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# Determination of Synchronous Generator Armature Leakage Reactance Based on Air Gap Flux Density Signal

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Preliminary communication

A new procedure for the armature leakage reactance of the synchronous generator determination is proposed. The procedure is based on measurement of the phase displacement between the fundamental harmonics of the armature voltage and the generator air gap flux density. The accuracy of the algorithm for the phase displacement determination, which is based on Discrete Fourier Transform (DFT), is proved by the laboratory measurement first. After that, measurements on the synchronous generator in HEPP Peruća on Cetina River in Croatia are performed. Two sensors are used for the air gap flux density measurement: the Hall sensor and the measuring coil. To prove the proposed procedure, on the same generator the applied voltage test with rotor removed is performed, too. It is proved that a measuring coil can not be used as the flux density sensor because of the system error due to the air gap field harmonics. On the other hand, when the Hall sensor is used one should pay attention to the voltage noise due to the armature reaction fundamental harmonic. This voltage noise can be minimized by performing measurement at a particular operating point of the generator. The theoretical criterion for determination of this particular operating point is defined in this paper.

**Key words:** synchronous generator, armature leakage reactance, on-line parameter identification

## 1 INTRODUCTION

The reluctance of the leakage paths of transformers and rotating electrical machines is difficult to express and impossible to measure. Thus, the leakage reactances are difficult to determine. In fact, a unique determination of the transformer leakage reactances cannot be made by tests. On the other hand, the stator winding leakage reactance of rotating electrical machines can be determined by the applied voltage test with rotor removed. The procedure for the armature leakage reactance determination of the synchronous machine is defined by IEC Standard, [1]. There are very few papers dealing with this topic [2, 3]. The method described in [2] is based on measurements which are too difficult to be performed out of the laboratory condition. In one word, the armature voltage versus armature current characteristic of the un-load and unexcited generator should be performed. In [3] the armature leakage reactance is estimated together with the other parameters of the synchronous generator mathematical model, but the accuracy of the procedure is not even tested at the labo-

ratory conditions. It is common in papers dealing with on-line identification of the synchronous generator parameter that the armature leakage reactance is not determined separately. On the other hand, a methodology described in [6] includes a procedure for separately determination of the armature leakage reactance.

In this paper a possibility for the armature leakage reactance identification is considered, which, according to the methodology described in [6], is based on measurement of the armature voltage, armature current and phase displacement between the fundamental harmonics of the armature voltage and the air gap flux density. The measurement results obtained by tests performed on the generator in HEPP Peruća are presented, including the applied voltage test with rotor removed. It is proved that a measuring coil can not be used as the flux density sensor since the air gap field harmonics cause the phase displacement measurement error. Therefore the Hall sensor is used for the flux density measurement.

## 2 IDENTIFICATION PROCEDURE

The determination of the armature leakage reactance is proposed in [6] to be the first step of the on-line parameter identification procedure for synchronous hydrogenerators. Theoretically, it is possible to determine the armature leakage reactance by the measurement system which is a part of the modern hydrogenerator monitoring system, [6]. For this purpose the armature voltage and current signals and the signal of the air-gap flux density sensor at a steady state operating point should be recorded. This is based on the armature winding phasor diagram shown in Figure 1 where the armature resistance is neglected.

Applying the sinus theorem on the triangle *oab* in Figure 1, the following expression for the armature leakage reactance can be written

$$X_{\sigma s} = \frac{U_s \sin \beta}{I_s \cos(\varphi_s + \beta)}. \quad (1)$$

The armature current ( $I_s$ ) and phase angle ( $f_s$ ) in the above relationship can be replaced by the active and reactive power. Thus, relationship (1) in per unit becomes

$$X_{\sigma s} = \frac{U_s^2 \operatorname{tg} \beta}{P - Q \operatorname{tg} \beta}. \quad (2)$$

The armature voltage ( $U_s$ ), active power ( $P$ ) and reactive power ( $Q$ ) can be obtained from the sig-

