MATHEMATICAL COMMUNICATIONS **12**(2007), 183-190

A class of Siamese twin Menon designs

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Abstract. *A* $\{0,\pm 1\}$*-matrix* S *is called a Siamese twin design sharing the entries of* I*, if* $S = I + K - L$*, where* I, K, L *are non-zero* $\{0,1\}$*-matrices and both* $I + K$ *and* $I + L$ *are incidence matrices of symmetric designs with the same parameters. Let* p *and* $2p-1$ *be prime powers and* $p \equiv 3 \pmod 4$*. We describe a construction of a Siamese twin Menon design with parameters* $(4p^2, 2p^2 - p, p^2 - p)$*, yielding a Siamese twin Hadamard design with parameters* $(4p^2 - 1, 2p^2 - 1, p^2 - 1)$*.*

Key words: *Hadamard matrix, symmetric design, Menon design, Siamese twin design*

AMS subject classifications: 05B20, 05B05

Received February 8, 2007 Accepted June 28, 2007

1. Introduction

A symmetric (v, k, λ) design is a finite incidence structure $(\mathcal{P}, \mathcal{B}, I)$, where $\mathcal{P}$ and $\mathcal{B}$ are disjoint sets and $I \subseteq \mathcal{P} \times \mathcal{B}$, with the following properties:

1. $|\mathcal{P}| = |\mathcal{B}| = v$;

2. every element of $\mathcal{B}$ is incident with exactly k elements of $\mathcal{P}$;

3. every pair of distinct elements of $\mathcal{P}$ is incident with exactly λ elements of $\mathcal{B}$.

The elements of the set $\mathcal{P}$ are called points and the elements of the set $\mathcal{B}$ are called blocks.

A Hadamard matrix of order m is an $(m \times m)$ matrix $H = (h_{i,j})$, $h_{i,j} \in \{-1, 1\}$, satisfying $HH^T = H^T H = mI_m$, where I_m is an $(m \times m)$ identity matrix. A Hadamard matrix is regular if the row and column sums are constant. It is well known that the existence of a symmetric $(4u^2, 2u^2 - u, u^2 - u)$ design is equivalent to the existence of a regular Hadamard matrix of order $4u^2$ (see [10, Theorem 1.4]). Such symmetric designs are called Menon designs.

A $\{0,\pm 1\}$-matrix S is called a Siamese twin design sharing the entries of I, if $S = I + K - L$, where I, K, L are non-zero $\{0,1\}$-matrices and both $I + K$ and $I + L$ are incidence matrices of symmetric designs with the same parameters. If $I + K$

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and $I + L$ are incidence matrices of Menon designs, then S is called a Siamese twin Menon design. Some infinite classes of Siamese twin Menon designs obtained from Bush-type Hadamard matrices are described in [3], [4], [5] and [6]. These Siamese twin Menon designs have parameters

$$v = 36(49^m + 49^{m-1} + \cdots + 49 + 1),\ k = 21(49)^m,\ \lambda = 12(49)^m,$$
$$v = 100(121^m + 121^{m-1} + \cdots + 121 + 1),\ k = 55(121)^m,\ \lambda = 30(121)^m,$$
$$v = 324(361^m + 361^{m-1} + \cdots + 361 + 1),\ k = 171(361)^m,\ \lambda = 90(361)^m,$$

where m is a positive integer. Construction of a series of Siamese twin designs with parameters

$$v = 4p^2(q^{m+1} + \cdots + q + 1),\ k = (2p^2 + p)q^{m+1},\ \lambda = (p^2 + p)q^{m+1},$$

where $p = 53208$, $q = 106417$, and m is any positive integer, is described in [7]. In [2] the author describes a construction of Siamese twin designs with parameters $(4(p+1)^2, 2p^2 + 3p + 1, p^2 + p)$, whenever p and $2p + 3$ are prime powers and $p \equiv 3\ (mod\ 4)$.

