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SHEARING STRENGTH TESTING ON »ROBERTSON RESEARCH« AND »AMP-1«

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Paper describes shearing machines »Robertson Research« and shearing machine »AMP-1« for »softer« rocks. Maximal horizontal and vertical power for »AMP-1« is 400 kN for sample surface between 100 and 200 cm^2, and can be used for shear tests along the plane of discontinuity of very hard rocks. Average content of $CaCO_3$ in tested samples is 71,39%.

Ključne riječi: Smicanje uzoraka, Uređaj za smicanje, Laboratorijska ispitivanja stijena

U radu su opisani uređaji za smicanje »Robertson Research« i uređaj »AMP-1« za smicanje »mekših« stijena. Maksimalna horizontalna i vertikalna sila za »AMP-1« iznosi 400 kN za površinu uzoraka između 100 i 200 cm^2, a može se koristiti i za ispitivanje smicanja po plohi diskontinuiteta vrlo čvrstih stijena. Prosječni sadržaj $CaCO_3$ u ispitanim uzorcima iznosi 71,39%.

Introduction

Among the many types of investigations of mechanical properities of rock samples, shear strength tests play an important role, since they provide solutions to many problems in rock mechanics, soil mechanics, engineerint geology, complex structure stability calculations, and also for static and dynamic conditions.

The assessment of rock slope problems cannot be evaluated without the knowledge of shear strength parameters. The comprehension of shear strength parameters is necessary during the design of open pits, underground openings and theoretical analysis of rock stress and strain for underground mine construcions.

Historic development of material failure theory is connected to with the first experiments in this field. In the year 1500 Leonardo da Vinci tests the strength of wire, Galileo Galilei in 1638 performed tensile strength tests on various materials, Robert Hook in 1678 describes his experiments on wire, and Edmé Mariotte in 1690 performs tensile strength tests on wood.

The first laboratory was founded in 1871 at the Polytechnic institute in Munich. Its first director was Johann Bauschinger (1833—1893) who was a mechanics professor at the institute where a compression machine (1000 kN) was installed. Bauschinger constructed a mechanic extensiometer for measurement of small deformations which allowed precise measurement of unit extension in the order of magnituede 1×10^{-6}. An experiment on coal was first performed in 1875 in Germany (Timoshenko, 1965). In Berlin in 1871 A. Martens founded a material testing laboratory, and in 1907 he is the first to presuppose that changes of sample dimensions do not change its strength parameters.

Hypothesis of failure

The strength of a rock is a complex function of a number of parameters, ans in a general sense is dependent on normal effective stress, porosity, cohesion, internal angle of friction, mineral and granulometric composition, previous load history, temperature, deformation, structure, type of fluid in pores, etc. These characteristics do not have to be mutually independent. The influence of each single parameter is not well defined well in a quantitative functional sense.

The mechanical properties of technical materials evaluated with testing machines in which the samples are exposed to simple strain. Most data about resistivity of metals is obtained from tensile strength tests, while materials such as stones and concrete are exposed to compression tests.

Criterion of failure

The hypothesis of the maximum normal stress, also termed the Rankin hypothesis, stresses that failure occurs when the maximum i. e. the least principal stress exceeds a certain value. For tensile materials this means that yield in an element of a strained body begins when the maximum stress equals the yield point limit of material exposed to tensile strength tests or when the least stress is qual to the yield limit obtained during compression tests. It has also been proved that homogeneous and isotropic material of poor capabilities to resist axial stress can without becoming viscous withstand large hydrostatic stress. This obviously implies that the value of normal stress alone does not define conditions at which material becomes viscous or when failure of material occurs.

The hypothesis of failure ascribed to Saint Venant is termed the hypothesis of the maximum deformation. This theory presumes that tensile material becomes viscous when the maximum dilatation equals the dilatation limit realized during tensile strengths tests, or when the least deformation is correspondent to the deformation at the yield limit obtained by compression tests. Experimental results on tensile materials are in much better agreement with the hypothesis of maximum shear stress. According to this hypothesis yield begins when the maximum shear strain in a material is equal to the maximum shear strain at the viscosity limit achieved by tensile strength tests. Maximum shear strain is equal to the half of the difference between maximum and least pricipal stress i. e. it is equal to half the normal strain obtained by uniaxial tensile strength tests. During machine desing the maxiumum shear stress hypothesis is used. Experiments performed by J. J. Guesta confirmed this hypothesis (Timoshenko, 1966).