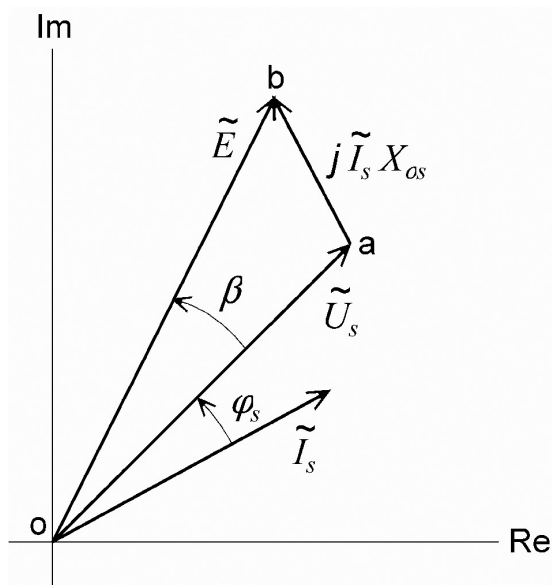


Fig. 1 The armature winding phasor diagram with the armature resistance neglected

nals of the line voltages and currents, which are recorded by mentioned measurement system, using the following equations

$$U_s = \frac{1}{3} \sqrt{u_{ab}^2 + u_{bc}^2 + u_{ca}^2} \quad (3)$$

$$P = i_a u_{ab} - i_c u_{bc} \quad (4)$$

$$Q = \frac{1}{\sqrt{3}} (u_{ab} i_c + u_{bc} i_a + u_{ca} i_b).$$

Thus, the problem is reduced to determination of the displaced angle  $\beta$  between the armature voltage and electromotive force (EMF). The angle  $\beta$  can be obtained by determining the displaced angle between the fundamental harmonics of two measured signals: (1) the armature voltage signal and (2) the flux density signal, at the no-load and load condition, respectively. This will be truth under one condition: the angle between the fundamental harmonics of the EMF and the flux density signal ( $\alpha_0$ ) does not depend on the generator operating point.

The angle  $\alpha_0$  is determined by the flux density sensor position in relation to the considered phase axis. For instance, if the flux density sensor is placed in the *as*-axis, the fundamental harmonics of the  $u_{as}$  voltage and the flux density signal at no-load condition will be displaced by 90 electrical degrees, i.e.  $\alpha_0 = 90^\circ$ . It is clear that any phase or line voltage can be used for the angle  $\beta$  determination and, after the flux density sensor is glued onto the lamination stator surface,  $\alpha_0$  depends on a chosen voltage signal only. When  $\alpha_0$  is obtained, the displaced angle ( $\alpha_1$ ) between the same signals at load condition should be determined. After that we can calculate the displaced angle between the armature voltage and EMF

$$\beta = \alpha_1 - \alpha_0. \quad (5)$$

Figure 2 shows the waveforms of the armature line voltage ( $u$ ) and the generator air gap flux density ( $B$ ) at no-load and load condition, recorded by the hydrogenerator monitoring system in HEPP Peruća where the Hall sensor is used for the air gap flux density monitoring. One can see that the armature line voltage practically contains the fundamental harmonic only.

It is important to note that the difference between  $\alpha_1$  and  $\alpha_0$  is very small (a few degrees) what emphasises the problem of the angle  $\beta$  determination accuracy. In order to solve this problem the sliding window procedure is applied. After filtering both signals, the common sliding window is