Recently, the notion of Siamese twin designs have been generalized, and the concept of Siamese combinatorial objects have been introduced (see [8]). Beside Siamese twin designs, other Siamese combinatorial structures have been studied, such as Siamese colour graphs, Siamese association schemes, and Siamese Steiner designs.

Let p and $2p - 1$ be prime powers and $p \equiv 3\ (mod\ 4)$. Then there exists a symmetric design with parameters $(4p^2, 2p^2 - p, p^2 - p)$ (see [1]). In this article twins of Menon designs of the designs described in [1] are constructed, which leads us to a series of Siamese twin Menon designs. Parameters of the Siamese twin designs constructed in this article do not belong to any of the known series of Siamese twin designs.

In order to make this article self-contained, in the next section we state some facts about developments of Paley difference sets and Paley partial difference sets which can be found in [1] and [2].

2. Nonzero squares in finite fields

Let p be a prime power, $p \equiv 3\ (mod\ 4)$ and F_p a field with p elements. Then a $(p \times p)$ matrix $D = (d_{ij})$, such that

$$d_{ij} = \begin{cases} 1, \text{ if } (i - j) \text{ is a nonzero square in } F_p, \\ 0, \text{ otherwise.} \end{cases}$$

is an incidence matrix of a symmetric $(p, \frac{p-1}{2}, \frac{p-3}{4})$ design. Such a symmetric design is called a Paley design (see [9]). Let $\overline{D}$ be an incidence matrix of a complementary symmetric design with parameters $(p, \frac{p+1}{2}, \frac{p+1}{4})$. Since -1 is not a square in F_p, D is a skew-symmetric matrix. Further, D has zero diagonal, so $D + I_p$ and $\overline{D} - I_p$ are incidence matrices of symmetric designs with parameters $(p, \frac{p+1}{2}, \frac{p+1}{4})$ and

$(p, \frac{p-1}{2}, \frac{p-3}{4})$, respectively. Matrices D and $\overline{D}$ have the following properties:

$$D \cdot \overline{D}^T = (\overline{D} - I_p)(D + I_p)^T = \frac{p+1}{4} J_p - \frac{p+1}{4} I_p,$$

$$[\, D \mid \overline{D} - I_p \,] \cdot [\, \overline{D} - I_p \mid D \,]^T = \frac{p-1}{2} J_p - \frac{p-1}{2} I_p,$$

$$[\, D \mid D \,] \cdot [\, D + I_p \mid \overline{D} - I_p \,]^T = \frac{p-1}{2} J_p,$$

$$[\, \overline{D} \mid D \,] \cdot [\, \overline{D} - I_p \mid \overline{D} - I_p \,]^T = \frac{p-1}{2} J_p,$$

where J_p is the all-one matrix of dimension $(p \times p)$.

Let $\Sigma(p)$ denote the group of all permutations of F_p given by

$$x \mapsto a\sigma(x) + b,$$

where a is a nonzero square in F_p, b is any element of F_p, and σ is an automorphism of the field F_p. $\Sigma(p)$ is an automorphism group of symmetric designs with incidence matrices D, $D + I_p$, $\overline{D}$ and $\overline{D} - I_p$ (see [9, p. 9]). If p is a prime, $\Sigma(p)$ is isomorphic to a semidirect product $Z_p : Z_{\frac{p-1}{2}}$.

Let q be a prime power, $q \equiv 1 \ (mod\ 4)$, and $C = (c_{ij})$ be a $(q \times q)$ matrix defined as follows:

$$c_{ij} = \begin{cases} 1, \text{ if } (i-j) \text{ is a nonzero square in } F_q, \\ 0, \text{ otherwise.} \end{cases}$$

C is a symmetric matrix, since -1 is a square in F_q. There are as many nonzero squares as nonsquares in F_q, so each row of C has $\frac{q-1}{2}$ ones and $\frac{q+1}{2}$ zeros. Let $i \neq j$ and $C_i = [c_{i1} \ldots c_{iq}]$, $C_j = [c_{j1} \ldots c_{jq}]$ be the i^{th} and the j^{th} row of the matrix C, respectively. Then

$$C_i \cdot C_j^T = \begin{cases} \frac{q-1}{4}, & \text{if } c_{ij} = c_{ji} = 0, \\ \frac{q-1}{4} - 1, & \text{if } c_{ij} = c_{ji} = 1. \end{cases}$$

