APPRAISING MOBILE MATHS APPS: THE TPACK MODEL

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Abstract

The purpose of this study was to develop an instrument for appraising educational apps in mathematics education. The instrument allows mathematics related apps to be analysed based on the three aspects of the TPACK (technological pedagogical content knowledge) model, namely, content, technology and pedagogy. Four sub-scales were created with the first one examining the app role according to the type of task promoted: explorative, productivity and/or instructive. The second sub-scale appraises the degree of cognitive involvement when a learner interacts with the app. The third and fourth sub-scale deals with general pedagogical and operational affordance. The instrument framework was piloted and subsequently trialled with ten school teachers and mathematics educators to ensure content validity. It was further endorsed with examples of educational apps currently available in the context of the secondary curriculum.

Introduction

This article describes the conceptual framework underpinning the design of an instrument aimed at assisting teachers in appraising mobile apps related to the teaching and learning of school mathematics. In the past 30 years, technology has changed. Apart from the change in technology, learners' profile has changed a lot. Today's learners are mobile. They demand access to the learning material and information anytime and anywhere. Use of mobile devices such as tablets and smart phones to access information is wide spread. This makes it critical for teachers at all levels to re-examine how learning materials are designed and delivered for the new generation of mobile learners (Ally, 2007).

Various instruments which mostly appear on the WWW have been developed to appraise the quality of educational apps but they do not provide evidence of being grounded in educational theory and do not discuss their conceptual constructs (Watlington, 2011). Besides their ad-hoc design, most of them present a uni-dimensional structure foundation and are not discipline specific (Kearney, Schuck, Burden & Aubusson, 2012). This paper describes the rationale for an instrument based on the TPACK (technological pedagogical content knowledge) model initially elaborated by Mishra and Koehler (2006).

It is only in the past decade or so that researchers in the area of mobile learning (henceforth referred to as m-learning) seriously considered the need for some theoretical framework for m-learning. As discussed in the literature review section, there are several frameworks around learning through mobile technologies. Different frameworks provide different contexts for m-learning. The literature around m-learning identifies the correlation between the role of mobile technology in learning, that is, how mobile devices can help learners and enhance and enrich their learning experience.

The literature review in this paper examines a number of theoretical considerations on m-learning. It reviews some quality design principles introducing the TPACK model as the theoretical framework to embed those attributes. The resulting maths app appraisal instrument (please see Appendix) reflects such criteria for assessing mobile applications in primary and secondary mathematics education within a pedagogical and operational context.

Literature review

Use of mobile learning applications

In general, mobile handheld devices differ from other tools such as laptops because the latter, although portable, are typically not small and light enough to carry around. The term mobile devices is commonly applied to smartphones and tablet PCs although other portable hardware can fit into that category devices such as CD-ROMs and DVDs, flash storage devices/drives, Global Positioning Systems (GPS), laptops or notebooks, mobile computers, MP3 players, Personal Data Assistants (PDAs), portable media players and portable video game devices.

Mobile applications, commonly know as apps, can provide more or less structure to facilitate or scaffold the collection and presentation of data by students or groups of students. An app is an application capable of running in mobile devices. These self-contained programs are endowed with various technical and pedagogical affordances. For example, they are multimedia based with audio, image and/or animation functionalities.

In addition, some apps automatically aggregate and visualize data about students' learning (e.g. their responses to questions) for teachers to examine (Vahey, Roschelle & Tatar, 2007). Their capacity of representing complex mathematical concepts, process and procedures has been highlighted for an increasing body of research in the past ten years (Handal, El-Khoury, Cavanagh & Campbell, 2013). At low cost or sometimes free of charge, these applications are linked to the internet allowing multiple learning and teaching experiences such as simulations, collaboration, document-sharing, online testing, audio/video-recording, m-blogging, surveying, presentations, note-taking, digital-story telling, social networking, email and geo blogging.

Mobile applications differ on how rich (complex) or lean (less complex) their contents are conveyed. Leaner applications only present a limited set of content, typically well-structured to facilitate certain kinds of behaviour and communication. Also, mobile applications, or more precisely, the activity built around mobile technologies, could differ on the degree of interactivity required between students and the tools (Parsons, Ryu & Cranshaw, 2007). Some activities require students to interact more intensively with the tools. Again, some activities require students to interact more with their peers rather than with the tools (Geddes, 2004).

M-learning frameworks

Mobile learning is an instructional mode that results from the interface between individuals and handheld technologies creating a specific educational environment. Various authors have suggested the advantages brought by mobile devices into school education which, in a way, make m-learning different from other instructional delivery modes (Traxler, 2009). Their ubiquity and mobility make m-learning more situated and unique. Other particular m-learning features include connectivity scope and structure, data collection by students, student data aggregation, content richness, interactivity and collaboration (Peters, 2005; Geddes, 2004; Parsons, Ryu & Cranshaw, 2007).

With connectivity scope, mobile devices and applications can be set to allow local communication within the classroom, or narrower still, within groups, through Bluetooth or Wi-Fi. Alternatively, they can be set to allow communication with others beyond the classroom and access information on the internet as collaboration is considered an important aspect of the m-learning. Similarly, students can be connected directly only to the teacher (i.e. to a central device that the teacher has access to), and indirectly to other students via the teacher. Alternatively, students can be interconnected directly to one another (Roschelle, Vahey, Tatar, Kaput & Hegedus, 2003).

A number of theoretical models have been developed to explain m-learning as an instructional approach. Their attributes are useful to characterise quality m-learning design principles. These design principles can be applied to the learning situation itself as well as in the construction of effective educational apps.

Most authors agree that an m-learning framework, should be able to describe pedagogy along with mobile technologies (Koehler & Mishra, 2008; Roschelle, Rafanan, Estrella, Nussbaum & Claro, 2010; Roschelle, Shechtman, Tatar, Hegedus, Hopkins, Empson, Knudsen & Gallagher, 2010).

