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# The use of fuzzy logic in coordinating investment projects in the public sector*

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***Abstract***

*Politics and the professions often have different standpoints on development planning and investment decisions in the public sector. Opposing views, lack of understanding and an inability to accept compromises in the planning and execution of investments frequently cause deviations which ultimately reveal themselves in negative economic consequences. Effective and successful planning and execution of investments in the public sector is one of the key development tasks in a state or local community. For this reason we have undertaken a detailed investigation of the issue with the goal of devising a tool in the form of a model of a system for support in decision making. In order to make it easier for appraisers and decision makers to express their opinions in a natural and individual manner, we have introduced an appraisal with linguistic values through the use of fuzzy logic. The purpose of this article is to present the characteristics and structure of the fuzzy model for coordinating opinions, and the progress and results of its use in the case of two Slovenian municipalities.*

***Key words***: *investment project, multiple-criteria decision making, public sector, linguistic variable, fuzzy logic.*

***JEL classification:*** *C65, D81*

## 1. Introduction

In this article we present the results of our research concerning the question of investment decision making in public administration, with special emphasis on lo-

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cal government. Investment activity in the local sector involves long-term capital investments. For this reason the local government must have a clear picture of the needs and possibilities of the long-term distribution of capital sources. It must have a specific picture of each investment project along with an overview of all projects, which is achievable only through a comprehensive approach to planning and implementing the projects (Sever, 2004). In the coordination of public interests we regularly come across opposing views and assertions of various needs, demands and desires. Developed democracies are aware of the current dilemmas of indirect democracy and strive to solve this problem by stimulating direct decision making by citizens or residents. At this point, however, decisions on public affairs are still made mostly in a confrontation between the professions and politicians.

Investment decisions are influenced by the needs and opinions of a large number of stakeholders (citizens, civil society, municipal administrations, professional services), which increases the risk of failure in the planning, execution and exploitation of the investment project. It is our intention to demonstrate that with the correct approach in public organisations and communities it is possible to establish a decision-making procedure that includes the process of opinion coordination among various decision makers. By this, even in an environment in which the level of cooperation and acceptance of compromise might otherwise be low, a coordination process through which decisions are made in a dialogue between professional arguments and political or stakeholders' demands and desires can be established. By monitoring the deviations, the crucial discrepancies in the appraisals of various individuals and groups can be revealed. By analysing the patterns that appear in the results, the understanding of the decision-making body's preferences can be deepened.

We have focused our research on the question of optimum choice of investment projects in a local community burdened by various circumstances that can result in the municipality's inopportune investment orientation (Benčina and Devjak, 2004). Decision making in municipalities takes place successively with two groups of participants. Professional services assess the investment projects and merge them into investment options according to professional criteria. The proposals are then revised and approved by the mayor and forwarded to the municipal council, which decides independently and autonomously. The decision makers are confronted with various difficulties caused by an unsystematic approach. They use various criteria in making appraisals and often rely to a great extent on their intuition. The appraisal methods are frequently obscure, and it is difficult to identify the causes of discrepancies. Communication between groups is usually limited to an exchange of views and demands, and often the position of power is used. Political decision makers are reluctant to take professional arguments into consideration, while professionals tend to disregard the political circumstances (Benčina, 2003); however, an optimum decision is achieved only if all the opinions and comments are dealt with in the decision-making process.

We have therefore been seeking a solution to the issue of making optimal decisions on investments in local government, in the phase of preparing the investments as well as in the phase of initiating their realisation and financing. In this we had to take into account the fact that we would be introducing a new element in an environment where the decision makers were less acquainted with the methods of decision making, for which reason the solution had to be simple and had to allow users an intuitive understanding of the tools, procedures and results. The solution would have to establish a process that allowed confrontation and coordination of diverse opinions and interests on the professional and political levels, in professional-political, as well as in professional-professional and political-political relations.

We presumed that there would be a reasonable level of professional knowledge available and that professional opinions and proposals would be based on professional arguments, and that political opinions and desires would actually reflect the needs and desires of the electorate. We could thus justifiably expect that the coordinated decisions would be the better ones. An investment project with strong professional and political support would then have a better chance of being effectively executed and of yielding the expected results after being put into use (Devjak, 1999).