The matrix $\overline{C} - I_q$ has the same property. Let $i \neq j$ and $\overline{C}_i = [\overline{c}_{i1} \ldots \overline{c}_{iq}]$, $\overline{C}_j = [\overline{c}_{j1} \ldots \overline{c}_{jq}]$ be the i^{th} and the j^{th} row of the matrix $\overline{C}$, respectively. Then

$$\overline{C}_i \cdot \overline{C}_j^T = \begin{cases} \frac{q-1}{4}, & \text{if } \overline{c}_{ij} = \overline{c}_{ji} = 0, \\ \frac{q-1}{4} + 1, & \text{if } \overline{c}_{ij} = \overline{c}_{ji} = 1. \end{cases}$$

The matrix $C + I_q$ has the same property. Further,

$$C \cdot (C + I_q)^T = \overline{C} \cdot (\overline{C} - I_q)^T = \frac{q-1}{4} J_q + \frac{q-1}{4} I_q,$$

$$C \cdot (\overline{C} - I_q)^T = \frac{q-1}{4} J_q - \frac{q-1}{4} I_q,$$

$$(C + I_q) \cdot \overline{C}^T = \frac{q+3}{4} J_q - \frac{q-1}{4} I_q,$$

$$[\,C \mid C+I_q\,]\cdot[\,C \mid C+I_q\,]^T = \frac{q-1}{2}J_q + \frac{q+1}{2}I_q,$$

$$[\,\overline{C} \mid \overline{C}-I_q\,]\cdot[\,\overline{C} \mid \overline{C}-I_q\,]^T = \frac{q-1}{2}J_q + \frac{q+1}{2}I_q,$$

$$[\,C \mid C+I_q\,]\cdot[\,\overline{C} \mid \overline{C}-I_q\,]^T = \frac{q+1}{2}J_q - \frac{q+1}{2}I_q.$$

$\Sigma(q)$ acts as an automorphism group of incidence structures with incidence matrices C, $C+I_q$, $\overline{C}$ and $\overline{C}-I_q$.

For the proof of the properties of the matrices C and D listed in this section we refer the reader to [2].

3. Siamese twin Menon designs

Let $H=(h_{ij})$ and K be $m\times n$ and $m_1\times n_1$ matrices, respectively. Their Kronecker product is an $mm_1\times nn_1$ matrix

$$H\otimes K = \begin{bmatrix} h_{11}K & h_{12}K & \dots & h_{1n}K \\ h_{21}K & h_{22}K & \dots & h_{2n}K \\ \vdots & \vdots & & \vdots \\ h_{m1}K & h_{m2}K & \dots & h_{mn}K \end{bmatrix}$$

Following the notation used in [1], for $v\in N$ we denote by j_v the all-one vector of dimension v, by 0_v the zero-vector of dimension v, and by $0_{v\times v}$ the zero-matrix of dimension $v\times v$.

Let p and $2p-1$ be prime powers and $p\equiv 3\ (mod\ 4)$. Put $q=2p-1$. Then $q\equiv 1\ (mod\ 4)$. Let D, $\overline{D}$, C and $\overline{C}$ be defined as above. Define a $(4p^2\times 4p^2)$ matrix M_1 in the following way:

$$M_1 = \left[\begin{array}{c|c|c|c}
0 & 0_q^T & j_{p\cdot q}^T & 0_{p\cdot q}^T \\ \hline
0_q & 0_{q\times q} & (\overline{C}-I_q)\otimes j_p^T & \overline{C}\otimes j_p^T \\ \hline
j_{p\cdot q} & C\otimes j_p & \begin{array}{c}(C+I_q)\otimes D\\ +\\ \overline{C}\otimes(\overline{D}-I_p)\end{array} & \begin{array}{c}C\otimes D\\ +\\ (\overline{C}-I_q)\otimes\overline{D}\end{array} \\ \hline
0_{p\cdot q} & (C+I_q)\otimes j_p & \begin{array}{c}C\otimes(D+I_p)\\ +\\ (\overline{C}-I_q)\otimes(\overline{D}-I_p)\end{array} & \begin{array}{c}(C+I_q)\otimes(\overline{D}-I_p)\\ +\\ \overline{C}\otimes D\end{array}
\end{array}\right]$$