A commonality across these frameworks is their multi-dimensionality allowing for complex realities within the m-learning construct (see Table 1).

Table 1: Main m-Learning frameworks dimensions

Authors	*Dimensions*
Danaher, Gururajan and Hafeez-Baig (2009)	Engagement, presence and flexibility
Koole (2009)	Device aspects, learner aspects and social aspects.
Kearney, Shuck, Burden and Aubusson (2012)	Personalisation, authenticity, and collaboration
Peng, Su, Chou and Tsai (2009)	Learners and tools, pedagogical methods (constructivism and lifelong learning theories), a vision
Pachler, Cook and Bachmair (2010)	Structures, agency and cultural practices
Koehler and Mishra (2008)	Knowledge, pedagogy and technology

A review of the above frameworks reveals that the pedagogy and theories of teaching and learning may need to change in the perspective as a result of the emergence of m-learning particularly on mobile literacy. Some of the themes coming from those frameworks and related literature include a new literacy where participation is considered as a part of cultural practice (Pachler, Cook & Bachmair, 2010). Also, teaching and learning is becoming more informal (Seipold & Pachler, 2011) with elements of situatedness, collaboration and problem-solving along with strong focus on knowledge building (Geddes, 2004) and meaning-making (Roschelle et al., 2010a). The authors see the notion of mobility not just as *moving* (Traxler, 2009). Mobility is seen in context with space, time, activity, relationships, curriculum and engagement (Kearney, Shuck, Burden & Aubusson, 2012; Pachler, Cook & Bachmair, 2010). Users are encouraged to generate their own content and context for example aided by the mobile devices that allow ubiquity, choice and knowledge appropriation (Pachler, Cook & Bachmair, 2010).

The TPACK model

The TPACK model developed by Koehler and Mishra (2008) is described below with its three dimensions: technology, pedagogy and content. While recognising the advantages of the aforementioned models in terms of their various dimensions, this study chose the TPACK framework as the main theoretical framework to underpin the design of an app appraisal instrument. In arriving to at such a decision the authors considered TPACK capacity as a theoretical tool to include the subject area and specific mathematical concepts and processes.

Several instruments have been developed using the TPACK framework in order to examine a wide range of variables in the context of mathematics education. These include assessment (Schmidt, Baran, Thompson, Mishra, Koehler & Shin, 2009), students' achievement (Lyublinskaya & Tournaki, 2011), teacher education (Lee & Hollebrands, 2008), teachers' eLearning skills, professional development (Niess, van Zee & Gillow-Wiles, 2011), teachers' attitudes towards technological and pedagogical skills (Handal, Campbell, Cavanagh & Petocz, 2012), curriculum development (Niess, Ronau, Shafer, Driskell, Harper, Johnston, Browning, Özgün-Koca & Kersaint, 2009), among others.

Considering TPACK's use in previous research and through the development of these instruments it was considered that TPACK be the best instrument to use in this situation. For example, the FRAME Model by Koole suggests that "mobile learning experiences are viewed as existing within a context of information" (Koole, 2009, p.26) thus, the learner is consuming and creating information. The limitation of this model is that there is no pedagogical inclusion as its three parts include device, learner and social aspects, but there is no aspect for the teacher or teaching. Similarly, the framework of ubiquitous knowledge construction proposed by Peng et al. (2009) considers learners, tools, and learning theories such as constructivism but does not directly address pedagogy. However, the authors do advocate that, "Educators should take a proactive stance towards emerging technology and become integrally involved in the development, as well as the evaluation, of pedagogically sound educational tools." (p.178). The instrument developed in the present study is designed to facilitate such a process.

Kearney et al. (2012) and Pachler, Cook and Bachmair (2010) both provide a pedagogical perspective on mobile learning to assess lesson activities and pedagogical approaches from a socio-cultural perspective. Kearney et al. (2012) identify three characteristics of m-learning pedagogy in their model: personalisation (learner agency and control), authenticity (situated learning experiences), and collaboration (connections to people and resources). Pachler, Cook and Bachmair's (2010) model is based on agency (students' ability to engage with technology), cultural practices (norms and practices of students' everyday lives) and socio-cultural and technological structures. However, both studies are not primarily concerned with the evaluation of tools and devices so while they offer important insights into pedagogical practices the models they propose are not of direct relevance to the present study.

Danaher, Gururajan and Hafeez-Baig (2009) is a framework based on mobile learning and teaching environments at university level and it uses three items which are engagement, presence and flexibility. This is limited in that the context and technology are not taken into account. The authors of the model acknowledge that there are future research directions for their model and although their research suggests there are strategies that work in fostering student engagement and flexibility in using mobile learning in teaching they realise their model has some limitations.

TPACK constitutes a conceptual framework that is valuable because it integrates three dimensions in using ICTs in teaching and learning, namely, pedagogical knowledge, technological knowledge and disciplinary content. Pedagogical knowledge (PK) represents teachers' understanding of evidence-based quality teaching as well as expertise aiming at enhancing students' experiences and therefore learning. In turn, technological knowledge (TK) represents those operational capabilities that teachers need to deploy technology. Content knowledge (CK) stands for teachers' acquaintance with the subject matter, more specifically, expertise in a particular branch of learning that qualifies them as professional in the field.

The interaction among PK, CK and TK renders three singular constructs: technological pedagogical knowledge (TPK), technological content knowledge (TCK) and pedagogical content knowledge (PCK). TPK is knowledge about the link between technologies and pedagogy, that is, the selection and application of technology in the context of a particular instructional approach. For example the ability to use technology to develop students' research skills, or using it to provide students with alternative forms of assessment. TCK deals with understandings about using a specific technology in a mathematical context such as making calculations on a spreadsheet or using computer algebra software. Furthermore, PCK represents the integration of pedagogy and content such as the ability to teach mathematics effectively to schools students.