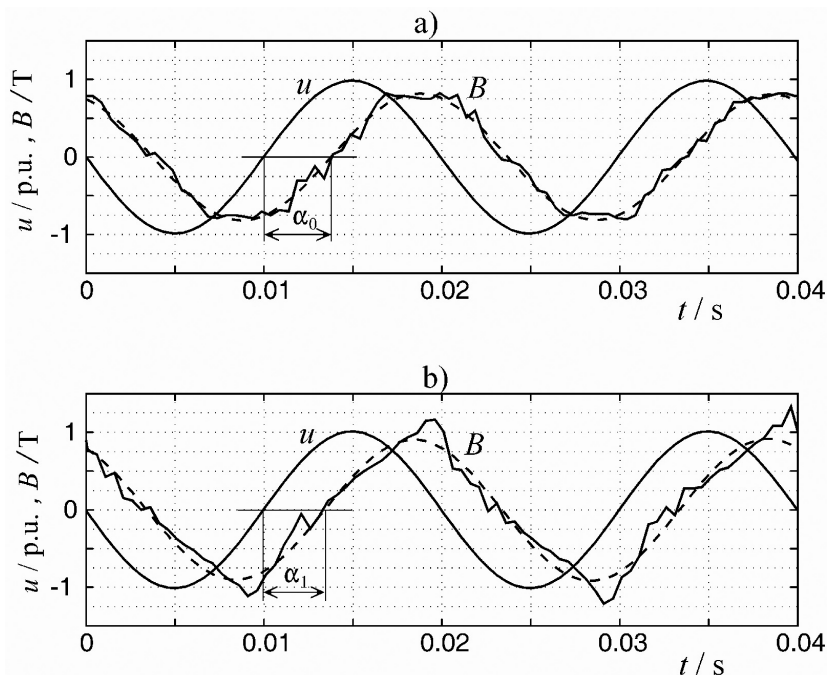


Fig. 2 Armature line voltage and air-gap flux density waveforms of the 26 MVA generator in HEPP Peruća: a) no-load condition, b) load condition

defined and, for data captured inside the window, Discrete Fourier Transform (DFT) is applied [7]. In that way the displaced angle between two considered signals is obtained. This angle corresponds to the sample in the middle of the sliding window. By shifting the whole window by one sample point, the displaced angle for the next sample point is obtained. In that way, the angle  $\beta$  time response could be obtained but this is not the subject of this work.

Described procedure is tested theoretically by applying it for determination the displaced angle between two known sinus functions and the perfect accuracy is obtained. An experimental testing of described procedure is performed too. For that purpose the reactor with known parameters is fed with a current, which is measured, and the voltages in front of and behind the reactor are recorded. By applying described procedure the displaced angle between these two voltages is obtained. Based on the measured current and displaced angle between two mentioned voltages the reactor reactance is calculated, which matches very good with the result obtained by U-I method, [8]. The discrepancy of the results for the reactor reactance is less than 2 % and the measured displaced angle has been about 2 degree only. This doubtlessly confirms a high accuracy of the described procedure for displaced angle determination.

By applying described procedure to the signals of the armature line voltage and air-gap flux density at no-load condition, the angle  $\alpha_0$  (which is constant) is determined first. After that, the same procedure should be applied at load condition in order to determine the angle  $\alpha_1$  and according to (5) the angle  $\beta$ .

### 3 ANALYSIS OF MEASUREMENT RESULTS IN HEPP PERUĆA

In order to monitor the excitation winding condition, a measuring coil is commonly used as the air-gap flux density sensor in the generator monitoring system. Unfortunately, this type of sensor can not be used for the angle  $\beta$  determination because of the system error due to the armature reaction space harmonics, [8]. As it is shown in [8], based on some theoretical investigation, this problem could be solved if the Hall sensor (instead of the measuring coil) was used for the flux density monitoring. Because of that, besides the measurement coil, the Hall sensor is glued onto the lamination stator surface of the HEPP Peruća generator, too.

