An Experimental and Theoretical Study on the Dilatancy of Sand and Clays

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Summary: Rowe's (1962) stress–dilatancy relationship for sand, and Schofield and Wroth (1968) (Roscoe *et al.*, 1963), and Roscoe and Burland (1968) energy equations for soft clays are classical work in describing the stress-dilatancy behaviour of sand and clay. However, the stress-dilatancy behaviour of soils is affected by the stress and strain paths followed by the soil and it is difficult to use a single equation to model the stress-dilatancy behaviour of all soils. In this paper, the historical development of the stress-dilatancy theories is briefly reviewed. The generalised stress-dilatancy behaviour of clay and sand is presented. Comparisons between the experimental observations and the theoretical predictions are made. The suitability of different stress-dilatancy theories to different soils under different testing conditions is examined. The data presented can also be used to examine other stress-dilatancy theories proposed by other researchers.

INTRODUCTION

Dilatancy of soils is an important phenomenon that differentiates the plastic behaviour of soil from that of metal. It is also one of the deformation characteristics of soil that cannot be modelled properly by the classical plasticity that was developed essentially for metals. Whether the dilatancy behaviour of soil can be modelled properly has been an important criterion in examining the usefulness of any plastic models for soil. Since the pioneer work of Osborn Reynolds, more experimental studies on the stress-dilatancy behaviour of soil have been made. It is generally understood now that the normally and lightly overconsolidated clays as well as loose sand develop positive pore pressure during undrained shear or tend to reduce in volume under drained conditions. On the other hand, heavily over consolidated clays and dense to very dense sand develop negative pore pressures during shear and tend to increase in volume under drained conditions. Based on the experimental observations, several stress-dilatancy theories have been developed. The critical state concepts of the work of Roscoe, Schofield, Wroth, Pooroshasb, Thurairajah and Burland in the development of a successful stress-strain theory foe soils obtained the plastic dilatancy ratio in terms of an energy dissipative function, which used the critical state parameter, M dependent on the angle of friction of the soils. Such an expression has its origin in the work of Taylor at MIT on the interlocking behaviour of sand and the boundary energy corrections for shear in the direct shear apparatus. Rowe (1957) at the University of Manchester worked in parallel with the Roscoe Group at Cambridge developed the most successful stress-dilatancy relation. Experimental behaviour of soils in the laboratory and in the field indicated that the Cambridge dilatancy relation modelled the behaviour of normally consolidated clay and loose sand and the Rowe's (1962) stress- dilatancy relation is more appropriate in the case of dense sand.

The work of Drucker, Poorooshasb and Caladine paved the way to relate the plastic dilatancy ratio to yield locus and plastic potentials and this in turn helped to derive an undrained stress path, which indicated the magnitude of the development of pore pressures in clayey soils. Extensive experimental studies performed by Balasubramaniam (1969), favoured a non-associated flow rule of the type originally formulated by Roscoe and Pooroshasb (1963), and helped to explain the plastic shear strains of overconsolidated clays within the state boundary surface. Based on the work of Loudon (1967), Roscoe and Burland proposed a second set of constant deviator stress yield loci within the state boundary surface, to obtain the undrained component of the plastic shear strain which needs to be added to the strains from the volumetric yield loci obtained from an associated flow rule to fully describe the shear strains for stress paths of the non radial type in which the stress ratios increase or decrease monotonically during drained and undrained shear. Pender (1978) was the first to develop a plastic dilatancy relation for overconsolidated clays which can capture the development of positive pore pressures in normally and lightly overconsolidated clays and negative pore pressures in heavily overconsolidated clays. The extensive experimental work carried out by Balasubramaniam and his co-workers at AIT on the stress-strain behaviour of soft Bangkok clay below the state boundary surface seem to support in most instances the work of Pender (1978), while various types of refinements seem better in modelling the detailed behaviour. These data are presented here on clays in the hope that this work would be helpful in developing better stress strain models in line with the work of Puzrin and Houlsby (2001), Collins and Kelly (2002) and others. The stress-dilatancy responses of Sydney sand to various stress and strain paths are presented. A general stress-dilatancy relationship is also proposed based on the experimental observations.