Figure 1 shows the various elements of the TPACK model.

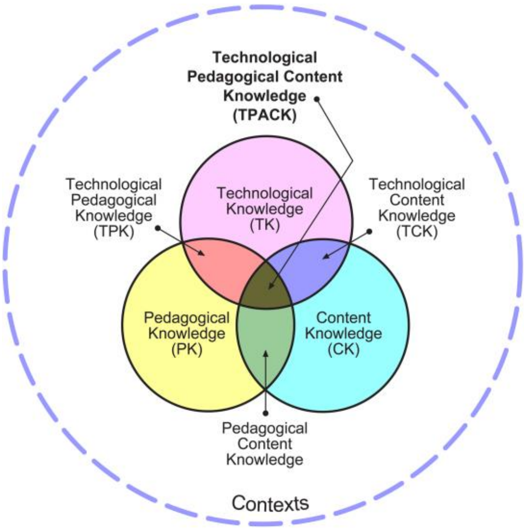

Figure 1: the TPACK model
Source: http://tpack.org

The intersection of these three fields yields the area known as technological pedagogical content knowledge (TPACK). It represents the full and seamless blending of knowledge about technology along with the appropriate deployment of suitable pedagogies related to a specific learning objective within the school mathematics curriculum. Such space provides a reflective place to explore how the three dimensions interact with each other to ensure that learning and teaching with technology and within knowledge content takes place at its highest level (Handal, Campbell, Cavanagh, Petocz & Kelly, 2013).

M-learning applications, commonly used as apps, can be comprehensively analysed through the TPACK model. Looking through the TPACK lenses, apps can become powerful tools in the hands of teachers and students. Teachers can use them for enacting effective curriculum experiences with great creativity and depth while students can actively engage in meaningful learning becoming producers rather than consumers of knowledge. Hence, the need to promote awareness of these tools within the school setting so that teachers and students can be cognizant of their benefits in teaching and learning. TPACK, due to its three dimensions, can become the vehicle through which apps can be appraised based on their own pedagogical affordances, technical capabilities and content delivery. This paper elaborates on these three themes.

On developing a TPACK model to appraise educational maths, this paper argues that pedagogical knowledge (PK) can be represented by the level of cognitive engagement facilitated as well as by the general instructional facilities offered by the app. In turn, the quality of technological knowledge (TK) embedded in an app as a piece of school software can be corresponded to their ability to evidence efficient interface design, navigation and control. Finally, the app ability to render the subject matter for specific mathematical purposes can be equated to the content knowledge (CK) component. The intersection itself from these three TPACK components leads to establishment of a *summa samarium* zone; where mathematical knowledge is creatively taught by the teacher and efficiently cognated by the student through the technology.

The Maths app appraisal instrument

The development of the maths appraisal instrument (see Appendix) was informed by the literature as outlined throughout the paper with emphasis on the TPACK model. The instrument is divided into four parts. The introduction requires teachers to identify the primary role of the app. There are four subscales. The first sub-scale dealing with the structure of each task through three item sets (e.g., explorative, productivity and informative apps). The other three sub-scales relate to cognitive engagement, pedagogical and operational issues. Table 2 shows the link between sub-scales and components from the TPACK.

Table 2: Relationship between TPACK components and sub-scales

Sub-scale	*TPACK component*
Task structure	Technological Pedagogical Content Knowledge (TPACK)
Cognitive engagement	Pedagogical Content Knowledge (PCK)
General Pedagogical issues	Technological Pedagogical Knowledge (TPK)
Operational issues	Technological Content Knowledge (TCK)

Responses to semantic items give the users the opportunity to select four options: *Always, To some extent, Never* and *Not applicable.* Instructions emphasise that there are no right or wrong answers. Icons of representative apps by task structure are shown to provide an element of visual imagery to respondents.

The maths appraisal instrument was validated with ten academics and secondary mathematics teachers from the Sydney area are to ensure clarity and meaning of the semantic items as well to guarantee content validity.

The task structure sub-scale

This sub-scale section refers to three main types of apps task structures, namely, exploration, production and practice and information. The TPACK notion was represented by the task structure sub-scale of the instrument because the semantic items describe a construct combining technology, pedagogy and content. Task structure elements relate to the teacher's deep knowledge about how best to use m-learning apps in developing students' understanding of the subject matter of mathematics. Identifying the task structure of the app raises the teacher's awareness of how the app's instructional roles can support and enhance different aspects of student learning. The task structure subscale therefore identifies the complexity of the inter-relationships between m-learning apps as technological tools, the mathematics content they include, specific teaching practices aligned to exploration, production, or practice and information, and student learning. Task structure brings into play an amalgam of the teacher's mathematical and technological content knowledge along with the choice of appropriate pedagogical approaches which the teacher selects based on the particular instructional role of the app.

In appraising m-learning apps it is vital to understand the instructional role that the each plays in mathematics education. Handal, El-Khoury, Campbell and Cavanagh (2013), based on Goodwin's work (2012), developed a framework to categorise apps for the type of task promoted as the learner interacts with the interface. The framework permitted a no "one-size-fits-all" approach to look at how apps can be delivered in the curriculum. Apps were assumed to have a particular instructional design structure depending on any of three instructional roles addressed (explorative, productivity and instructive roles).

The task structure sub-scale explores these three roles (Goodwin, 2012). The three groups were initially identified as common role characteristics. Explorative apps are useful for exploring and demonstrating mathematical models or concepts through manipulating objects that mimic or simulate complex physical situations (Botzer & Yerushalmy, 2007). These apps are designed to mirror a real-life situation and students can enter their own data as well as visualise changes in the model (Baya'a & Daher, 2009). Explorative apps embed a degree of ambiguity and uncertainty embedded in the task to encourage problem solving. The exploration is guided within a predetermined learning discovery framework which promotes personal investigation. Depending on the openness of the task, problem solving is actively promoted as well as students' research skills and their ability to conjecture, hypothesise and predict. Exploratory apps are very student-centred as students can pose their own problems and investigate possible solutions leading to deep learning.

