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Large-Scale Modes of the Tropical Atmosphere. Part I: Analytical Modeling of Convectively Coupled Kelvin Waves Using the Boundary-Layer Quasiequilibrium Approximation

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One way of modeling the convectively coupled Kelvin waves in an equatorial non-rotating atmosphere is presented. It implements a simple linear, analytical model using the boundary-layer quasiequilibrium approximation and wind-induced surface heat exchange. The dynamics of the model are based on the assumption that the vertical heating profile has the shape of the first baroclinic mode. The vertical velocity has two sinusoidal components of different vertical wavelengths. One component corresponds to deep convection and the imposed heating profile while the other component, with shallower vertical wavelength, defines the phase speed of the convectively coupled Kelvin wave.

The results of the model show fast Kelvin waves that resemble adiabatic modes with the vertical wavelength being twice the depth of the troposphere and convectively coupled Kelvin waves that are damped and propagate with phase speed of 18 m/s. Wind-induced surface heat exchange causes the instability of the convectively coupled Kelvin waves, but only for very long wavelengths.

The value of the model is that under the single dynamical assumption of the vertical heating profile and using the boundary-layer quasiequilibrium assumption it yields the observed phase speed for the convectively couple Kelvin waves.

Keywords: Kelvin waves, convection

1. Introduction

Understanding the physics of tropical atmosphere is a very challenging task. To this day, we still do not have many of the answers. What makes it so complex is that the primary physical mechanisms of middle latitudes do not exist in the tropics. The zeroth order balance in the middle latitudes, called the geostrophic balance, is the balance between the pressure gradients and the Coriolis force. The Coriolis parameter becomes much smaller as we approach the equator. Furthermore, in middle latitudes the biggest cause of inclement weather is the result of strong meridional temperature gradients, called baroclinic instability. The baroclinic instability is not only able to amplify the existing disturbance but is capable of generating the frontal zones. In the tropics the mid-tropospheric temperature is close to uniform, *i.e.* strong meridional temperature gradients do not exist.

In the tropics, the most important process is deep convection. Deep convection is generally associated with high surface temperature distribution (the higher the temperature, the stronger the deep convection), mainly high sea surface temperature (SST), as the ocean dominates the tropical area. The ocean is a major source of water vapor that is lifted into the atmosphere. The water vapor reaches the condensation level and creates clouds. Because of a substantial cloud cover in the tropics, the basic balance between the incoming solar radiation and outgoing longwave radiation is disturbed. When the clouds are stratiform and dense, the outgoing radiation is trapped in a similar effect to the greenhouse effect. Through the surface fluxes, wind comes into play through a mechanism called wind-induced surface heat exchange. The largest area of deep convection is called the intertropical convergence zone (ITCZ), a disturbance that exists around 8 to 12 °N and shifts to the south as we approach the Northern winter and exists all the way around the globe. In section 2, we give a short overview of the large-scale disturbances in the tropics. In section 3, we present a simple analytical model for convectively coupled Kelvin waves using the boundary-layer quasiequilibrium approximation, while section 4 gives the results and conclusions.

2. Short overview of large-scale disturbances in the tropics

Deep convection couples with large-scale disturbances. Satellite and other observations show that the large-scale disturbances go always in a form of a wave. The observed waves are the Madden-Julian oscillation (MJO), Kelvin waves, inertio-gravity waves, equatorial Rossby waves and mixed Rossby-gravity waves (Wheeler and Kiladis, 1999).

Since 1966 and Matsuno's work (Matsuno, 1966) on equatorial waves we know how all the waves except for MJO look in the ideal atmosphere where the flow is adiabatic. Matsuno developed a fundamental theory for the free gravity, Kelvin and Rossby modes based on linearized shallow water equations in a rotating atmosphere. The gravity waves are formed because the air parcels that get lifted in stable stratiform environment get pulled back to their original position by the force of gravity. The Kelvin waves are a type of gravity wave where the meridional velocity is zero. When coupled with convection they propagate eastward with a phase speed of 17 m/s whereas the free modes move with speed of 50 m/s. The inertio-gravity waves are gravity waves in a rotational atmosphere and propagate eastward and westward with a phase speed of 25–50 m/s. The Rossby waves are waves whose restoring force is the horizontal rotational force or Coriolis force that decreases as we approach the equator. Because of the conservation of angular momentum, as planetary vorticity caused by Coriolis force decreases as the air parcels move towards the equator, the relative vorticity has to increase which creates a cyclonic rotation and vice versa. The convectively coupled equatorial Rossby waves propagate westward with a phase speed of 5 m/s. The mixed Rossby-gravity waves propagate westward with speed of 20 m/s.