STRESS-DILATANCY BEHAVIOUR OF CLAY

One of the most popular stress-dilatancy relationships for clay is the one adopted in the Cam-clay model (Schofield and Wroth 1968). Based on the work hypothesis:

$$dW = p' d\varepsilon_v^p + q d\varepsilon_s^p = Mp' d\varepsilon_s^p \tag{1a}$$

We have:

$$\frac{q}{p'} = M - \frac{d\varepsilon_v^p}{d\varepsilon_s^p} \tag{1b}$$

where $M = 6\sin\varphi_{cs}/(3 - \sin\varphi_{cs})$. Under three dimensional conditions, the plastic work can be written as[1]:

$$dW = p' d\varepsilon_v^p + \beta q d\varepsilon_s^p \tag{2a}$$

where $$\beta = \frac{(3+\mu\nu^p)}{\sqrt{(3+\mu^2)(3+(\nu^p)^2)}} \tag{2b}$$

and μ and ν^p are Lode stress parameter and Lode plastic strain parameter respectively.

Then $$\beta\frac{q}{p'} = M - \frac{d\varepsilon_v^p}{d\varepsilon_s^p} \tag{2c}$$

Under axisymmetric conditions, $\beta = 1$ and Eq. (2c) degenerates to Eq. (1b). It should be noted that a modified relationship to Eq. (1) has also been proposed by Roscoe and Burland (1968).

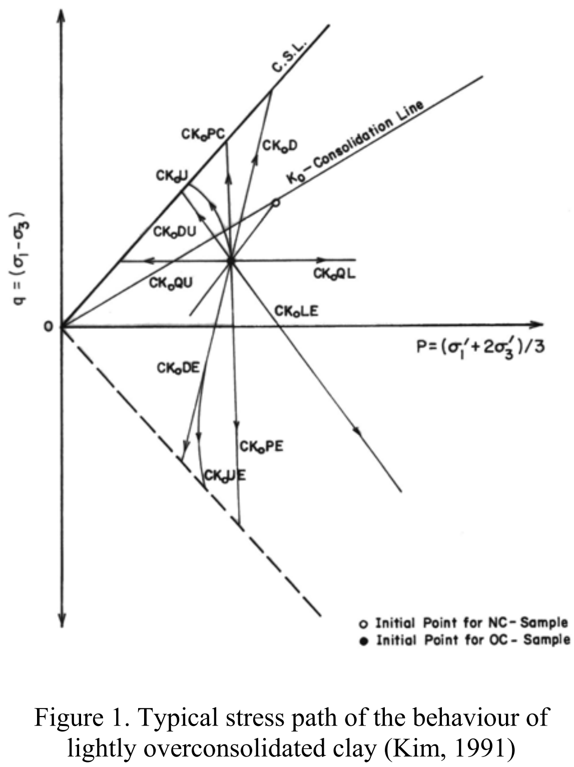

Figure 1. Typical stress path of the behaviour of lightly overconsolidated clay (Kim, 1991)