Productivity apps are more centred on the tool itself and embed an authoring aspect. These are apps useful for measuring and graphically representing objects or concepts in 2D/3D, collecting data, making calculations, or creating multimedia materials which make students producer of mathematical content (Franklin & Peng, 2008). Through these apps students can creatively come up with their own design and/or concept. These apps allow users to represent mathematical content by linking symbolic, numerical and graphical data. Usually, the app guides student in creating their own content/understandings. A great advantage of these apps is that they can represent or present mathematical content using a variety of digital tools (e.g., audio/video recording, measuring devices, etc). Frequently, app tools are intuitive and easy to use. Ideally, these tools would present an interpretive space for the learner to reflect on the activity done. At the highest level of instructional design, a productivity tool should encourage students assist to come up with new conceptual or procedural knowledge through hands-on experiences.

In turn, instructive apps are useful for practicing content through drill exercises, acquiring new skills through questions and answers (tutorials) or retrieving factual information which is a role traditionally supported by mathematical software (Handal, Handal & Herrington, 2006). Generally, these apps contain a variety of different activities/exercises and provide students with feedback with various degrees of meaning. It is expected that not only summative but formative feedback during the questions and answers process is provided to promote deep learning and help students in developing their maths problem solving skills. Ideally, instructive apps should engage students in critically analysing online content texts or images within real-life situations (Kearney & Maher, 2013). Similarly, a good instructive app would also require students to be able to demonstrate their mathematical understanding rather than engage in rote-learning text-like formats (Kurz, Middleton & Yanik, 2005). It should also let students acquire mathematical content in a variety of different ways with a non-linear navigation. Preferably, the content should be meaningful, fostering engagement and rich problem solving. Similarly, activities/exercises should cater for a range of student ability levels and should be graded and summary data provided (Handal & Herrington, 2003).

All of the above task structure concepts were embedded in the sub-scale items portraying the ideal combination of content, technology and pedagogy in specific mathematical educational contexts.

The cognitive engagement sub-scale

The Cognitive Engagement sub-scale section of the appraisal instrument was guided by the pedagogical content knowledge (PCK) because it leans more on general pedagogies of teaching rather than on subject specific matters. PCK is important as it teachers need to have both content knowledge and pedagogical knowledge when teaching. For the purpose of this sub-scale PCK is the teacher’s ability to appraise a maths education app based on its pedagogical and content capacity and for their capacity to foster student’s cognitive engagement. PCK is rendered in the sub-scale by the cognitive elements of the reviewed Bloom’s taxonomy (Anderson & Krathwohl, 2001).

The measure of a student’s cognitive engagement turns out to be a critical m-learning aspect as in many cases apps are of small educational value being the equivalent to a rote learning activity, with little problem-solving, or paralleling a bell-and-whistle multimedia spectacle barren from meaningful learning (Shuler, 2012). When learners are in m-learning situations, they would ideally interact with their mobile devices in a way that is pedagogically productive. M-learning should be student-centred and put the individual first because what is mobile is actually the learner not the device (Traxler, 2009). Educational technology, in general, should be used when no other teaching strategy can provide a better educational experience.

Hence it is crucial for the application/software to have high levels of cognitive interactivity to engage learners. Such levels of cognitive interactivity could be seen in the context of Bloom’s taxonomy where in learners are engaged at various levels of achievements. Due to its bearing in rendering differential assessment items as well as for its capacity to conceptualise curricular learning outcomes the Bloom's taxonomy has been extensively used in mathematics education (Webb, 2013)

Anderson and Krathwohl (2001) revised the Bloom's taxonomy (Figure 2) narrowing down to six domains, namely, remembering, understanding, applying, analyzing, evaluating and creating. The framework provides a context for measures of cognitive engagement which could be articulated smoothly to m-learning.

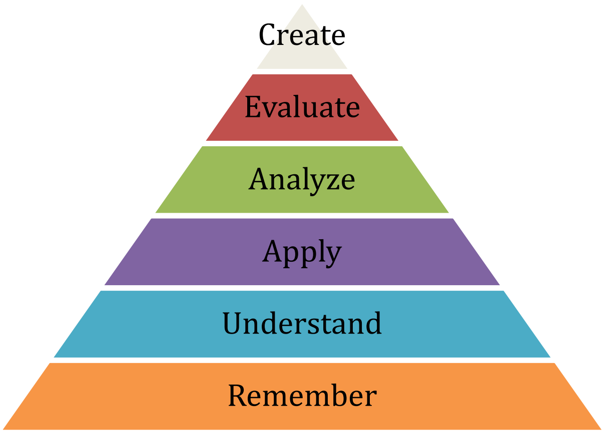

Figure 2: Revised Bloom's taxonomy

Such a scheme can be represented in terms of levels and definitions as follows reflecting the extent to which the app, in a math education context, encourages students to move from lower levels such as remembering facts to higher levels like creating knowledge. Table 3 represents this continuum of cognitive engagement related to the use of technology in mathematics education. Those definitions were incorporated in the cognitive engagement sub-scale.

