of 5,4 to $3{,}4 \times 10^3$ km.

The field of $\frac{\partial^2 |U|}{\partial y^2}$ indicates possible movements of cyclones away from their positions denoted on Fig. 3, since $\frac{\partial^2 |U|}{\partial y^2}$ values inside those regions are much greater than β and such a barotropic nonlinear shear disturbs a baroclinic development. The cyclones are supposed to move to the regions of smaller and positive $\frac{\partial^2 |U|}{\partial y^2}$ values (indicated by the arrow on Fig. 3d).

The expected development is denoted by a heavy dashed line on Fig. 4. It agrees relatively well with the real situation.

The same procedure has been performed for other days of the testing period — and the agreement with the real development is illustrated on Figs. 4 to 8. The results are encouraging.

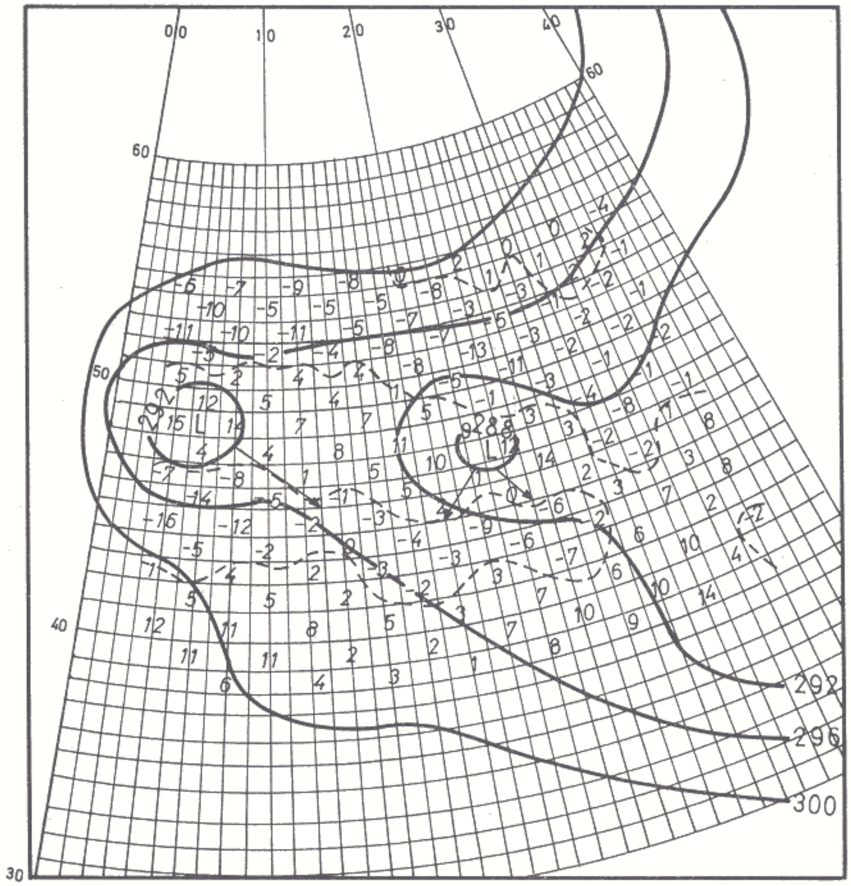

Figure 3d. $\partial^2 |U| / \partial y^2$ $[10^{-6} s^{-1} \varphi^{-1}]$, 14 March 1987. $\partial^2 |U| / \partial y^2 = 0$ is denoted by a dashed line; arrows indicate the expected direction of the perturbation movement.

5. Conclusion

The criterion (6), in spite of its simplicity and the assumptions included in its derivation, enables the study of the complex impact of barotropic and baroclinic instabilities. The results are comparable with similar investigations (Brown, 1968a, b; Simmons and Hoskins, 1980; James, 1987 etc.). Besides, (6) has some advantages in the sense that it uses real data of air flow, in contrast to other models, which work with chosen theoretical forms of zonal flow meridional profiles.

In spite of being a dominant mechanism of development, a baroclinic HI may be influenced by its usually much weaker barotropic companion. This influence indicates duality, i.e. it can disturb, as well as support, a baroclinic development depending on the magnitude od the nonlinear horizontal wind shear. Observations (Figs. 3 to 8) indicate that a barotropic „supporting" zone where

$0 < \frac{\partial^2 |U|}{\partial y^2} < \beta$ is narrow in comparison to prevailing regions of barotropic „disturbing“ effects. This dominant barotropic mechanism „pushes“ perturbations toward those regions where they can develop easier or live longer.