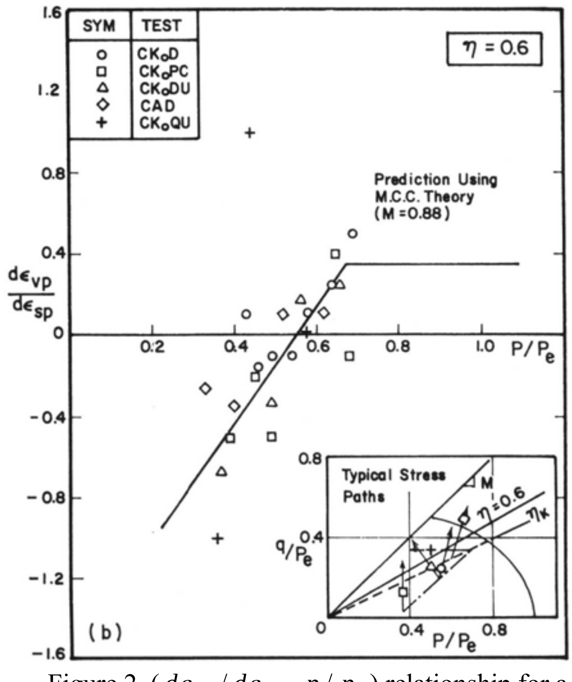

Figure 2. ($d\varepsilon_{vp} / d\varepsilon_{sp}$, p / p_e) relationship for a constant stress ratio line from the K_0 consolidation test.

Kim (1991) made a comprehensive study on the behaviour of soft Bangkok clay below the state boundary surface. He conducted triaxial consolidation and swelling tests (both isotropic and K_0 consolidation and swelling) as well as four series of undrained triaxial tests (three of them are in compression and one in extension)

[1] Under three dimensional conditions, $p = (\sigma_1 + \sigma_2 + \sigma_3)/3$, $q =[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]^{0.5}/\sqrt{2}$, $\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$, and $\varepsilon_s = \sqrt{2}\ [(\varepsilon_1 - \varepsilon_2)^2 + [(\varepsilon_2 - \varepsilon_3)^2 + [(\varepsilon_1 - \varepsilon_3)^2]^{0.5}/3$.

and ten series of drained tests (as shown in Figure 1). Subsequent students performed constant p tests and also stress probing types of tests.

The variation in the plastic dilatancy ratio, $d\varepsilon_{vp} / d\varepsilon_{sp}$ with p / p_e is shown in Figure 2, for a stress ratio η = q/p' = 0.6 as obtained from CID tests on lightly overconsolidated clays. Similar relationships were also established for the entire stress ratio from isotropic consolidation to the critical state both on the compression and extension sides. From these data, the variation of $\left| d\left(\frac{d\varepsilon_{vp}}{d\varepsilon_{sp}}\right) \Big/ d\left(\frac{p}{p_e}\right) \right|$ with $|\eta|$ for compression and extension tests are shown in Figure 3. This figure illustrates how the plastic strain increment vectors rotate within the state boundary surface with respect to stress ratio, η and the reduction of p / p_e. The general expression for the plastic dilatancy ratio on the compression side can be expressed as:

$$\frac{d\varepsilon_{vp}}{d\varepsilon_{sp}} = \left(\frac{M^2 - \eta^2}{2\eta}\right) - \left(\frac{M^2}{\eta} - \eta + \frac{3}{2}\right)\left(\frac{M^2}{M^2 + \eta^2} - \frac{p}{p_e}\right) \tag{3}$$