Table 3: Levels of cognitive engagement

Level	*Definition*
Remembering	retrieve and review mathematical concepts/skills/procedures
Understanding	demonstrate understanding of mathematical concepts/skills/procedures
Applying	apply their knowledge of mathematical concepts/skills/procedures in practical contexts
Analysing	critically analyse mathematical content in text, graphs and/or animations
Evaluating	appraise and justify mathematical ideas or products
Creating	construct new and meaningful mathematical ideas or products

The general pedagogical issues sub-scale

The General Pedagogical sub-scale was represented by the technological pedagogical content knowledge (TPK) component of the TPACK model (see Appendix). The ten sub-scale semantic items represent teachers' understanding of general pedagogical competences that technology should promote. It centres on instructional capabilities that the app would enhance to enrich the student experience and promote learning. In a way, these capabilities require linking technology and pedagogy at a more general level such as processes, practices and methods of teaching and learning. In an app context, TPK reminds the teacher to select those that facilitate the general outcomes of instruction.

For example, the sub-scale semantic items portray the idea that when students are encouraged to design their own problems they learn to think mathematically about the world around them. Such a competence moves them from being passive recipients of information to creators or co-sharers of a body of knowledge (Reys, Lindquist, Lambdin & Smith, 2008). There are documented instances when, for example, students are requested to create examination items or create an investigational project plan (Luxton-Reilly & Denny, 2010). This also leads to the issue of giving students control over their learning rather than placing such exercise in the hands of rigidly designed curricula and content usually portrayed by textbooks (Zoric, Cindric & Destovic, 2012).

The sub-scale items provide credit to cross-curricular knowledge. This is an important TPK element that adds to quality teaching because it gives the possibility to apply mathematical knowledge from a confined subject-matter niche into other branches of learning. There is certainly great pedagogical reward in extending students' mathematical knowledge across the school syllabus such as geography, history, science and the like (ACARA, 2012). Other aspects of good practice, acknowledged by the sub-scale, include the provision of differential activities for various levels of achievement through increasing levels of difficulty (Tucker, Singleton & Weaver, 2006). All the above attributes are applicable in m-learning when it is considered an instructional resource within the curriculum.

Some apps allow for collaboration such as in classroom learning response systems where students see what others share as well as their understanding and/or misunderstandings. This principle can also apply to group-based scenarios is similar to classroom response systems, in that the teacher presents short problems or multiple-choice questions using mobile devices. But instead of asking students to individually input their responses, the teacher gets students to work in groups to solve the problems. In addition, through collaborative data gathering, as acknowledged in this sub-scale, students can use mobile devices to collect, aggregate and present data. The analysis and presentation or visualization of the data is typically performed automatically by the device/application. This allows students to focus on discussing the meaning of the data/findings in the context of inquiry-based learning (Vahey, Roschelle & Tatar, 2007; Spikol & Eliasson, 2010). Finally, the general pedagogical issues sub-scale also allows examining the app capacity to show a reading level appropriate to the student's level as well as its ability for saving and keeping students' work in order to resume incomplete tasks or just simply to monitor performance.

The operational sub-scale

The Operational sub-scale of the appraisal instrument was informed by the TPACK Technological Content Knowledge (TCK) component (see Appendix). The TCK ensures teachers to have balance between students' capabilities around the use of technology (such as understanding navigation, what is expected in what fields and so on) and what s/he wants to achieve (for instance what kind of data he wants to gather). This leads to many more operational aspects that a teachers needs to be aware of while selecting the app. These aspects could include what is there in the app that encourages students to redo the task if it is not done correctly at first? Does the app allow any reinforcement? Does it allow for repeating the task? What is there in the app that allows students to self assess? More generally we could call it interface design combined with knowing the nature of the content and understanding of the learning goals. Even though we can separately correlate component of TPACK with various subscales, at broader level the components merge to paint a holistic picture.

The ten sub-scale semantic items deal with the app technical and operational technical affordances. Very little research has been conducted in this area with most of the perspectives coming from the literature on evaluating general educational software (Watlington, 2011).

There are also calls for letting students alter its settings to customise the app to their needs and be provided with helpful technical instructions to the user (Rosenthal-Tolisano, 2012). These additional features might include a Help function and a supporting web page providing additional useful information. Instructional designers also suggest checking for the app capacity to easily importing a range of media (audio, video, image, text, animations) and presenting an uncluttered display which is visually stimulating. New mobile functionalities now allow an interface with the broader online environment (e.g., Facebook, wiki, blog, Twitter) and allows file sharing, streaming of content and/or online communications (Schrock, 2012; Shuler, 2012).

Conclusions

The rapid inroads of mobile apps into the school maths curriculum during the past ten years made more compelling the need to evaluate systematically the deployment of those applications in teaching and learning. Mobile devices like smartphones and tablets began making a strong presence in school settings as personal tools for communicating and accessing information instantaneously. Later, due to its ubiquitousness and multimedia capabilities, these devices, once born for more general purposes, have become an essential element of school life.

Their integration with the curriculum is gaining momentum as their pedagogical affordances are being explored, discovered and utilized more systematically. Such is their popularity that within a short period of time these devices are aggressively competing for curricular space with long-standing tools such as laptops, desktops let alone the traditional computer lab.

The coming of mobile devices have brought, however, an astonishing number of apps into the market. It is claimed now that, as we write, over a million applications have been developed only on the Apple platform (148AppsBiz, 2012). In such a short period of time academics have also advanced our understanding as how these devices and their applications can be productively utilized to enhance the student experience. This has resulted in the formulation of various frameworks emphasizing diverse m-learning conceptual models whose empirical implications and validation remains a challenge for future researchers.

This study is the first known attempt to develop an instrument for appraising educational maths apps. The four sub-scales semantic items were drawn from the literature and validated with maths educators from schools and universities (see Appendix). A distinctive feature of the instrument was the appraisal of educational apps according to their instructional role in maths education. The instrument also characterised various levels of cognitive engagement, pedagogical issues as well as surface features, interface design, navigation and control.

The TPACK model was chosen as the conceptual framework because of its potential to integrate technological content knowledge (TCK), technological pedagogical knowledge (TPK) and pedagogical content knowledge (PCK). Due to its various dimensions the model lends itself well to understand the instructional design of an app from multiple technical pedagogical dimensions. As a result, the instrument embeds an evidence-based methodology acknowledging an app capacity to render differential degrees of task structure, cognitive engagement, pedagogical and operational affordances.