„Supporting“ zones coincide with inflexional parts of the zonal flow meridional profiles. Such profiles therefore make a very useful diagnostic tool, since they can help an estimation of the upper cyclone movement direction.

The calculation of $\frac{\partial \overline{T}}{\partial y}$ throughout our test period proves Stone's (1977) findings that their variations about the critical value $\left(\frac{\partial \overline{T}}{\partial y}\right)_c$ are small.

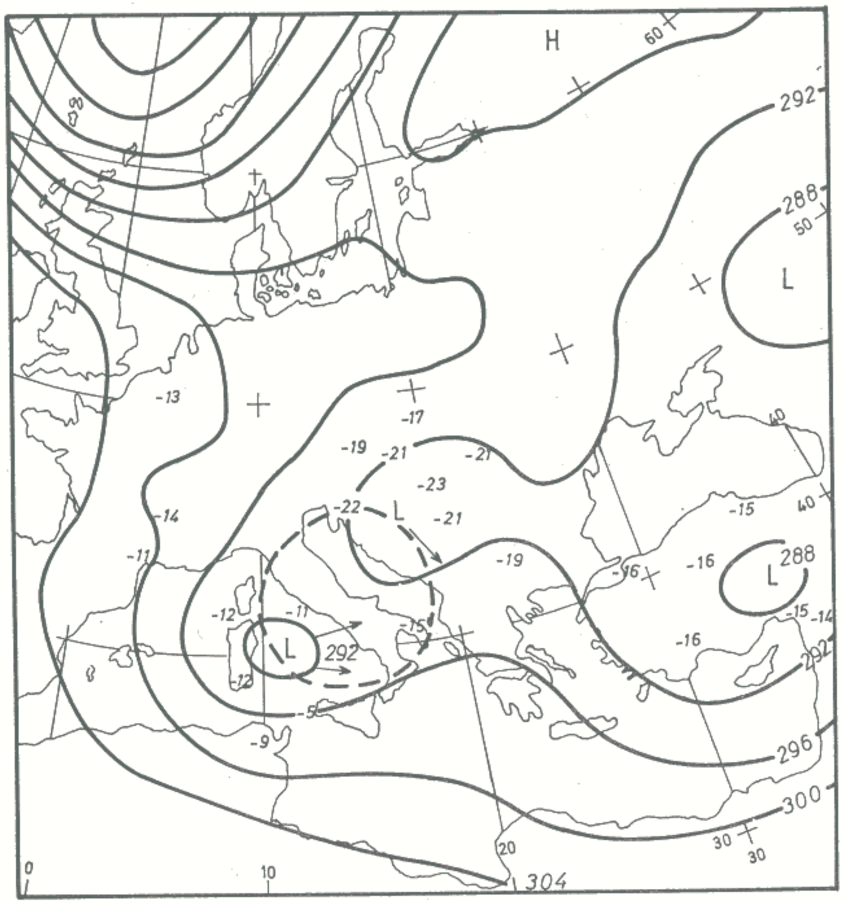

Figure 4. AT 700 (gpdam) OO UTC, 15 March 1987. Temperature data are denoted. The thick dashed line indicates the expected position of the perturbation, based on a previous day diagnosis. Arrow indicates the expected further movement.

Criterion (6) has a diagnostic character. Still, its application indicates that the process of macroperturbation development lasts at least about one day (see also Simmons and Hoskins, 1980).