The author proved in [1], using the properties of the matrices D, $\overline{D}$, C and $\overline{C}$ listed in *Section 2.*, that the matrix M_1 is the incidence matrix of a symmetric design with parameters $(4p^2, 2p^2-p, p^2-p)$ having an automorphism group isomorphic to $\Sigma(p)\times\Sigma(2p-1)$. In a similar way one proves that the matrix

$$M_2 = \left[\begin{array}{c|c|c|c}
0 & 0_q^T & j_{p\cdot q}^T & 0_{p\cdot q}^T \\ \hline
0_q & 0_{q\times q} & (\overline{C}-I_q)\otimes j_p^T & \overline{C}\otimes j_p^T \\ \hline
j_{p\cdot q} & C\otimes j_p & \begin{array}{c}(C+I_q)\otimes D\\ +\\ \overline{C}\otimes(\overline{D}-I_p)\end{array} & \begin{array}{c}C\otimes(\overline{D}-I_p)\\ +\\ (\overline{C}-I_q)\otimes(D+I_p)\end{array} \\ \hline
0_{p\cdot q} & (C+I_q)\otimes j_p & \begin{array}{c}C\otimes\overline{D}\\ +\\ (\overline{C}-I_q)\otimes D\end{array} & \begin{array}{c}(C+I_q)\otimes(\overline{D}-I_p)\\ +\\ \overline{C}\otimes D\end{array}
\end{array}\right]$$

is also the incidence matrix of a symmetric $(4p^2, 2p^2-p, p^2-p)$ design. It is easy to see that $M_2 \cdot J_{4p^2} = (2p^2-p)J_{4p^2}$. We have to prove that $M_2 \cdot M_2^T = (p^2-p)J_{4p^2} + p^2 I_{4p^2}$. Using properties of the matrices D, $\overline{D}$, C and $\overline{C}$ which we have menitioned before, one computes that the product of block matrices M_2 and M_2^T is:

$$M_2 \cdot M_2^T = \left[\begin{array}{c|c|c|c}
pq & (p^2-p)j_q^T & (p^2-p)j_{pq}^T & (p^2-p)j_{pq}^T \\ \hline
(p^2-p)j_q & \begin{array}{c}(p^2-p)J_q\\ +\\ p^2 I_q\end{array} & (p^2-p)J_{q\times pq} & (p^2-p)J_{q\times pq} \\ \hline
(p^2-p)j_{pq} & (p^2-p)J_{pq\times q} & \begin{array}{c}(p^2-p)J_{pq}\\ +\\ p^2 I_{pq}\end{array} & (p^2-p)J_{pq\times pq} \\ \hline
(p^2-p)j_{pq} & (p^2-p)J_{pq\times q} & (p^2-p)J_{pq\times pq} & \begin{array}{c}(p^2-p)J_{pq}\\ +\\ p^2 I_{pq}\end{array}
\end{array}\right]$$

where $J_{m\times n}$ is the all-one matrix of dimension $m \times n$. Thus,

$$M_2 \cdot M_2^T = (p^2-p)J_{4p^2} + p^2 I_{4p^2}$$

which means that M_2 is an incidence matrix of a symmetric design with parameters $(4p^2, 2p^2-p, p^2-p)$ having an automorphism group isomorphic to $\Sigma(p)\times\Sigma(2p-1)$. Incidence matrices M_1 and M_2 share the entries of