We further believed that a solution to the issue would bring progress into environments in which professional knowledge was lacking or subject to political influence, and where political opinions were formulated without consideration for the electorate's views or even contrary to them. Two facts spoke in favour of this assertion. In the process of coordination, the participants can interact and come to incomparably better decisions through dialogue, despite a disadvantageous starting position. Even in the absence of political will for cooperation, where decisions are accepted on the basis of the influence of the advocates of a particular alternative, the process contributes to interrelations and understanding of responsibility.

In general, the extent of development and implementation of decision support systems based on fuzzy logic is rising. Researchers are solving different business problems using different approaches.

We have been witnessing a rapid increase of the use of fuzzy logic in decision-making systems. Various approaches are gaining ground in development of such systems, in cases of similar solutions we have limited ourselves to systems supporting decision-making, based on the use of fuzzy logic. We have focused on solutions in the area of public sector and general solutions for group decision support. Zhou presents a fuzzy system for group decision support that comprises six modules for brainstorming support, a tool for multiple criteria and fuzzy multiple criteria decision-making, a tool for presentation of decision-makers' preferences, an instrument for transformation or preference unification, a modul for preference aggregation and a tool for system administration (Zhou, 2002). In this case, a general approach is used that does not deal with special needs of various decision-makers and has three flaws regarding the requirements of the system in question: it does not offer any presentation of results, it does not provide a system of coordination among decision-makers, and preference aggregation is based on ordered weighted averaging (OWA) operators with fuzzy quantifiers which are somewhat more difficult to implement. Web-based group decision support for R&D project outcome assessment in government funding agencies (Ma, 2002) supports assessment of project outcomes on the Web against multiple criteria. The model comprises assessment with linguistic values, aggregation of individual fuzzy preferences on criteria (weights) with OWA operators, and aggregation of project outcome assessment through defuzzification of fuzzy numbers. In comparison to our solution this model also deals with criteria weights. It introduces two ways of treatment of fuzzy values, which increases the complexity of implementation. It does not deal with the presentation of the results nor with coordination of opinions. A consensus model for multiperson decision-making problems defines two consensus criteria: the consensus measure, which indicates the agreement between experts' opinions, and a measure of proximity to find out how far the individual solutions are from a collective solution (Herrera-Viedma, 1999). This model is based on the use of linguistic values aggregated with OWA operators. Being designed for experts, it does not pay any attention to presentation and explanation of results to the assessors. Galinec and Vidovič used a fuzzy rule based system approach for Evaluating Personnel in Project Management (Galinec, 2006). This approach is very clear and efficient if there is an appropriate number of criteria with not too many linguistic assessment values, whereas with more complex structures the multitude of logical rules becomes uncontrollable. The field of fuzzy decision-making is very well represented in literature and it covers a wide area of usage. An interesting hybrid approach can be observed in the field of marketing, combining the Delphi group decision method, fuzzy logic and expert systems (Li, 2002). The area of public sector is less represented with prevailing systems for expert assessment. Some cases of use in the field of public interest are: quality function deployment (Büyüközkan, 2007), supplier selection (Jain, 2007), selection of advanced technology (Choudhury, 2006), freshwater inflows (Ji, 2006), housing construction (Perng, 2005), environmental assessment (Tran, 2002b), and budgeting of the health care system (Mosmans, 2002).

In the given frame our solution is based on the achieved research results in various fields and, focusing on the area less represented in literature, combines methods and approaches into a system that solves specific problems of decision-making and participation in decision-making in public sector. Especially distinctive is the paradigm of simplicity and comprehensibility which alone makes possible a successful assertion of the method in the field of our interest.

The contents of this article are presented in five chapters. We will give a short account of the events in the environment from which the problems that have drawn us towards developing the aforementioned solution have stemmed. We will then connect the characteristics and particularities of this procedure of decision making in the public sector with the theory and practice of multiple-criteria decision making, and present arguments that support the choice of approaches and methods used in the model. This will be followed by a presentation of the appraisal and optimal-choice model. Prior to the conclusion, we will report on the use of the solution in applying the model in municipalities. We will finally conclude with an overview of the advantages and limitations of the presented approach and the possibilities of using this solution in other environments. The input data are given in Appendix I.