The next stage of this research will consist of a qualitative study to determine teachers' inter-rater reliability of the instrument using a larger sample. It would also look at other understandings that teachers bring to the process of selecting an appropriate app through observations and interview studies. As such, the prospective study will bring more closely the environment and context variables within the research equation.

References

148AppsBiz. (2012). *App Store Metrics*. Retrieved on April 2, 2013 from http://148apps.biz/app-store-metrics/

Ally, M. (2007). Guest editorial - M-learning. *The International Review of Research in Open and Distance Learning,* 8(2). Retrieved April 1, 2013 from: http://www.irrodl.org/index.php/irrodl/article/view/451/918

Anderson, L. W., & Krathwohl, D. R. (Eds.). (2001). *A taxonomy for learning, teaching and assessing: A revision of Bloom's Taxonomy of educational objectives*. New York: Longman.

Australian Curriculum Assessment and Reporting Authority (ACARA). (2012). *General capabilities in the Australian curriculum.* ACARA. Retrieved April 1, 2013 from: http://www.australiancurriculum.edu.au/GeneralCapabilities/General%20capabilities.pdf

Baya'a, N., & Daher, W. (2009, April). Students' perceptions of mathematics learning using mobile phones. In *Proceedings of the International Conference on Mobile and Computer Aided Learning* (Vol. 4, pp. 1-9).

Botzer, G., & Yerushalmy, M. (2007, December). Mobile application for m-learning. In *Proceedings of the International Conference on Cognition and Exploratory Learning in Digital Age* (pp. 7-9).

Danaher, P., Gururajan, R., & Hafeez-Baig, A. (2009). Transforming the practice of m-learning: promoting pedagogical innovation through educational principles and strategies that work. In H. Ryu & D. Parsons (Eds.), *Innovative M-learning: Techniques and Technologies*. Hershey IGI Global.

Franklin, T., & Peng, L. W. (2008). Mobile math: Math educators and students engage in m-learning. *Journal of Computing in Higher Education*, *20*(2), 69-80.

Geddes. (2004). *M-learning in the 21st century: Benefit for learners*. The Knowledge Tree, 6. Retrieved April 1, 2013 from: http://pre2005.flexiblelearning.net.au/knowledgetree/edition06/download/geddes.pdf

Goodwin, K. (2012). *Use of tablet technology in the classroom*. NSW Department of Education and Communities.

Handal, B., Handal, P., & Herrington, T. (2006). Evaluating maths education websites: Teachers tools. *Australian Primary Mathematics Classroom*, *11*(2), 8-14.

Handal, B., & Herrington (2003). Re-Examining categories of computer-based learning in mathematics education. *Contemporary Issues in Technology and Teacher Education*, *2*(1). Retrieved April 1, 2013 from: http://www.citejournal.org/vol3/iss3/mathematics/article1.cfm

Handal, B., Campbell, C., Cavanagh, M., Petocz, P., & Kelly, N. (2013). Technological pedagogical content knowledge of secondary mathematics teachers. *Contemporary Issues in Technology and Teacher Education*, *13*(1). Retrieved April 1, 2013 from: http://www.citejournal.org/vol13/iss1/mathematics/article1.cfm

Handal, B., & Herrington, A. (2003). Re-examining categories of computer-based learning in mathematics education. *Contemporary Issues in Technology and Teacher Education*, *3*(3), 275-287.

Handal, B., Handal, P., & Herrington, A. (2003). Training teachers in evaluating educational tutorial software. *Electronic Journal for Technology in Education*, *2*(1). Retrieved April 1, 2013 from: http://ejite.isu.edu/Volume2No1/handal.htm

Handal, B., El-Khoury, J., Cavanagh, M., & Campbell, C. (2013). A framework for categorising mobile learning applications in mathematics education. Proceedings of the *Australian Conference on Science and Mathematics Education* (pp. 142-147). Canberra: IISME.

Kearney, M., & Maher, D. (2013). M-learning in maths teacher education: Using iPads to support pre-service teachers' professional development. *Australian Educational Computing*, *27*(3), 76.

Kearney, M., Schuck, S., & Burden, K. (2010). Locating m-learning in the third space. In *Proceedings of mlearn2010: 10th world conference on mobile and contextual learning*, (Eds. M. Montebello, V. Camilleri & A. Dingli), *University of Malta, Valetta* (pp. 108-115).

Kearney, M., Shuck, S., Burden, K. & Aubusson, P. (2012). Viewing mobile learning from a pedagogical perspective. *Research in Learning Technology, 20*, 1-17.

Koehler, M. J., & Mishra, P. (2008). Introducing TPCK. In The AACTE Committee on Innovation and Technology (Eds.), *Handbook of technological pedagogical content knowledge for educators* (pp. 1-29). New York, NY: Routledge.

Koole, M. L. (2009). A Model for Framing M-learning. In A. Mohamed (Ed.), *M-learning: Transforming the Delivery of Education and Training* (pp. 9-24). AU Press, Athabasca University.

Kurz, T. L., Middleton, J. A., & Yanik, H. B. (2005). A taxonomy of software for mathematics instruction. *Contemporary Issues in Technology and Teacher Education* [Online serial], *5*(2). Retrieved April 1, 2013 from: http://www.citejournal.org/vol5/iss2/mathematics/article1.cfm

Lee, H., & Hollebrands, K. (2008). Preparing to teach mathematics with technology: An integrated approach to developing technological pedagogical content knowledge. *Contemporary Issues in Technology and Teacher Education*, *8*(4), 326-341.

Luxton-Reilly, A., & Denny, P. (2010). Constructive evaluation: a pedagogy of student-contributed assessment. *Computer Science Education*, *20*(2), 145-167.