The measurement results obtained at some few steady state operating points of the considered generator are given in Table 1. In the last row of the Table 1 the data for the displaced angle between

Table 1 Measurement results for armature leakage reactance by means Hall sensor

$U_s/\text{kV}$	$10.52/\sqrt{3}$	$10.66/\sqrt{3}$	$10.73/\sqrt{3}$	$10.79/\sqrt{3}$	$10.92/\sqrt{3}$
$P/\text{MW}$	20.7	20.9	20.8	20.9	20.7
$Q/\text{MVAr}$	0.52	3.25	4.42	5.60	8.84
$\beta_{HS}/^\circ \text{ el.}$	7.1	6.4	5.8	5.7	5.1
$X_{os}/\Omega$	0.66	0.62	0.58	0.57	0.53
$\beta_{MC}/^\circ \text{ el.}$	4.9	4.5	4.3	4.2	3.8

the armature voltage and EMF ( $\beta_{MC}$ ) obtained by means of measuring coil are given. One can see that  $\beta_{MC}$  is less than the displaced angle measured by means Hall sensor ( $\beta_{HS}$ ). This proves the conclusion from [8] – the measuring coil, which is commonly used for the excitation winding condition monitoring, can not be used for the angle  $\beta$  determination.

It should be noted from data given in Table 1 that the armature leakage reactance depends on the operating point, i.e. on the reactive power. This effect is not caused by saturation since the armature leakage reactance is not saturated at normal operating condition, what is proved in the considered case by means of the removed rotor test (see Table 2). Hence, one can conclude that the armature leakage reactance dependence on the operating point, according to data in Table 1, is caused by an inaccuracy of the angle  $\beta_{HS}$ .

In order to prove that, during dismounting of Generator 1 in HEPP Peruća, the applied voltage test with rotor removed is performed and the armature leakage reactance is determined. According to [1], three-phase rated frequency symmetrical voltage is applied to the armature winding when the rotor being removed. When the armature resistance is negligible, the armature voltage can be expressed as

$$U_s = X I_s = (X_{os} + X_b) I_s \quad (6)$$

where  $X_b$  is the reactance due to flux over the armature active surface created by the armature winding in the space which is normally occupied by the rotor. This reactance can be determined by means of a search coil by using the following equation

$$X_b = \frac{U_c}{I_s} \cdot \frac{N_s k_{ws}}{N_c} \quad (7)$$

where  $U_c$  is the search coil voltage and  $I_s$  is the armature current and  $N_s$  is the number of armature winding turns and  $k_{ws}$  is the armature winding factor and  $N_c$  is the number of the search coil turns.

According to [1], a search coil is placed over teeth; the length of the coil is equal to the full armature core length; the width of the coil is equal to the pole pitch. The end parts are stretched by bracing wires towards the machine axis along the radius in the plane of the end armature core teeth to remove them from the influence of the leakage fluxes around the armature and winding.

In order to perform the test, the buses of the Generator 1 and the Generator 2 were connected together and armature winding of the Generator 1 (with rotor removed) was supplied by Generator 2. In that way the generator protection systems had to be put out of the service during the test. By hand adjusting the field current of the Generator 2, the armature current of the Generator 1 was increased step by step to the value slightly over rated current. Using the generator data ( $m_s = 3$ ,  $Q_s = 348$ ,  $p = 16$ ) the product  $N_s k_{ws}$  where calculated:  $N_s k_{ws} = 110.8$ . The search coil with one turn was used ( $N_c = 1$ ).

The measurement results are presented in Table 2. One can see that, as it is expected, the armature leakage reactance, in the considered area, does not depend on the armature current. In addition, it should be noted that the mean value of the measurement results for the  $X_{os}$  in Table 2 (0.75  $\Omega$ ) matches very well with the manufacturer data for armature leakage reactance which amounts 0.725  $\Omega$ . On the other hand, discrepancy between this mean value of the  $X_{os}$  and the mean value of the results shown in Table 1 (0.59  $\Omega$ ) equals 21%. But

Table 2 Measurement results obtained by applied voltage test with rotor removed

$U_s/\text{V}$	$I_s/\text{A}$	$U_c/\text{V}$	$X/\Omega$	$X_b/\Omega$	$X_{os}/\Omega$
1185	573	2.30	1.195	0.445	0.750
1460	927	3.75	1.203	0.448	0.755
1930	1080	4.35	1.196	0.446	0.750
2235	1305	5.30	1.198	0.450	0.748
3330	1623	6.55	1.186	0.447	0.739

the results dependence on the generator reactive power (see Figure 3) shows that the accuracy of the presented procedure could be improved if it was performed at an operating point with capacitive reactive power. Since carrying on with the measurements was not possible, because of the previously mentioned dismantling of the Generator 1, here we can analyze this problem only theoretically.