$$I = \left[\begin{array}{c|c|c|c}
0 & 0_q^T & j_{p\cdot q}^T & 0_{p\cdot q}^T \\ \hline
0_q & 0_{q\times q} & (\overline{C}-I_q)\otimes j_p^T & \overline{C}\otimes j_p^T \\ \hline
j_{p\cdot q} & C\otimes j_p & \begin{array}{c}(C+I_q)\otimes D\\ +\\ \overline{C}\otimes(\overline{D}-I_p)\end{array} & (\overline{C}-I_q)\otimes I_p \\ \hline
0_{p\cdot q} & (C+I_q)\otimes j_p & C\otimes I_p & \begin{array}{c}(C+I_q)\otimes(\overline{D}-I_p)\\ +\\ \overline{C}\otimes D\end{array}
\end{array}\right]$$

Thus, the following theorem holds

Theorem 1. *Let p and $q = 2p-1$ be prime powers and $p \equiv 3 \ (mod\ 4)$. Further, let the matrices D, $\overline{D}$, C, $\overline{C}$ and I be defined as above. Then the matrix*

$$S = \left[\begin{array}{c|c|c|c} 0 & 0_q^T & j_{p\cdot q}^T & 0_{p\cdot q}^T \\ \hline 0_q & 0_{q\times q} & (\overline{C}-I_q)\otimes j_p^T & \overline{C}\otimes j_p^T \\ \hline j_{p\cdot q} & C\otimes j_p & \begin{array}{c}(C+I_q)\otimes D \\ + \\ \overline{C}\otimes(\overline{D}-I_p)\end{array} & \begin{array}{c}C\otimes(D-\overline{D}+I_p) \\ + \\ (\overline{C}-I_q)\otimes(\overline{D}-D)\end{array} \\ \hline 0_{p\cdot q} & (C+I_q)\otimes j_p & \begin{array}{c}C\otimes(D+2I_p-\overline{D}) \\ + \\ (\overline{C}-I_q)\otimes(\overline{D}-I_p-D)\end{array} & \begin{array}{c}(C+I_q)\otimes(\overline{D}-I_p) \\ + \\ \overline{C}\otimes D\end{array} \end{array}\right]$$

is a Siamese twin design with parameters $(4p^2, 2p^2-p, p^2-p)$ sharing the entries of I.

4. Siamese twin Hadamard designs

From each Hadamard matrix of order m with $m \equiv 0 \ (mod\ 4)$, one can obtain a symmetric $(m-1, \frac{1}{2}m-1, \frac{1}{4}m-1)$ design, by normalizing and deleting the first row and column and changing all entries -1 to 0 (see [9]). Also, from any symmetric $(m-1, \frac{1}{2}m-1, \frac{1}{4}m-1)$ design one can recover a Hadamard matrix. Symmetric designs with parameters $(m-1, \frac{1}{2}m-1, \frac{1}{4}m-1)$ are called Hadamard designs.

Let M_1 and M_2 be the matrices from the previous section. Further, let H_1 and H_2 be regular Hadamard matrices corresponding to the incidence matrices M_1 and M_2, respectively. By normalizing and deleting the first row and column, these Hadamard matrices lead to the following incidence matrices of Hadamard designs:

$$N_1 = \left[\begin{array}{c|c|c} J_{q\times q} & (\overline{C}-I_q)\otimes j_p^T & C\otimes j_p^T \\ \hline C\otimes j_p & \begin{array}{c}C\otimes\overline{D} \\ +\ I_{p\cdot q}\ + \\ (\overline{C}-I_q)\otimes(D+I_p)\end{array} & \begin{array}{c}C\otimes D \\ + \\ (\overline{C}-I_q)\otimes\overline{D}\end{array} \\ \hline (\overline{C}-I_q)\otimes j_p & \begin{array}{c}C\otimes(D+I_p) \\ + \\ (\overline{C}-I_q)\otimes(\overline{D}-I_p)\end{array} & \begin{array}{c}C\otimes(D+I_p) \\ +\ I_{p\cdot q}\ + \\ (\overline{C}-I_q)\otimes\overline{D}\end{array} \end{array}\right]$$