Lyublinskaya, I., & Tournaki, N. (2011). The effects of teacher content authoring on TPACK and on student achievement in algebra: Research on instruction with the TI-Nspire™ Handheld. *Educational Technology, Teacher Knowledge, and Classroom Impact: A Research Handbook on Frameworks and Approaches*, 295.

Mishra, P., & Koehler, M.J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, *108*(6), 1017-1054.

Niess, M. L., Ronau, R. N., Shafer, K. G., Driskell, S. O., Harper S. R., Johnston, C., Browning, C.,. Özgün-Koca, S. A., & Kersaint, G (2009). Mathematics teacher TPACK standards and development model. *Contemporary Issues in Technology and Teacher Education*, *9*(1), 4-24.

Niess, M. L., van Zee, E. H., & Gillow-Wiles, H. (2011). Knowledge growth in teaching mathematics/science with Spreadsheets: Moving PCK to TPACK through online professional development. *Journal of Digital Learning in Teacher Education*, *27*(2), 42-52.

Pachler, N., Cook, J., & Bachmair, B. (2010). Appropriation of mobile cultural resources for learning. *International Journal of Mobile and Blended Learning*, *2*(1), 1-21. .

Parsons, D., Ryu, H., & Cranshaw, M. (2007). A design requirements framework for m-learning environments. *Journal of Computers*, *2*(4), 1-8.

Peng, H., Su, Y., Chou, C., & Tsai, C. (2009). Ubiquitous knowledge construction: m-learning re-defined and a conceptual framework. *Innovations in Education and Teaching International*, *46*(2), 171-183.

Peters, K. (2005). *Learning on the move: Mobile technologies in business and education.* Australian Flexible Learning Framework. Retrieved 1 April 2013: http://www.flexiblelearning.net.au/projects/resources/2005/Research.htm

Reys, R. E., Lindquist, M. M., Lambdin, D. V., Smith, N. L. (2008). *Helping Children Learn Mathematics* (9th Ed.). New York, NY: John Wiley & Sons, Inc.

Rosenthal-Tolisano, S. (2012.) *iPad app evaluation for the classroom.* Retrieved April 1, 2013 from: http://www.scribd.com/doc/94980508/iPad-App-Evaluation-for-the-Classroom

Roschelle, J., Rafanan, K., Estrella, G., Nussbaum, M., & Claro, S. (2010a). From handheld collaborative tool to effective classroom module: Embedding CSCL in a broader design framework. *Computers & Education*, *55*(3), 1018-1026. DOI: http://dx.doi.org/10.1016/j.compedu.2010.04.012

Roschelle, J., Shechtman, N., Tatar, D., Hegedus, S., Hopkins, B., Empson, S., Knudsen, J. & Gallagher, L. P. (2010b). Integration of technology, curriculum, and professional development for advancing middle school mathematics: Three large-scale studies. *Journal of Educational and Behavioral Statistics*, *47*(4), 833-878.

Roschelle, J., Vahey, P., Tatar, D., Kaput, J., & Hegedus, S. J. (2003, July). Five key considerations for networking in a handheld-based mathematics classroom. In *Proceedings of the 2003 Joint Meeting of PME and PMENA* (Vol. 4, pp. 71-78).

Schmidt, D. A., Baran, E., Thompson, A. D., Mishra, P., Koehler, M. J., & Shin, T. S. (2009). Technological pedagogical content knowledge (TPACK): The development and validation of an assessment instrument for preservice teachers. *Journal of Research on Computing in Education*, *42*(2), 123.

Schrock, K. (2012). Critical evaluation of a content-based iPad/iPod app. Retrieved April 1, 2013 from: http://kathyschrock.net/pdf/evalipad.pdf

Seipold, J., & Pachler, N. (2011). Evaluating m-learning practice towards a framework for analysis of user-generated contexts with reference to the socio-cultural ecology of m-learning. *MedienPädagogik*, 19, 1-13.

Shuler, C. (2012). *iLearn II; An analysis of the education category of the iTunes App Store.* New York: The Joan Ganz Cooney Center at Sesame Workshop. Retrieved April 1, 2013 from: http://joanganzcooneycenter.org/upload_kits/ilearnii.pdf

Spikol, D., & Eliasson, J. (2010, April). Lessons from designing geometry learning activities that combine mobile and 3D tools. In *Wireless, Mobile and Ubiquitous Technologies in Education (WMUTE), 2010 6th IEEE International Conference on* (pp. 137-141). IEEE.

Traxler, J. (2009). Learning in a mobile age. *International Journal of Mobile and blended Learning*, *1*(1), 1-12.

Tucker, B.F., Singleton, A.H., & Weaver, T.L. (2006) Creating lessons that meet the needs of a diverse classroom. In Tucker, B.F., Singleton, A.H., Weaver, T.L *Teaching mathematics to all children* (2nd Ed.) (pp. 29-44) New Jersey, USA: Pearson.

Vahey, P., Roschelle, J., & Tatar, D. (2007). Using handhelds to link private cognition and public interaction. *Educational Technology*, *47*(3), 13-16.

Watlington, D. (2011, March). Using iPod Touch and iPad educational apps in the classroom. In *Society for Information Technology & Teacher Education International Conference* (Vol. 2011, No. 1, pp. 3112-3114).

Webb, D.C. (2013). Bloom's taxonomy in mathematics education. In Steve Lerman (Editor-in-Chief), *Encyclopedia of Mathematics Education.* SpringerReference. Retrieved April 1, 2013 from: http://www.springerreference.com/docs/html/chapterdbid/313196.html

Zoric, I., Cindric, M., & Destovic, F. (2012). Traditional and contemporary approach to teaching mathematics. *TTEM- Technics Technologies Education Management*, *7*(3), 882-888.