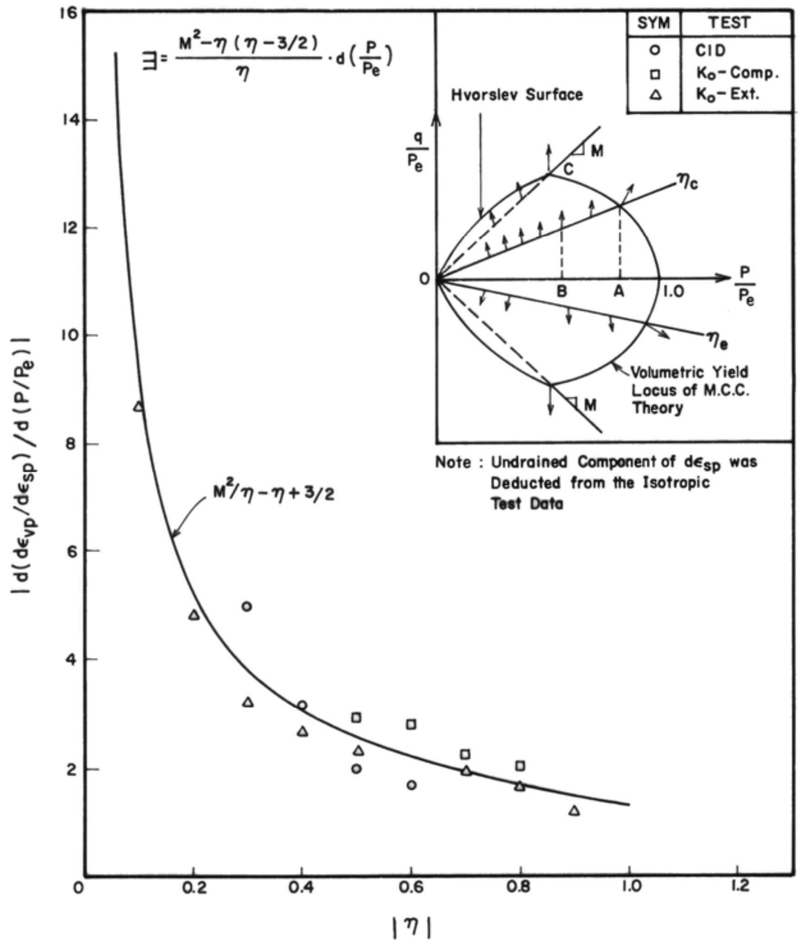

Figure 3. Variation of the ($d\varepsilon_{vp} / d\varepsilon_{sp}$, p / p_e) plot with the stress ratio within the state boundary surface.

This equation will give the same plastic dilatancy ratio as the Roscoe and Burland (1968) theory for normally consolidated clays and for the overconsolidated states will have the ratio at any particular stress ratio progressively reduce with decreasing value of p / p_e. The stress paths within the SBS for the stress probing experiment is shown in Figure 4 in the (q/p$_e$, p/p$_e$) plot. Constant values of $\sqrt{\Delta q^2 + \Delta p^2}$ were obtained on each stress path as shown in Figure 5a and the strain paths in ($\varepsilon_v , \varepsilon_s$) plot are shown in Figure 5b. The strain paths were initially circular and then distorted in shape to somewhat like ellipses.

The dilatancy ratio of graded rockfill, lime treated clay and cement treated clay are shown in Figure 6. These relationships are similar in shape to the prediction of Roscoe and Burland (1968) but different in magnitude.