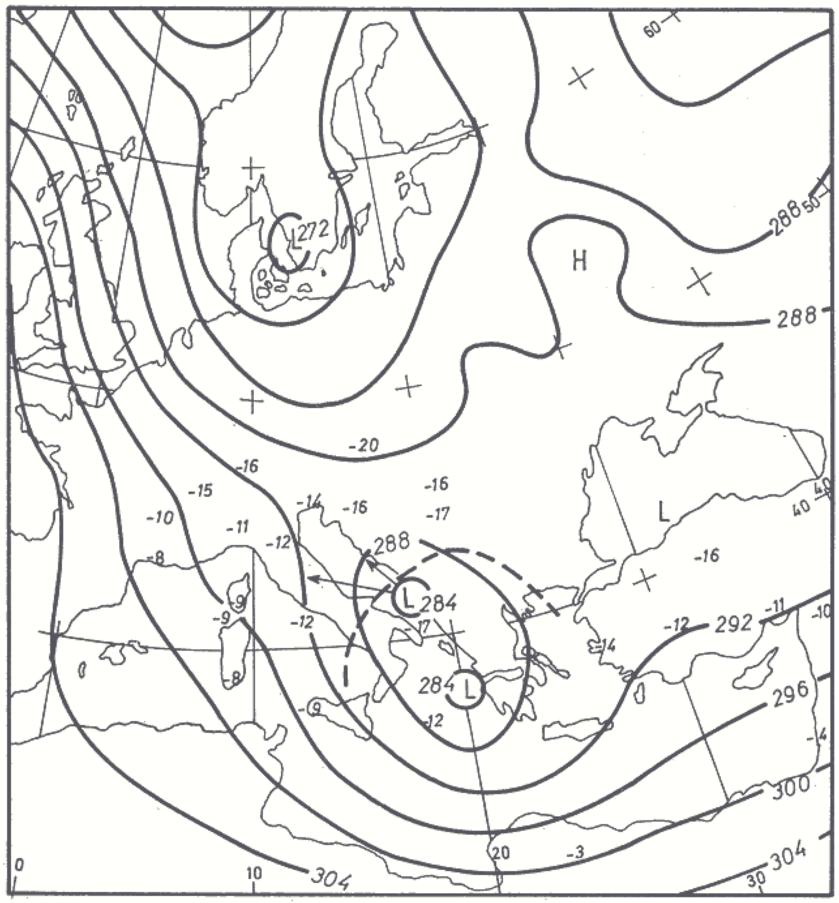

Figure 5. AT 700 (gpdam) OO UTC, 16 March 1987. Explanations the same as on the figure 4.

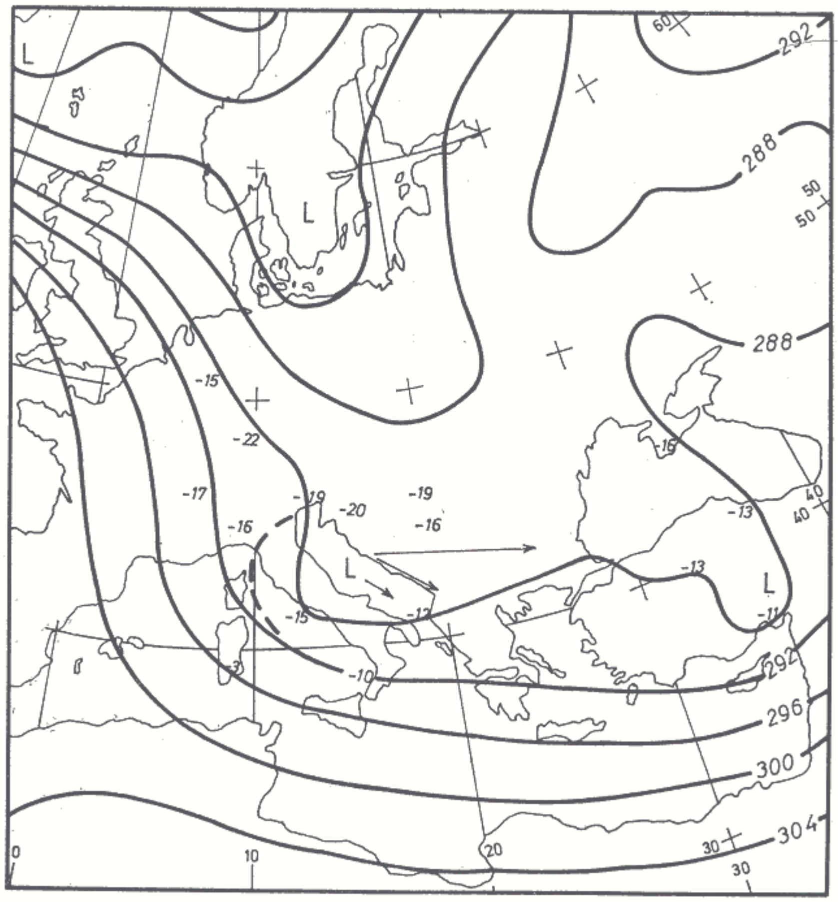

Figure 6. AT 700 (gpdam) OO UTC, 17 March 1987. Explanations the same as on the figure 4.