Acknowledgement

All images were sourced from the Apple Store at www.apple.com/itunes

Appendix

Appraising Maths Apps

Maths apps are created to serve specific roles in teaching and learning across the school curriculum. Depending on their role maths apps can be classified either as explorative, productive or instructive, or as a combination of one or more of these.

Explorative apps: for exploring and demonstrating mathematical models or concepts through manipulating objects that mimic or simulate complex physical situations, e.g.:

Sketchpad Explorer

Move the Turtle

Weighing

Productivity apps: for measuring and graphically representing objects or concepts in 2D/3D, collecting data, making calculations, or creating multimedia materials, e.g.:

Protractor

Stopwatch

GeoBoard

Instructive apps: for practicing content through drill exercises, acquiring new skills through questions and answers (tutorials), or retrieving factual information, e.g.:

Math Dictionary

Mathemagics

Math Paradise

Instructions:

1. Investigate the app thinking about its role in teaching and learning mathematics – there are no right or wrong answers.
2. Choose any of the three roles described above– *You can choose a combination where roles overlap.*
3. Go to the relevant section(s) next page 6 and 7 where specific issues are presented for your appraisal
4. Complete items on following page.

Step 1	Choose the app role(s)
Step 2	Explorative app → Go to next page: Section 3
	Productivity app → Go to next page: Section 4
	Instructive app → Go to next page: Section 5
Step 3	Complete items on following pages 4 and 5

Task Structure *Circle any of the three roles outlined below – You can choose a combination where roles overlap.*

SECTION 1: EXPLORATIVE APPS *(please check one of the options for each row)*	**Always**	**To some extent**	**Never**	**Not applicable**
App closely mirrors a model or real-life situation	☐	☐	☐	☐
Students can enter their own data and observe changes in the model	☐	☐	☐	☐
Exploration is guided within a predetermined learning discovery framework	☐	☐	☐	☐
Tasks are goal oriented driving student interest and curiosity	☐	☐	☐	☐
There are elements of ambiguity and uncertainty fostering personal investigation	☐	☐	☐	☐

If you are not doing any other section please continue to next page

SECTION 2: PRODUCTIVITY Apps *(please check one of the options for each row)*	**Always**	**To some extent**	**Never**	**Not applicable**
App lets students to creatively come up with their own design and/or concept	☐	☐	☐	☐
App allows representing maths content by linking symbolic, numerical and graphical data	☐	☐	☐	☐
Students are guided in creating their own content/understandings	☐	☐	☐	☐
Students can represent or present maths content using a variety of different tools (e.g., audio/video recording, measuring devices, etc)	☐	☐	☐	☐
App tools are intuitive and easy to use	☐	☐	☐	☐

If you are not doing any other section please continue to next page

SECTION 3: INSTRUCTIVE Apps *(please check one of the options for each row)*	**Always**	**To some extent**	**Never**	**Not applicable**
App contains a variety of different activities/exercises	☐	☐	☐	☐
Appropriate feedback is provided to students	☐	☐	☐	☐
Activities/exercises cater for a range of student ability levels	☐	☐	☐	☐
Content is meaningful, fostering engagement and rich problem solving	☐	☐	☐	☐
App contains activities/exercises that are graded and summary data is provided	☐	☐	☐	☐

If you are not doing any other section please continue to next page

Cognitive Involvement

The app encourages students to … *(please check one of the options for each row)*	**Always**	**To some extent**	**Never**	**Not applicable**
retrieve and review maths concepts/skills/procedures (Remembering)	☐	☐	☐	☐
demonstrate understanding of maths concepts/skills/procedures (Understanding)	☐	☐	☐	☐
apply their knowledge of maths concepts/skills/procedures in practical contexts (Applying)	☐	☐	☐	☐
critically analyse maths content in text, graphs and/or animations (Analysing)	☐	☐	☐	☐
appraise and justify maths ideas or products (Evaluating)	☐	☐	☐	☐
construct new and meaningful maths ideas or products (Creating)	☐	☐	☐	☐

General Pedagogical Issues

The app … *(please check one of the options for each row)*	**Always**	**To some extent**	**Never**	**Not applicable**
permits students to pose their own problems	☐	☐	☐	☐
allows for differentiation through sequentially designed degrees of difficulty	☐	☐	☐	☐
gives students control over their learning	☐	☐	☐	☐
delivers content in an appealing and motivating way according to the age group	☐	☐	☐	☐
provides meaningful teaching and learning guidelines	☐	☐	☐	☐
integrates maths with content from other Key Learning Areas	☐	☐	☐	☐
allows students to collect and record their own data	☐	☐	☐	☐
shows a reading level appropriate to the student's level	☐	☐	☐	☐
saves and keeps students' work	☐	☐	☐	☐
provides opportunities for collaboration	☐	☐	☐	☐

Continue to next page …

Operational Issues

The app … *(please check one of the options for each row)*	**Always**	**To some extent**	**Never**	**Not applicable**
has an intuitive and user friendly navigation	☐	☐	☐	☐
contains helpful technical instructions to the user and/or a Help function	☐	☐	☐	☐
lets students alter its settings to customise the app to their needs	☐	☐	☐	☐
allows file sharing, streaming of content and/or online communications	☐	☐	☐	☐
is flexible permitting students to move in different directions	☐	☐	☐	☐
has a supporting Web page providing additional useful information	☐	☐	☐	☐
easily works with a range of media (audio, video, image, text, animations)	☐	☐	☐	☐
can interface with social media tools (e.g., Facebook, wikis, blogs, Twitter, YouTube)	☐	☐	☐	☐
presents an uncluttered display which is visually stimulating	☐	☐	☐	☐
permits a student leave at any time and begin where he or she left	☐	☐	☐	☐

Write here any other comments you might have about the quality of your maths app.

-END OF THE APPRAISAL-