The greatest variability observed in the tropics is due to the Madden-Julian oscillation (MJO). In 1971 Madden and Julian (Madden and Julian, 1971) discovered the MJO. They analyzed the dataset of rawinsonde data for tropical stations where the longest available data of 10 years was available for Canton Islands (2.8 °S, 171.7 °W). Coherence analysis showed a strong coherence between the surface pressure and zonal wind and temperature at various height levels over a broad period range maximizing between 41 to 53 days. Phase angle analysis of those fields with the fields in Balboa (9 °N, 79.6 °W) showed the eastward propagation of the oscillation.

Since 1971, observation and theoretical studies have tried to shine some light onto the MJO. From the observational studies, the most recent one being by Kiladis et al. (2005), we know that the MJO is generally proceeded by low level convergence moistening and upward motion in the lower troposphere, while upper troposphere descent, cooling, and drying still prevail. Low level moistening and warming due to shortwave radiation and surface fluxes initially results in destabilization of the boundary layer and development of shallow convection. This is then followed by gradual and then at the onset of deep convection by more rapid lifting of the moisture into the middle troposphere by congestus clouds. After the passage of heaviest rainfall, stratiform precipitation follows accompanied by westerly wind bursts. The disturbance moves with the phase speed of 5–10 m/s and it takes 30–60 days to travel around the globe. The theoretical studies have proposed numerous models for the MJO. As it is the disturbance that propagates eastward it has been suggested that it is some form of the Kelvin wave. It exhibits somewhat cyclonic behavior in zonal wind at 850 hPa so it was thought that it could be a Rossby-Kelvin wave. The ocean-atmosphere coupling has been suggested to be of the most importance as well as the cloud-radiation interactions. These are just some of many ideas that were explored during the past decades. Unfortunately, none have succeeded. As of today, we still do not know what is the governing mechanism for the MJO. What is its restoring force? General circulation models do a very poor job forecasting the MJO. They either fail to model it all together or when they do, it is either too strong or too weak. What we lack is a fundamental theory that answers the question of governing mechanism of the MJO. What we also lack is a theory that would capture the coupling of Matsuno modes with deep convection, *i.e.* the reason for the observed phase speeds, which are less than those for the free modes as well as the reason for their instabilities. Still, forecasting of convectively coupled Matsuno modes gives much better results than forecasting of the MJO, but the models rely on imposed vertical wavelengths of the modes, the right phase speed of their propagation is a result of an a priori assumption rather than calculated from dynamical hypothesis.

There have been two main streams in modeling the coupling between the large-scale disturbances and deep convection. One is based on convergence hypothesis and implies that primarily through low-level moisture convergence we excite the instability in the large-scale wave motion. The other one is based on convective available potential energy (CAPE) and implies that increased CAPE results in increased precipitation. Implementing those ideas into the models failed to model the MJO. Raymond (2000) proposed a simple direct relationship between the precipitation rate and the water vapor mixing ratio, which simply says that precipitation rate directly increases with precipitable water or the vertically integrated mixing ratio. In their articles, Fuchs and Raymond (2002, 2005) implemented that idea on a simple model for large-scale modes and got an unstable mode usually referred to as the moisture mode (Sobel and Horinouchi, 2000 and Sobel et al., 2001), which might resemble the MJO.

In this paper, we would like to show one way of modeling the convectively coupled Kelvin waves. It implements a simple linear, analytical model using the boundary-layer quasiequilibrium approximation (BLQ) (for details on BLQ see Raymond, 1995 and Emanuel, 1995). The method succeeds in modeling the convectively coupled Kelvin wave without a priori adjusting its vertical wavelength.