The criterion of plasticity for isotropic materials is given in simple form as $f(s_1, \sigma_2, \sigma_3) = 0$, where the principal stresses σ_1, σ_2, and σ_3 have an equal influence on yield. Tests performed on ansitropic materials show that yield is independent of uniform triaxial pressure or dilatation. Due to this the yield surface in a coordinate system $(0, \sigma_1, \sigma_2, \sigma_3)$ is a symmetrical body with an axis of symmetry $\sigma_1 = \sigma_2 = \sigma_3$ (Fig. 1). The plane passing through

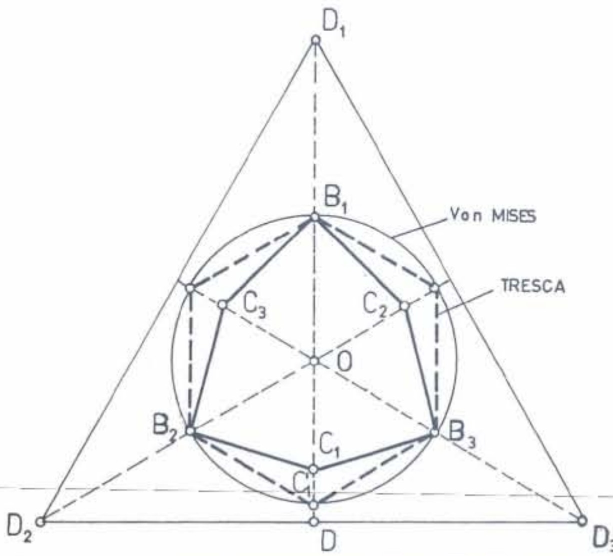

Fig. 2. Flow criterion of deviatoric plane vertically at the line $\sigma_1 = \sigma_2 = \sigma_3$

Sl. 2. Kriterij tečenja u devijatorskoj ravnini, okomito na pravac $\sigma_1 = \sigma_2 = \sigma_3$