## 2. Model

In solving the problem, we have used several coordinated and related approaches and methods. In essence this is a question of decision making with many appraisers participating in assessing several alternatives. By appraisal and calculation of the results, we can come closer to the needs and desires of a specific environment in a local community. Processing the results should not be too demanding, and the results must be clearly understandable. The solution should establish a transparent process of joint decision making within which continuous cooperation and exchange of views among the participants will take place, with the aim of achieving an optimum coordinated solution, from both professional and political points of view.

To this end, we have used a fuzzy logic approach suitable for solving such issues in environments in which the problems are not clearly defined and can be identified only by coarsely delineated attributes. The tool for realising the model in practice is a software solution which, together with the model, forms a system for decision-making support.

In accordance with the methods of multiple-criteria decision making we have chosen a combination of multiple-attribute and group decision making. The definition of a multiple-attribute decision problem encompasses the following:

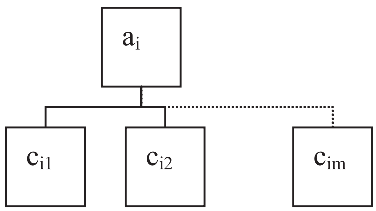
- a set of attributes (parameters, factors, viewpoints, views, ranges) $C = \{c_1,...,c_n\}$;
- a set of alternatives (possibilities, projects, scenarios, actions, goals, purposes) $A = \{a_1,...,a_m\}$;
- specific information in each pair $(a_i, c_j)$, $i \in \{1,..., n\}, j \in \{1,..., m\}$ ascertaining the relative importance of each attribute $c_j$ – weight $w_j$;
- suitability $r_{ij}$, which is the decision maker's appraisal of the alternative $a_i$ with regard to the attribute $c_j$;

- the merging function U, by which the appraisals of the criteria $c_j$, $j \in \{1,..., m\}$ of an alternative $a_i$ - $r_{ij}$ are aggregated into an alternative appraisal;
- in group decision making, the given alternatives are appraised by the set of individuals $D = \{d_1,...,d_k\}$.

The entire appraisal model is thus three-dimensional:

- the basic structures are decision trees[2] for each individual alternative, where the appraisals of the attributes $c_{ij}$, $j \in \{1,..., m\}$ (leaves) join into the appraisal of alternative $a_i$ by the aggregating function $U_a$:

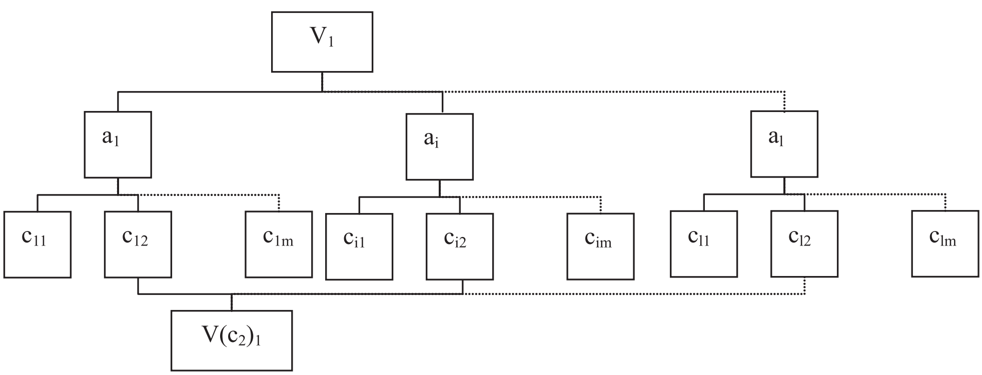
Figure 1: Decision tree for an alternative




Source: Author

- the individual alternatives ai join into subsets of the set *A*, $V \in P(A)$ and the appraisal of the subset of alternatives is given by aggregating function Uv,:

Figure 2: Decision tree for a variant




Source: Author

[2] For the sake of simplicity the decision trees are presented in the simplest one-level version; however, the depth of the decision trees in the model is limited only for practical reasons.

- for both dimensions mentioned above the appraisals of individual appraisers $d_l$ are joined into group appraisals by joining individual appraisers into groups (subsets of the set of the assessors D) $G \in P(D)$ and using an aggregating function $U_G$.