From (2), the displaced angle  $\beta$  may be expressed:

$$\beta = \arctg \frac{PX_{\sigma s}}{U_s^2 + QX_{\sigma s}} \tag{8}$$

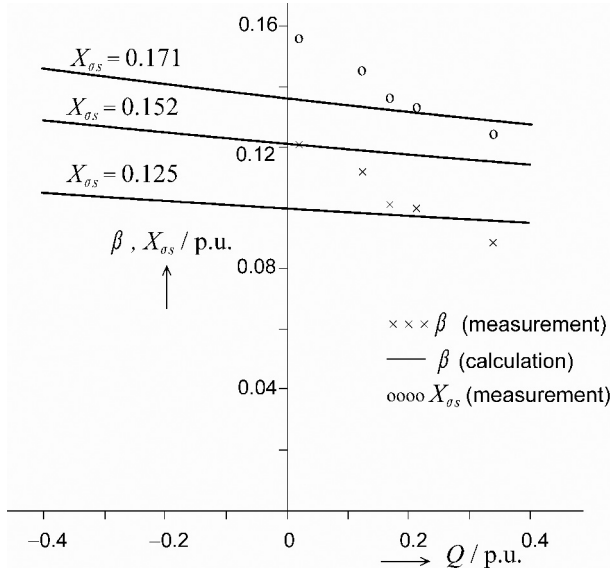


Fig. 3 Measurement results for  $X_{\sigma s}$  and  $\beta$  and comparison with calculated  $\beta$  using (8)

The measurement results from Table 1 are shown in Figure 3 in comparison with theoretical characteristics  $\beta(Q)$  obtained by (8) for three different values of the armature leakage reactance, at rated active power ( $P = 0.8$ ) and rated voltage ( $U = 1$ ). One can see that the measured  $\beta$  depends on reactive power much stronger than the theoretical one. This must be because of measurement noise which is caused by an induced voltage in the output circuit of the Hall-sensor due to the fundamental harmonic of the air-gap flux density.

The measurement noise caused by induced voltage in the output circuit of the Hall-sensor can be minimised by selection of the generator operating point. In order to select this operating point it is convenient to consider the phase angle between

the Hall-sensor output voltage (without noise  $\tilde{E}_H$ ) and the noise voltage caused by the armature reaction fundamental harmonic ( $\Delta\tilde{E}_{Ha}$ ). Namely, the influence of the noise voltage caused by the field current upon the phase angle of the Hall-sensor output voltage may be neglected, because it is practically nullified by the signals subtraction at any operating point and at no-load condition.

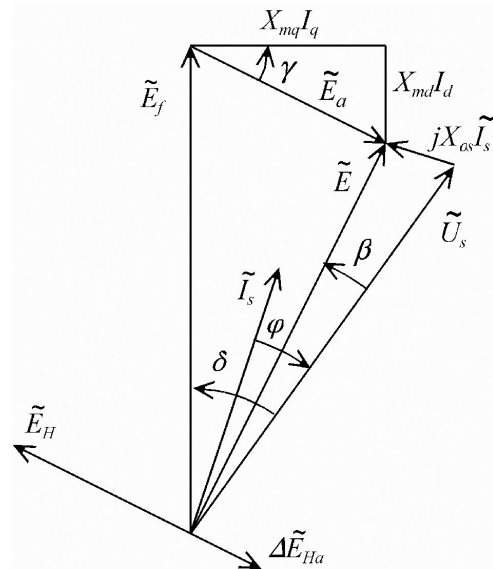


Fig. 4 Phasor diagram at generator operating point at which influence of  $\Delta\tilde{E}_{Ha}$  upon angle  $\beta$  measurement is vanished