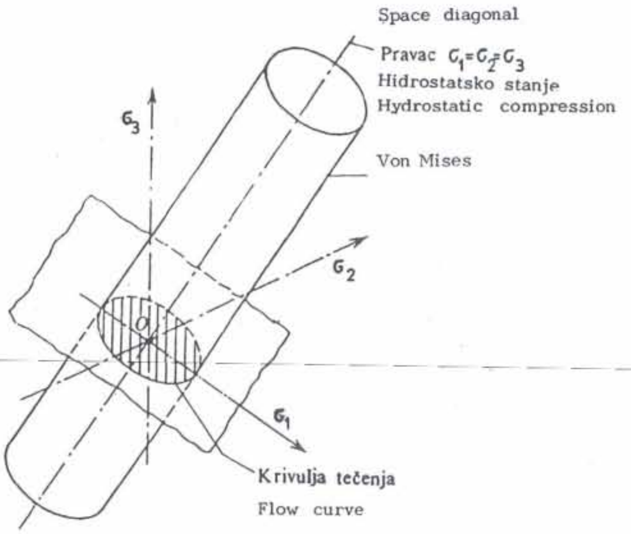

Fig. 1. Surface of the flow against Von Mises criterion

Sl. 1. Površina tečenja prema Von Misesovu kriteriju

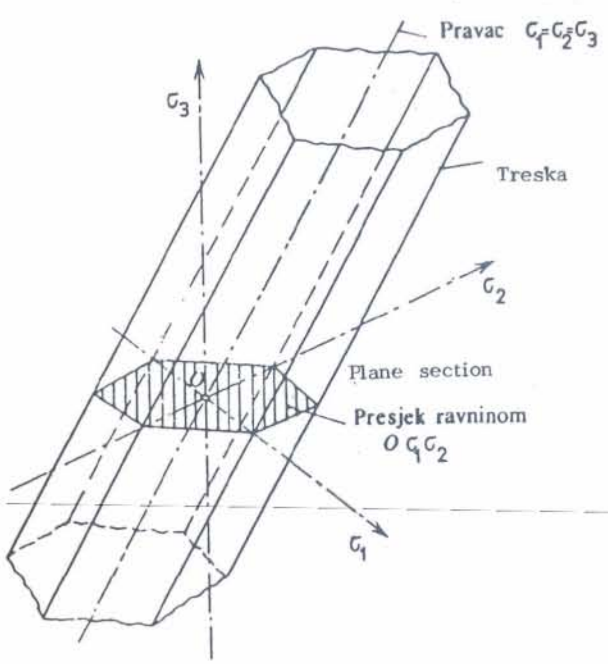

Fig. 3. Surface of flow against Tresca criterion

Sl. 3. Površina tečenja prema Trescinu kriteriju

the inception is termed deviatory plane. The intersection curve between the deviatory plane and the yield surface is called the yield curve. The curve does not pass through the inception point and is convex in shape. The axes S_1, S_2, and S_3, are projected σ_1, σ_2, and σ_3 on the deviatory plane and present the principal values of the deviatory parts of the stress tensor. The possible curves are circular (Von Mises) or hexagon (Tresca) in shape (Fig. 2). The Tresca yield curve is a regular hexagonal cylinder (Fig. 3), whose intersection plane with the plane $0, \sigma_1, \sigma_2$, and σ_3 is an elongate hexagon. A circle is the von Mises criterion, whose yield surface is a circular cylinder. The cylinder intersects the plane $0, \sigma_1, \sigma_2$, and σ_3 as an ellipse.

The Mohr-Coulumb criterion defines the dependence of critical stress and the circular component of the stress tensor and is applicable to materials such as rocks and soils. The representation of the Mohr-Coulumb criterion is a hexagonal pyramid around the axis $\sigma_1 = \sigma_2 = \sigma_3$. An approximation to the Mohr-Coulumb criterion was presented by Drucker and Prager as a modification of the Von Mises criterion of yielding (Owen and Hinton, 1980). The influence of the hydrostatic stress component on yielding by inclsusion of the

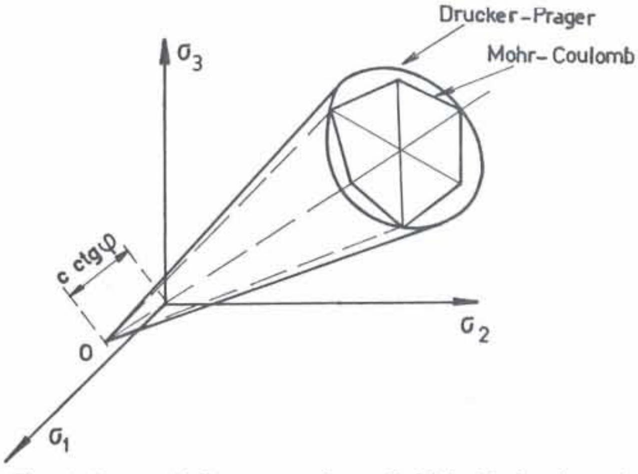

Fig. 4. Geometrical presentation of Mohr-Coulomb and Drucker-Prager criterion of plastic failure
Sl. 4. Geometrijska prezentacija Mohr-Coulomb i Drucker-Prager kriterija plastičnog loma

additional term in the Von Mises expression (Fig. 4) to give:

$$aJ_1 + (J_2')^{1/2} = k' \qquad (1)$$

a — the value of this coefficient is dependent on whether the cylinder is placed inside, or outside the points of the Mohr-Coulumb hexagon
k' — material parameter
J_1 — the first stress infariant, MPa: $J_1 = \sigma_{ii}$
J_2' — the second stress invariant, MPa:

$$J_2' = \frac{1}{2}\sigma_{ij}'\,\sigma_{ij}'$$

Hoek and Brown (1980) uses their experience both in the theory and practical aspects of rock behaviour in order to develop an empirical link between principal stresses defining rock failure:

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_c\sigma_3 + s\sigma_c^2} \qquad (2)$$

σ_1 — maximal normal stress, MPa
σ_3 — minimal normal stress, MPa
σ_c — uniaxial strength, MPa
m and s — constants that depend on the condition of the rock mass.

Griffith criterion of failure

This criterion is based on the presumption placed by Griffith (1921) that failure occurs due to stress concentration in the apex of fissures which are presumed to reach deep into the material and failure occurs when maximum stress near the tip of the most favorably orientated fissure reaches the value characteristic for the material (Jaeger, 1968).

The Griffith parabola eyuation is written as:

$$\tau^2 = 4\,T_o(\sigma + T_o) \qquad (3)$$

T_o — uniaxial tensile strength, MPa.

This criterion ignores the fact that the fissures can close if the stress is high enough. If the fissures close it can be expected that friction will occur between the close surfaces. By taking into consideration these corrections a modified Griffith criterion was proposed by McClintock and Walsh, and Brace (according to Jaeger and Cook, 1968).

