# Modeling and Optimization of Planar Microcoils

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Abstract. Magnetic actuation has emerged as a useful tool for manipulating particles, droplets and biological samples in microfluidics. A planar coil is one of the suitable candidates for magnetic actuation and has a high potential to be integrated in digital microfluidic devices. A simple model of microcoils is needed to optimize their use in actuation applications. This paper first develops an analytical model for calculating the magnetic field of a planar microcoil. The model was validated by experimental data from microcoils fabricated on printed circuit boards (PCB). The model was used for calculating the field strength and the force acting on a magnetic object. Finally, the effect of different coil parameters such as the magnitude of the electric current, the gap between the wires and the number of wire segments are discussed. Both analytical and experimental results show that smaller gap size between wire segments, more wire segments and a higher electric current can increase both the magnitude and the gradient of the magnetic field, and consequently cause a higher actuating force. The planar coil analyzed in the paper are suitable for applications in magnetic droplet-based microfluidics.

# 1. Introduction

Recently, magnetism has found many applications in microfluidics. Typical examples are magnetohydrodynamic micropumps [1, 2], ferrofluid droplets for sample transport in small scales [3, 4, 5], magnetically doped polydimethylsiloxane (PDMS) as actuating material [6] and magnetic micro particles for improved mixing [6]. In life sciences, magnetism also has been used for trapping, sorting, separating cells and transporting biological objects [7, 8, 9]. Magnetic particles have been used for DNA separation as an alternative for gel matrix [10, 12].

Using magnetic force for manipulation of particles or droplets offers a number of advantages. Objects can be transported or manipulated by external magnets or coils that are not in contact with the fluid. In spite of the electric field, the effect of magnetic field on particles is generally not affected by surface charges, pH-level, ionic concentrations or temperature [7].

Yamaguchi [11] gave a review on magnetic films for planar inductive components such as planar coils. Analytical formulations of inductance for standard geometries such as meander-shaped coil, parallel lines, spiral rectangular coil and concentric rectangular coil were given. MEMS applications of microcoils were mentioned but no calculation of the magnetic field was reported in this review. Microcoils have a great potential to be used in miniaturized devices as functional components such as valves [13], actuators for magnetic particles [14, 15], relays [16] and even sensors for near-surface material properties [17]. Microcoils have been used for digital microfluidics, where magnetic droplets are positioned and transported by external magnetic fields [3, 4, 7]. The magnitude of the magnetic flux can be well controlled by the current passing through the coil. The current itself is controlled by external electronic circuits, micro controllers or personal computers (PC). Thus, microcoils allow further flexibilities in designing magnetically-actuated droplet-based microfluidic devices. The other benefit of planar microcoils is the existence of an analytical formulation for calculating the strength and the flux of the magnetic field. Such a model enables the prediction of the device behavior and the optimization of its design. The first step towards using planar coils in microfluidic devices is the development of a simple and effective model for the resulting magnetic field.

Busch-Vishniac discussed the use of magnetically driven microactuators and the possibility of generating magnetic forces that are strong enough for actuation [18]. Recently several works have been reported on modeling of magnetic actuators. Ko et al. reported the numerical analysis for a micro machined magnetic actuator using finite element method to identify critical parameters that affect the performance of a microactuator. Their numerical results indicate that the dimensions of the core and the magnetic material have an influence on the performance of planar actuators [19]. Lin et al. used a numerical model with FARADAY software (Integrated Engineering Software Inc., Canada) to study the effect of coil parameters in stimulating live tissues [20]. This work identifies the governing parameters of the coil design to reach an optimum field

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penetration. The parameters are coil configuration, diameter, and the number of turns. Feustel et al. reported a numerical simulation for optimization of planar electromagnetic actuators [21]. Garcia et al. modeled the magnetic field of only one straight segment of current carrying wire with a finite length [22] and proposed a numerical procedure for calculating the magnetic field of the coils. A number of studies were also conducted for calculating the inductance and impedance of electromagnetic structures. Hurley et al. calculated the impedance for planar structures with spiral windings [23]. Mohan et al. developed simple expressions for inductance of spiral coils [24]. Engel-Herbert and Hesjedal present an analytical solution of a bar-shaped permanent magnet [25]. Babic and and Akyel [26] presented analytical formulation of the magnetic field of a threedimensional (3-D) conductor, that has a rectangular cross section. This work does not have an explicit solution for a certain coil shape such as the spiral coil investigated in our present paper. Nandy et al. [27] formulated the analytical solution for the dynamics of a magnetic particle trapped by the field of a single current conducting line. Ramadan et al. [28] reports numerical results with a commercial software (TOSCA) and experimental results using superconducting quantum interference device (SQUID) of meander-shaped coils and spiral-shaped circular planar coil. Misakian [29] reports the analytical formulation for the magnetic field of a closed rectangular loop.