3. Boundary-layer quasiequilibrium approximation

It turns out that there is a way of getting the right dynamics of the convectively coupled Kelvin wave through the boundary-layer quasiequilibrium approximation (BLQ). Together with BLQ we are going to include wind-induced surface heat exchange (WISHE), the mechanism that will make the Kelvin mode unstable, but only for long wavelengths. We place the latent heating source at the upper troposphere (Straub and Kiladis, 2002) and the boundary layer at the height 2 km. The boundary layer height, z_d, is the height of the maximum potential temperature and the minimum convective inhibition (CIN). CIN is associated with the strength of convection, which can be measured through the downdrafts. That implies including buoyancy perturbation, b_d, scaled from dry entropy perturbation at 2 km into the equations. The cloud-radiation effects are treated as constant and we shall eventually ignore them as Fuchs and Raymond (2002, 2005) showed the robustness of Kelvin mode to cloud-radiative instability. We use the BLQ approximation and write the equation for scaled moist entropy, e_d, in the boundary layer:

$$\frac{\partial e_d}{\partial t} = \overline{S}_{ed} = \overline{S}_{eR} + \frac{F_{es} - F_{et}}{d} \tag{1}$$

where the index d means in the boundary layer. $\overline{S}_{ed}$ is the moist entropy source term. $\overline{S}_{eR}$ is the averaged radiation source term that we will ignore as small in planetary boundary layer (PBL), F_{es} is a scale flux at the surface that will include WISHE, while F_{et} is a small scale flux at the top of the boundary layer that will represent the downdrafts. d is the depth of the boundary layer. The moist static stability Γ_e is zero as in PBL the air is well mixed and $dS_0/dz = 0$. S_0 is the mean moist entropy.

Defining the scaled moist entropy at the top of the boundary layer as $e_t = e_{sat}$ (2 km), where index sat means saturated and using the equality $e_{sat} = b_t + q_{sat}$, q being the scaled mixing ratio, after the perturbation method we are left with: $e'_{sat} = b'_t$ where index t means the top of the PBL. We now express F_{et} through vertically integrated scaled buoyancy source term S_b:

$$F'_{et} \equiv D\int_0^h S_b dz \tag{2}$$

where h is the top of the troposphere. In further write-up we do not specify the perturbation values specifically, *i.e.* $F'_{et} = F_{et}$. The equation (2) states that the downdrafts are proportional to the heating due to the precipitation with a constant D. Therefore, D is a ratio between the heating due to the precipitation and entropy flux out of the boundary layer and is approximated through mass fluxes. The moist entropy is higher at the top of the boundary layer than in the middle of the troposphere, therefore higher entropy goes up while the lower entropy comes down via the downdrafts. It then lowers the boundary layer moist entropy and enhances the convection.

Combining (1) and (2) we get:

$$\frac{de_d}{dt} = \frac{F_{es} - F_{et}}{d} = \delta u_s - \frac{D}{d}\int_0^h S_b dz \tag{3}$$

where $\delta = C\eta\Delta q/d$ is WISHE parameter. C is the transfer coefficient, $\eta = -u_0 / u_{eff}$ is the negative ratio between the velocities of the ocean relative to the ambient air and the effective wind. For strong ambient easterly winds that we choose in WISHE $\eta = -1$. u_s is the surface wind velocity and Δq is the scaled difference between the saturation mixing ratio at the sea surface temperature and the subcloud mixing ratio. The vertically integrated buoyancy source term can be written as:

$$\int_0^h S_b dz = A(e_d - e_t) = A(e_d - b_t) \tag{4}$$

This is our BLQ approximation and A is a constant. If we derive it with respect to time and use the equation (3) we get:

$$\frac{d}{dt}\int_0^h S_b dz = A\left(\frac{de_d}{dt} - \frac{db_t}{dt}\right) = A\left(\delta u_s - \frac{D}{d}\int_0^h S_b dz - \frac{db_t}{dt}\right) \tag{5}$$

$B = \int_0^h S_b dz$ where $S_b(z) = B\eta(z)$. We assume a simple heating profile such that $\eta(z) = E\, sin(m_0 z)$ where $m_0 = \pi/h$, h being the depth of the troposphere. This heating profile is saying, basically, that the heating has a maximum in the middle of the troposphere. If we normalize $\eta(z)$ by requiring that its integral through the troposphere is one, we get that $E = \pi/2h$. The differential equation for B is:

$$\frac{dB}{dt} + \frac{AD}{d}B = A\delta u_s + w_d A\Gamma_B - AS_{bd} \tag{6}$$

where we call $\tau = d/AD$ boundary relaxation time. B has $\exp(-i\omega t)$ dependence in time, w_d and u_s are given by equations:

$$w_d(z) = \frac{BE}{\Gamma_B(1-\Phi^2)}\left[\sin(m_0 z_d) + \Phi \exp\left(-i\frac{\pi}{\Phi}\right)\sin(m z_d)\right] \tag{7}$$

$$u_s = \mathrm{i}\,\frac{m_0 BE}{k\Gamma_B(1-\Phi^2)}\left[1 + \exp\left(-i\frac{\pi}{\Phi}\right)\right] \tag{8}$$

where Γ_B is Brunt-Vaisala frequency squared, $m = \Phi m_0$, Φ being the dimensionless phase speed. The equation (7) comes from satisfying the equation of motion, hydrostatic equation, continuity equation and the buoyancy equation with the imposed vertical heating profile of first baroclinic mode, for more details see Fuchs and Raymond (submitted 2006). It is consisted of two sinusoidal components where the first one corresponds to the deep convective component while the second one stands for the shallower component of the mode. The equation (8) is simply the surface zonal wind obtained from the continuity equation where the equation (7) is used for vertical velocity. We are now ready to write the dispersion relation:

$$\kappa\Phi^3 + i[1 + \tilde{A}\sin(m_0 z_d)]\Phi^2 - \kappa\Phi - i +$$

$$i\tilde{A}\Phi\exp\left(-i\frac{\pi}{\Phi}\right)\sin\left(\frac{\pi}{\Phi}\frac{z_d}{h}\right) - \frac{\Lambda}{\kappa}\tilde{A}\left[1 + \exp\left(-i\frac{\pi}{\Phi}\right)\right] = 0 \tag{9}$$

where z_d is 2 km, $\Phi = \Omega/\kappa$ is nondimesionilized phase speed, $\Omega = \omega\tau$. The nondimesionilized horizontal wavenumber is $\kappa = \tau k \Gamma_B^{1/2} / m_0$. Nondimensional constant $\tilde{A}$ is: $\tilde{A} = AE\tau$, and WISHE parameter is: $\Lambda = \delta\tau/\Gamma_B^{1/2}$. We solve the

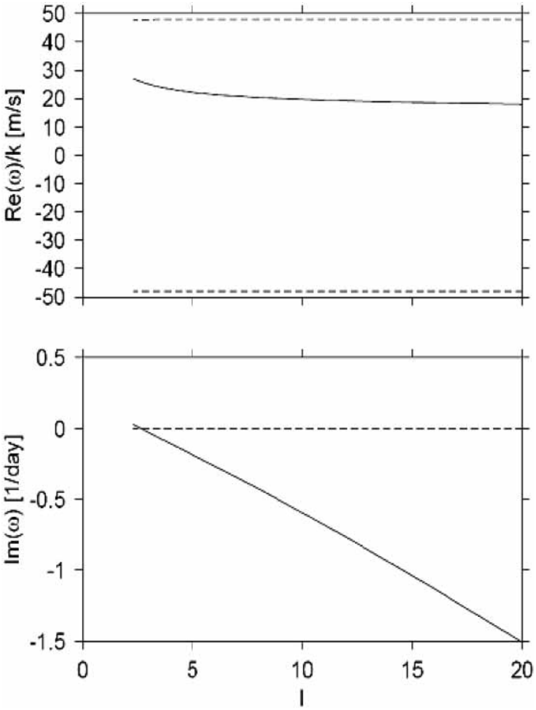

Figure 1. Dispersion curves for free (dashed line) and coupled Kelvin mode (solid line) as a function of planetary wavenumber l. The upper panel shows the dimensional phase speed while the lower plot shows the growth rate of the modes.

dispersion relation for Φ numerically using the Newton's method for the range of planetary wavenumber l from 2 to 20. The planetary wavenumber can easily be obtained from the dimensionless wavenumber. The real part of the solution represents the phase speed while the imaginary part represents the growth rate. Because of the defined dependence of the fields in time the positive imaginary part corresponds to growing modes and negative to decaying or damped modes.

4. Results and conclusions

The results to a dispersion relation (9) are shown in figure 1. The top panel shows the phase speed while the bottom one shows the growth rate. The dashed line represents the free Kelvin modes that are damped slightly and travel with a phase speed of 48 m/s. The solid line represents the coupled Kelvin mode that for the planetary wavenumber l from 10 to 20 (horizontal wavelength 4000 to 2000 km) propagates eastward with a phase speed of

18 m/s which is in agreement with the observations, Straub and Kiladis (2002). However, it is damped for the realistic wavelengths and unstable only for long wavelengths because of the WISHE mechanism.