»Robertson Research« shear strength testing apparatus

The adventages of this test machine are, its easy handling in field work, lightweight, and a very simple force inflicting system. The hydraulic links are designed for fast and easy connecting. The main advantage of this device is ists capability of testing samples of irregular shape. To determine the force value during test runs the machine is equipped with measuring instruments scaled with 10, 25 and 50 kN. During tests a corresponding measuring instrument is used (Fig. 5) according to the sample type. The drawbacks observed during handling and testing is that the preparation of samples and placing into holders is time-consuming, also the sample section surface is limited to 25 cm².

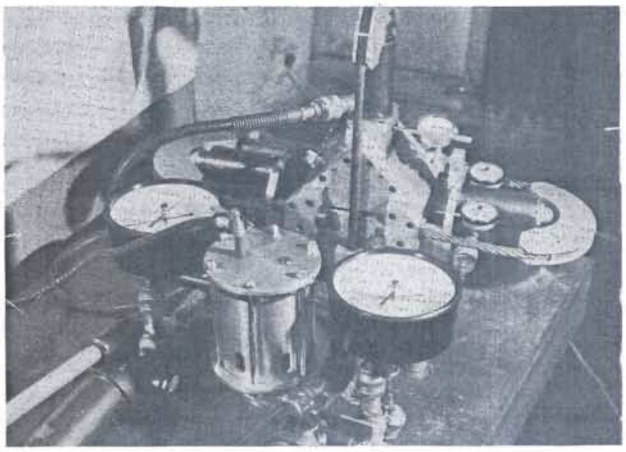

Fig. 5. Shearing machine »Robertson Research«
Sl. 5. Uređaj za smicanje »Robertson Research«

Shear strength testing in the »Robertson Research« testing apparatus

The test results are displayed in Table 1, and their elaboration was performed by the least square method due to the relatively narrow normal strss (σ) range. The series of points $(\sigma_1\,\tau_1)$ .... $(\sigma_n\,\tau_n)$ are best described the linear fit:

$$\tau = c + \sigma \mathrm{tg}\varphi \qquad (4)$$

for which the constants tgφ and c are claculated from the system of equations:

$$\Sigma\tau_{1\,=\,\mathrm{tg}\varphi\Sigma\sigma 1} + nc \qquad (5)$$

$$\Sigma\sigma_i\tau_i = \mathrm{tg}\varphi\Sigma\sigma_i^2 + c\Sigma\sigma_i$$

The line (4) defined with the system of equations (5) is termed as the first degree regression curve. The whole calculation was performed by tab-

Table 1. Results of shear tests — »Robertson Research«
Tablica 1. Rezultati ispitivanja uzoraka na smicanje — »Robertson Research«

Sample No *Oznaka uzorka*	Normal stress *Normalno naprezanje* σ(MPa)	Shear strength *Tangencijalno naprezanje* τ(MPa)	στ	σ^2	Surface of the cut *Površina presjeka* (m^2)
7—7	18,28	20,43	373,46	334,16	0,000930
9—3	15,14	18,70	283,12	229,22	0,001123
9—4	12,80	14,80	189,44	163,84	0,001250
10—2	12,71	16,24	206,41	161,54	0,001416
10—4	9,08	12,72	115,50	82,45	0,001376
10—5	10,23	13,16	134,63	104,65	0,001368
10—6	11,28	14,29	161,19	127,24	0,001330
18—5	14,26	16,93	241,42	203,35	0,001122
n = 8	∑ 103,78	∑ 127,27	∑ 1705,17	∑ 1406,45	

luation. The cohesion value was calculated from the expression:

$$c = \frac{\Sigma_i^2 \Sigma\tau_i - \Sigma\sigma_i \Sigma\sigma_i\tau_i}{n\,\Sigma\sigma_i^2 - (\Sigma\sigma_i)^2} \tag{6}$$

$$c = 4{,}23 \text{ MPa}$$

The angle of friction is obtained from:

$$\mathrm{tg}\varphi = \frac{n\,\Sigma\sigma_i\Sigma\tau_i - \Sigma\sigma_i\Sigma\sigma_i\tau_i}{n\,\Sigma\sigma_i^2 - (\Sigma\sigma_i)^2} \tag{7}$$

$$\varphi = 41°51'14''$$

The results are given in Table 1 and on the σ-τ diagram in Fig. 6. From expression (4) the linear regression equation can be written as:

$$\tau = 4{,}23 + 0{,}90\sigma \tag{8}$$

Shear testing apparatus AMP-1

The device consists of a frame, hydraulic cylinders, cylindrical bearings and a sample holder. The maximum working pressure of the inbuilt hydraulic system is 445 bars (Fig. 7). To detect the force value and its change rate two dial gages were monuted. The forces are calculated over the surface of the pressure cylinder. The value of verticals stress (σ) is measured by a membrane gage with a scale between 0—600 bars, and the horizontal stress (τ) is measured with an electronic gage. The measurement range is from 0 to 600 bars, with an accuracy class of 0,5 and accepted deviation of ± 2 bars (Fig. 8). The displacement of the sample is measured with a mechanical comperator, with sensitivity of 1/100 mm or by a LVDT (Linear Variable Displacement Tranducer) sensor.

Shear strength testing in the AMP-1 apparatus

Data for each sample is in the testing record, which contains the data on displacement furfaces, values of vertical force, horizontal force increase, the size of displacement with increasing force, and the values of normal and tangential stress. This data was used for plotting a value for each sample on the σ-τ diagram (Fig. 6). The whole calculation was performed by tabulation (Table 2). The average surface of tested samples is 100,4 cm^2 (Petzel, 1992). The average content of $CaCO_3$ in the samples is 71,39% (ranging from 67,96 to 75,58%) determined by calcimetric analysis.

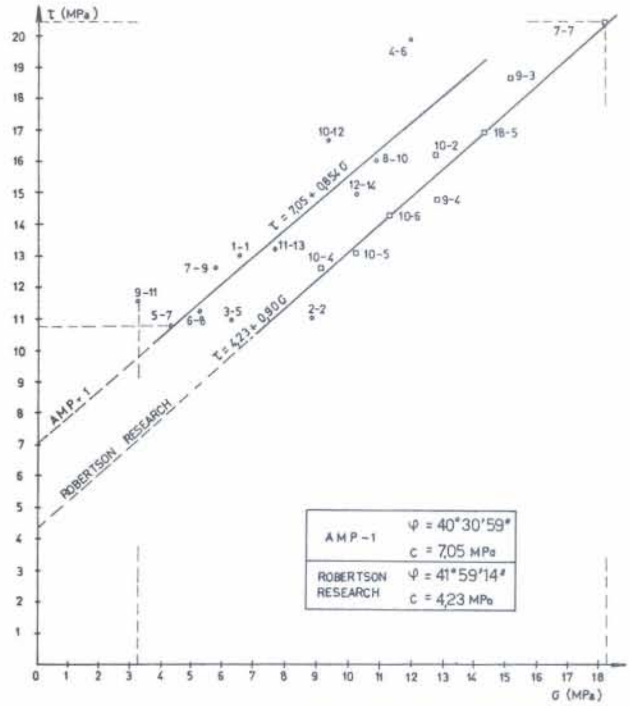

Fig. 6. Diagram of normal stress and shear strength
Sl. 6. Diagram normalnog i tangencijalnog naprezanja