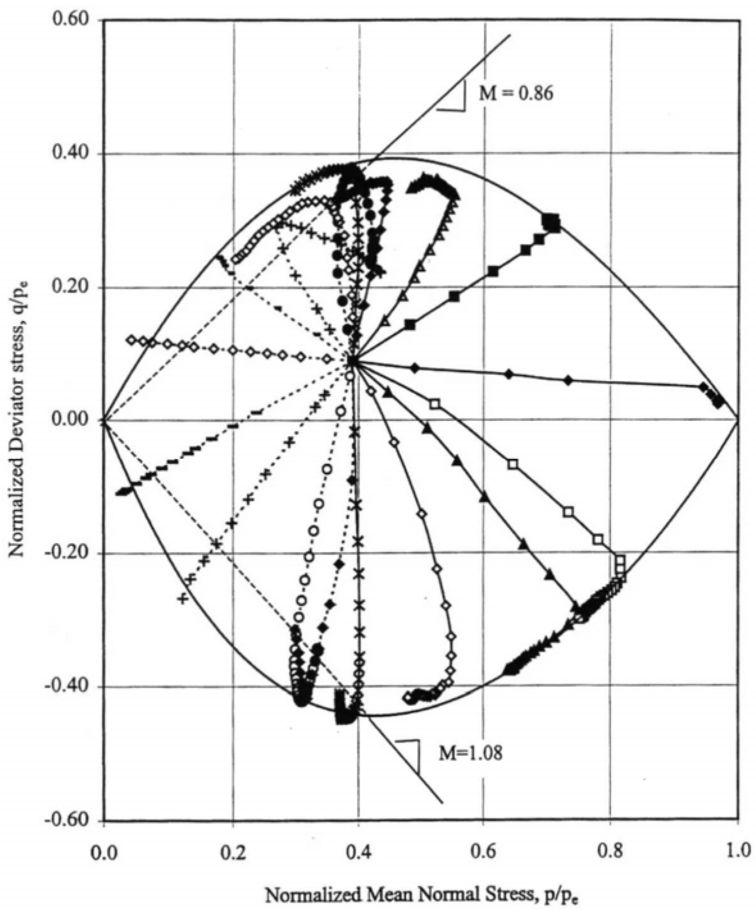

Figure 4. Stress probe paths in (q/p_e , p/p_e) plot (after Khan, 1999).

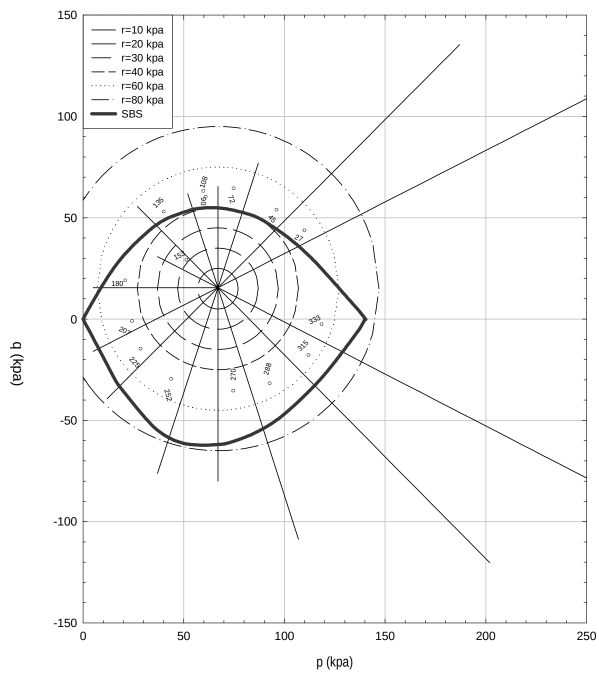

Figure 5a. Stress increment circles (after Pornpong, 2001)

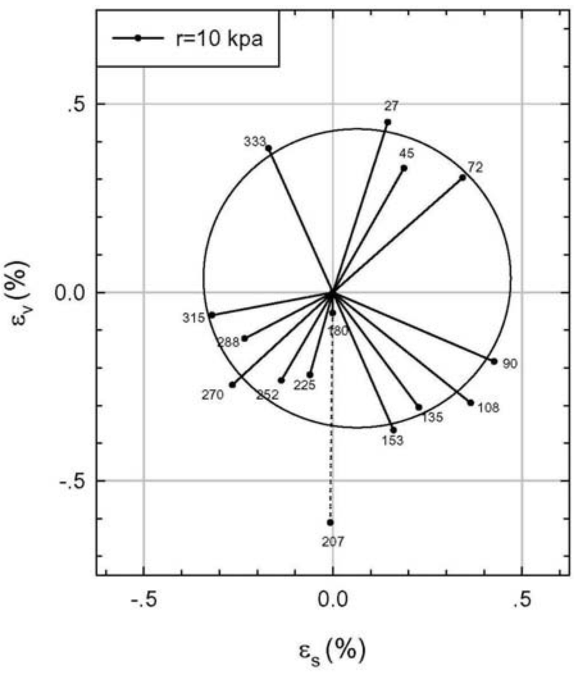

Figure 5b. (ε_v , ε_s) plot for $\sqrt{\Delta p^2 + \Delta q^2}$ = 10 kPa with fitting (after Pornpong, 2001).