According to the phase diagram shown in Figure 4, which is depicted by assuming that the Hall-sensor position corresponds with the referent phase axis, the influence of the noise voltage upon the phase angle of the Hall-sensor output voltage will be minimal at the operating point at which the phasor of the generator EMF ( $E$ ) is vertical to its component due to the armature reaction ( $\tilde{E}_{a2}$ ), since then the phasor  $\Delta\tilde{E}_{Ha}$  is collinear with the  $\tilde{E}_H$ . At this condition it can be written:

$$\text{tg} \gamma = \text{tg}(\delta - \varphi) = \frac{X_{md}I_d}{X_{mq}I_q} \tag{9}$$

and if it is noted that:

$$\frac{I_d}{I_q} = \text{tg}(\delta + \varphi) \tag{10}$$

than, by substituting (10) into (9), the condition for the minimal influence of the noise voltage upon the angle  $\beta$  measurement may be expressed as:

$$\operatorname{tg}(\delta - \beta) = \frac{X_{md}}{X_{mq}} \operatorname{tg}(\delta + \varphi). \quad (11)$$

When the generator parameters are known, the angles  $\varphi$ ,  $\delta$  and  $\beta$  depend on the generator operating point only, i.e. on the generator voltage ( $U_s$ ), active power ( $P$ ) and reactive power ( $Q$ ). Thus, the generator operating point which satisfies the condition (11) can be determined by numerical calculation using the following algorithm:

1. Choose the values for  $P$  and  $U_s$
2. For the given area of the angle  $\delta$ , calculate  $Q(\delta)$  in per unit, [9]:

$$Q = \frac{U_s E_f}{X_d} \cos \delta - \frac{(U_s \cos \delta)^2}{X_d} - \frac{(U_s \sin \delta)^2}{X_q} \quad (12)$$

where

$$E_f = \frac{X_d}{U_s \sin \delta} \left[ P + \frac{U_s^2}{2} \left( \frac{1}{X_q} - \frac{1}{X_d} \right) \sin 2\delta \right]. \quad (13)$$

3. For each one  $Q(\delta)$ , calculate the angle  $\beta$  by using (8) and the generator phase angle:

$$\varphi = \operatorname{arctg} \frac{Q}{P}.$$

4. For each one  $Q(\delta)$ , calculate the left side and the right side of (11).

5. Determine the cross point of the curves:

$$y_1(Q) = \operatorname{tg}(\delta - \beta)$$

and

$$y_2(Q) = \operatorname{tg}(\delta + \varphi) \frac{X_{md}}{X_{mq}}.$$

The characteristics  $Q(\delta)$  of the considered generator in HEPP Peruća for rated voltage and different values of the active power are shown in Figure 5, in the area limited by  $I_s \leq I_N$ . For all considered active powers, the operating points, at which the condition (11) is satisfied, are in the capacitive area of the generator operating diagram. Since the reactive powers at that operating points are relatively small it is easy to establish them by decreasing the field current after setting the generator active power. It is important to note that the magnitudes of the field current at the considered operating points do not much differ from the field current at no-load condition. Thus the noise voltage

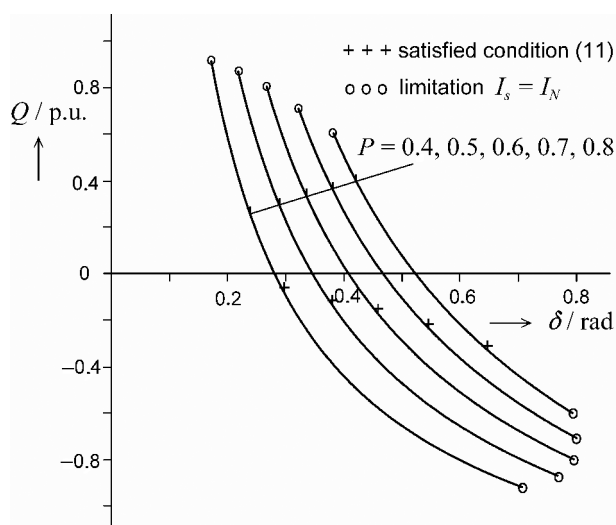


Fig. 5 Characteristics  $Q(\delta, P)$  of considered generator with depicted operating points which satisfied condition (11)