Many of the previous works are based on numerical solutions, and no detailed parametric study was reported. Our contribution in this paper is extending the simple analytical model for the magnetic field of a single straight wire to a spiral-shaped rectangular planar coil, which is the most common geometry for the implementation of magnetic microactuators. This model can then be used for the calculation of the strength, the flux and the resultant force of the magnetic field generated by the microcoil. A parametric analysis is carried out to discuss the influence of the different coil parameters on the induced magnetic field. The results presented in this paper help to maximize the field strength and the force for applications in magnetically-actuated droplet-based microfluidic devices.

# 2. Model of planar microcoils

# 2.1. Magnetic field of a straight wire segment with finite length

Figure 1(a) shows the wire arrangement of a planar rectangular coil. Each straight segment of the coil is termed here as a wire segment. The gap size g is defined as the center to center distance between two neighboring parallel segments. Four consecutive wire segments form a turn. Thus, if the number of segments is n, the number of turns is approximately n/4. As shown in FIg. 1, planar coils are composed of a series of current-carrying wires with finite length, Figure 1(a). To calculate the magnetic field of a planar coil, the magnetic field created by each wire of a finite length should be obtained in the the calculation domain. The superposition of the magnetic fields of all these segments results in the total magnetic field of the coil.



**Figure 1.** Wire arrangement for modeling a planar coil: (a)Wire arrangement of a planar rectangular coil. (b) Arrangement of a horizontal wire segment with finite length. (c) Coordinate transformation to formulate the magnetic field of a vertical segment of wire.

Applying Biot-Savart's law to a segment of wire carrying current I, parallel to x - z plane results in [22]:

$$\begin{aligned}
H_x &= 0 \\
H_y &= -\frac{zI}{4\pi} \int_{x_1}^{x_2} \frac{\mathrm{d}l}{[\sqrt{(x-l)^2 + (y-a)^2 + z^2}]^3} \\
&= -\frac{zI}{4\pi[(a-y)^2 + z^2]} \times \left[\frac{x_2 - x}{\sqrt{(x_2 - x)^2 + (a-y)^2 + z^2}} \\
&-\frac{x_1 - x}{\sqrt{(x_1 - x)^2 + (a-y)^2 + z^2}}\right] \\
H_z &= \frac{(y-a)I}{4\pi} \int_{x_1}^{x_2} \frac{\mathrm{d}l}{[\sqrt{(x-l)^2 + (y-a)^2 + z^2}]^3} \\
&= -\frac{(a-y)I}{4\pi[(a-y)^2 + z^2]} \times \left[\frac{x_2 - x}{\sqrt{(x_2 - x)^2 + (a-y)^2 + z^2}} \\
&-\frac{x_1 - x}{\sqrt{(x_1 - x)^2 + (a-y)^2 + z^2}}\right]
\end{aligned}$$
(1)

where x, y, z are the coordinates of the point where the magnetic field is to be calculated,  $x_1$  and  $x_2$  are the x coordinate of the two ends of the segment of wire, I is the electric current passing through the wire, a is the distance of the wire from the x - z plane and dl is the differential spatial element on the wire segment, Figure 1(b).

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Equation (1) describes the magnetic field for the horizontal segments of a planar coil. The magnetic field of the vertical segments can be calculated in the similar manner. The calculation can be simplified by a coordinate transformation. According to Figure 1(c), (1) can be formulated for a vertical segment using the following coordinate transformation:

$$(x, y, z) \to (y, x, -z) \tag{2}$$

After the coordinate transformation, the magnetic field components of a vertical segment can be determined as:

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$$H_{x} = \frac{zI}{4\pi[(a-x)^{2}+z^{2}]} \times \begin{bmatrix} \frac{y_{2}-y}{\sqrt{(y_{2}-y)^{2}+(a-x)^{2}+z^{2}}} \\ -\frac{y_{1}-y}{\sqrt{(y_{1}-y)^{2}+(a-x)^{2}+z^{2}}} \end{bmatrix}$$

$$H_{y} = 0 \qquad (3)$$

$$H_{z} = -\frac{(a-x)I}{4\pi[(a-x)^{2}+z^{2}]} \times \begin{bmatrix} \frac{y_{2}-y}{\sqrt{(y_{2}-y)^{2}+(a-x)^{2}+z^{2}}} \\ -\frac{y_{1}-y}{\sqrt{(y_{1}-y)^{2}+(a-x)^{2}+z^{2}}} \end{bmatrix}.$$

# 2.2. Magnetic field of a rectangular planar coil

The magnetic field vectors of all segments of a rectangular planar coil can be determined by superposition of the field vectors of all the horizontal and vertical segments:

$$H_{x,\text{coil}} = \sum_{i=1}^{n} H_{x,i}, \ H_{y,\text{coil}} = \sum_{i=1}^{n} H_{y,i}, \ H_{z,\text{coil}} = \sum_{i=1}^{n} H_{z,i},$$
(4)

where n is the total segment number of the coil. In (1) and (3), I is the current passing through all wire segments with consideration of its relative direction in each segment.

The magnetic flux  $\mathbf{B}$  can subsequently be calculated as:

$$\mathbf{B} = \mu_0 \mathbf{H} (1 + \chi_m) \tag{5}$$

where  $\chi_{\rm m}$  is the magnetic susceptibility of the medium in which the magnetic field is to be calculated and  $\mu_0 = 4\pi \times 10^{-7}$  is the permeability of free space.

The above model was implemented in Matlab (MathWorks Inc, USA). Figure 2 shows the typical results of the model for a rectangular coil. The field strength was calculated for a line running along the x-axis and through the center of the coil.

The modeling results show that the total magnetic field of a planar coil has almost the same trend and magnitude as the z-component of its magnetic field strength. To validate the model, the z-component of the magnetic field in of a microcoil with the same parameters as in the model was measured by a gaussmeter (GM05, Hirst magnetic instrument, UK). Details on the fabrication of the coil and the experimental setup are



Figure 2. Modeling results for magnetic field distribution of a single planar coil on the central line of the coil (n=47,  $g=200 \ \mu \text{m}$ , I=0.3A,  $y=0 \ \text{mm}$ ,  $z=1 \ \text{mm}$ ).



Figure 3. Comparison between the modeling result and the experimental result of the *z*-component of the magnetic field of a planar coil. The magnetic flux was measured with a Gaussmeter and converted into to magnetic field strength using (5) (n = 94,  $g = 200 \ \mu\text{m}$ , I = 0.55 A,  $\chi_{\text{m}} = 0$ ,  $\mu_0 = 4\pi \times 10^{-7}$ , y = 0 mm, z = 1 mm).

reported later in Section 3. Both theoretical and experimental results of a micro coil with 94 segments,  $200-\mu m$  gap and a current of 0.55 A are shown in Figure 3.

# 2.3. Influence of coil parameters on its magnetic field and magnetic force

In this section, the developed analytical model is used to study the influence of coil parameters on its magnetic field. The gap between the wires, the number of wire segments and the current magnitude are varied, and their influences on the resultant magnetic field and magnetic force is discussed. The force acting on a magnetic object located in a magnetic field can be expressed as [30]:

$$\mathbf{F} = \frac{V\Delta\chi}{\mu_0} (\mathbf{B}.\nabla)\mathbf{B}$$
(6)

where V is the volume of the object,  $\mu_0$  is the magnetic permeability of the free space and  $\Delta \chi$  is the difference between magnetic susceptibility of the object and the surrounding medium. Extending (6) for each force component leads to:

$$F_{x} = \frac{V\Delta\chi}{\mu_{0}} \left( B_{x} \frac{\partial B_{x}}{\partial x} + B_{y} \frac{\partial B_{x}}{\partial y} + B_{z} \frac{\partial B_{x}}{\partial z} \right)$$

$$F_{y} = \frac{V\Delta\chi}{\mu_{0}} \left( B_{x} \frac{\partial B_{y}}{\partial x} + B_{y} \frac{\partial B_{y}}{\partial y} + B_{z} \frac{\partial B_{y}}{\partial z} \right)$$

$$F_{z} = \frac{V\Delta\chi}{\mu_{0}} \left( B_{x} \frac{\partial B_{z}}{\partial x} + B_{y} \frac{\partial B_{z}}{\partial y} + B_{z} \frac{\partial B_{z}}{\partial z} \right)$$
(7)