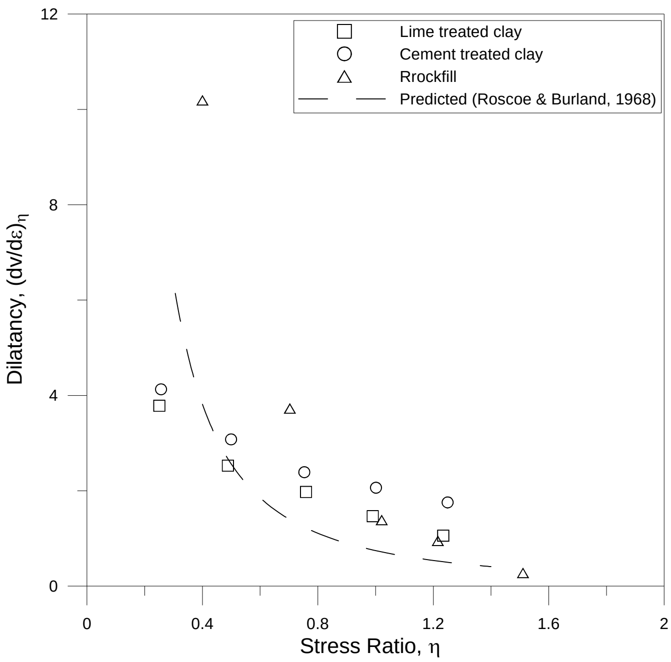

Figure 6. Variation of dilatancy with stress ratio

STERSS-DILATANCY BEHAVIOUR OF SAND

One inherent limitation of the Cam-clay stress-dilatancy relationship is the condition that when η = M, $d\varepsilon_v = 0$, even though it is not at the critical state. This apparently does not apply to dense sand. For sand, Rowe's stress-dilatancy equation (Rowe, 1962; 1966) is more frequently used. Rowe's stress-dilatancy equation is written as:

$$\frac{\sigma'_1}{\sigma'_3} = K\left(1 - \frac{d\varepsilon_v}{d\varepsilon_1}\right) \tag{4}$$

where K is related to an angle of soil friction. Eq. (4) inherently assumes that the soil is rigid plastic so the stress-dilatancy behaviour is not affected by the stress level. To overcome this limitation, a modified Rowe's stress dilatancy equation has been proposed by Wan and Guo (1988), which retains the format of Eq. (3), with K being calculated as:

$$K = \frac{1+\sin\phi^*}{1-\sin\phi^*} \qquad \text{and} \qquad \sin\phi^* = (e/e_{cr})^{\alpha}\sin\phi_{cv} \tag{5}$$

where φ^* is the friction angle mobilized along a certain macroscopic plane which evolves during deformation history, φ_{cr} and e_{cr} are the friction angle and void ratio at the critical state.

Rowe's stress-dilatancy equation has been examined using experimental data by several researchers. Lee (1966) and Lo and Lee (1990) concluded that Eq. (3) could describe the behaviour of dense sand in consolidated drained (CD) triaxial tests well. However, Lee (1966) and Lee and Ingles (1968) also showed that the equation is not applicable to unloading and reloading and the stress-dilatancy relationship for the post-peak region is different from that for the pre-peak region. This is further proven by Chu et al. (1992) and a post-peak stress-strain relationship has been established by Chu et al. (1992). Using data obtained from constant stress ratio (σ_1'/σ_3') and constant stress increment ratio ($d\sigma_1$'/$d\sigma_3$') path tests, Lo and Lee (1990) further observed that in general, Eq. (3) is not applicable to describing the complete stress-dilatancy behaviour of sand along σ_1'/σ_3'and $d\sigma_1$'/$d\sigma_3$' paths. It should also be pointed out that Rowe's stress-dilatancy relationship calibrated using triaxial CD tests may not be applicable to drained tests under three-dimensional conditions. The stress dilatancy responses of dense Sydney sand to a series of constant b (b = $(\sigma_2 - \sigma_3)/(\sigma_2 - \sigma_3)$) true-triaxial tests are presented in Figure 7. It can be seen that the stress-dilatancy relationship is affected by the b value. In general, the larger the b, the higher the stress ratio for a given dilatancy ratio. For drained tests, the stress ratio σ_1'/σ_3' and q/p' is convertible. The Rowe's stress-dilatancy Equation fits the test for b = 0 closely, but is unable to model the other tests with other b values (Chu 1991).