in the Hall-sensor output circuit due to field current practically does not influence upon the accuracy of the angle  $\beta$  determination, for the noise voltage signals at operating point and at no-load condition are subtracted if the operating point satisfied condition (11).

#### 4 CONCLUSION

In this paper a new procedure for the armature leakage reactance determination is presented which is based on the signals from the armature voltage, armature current and air gap flux density sensors recorded at the generator steady state operating point. In order to avoid the measurement error due to the air gap field harmonics, the Hall-sensor should be used for the flux density measurement and not the measuring coil. This is proved by the measurement results performed on the generator in HEPP Peruća. The measurement noise which is caused by an induced voltage in the output circuit of the Hall-sensor due to the fundamental harmonic of the air-gap flux density can cause a large error of the armature leakage reactance measurement result. This error can be minimized by performing measurement at the generator particular operating point. The theoretical criterion for this particular operating point determination is defined in this paper. For final proving of the proposed procedure the experimental investigation should be kept on, in order to eliminate the fundamental frequency noise voltage in the output circuit of the Hall-sensor.

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**Određivanje rasipne reaktancije armaturnog namota sinkronog generatora pomoću signala magnetske indukcije u zračnom rasporu.** Definiran je postupak za identifikaciju rasipne reaktancije armaturnog namota koji se temelji na mjerenju faznog pomaka između armaturnog napona i magnetske indukcije u zračnom rasporu generatora. Točnost algoritma za određivanje faznog pomaka, koji se temelji na primjeni diskretne Fourierove transformacije, provjerena je mjerenjima u laboratorijskim uvjetima. Nakon toga provedena su mjerenja na generatoru snage 26 MVA u HE Peruća na rijeci Cetini, Republika Hrvatska, pri čemu su korištena dva senzora magnetne indukcije: mjerni svitak i Hallova sonda. U svrhu eksperimentalne provjere istraživanog postupka na istom generatoru proveden je i pokus s izvađenim rotorom. Pokazano je da se u identifikacijskom postupku ne može koristiti mjerni svitak zbog sustavne pogreške koju uzrokuju prostorni harmonici polja u zračnom rasporu. S druge strane, kod primjene Hallove sonde velik utjecaj na točnost mjerenja može imati smetnja koju izaziva osnovni harmonik reakcije armature. U radu je dana teorijska analiza na temelju koje je pokazano da se može odabrati radna točka generatora u kojoj se ta smetnja minimizira.

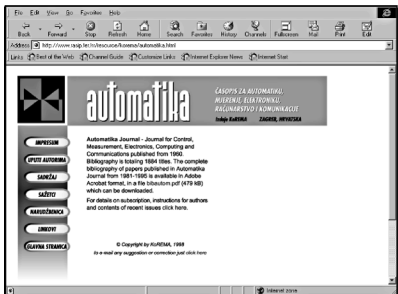
**Ključne riječi:** sinkroni generator, identifikacija parametara u pogonu, rasipna reaktancija armaturnog namota

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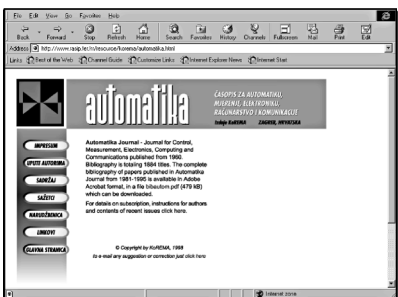
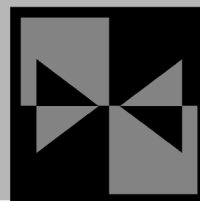
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