The Maxwell's equation

$$\frac{1}{\mu}\nabla \times \mathbf{B} = 0 \tag{8}$$

leads to

$$\frac{\partial B_y}{\partial z} = \frac{\partial B_z}{\partial y}, \ \frac{\partial B_x}{\partial z} = \frac{\partial B_z}{\partial x}, \ \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x}.$$
(9)

Thus, equation (7) can be rewritten for a planar coil with n segments as:

$$F_{x,\text{total}} = \frac{V\Delta\chi}{\mu_0} \sum_{i=1}^n \left( B_{x,i} \frac{\partial B_{x,i}}{\partial x} + B_{y,i} \frac{\partial B_{y,i}}{\partial x} + B_{z,i} \frac{\partial B_{z,i}}{\partial x} \right)$$

$$F_{y,\text{total}} = \frac{V\Delta\chi}{\mu_0} \sum_{i=1}^n \left( B_{x,i} \frac{\partial B_{x,i}}{\partial y} + B_{y,i} \frac{\partial B_{y,i}}{\partial y} + B_{z,i} \frac{\partial B_{z,i}}{\partial y} \right)$$

$$F_{z,\text{total}} = \frac{V\Delta\chi}{\mu_0} \sum_{i=1}^n \left( B_{x,i} \frac{\partial B_{x,i}}{\partial z} + B_{y,i} \frac{\partial B_{y,i}}{\partial z} + B_{z,i} \frac{\partial B_{z,i}}{\partial z} \right)$$
(10)

# 3. Experimental setup and results

#### 3.1. Experimental setup

Planar coils were fabricated by photolithography and etching on a double-sided PCB with precoated photoresist (Farnell, USA). The patterns of the coils were designed with AutoCAD (Autodesk Inc., USA) and printed on a transparent mask. Using a UV light exposure unit, the patterns were transferred to the photoresist of the PCB. After developing the photoresist, the PCB was etched by a ferric chloride solution. Coils with different numbers of wire segments and different gap sizes were fabricated. For a gap of 200  $\mu$ m, coils with 20, 48 and 94 segments were fabricated. For a segment number of



**Figure 4.** Validation experiments for the magnetic field of a micro coil: (a) microcoils with different parameters fabricated on a PCB, and (b) the experimental setup.

48, coils with different gap sizes of 200  $\mu$ m, 650  $\mu$ m and 1150  $\mu$ m were also fabricated. Figure 4(a) shows the PCB with coils characterized in our experiments.

Figure 4(b) shows the setup for the characterization measurement of the coils. Three sets of verification experiments were carried out for the coils. In the first set of experiments, the z-component of the magnetic field of a microcoil with a gap of 200  $\mu$ m, and 94 segments were measured at different currents. In the second set of experiments, the gap size was kept constant at 200  $\mu$ m while the segment number varies as 20, 48 and 94. The current in use was 0.4 A. In the third set of experiments, the segment number is fixed at 94 and the gap size varies as 200  $\mu$ m, 650  $\mu$ m and 1150  $\mu$ m. In this set of experiments, the current was fixed at 0.5 A. The same gaussmeter described in the previous section was used to measure the magnetic flux. The position of the sensor probe was adjusted by a precision positioner. The measured flux was converted to the field strength using (5) with  $\chi_m = 0$ ,  $\mu_0 = 4\pi \times 10^{-7}$  for air as the surrounding medium.

## 3.2. Results and discussions

The modeling and experimental results are presented and compared in Figures 5, 6 and 7. Figure 5 shows the theoretical and measured distribution of the z-component of the field strength along the x-axis. Figure 6 compares the data of the maximum field strength at the coil center. Figure 7 depicts the distribution of the term  $(\mathbf{B}.\nabla)\mathbf{B}_x$ , which according to 10 represents the x-component of the force on a magnetic particle at the given position. The results were taken along the x-axis through the coil center. The theoretical model shows that increasing the current through the planar coil increases both the magnitude and the gradient of the magnetic field, Figure 5(a). According to



**Figure 5.** Influence of coil parameters on the magnitude and the gradient of the magnetic fiel: (a) Field strength at different currents  $(n = 94, g = 200 \ \mu\text{m}, y = 0 \ \text{mm}, z = 1 \ \text{mm})$ . (b) Field strength with different segment numbers  $(I = 0.4 \text{ A}, g = 200 \ \mu\text{m}, y = 0 \ \text{mm}, z = 1 \ \text{mm})$ . (c) Field strength with different gap sizes  $(I = 0.5 \ \text{A}, n = 47, y = 0 \ \text{mm}, z = 1 \ \text{mm})$ .