For sand sheared along a σ_1'/σ_3' path, it is observed (Lo and Lee 1990; Chu 1991) that the strain increment ratio results in $d\varepsilon_v/d\varepsilon_1$ being constant. On the other hand, when sand is sheared along a strain path with constant strain increment ratio $d\varepsilon_v/d\varepsilon_1$, the stress path will approach a constant stress ratio σ_1'/σ_3' or q/p' (Chu 1991). Some examples for dense Sydney sand are shown in Figure 8. It is observed from all the stress path and strain path tests that there is a relationship between the stress ratio and the strain incremental ratio and the higher the dilatancy ratio, $-d\varepsilon_v/d\varepsilon_1$, the higher the stress ratio, q/p'. It has been established by Chu (1991, 1994) and Chu and Lo (1994) that this relationship is unique, irrespective of whether stress path or strain path control is used. This relationship can be written as:

$$\frac{q}{p'} = \alpha - \beta \frac{d\varepsilon_v}{d\varepsilon_1} \qquad \text{for} \qquad -\frac{d\varepsilon_v}{d\varepsilon_1} \geq -\left(\frac{d\varepsilon_v}{d\varepsilon_1}\right)_f \tag{6}$$

where α and β are two material constants. For dense Sydney sand, $\alpha = 1.4$ and $\beta = 0.47$. When $-d\varepsilon_v/d\varepsilon_1 \geq (d\varepsilon_v/d\varepsilon_1)_f$, q/p' will reach the failure stress ratio $(q/p')_f$ and will not change. For soil at different densities, $(d\varepsilon_v/d\varepsilon_1)_f$ and $(q/p')_f$ values will be different, but α and β are not sensitive to the variation of density. Chu and Lo (1994) have further proved that the above stress-dilatancy relationship is also applicable to three dimensional conditions when generalised stress and strain parameters are used.