$$N_2 = \left[\begin{array}{c|c|c} J_{q\times q} & (\overline{C}-I_q)\otimes j_p^T & C\otimes j_p^T \\ \hline C\otimes j_p & \begin{array}{c}C\otimes\overline{D} \\ +\ I_{p\cdot q}\ + \\ (\overline{C}-I_q)\otimes(D+I_p)\end{array} & \begin{array}{c}C\otimes(\overline{D}-I_p) \\ + \\ (\overline{C}-I_q)\otimes(D+I_p)\end{array} \\ \hline (\overline{C}-I_q)\otimes j_p & \begin{array}{c}C\otimes\overline{D} \\ + \\ (\overline{C}-I_q)\otimes D\end{array} & \begin{array}{c}C\otimes(D+I_p) \\ +\ I_{p\cdot q}\ + \\ (\overline{C}-I_q)\otimes\overline{D}\end{array} \end{array}\right]$$

Hadamard designs with the incidence matrices N_1 and N_2 admit an automorphism group isomorphic to $\Sigma(p) \times \Sigma(2p-1)$. Further, N_1 and N_2 share the entries of

$$I_1 = \left[\begin{array}{c|c|c} J_{q\times q} & (\overline{C} - I_q)\otimes j_p^T & C\otimes j_p^T \\ \hline C\otimes j_p & \begin{matrix} C\otimes\overline{D} \\ +\ I_{p\cdot q}\ + \\ (\overline{C} - I_q)\otimes(D + I_p) \end{matrix} & (\overline{C} - I_q)\otimes I_p \\ \hline (\overline{C} - I_q)\otimes j_p & C\otimes I_p & \begin{matrix} C\otimes(D + I_p) \\ +\ I_{p\cdot q}\ + \\ (\overline{C} - I_q)\otimes\overline{D} \end{matrix} \end{array}\right]$$

which proves the following theorem

Theorem 2. *Let* p *and* $q = 2p-1$ *be prime powers and* $p \equiv 3\ (mod\ 4)$. *Further, let the matrices* D, $\overline{D}$, C, $\overline{C}$ *and* I_1 *be defined as above. Then the matrix*

$$S_1 = \left[\begin{array}{c|c|c} J_{q\times q} & (\overline{C} - I_q)\otimes j_p^T & C\otimes j_p^T \\ \hline C\otimes j_p & \begin{matrix} C\otimes\overline{D} \\ +\ I_{p\cdot q}\ + \\ (\overline{C} - I_q)\otimes(D + I_p) \end{matrix} & \begin{matrix} C\otimes(D - \overline{D} + I_p) \\ + \\ (\overline{C} - I_q)\otimes(\overline{D} - D) \end{matrix} \\ \hline (\overline{C} - I_q)\otimes j_p & \begin{matrix} C\otimes(D + 2I_p - \overline{D}) \\ + \\ (\overline{C} - I_q)\otimes(\overline{D} - I_p - D) \end{matrix} & \begin{matrix} C\otimes(D + I_p) \\ +\ I_{p\cdot q}\ + \\ (\overline{C} - I_q)\otimes\overline{D} \end{matrix} \end{array}\right]$$

is a Siamese twin design with parameters $(4p^2-1, 2p^2-1, p^2-1)$ *sharing the entries of* I_1.

Parameters of Siamese twin designs belonging to the classes described in this paper, for $p \leq 100$, are given below.

p	$2p-1$	$4p^2$	Siamese twin Menon designs	Siamese twin Hadamard designs
3	5	36	(36,15,6)	(35,17,8)
7	13	196	(196,91,42)	(195,97,48)
19	37	1444	(1444,703,342)	(1443,721,360)
27	53	2916	(2916,1431,702)	(2915,1457,728)
31	61	3844	(3844,1891,930)	(3843,1921,960)
79	157	24964	(24964,12403,6162)	(24963,12481,6240)

Table 1. *Table of parameters for* $p \leq 100$

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