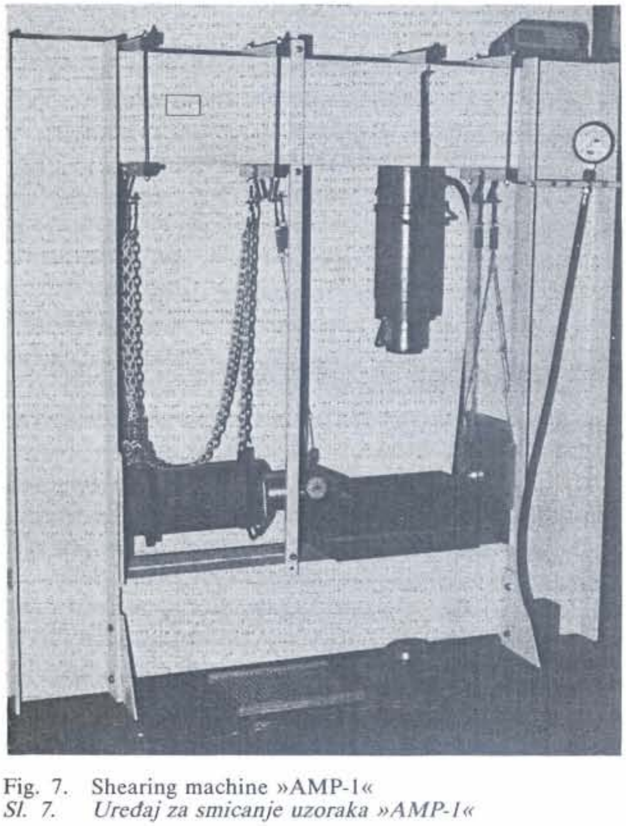

Fig. 7. Shearing machine »AMP-1«
Sl. 7. Uređaj za smicanje uzoraka »AMP-1«

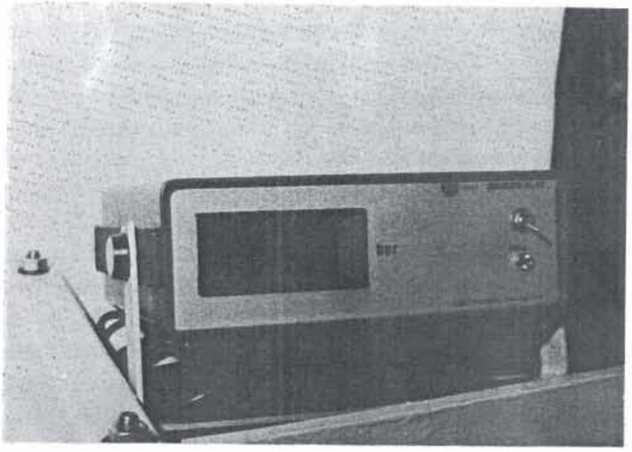

Fig. 8. Electronic manometar
Sl. 8. Elektronski manometar

The calculated value of cohesion equals $c = 7,05$ MPa, and the value of the angle of friction is $\varphi = 40°30'59''$. From expression (4) the linear regression equation can be written as:

$$\tau = 7,05 + 0,854\sigma \qquad (9)$$

Table 2. Results of shear test — »AMP-1«
Tablica 2. Rezultati ispitivanja uzoraka na smicanje — »AMP-1«

Sample No *Oznaka uzorka*	Normal stress *Normalno naprezanje* σ(MPa)	Shear strength *Tangencijalno naprezanje* τ(MPa)	στ	σ^2	Surface of the cut *Površina presjeka* (m^2)
1– 1	6,52	13,03	84,96	42,51	0,012560
2– 2	8,80	11,00	96,80	77,44	0,013280
3– 5	6,26	10,95	68,55	39,19	0,007475
4– 6	11,98	19,81	237,32	143,52	0,012685
5– 7	4,37	10,71	46,80	19,10	0,010695
6– 8	5,25	11,21	58,85	27,56	0,006675
7– 9	5,73	12,60	72,20	32,83	0,007145
8–10	10,84	16,00	173,44	117,51	0,008035
9–11	3,21	11,56	37,11	10,30	0,007275
10–12	9,37	16,69	156,39	87,80	0,013730
11–13	7,71	13,22	101,93	59,44	0,010610
12–14	10,20	14,95	152,49	104,04	0,010315
n = 12	∑ 90,24	∑ 161,73	∑ 1286,84	∑ 1761,24	

Result analysis and conclusion

The results obtained from AMP-1 apparatus are plotted on the σ-τ diagram (Fig. 6), and the corresponding linear equation is given as:

$$\tau = 7,05 + 0,854\sigma \qquad (10)$$
$$c = 7,5 \text{ MPa} \text{ — cohesion}$$
$$\varphi = 40°30'59'' \text{ — angle of friction}$$

The results can be compared with those obtained from the »Robertson Research« apparatus and the linear equation:

$$\tau = 4,23 + 0,90\sigma \qquad (11)$$
$$c = 4,23 \text{ MPa} \text{ — cohesion}$$
$$\varphi = 41°51'14'' \text{ — angle of friction}$$

From the plots on the diagram a slight difference in cohesion values can be observed, while the angle of friction is almost the same. Such results were to be expected due to the difference in sample surface size that is, in the AMP-1 apparatus they are larger, and the samples were not treated laterally. With the size of samples local decrepitude is avoided in each separate sample, and the test errors are reduced.

On the σ-τ diagram (Fig. 6) the area of normal stress ranges from 3,21 and 18,28 MPa, and the tangetial stress ranges from 10,71 and 20,43 MPa. The results of shear strength tests performed with the »Robertson Research« and the AMP-1 apparatuses can be considered as satisfactory. These investigations should be continued on different materials in order to evaluate the obtained results.

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