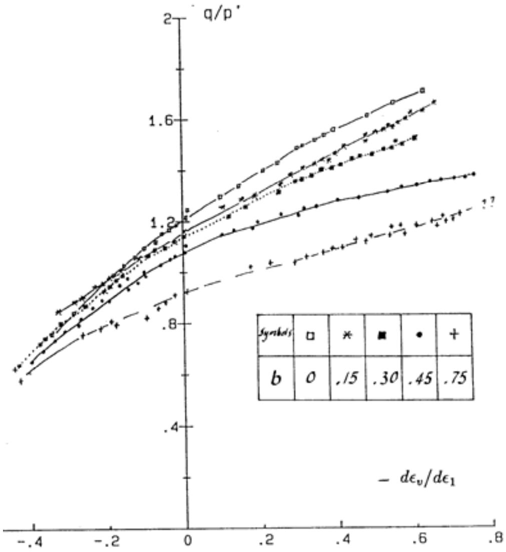

Figure 7. Stress-dilatancy behaviour obtained from constant b tests

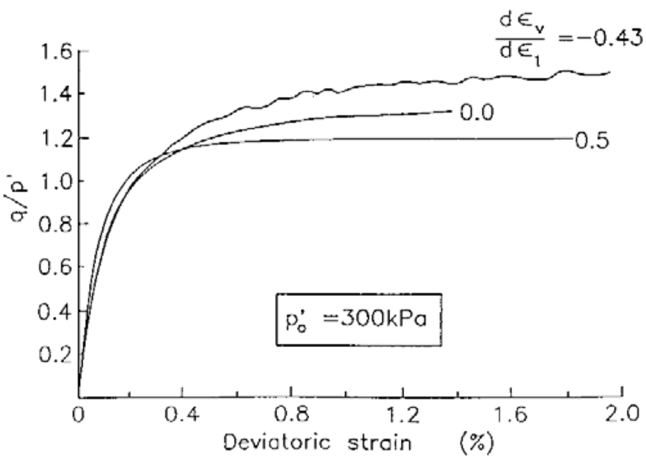

Figure 8. Stress ratio versus strain behaviour of dense sand obtained from constant strain increment ratio paths

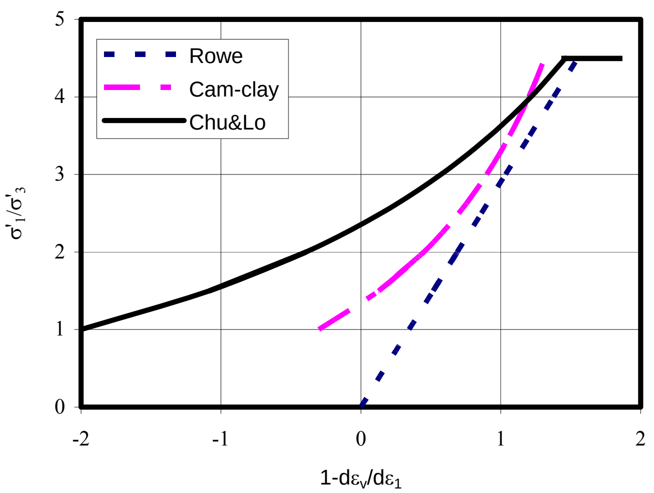

Figure 9. Comparison of different stress-dilatancy relationships

The stress-dilatancy relationship expressed in Eq. (4) is compared with the Rowe's stress-dilatancy equation, Eq. (2) and the stress-dilatancy equation derived in the Cam-clay model, Eq. (1b) in Figure 9 under axisymmetric conditions. For the convenience of comparison, q/p' has been converted into σ_1'/σ_3'. The critical state friction angle for Sydney sand is 32.48^0 and the corresponding $M = 1.3$. $K = 2.9$ is used for Eq. (2) (Chu 1994). It can be seen from Figure 9 that the three stress-dilatancy relationships are all different. Rowe's stress-dilatancy equation represents the CD behaviour of dilative sand, whereas the Cam-clay equation depicts the behaviour of

NC or moderately OC clay. The stress-dilatancy equation of Chu and Lo (1993) is only applicable to the asymptotic state obtained along constant σ_1'/σ_3' or constant $d\varepsilon_v/d\varepsilon_1$ paths, which include undrained and drained paths. Despite of the differences, the 3 curves appear to merge at the failure point, indicating that failure state is independent of the stress or strain paths leading to failure. The horizontal line in Figure 9 indicates that although in general, the stress ratio increases with the dilatancy ratio, the value is capped at the failure stress ratio.

CONCLUSIONS

Some classical stress-dilatancy theories are reviewed and examined using experimental data obtained for clay and sand. As the stress-strain behaviour of soil is greatly affected by the stress path and strain path adopted, no single theory can describe the stress-dilatancy behaviour of all soil. Along constant stress ratio σ_1'/σ_3', stress incremental ratio $d\sigma_1'/d\sigma_3'$, or strain increment ratio $d\varepsilon_v/d\varepsilon_1$ paths, the stress-dilatancy behaviour of soft Bangkok clay is different from that of Sydney sand. Nevertheless, stress-dilatancy relationships between the stress ratio and dilatancy ratio can be established for both clay (Figure 6) and sand (Figure 9). Although the theories of Roscoe and Burland (1968) and Rowe (1962) are able to predict some of those relationships for certain stress paths, each theory has its limitation and further experimental and theoretical studies on the stress-dilatancy behaviour of soil are still required.

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