The model of Fuchs and Raymond (2002, 2005) failed to produce the right phase speeds of the observed convectively coupled Kelvin wave, but it produced an unstable slow moisture mode that propagates eastward under the influence of WISHE and is stationary otherwise. In this model, we do not include the moisture budget, a simple parameterization where the precipitation rate directly increases with precipitable water with the moisture relaxation rate of one day, and so we cannot model the moisture mode. The convectively coupled modes or equatorial waves of the model from Fuchs and Raymond (2005) were: Kelvin waves, mixed Rossby-gravity waves, Rossby waves, and inertio-gravity waves that propagated with the same phase speed as the free adiabatic Matsuno's modes with vertical wavelength twice the depth of the troposphere. The model presented in section 3 for Kelvin modes tries to answer a question of reduced phase speeds of the convectively coupled waves, as we know them from the observations. It is done on a nonrotating atmosphere, which means that the modeled normal Matsuno's modes can only be the Kelvin modes. The only assumption that is made in a model is the fixed vertical heating profile that is assumed to have a vertical structure of the first baroclinic mode. The modeled modes are Kelvin modes with the same phase speed as the free adiabatic modes and convectively coupled Kelvin modes of the phase speed 18 m/s, which is in good agreement with the observations. The diabatic processes damp the Kelvin modes.

The forced vertical profile of the heating gives two sinusoidal components in vertical velocity perturbation, one that corresponds to the imposed deep convection profile and the other that is shallower and governs the phase speed of the convectively coupled Kelvin mode. This is an improvement from the existing parameterization for the phase speed of convectively coupled Matsuno modes, and then the Kelvin modes. The free adiabatic Kelvin wave phase speed is a function of vertical wavenumber and the dry static stability. Most models of today simply vary the vertical wavenumber and force the phase speed to be the observed one. Alternatively, we can replace the dry static stability with effective static stability (Neelin et al., 1987; Neelin and Held, 1987; Emanuel et al., 1994; Neelin and Yu, 1994) and reduce the phase speed of convectively coupled modes without reducing the vertical scale.

Although the convectively coupled Kelvin waves account for second largest outgoing longwave radiation (OLR) variance after the Madden-Julian oscillation they are stable in this model. This problem remains a challenge.

This model gives us better theoretical understanding of the large-scale interactions with deep convection, enabling better understanding and parameterization of the numerical models which can in the future lead to better forecast of the tropics.

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SAŽETAK

Dugoperiodički modovi u tropskoj atmosferi. Prvi dio: Analitičko modeliranje združenih Kelvinovih valova korištenjem kvaziravnotežne aproksimacije graničnog sloja

Željka Fuchs i Antun Marki

U radu se prezentira jedan način modeliranja konvektivno združenih Kelvinovih valova u ekvatorijalnoj nerotirajućoj atmosferi. Ono uključuje jednostavni linearni, analitički model koristeći kvaziravnotežnu aproksimaciju u graničnom sloju i površinsku izmjenu topline uzrokovanu vjetrom. Dinamika modela temelji se na pretpostavci da vertikalni profil grijanja ima oblik prvog baroklinog moda. Vertikalna brzina ima dvije sinusoidalne komponente različitih vertikalnih valnih duljina. Jedna odgovara dubokoj konvekciji i nametnutom profilu zagrijavanja, dok druga komponenta kraće vertikalne valne duljine definira faznu brzinu konvektivno združenog Kelvinovog vala.

Rezultati modela su brzi Kelvinovi valovi koji sliče adijabatskim modovima s vertikalnom valnom duljinom dvostruko većom od dubine troposfere i konvektivno združenih Kelvinovih valova koji su prigušeni i propagiraju faznom brzinom od 18 m/s. Površiinska izmjena topline uzrokovana vjetrom čini konvektivno združene Kelvinove valove nestabilnim samo za vrlo duge valove.

Vrijednost modela je u tome da pod jednostavnom pretpostavkom vertikalnog profila grijanja i korištenjem kvaziravnotežne aproksimacije graničnog sloja daje opaženu faznu brzinu konvektivno združenih Kelvinovih valova.

Ključne riječi: Kelvinovi valovi, konvekcija

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