(1) and (3), the magnetic field strength is linearly proportional to the magnitude of the electric current. Both analytical model and experimental data show that when the current increases from 0.1 A to 0.55 A, the maximum field strength at the center of the coil increases from 50 A/m to 800 A/m, Figure 6(a). Thus, a higher current will lead to larger magnetic force, Figure 7(a). From the relation of (10), the maximum magnetic force can be expected to be a square function of the actuating current. Besides its magnitude, the direction of the current also affects the magnetic field. If the direction of the current changes, the direction of magnetic field will be reversed.

It is not always possible to use a higher current to obtain a larger magnetic force,



Figure 6. Dependence of the maximum field strength at the coil center on (a) current  $(n = 94, g = 200 \ \mu\text{m}, y = 0 \ \text{mm}, z = 1 \ \text{mm})$ , (b) segment number  $(I = 0.4 \ \text{A}, g = 200 \ \mu\text{m}, y = 1 \ \text{mm})$ , and (c) gap size  $(I = 0.5 \ \text{A}, n = 47, y = 0 \ \text{mm}, z = 1 \ \text{mm})$ .

because a higher current also generates more heat in the coils. Exceeding a certain temperature limit, the heat can burn and destroy the coil. Another option to increase the magnetic field is increasing the number of wire segments in the coil. As shown in Figure 5(b), increasing the number of wire segments increases the strength of magnetic field but not significantly its gradient. The reason for the small change in field gradient is the larger coil size associated with the higher segment number. The more segments the coil has, the stronger is the field. Figure 6(a) shows that the relation between the field strength and the number of segments is not strictly linear. Sice the field gradient does not increase significantly with the number of wire segments, the maximum magnetic force caused by the coil is expected to be linearly proportional to the coil number as depicted in Figure 7(b).

The gap size between the coil wires also affects the magnetic field. Figure 5(c) shows that the smaller the gap is, the bigger is the magnitude and gradient of the magnetic field. The theoretical curve of the field strength is wavy when the gap size increases to 1000  $\mu$ m. At this large gap size, the fields generated by the wire segments do not overlap well and appear individually. The measurement can not capture thus wavy characteristics because of the relatively large size of the sensor chip on the order of 1 mm. The maximum field strength at the center of the coil is inversely proportional to the gap size, Figure 6(c). Thus, according to (10), the maximum magnetic force is expected to be inversely proportional the the square of the gap size, Figure 7(c). Thus, for actuation applications, the smaller the gap between coil segments, the larger is the magnetic force. The limitation for decreasing the gap is only the resolution of the micro fabrication process. Another advantage of decreasing the gap size is that more wires can be placed in each unit of length. Thus microcoils have the advantage of both large segment number and small gap size.



Figure 7. Magnetic force distribution in x direction on the line running through the center of the coil (a) as a function of current (b) as a function of wire number and (c) as a function of gap size between coil wires.

## 4. Conclusion

Magnetic actuation has shown itself as a promising tool for a variety of miniaturized devices such as microvalves, micropumps and droplet actuators. Planar microcoils have a huge potential for being used in magnetic digital microfluidic devices. We reported an analytical model for calculating the magnetic field of a planar rectangular coil. Based on this model, the force acting on a magnetic particle located in the field can be calculated. Microcoils with varying parameters were fabricated in a commercially available PCB using photolithography and wet etching. A series of experiments was carried out to validate the theoretical model. Analytical and experimental results show that smaller gap size between wire segments, large number of wire segments and higher electric current can increase the magnitude of the induced magnetic field. Considering practical applications of microcoils, decreasing the gap between the wire segments is the best solution for a maximum magnetic force. Although magnetic force is a volume-based force and is not favorable in microscale, coils fabricated by microtechnology may have very small gap size and a large number of wire segments, while the coil footprint is kept small. This fact promises the effective use of microcoils in actuation applications, especially in magnetic droplet-based microfluidics.

#### Acknowledgement

The authors would like to thank the Agency of Science, Technology and Research, Singapore (A\*Star, SERC Grant No. 0521010108) for their financial support.

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