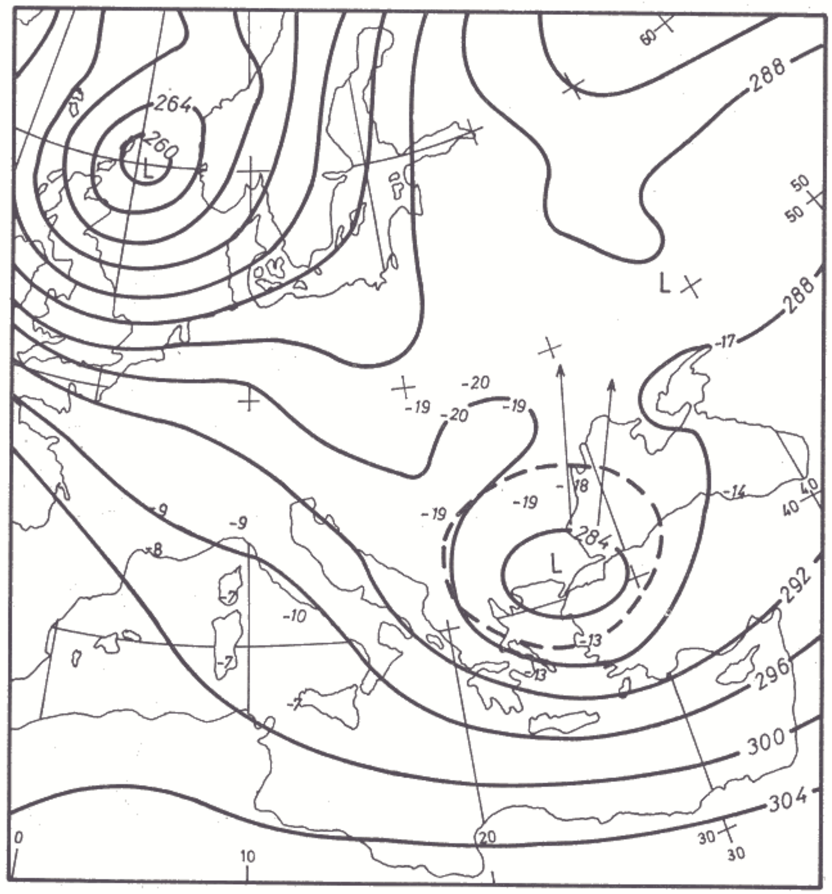

Figure 7. AT 700 (gpdam) OO UTC, 18 March 1987. Explanations the same as on the figure 4.

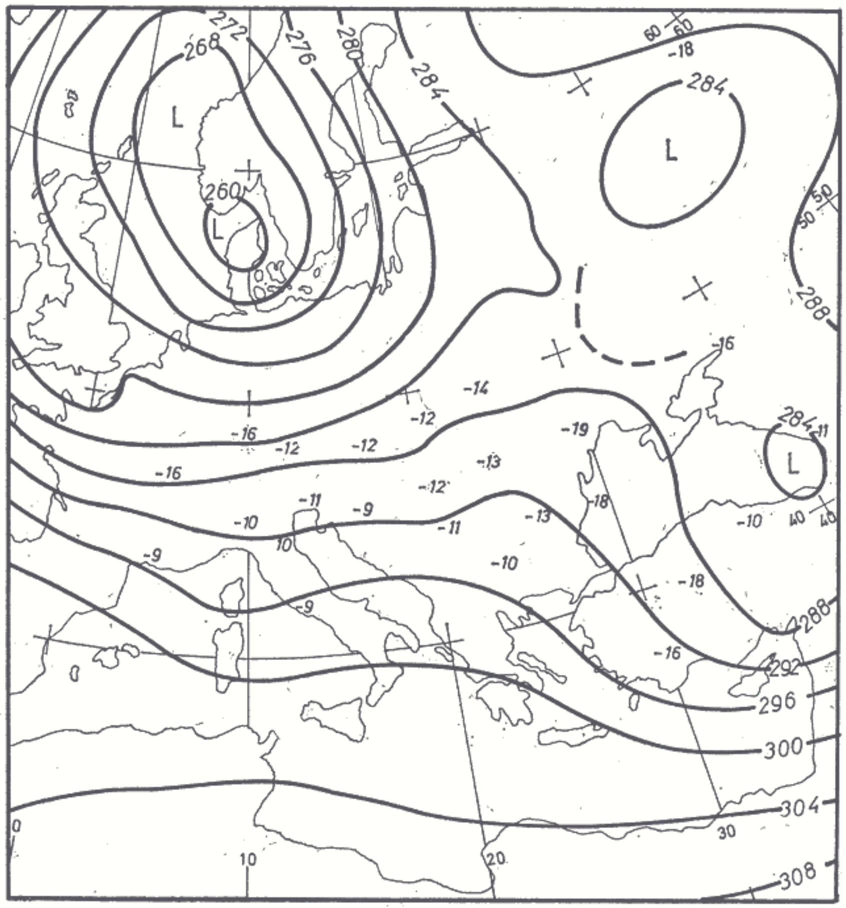

Figure 8. AT 700 (gpdam) OO UTC, 19 March 1987. Explanations the same as on the figure 4.

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