One thinks in terms of descriptive categories, for which reason assessment by descriptive values demands much less mental effort. An appraisal method that demands less mental effort will be more precise than a method that demands greater mental capacity (Zimmer, 1986). We can therefore claim that a descriptive appraisal is more precise than a numeric one. Additionally, a definition of the assessment by linguistic variables is easier for the appraiser (Chin and Ramachandran, 2000), (Herrera et al., 1996). These are undoubtedly sufficiently substantial arguments to support the use of linguistic appraisal and for upgrading the decision-making model by fuzzy logic methods (Zimmerman, 2001). In this environment we can also find suitable solutions for joining values, based on the mapping of linguistic values into fuzzy numbers and the use of aggregation operators for fuzzy numbers in making the calculation.

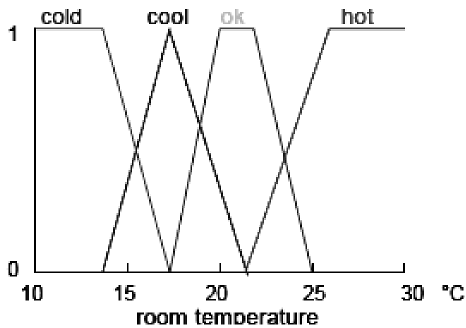
The starting points are Zadeh's definitions of fuzzy sets and linguistic variables. In classic mathematics, classes of objects have precisely defined criteria for membership; an object can take only two states – it either belongs or does not belong to the class. In the real world, more often than not classes of objects do not have precisely defined criteria for membership. For example, consider definitions of classes: "the class of all real numbers much greater than 1", "the class of beautiful women" or "the class of tall men" (Zadeh, 1965). Yet, the fact is that imprecisely defined classes play an important role in human thinking.

Definition 1: Fuzzy set (Zadeh, 1965)

Let X be a space of points (objects), with a generic element of X denoted by $x$. Thus, $X = \{x\}$. A fuzzy set (class) $\tilde{A}$ in X is characterised by a membership (characteristic) function $\mu_A(x)$ which associates with each point in X a real number in the interval [0,1], with the value of $\mu_A(x)$ at $x$ representing the "grade of membership" of $x$ in $\tilde{A}$. Thus the nearer the value of $\mu_A(x)$ to unity, the higher the grade of membership of $x$ in $\tilde{A}$. When A is a set in the ordinary sense of the term, its membership function can take only two values, 0 and 1, with $\mu_A(x) = 1 \text{ or } 0$ according to whether $x$ does or does not belong to A. Thus, in this case $\mu_A(x)$ reduces to familiar characteristic functions of set A.

This can be illustrated by the simple example of the definition of temperature where the membership levels of temperatures in classes differ from 0 to1 (Fig. 3).

Figure 3: Fuzzy temperature classes




Source: Stelzer, 2003

As seen in Fig. 3, it is useful to give names to fuzzy sets. In general the motivation for the use of words or sentences rather than numbers is that linguistic characterisation is less specific than numerical characterisation. Through the use of Zadeh's extension principle, a connection between the world of words and world of numbers can be established, and in this way the possibility of numerical designation of linguistic values becomes wide open.

Definition 2: Linguistic variable (Zadeh, 1975)

A linguistic variable is defined by a quintuple $(\mathcal{H}, T(\mathcal{H}), U, G, \widetilde{M})$ in which $\mathcal{H}$ is the name of the variable; $T(\mathcal{H})$ (or simply T) is the term set of $\mathcal{H}$, that is, the set of names for linguistic values $\mathcal{H}$ with each value being a fuzzy variable denoted generically by $X$ and ranging over a universe of discourse $U$ which is associated with the base variable $u$; $G$ is a syntactic rule (which usually has the form of grammar) for generating names $X$ of values of $\mathcal{H}$; and $M$ is a semantic rule for associating each $X$ with its meaning $\widetilde{M}(X)$, which is a fuzzy subset of $U$. A particular $X$, that is, a name generated by $G$ is called a term. A term consisting of a word or words which function as a unit (i.e. always occur together) is called an atomic term. A concatenation of components of a composite term is a subterm.

An example of a term set is:

$$T = \{Reject, Lowest, Very\ Low, Low, Middle, High, Very\ High, Highest, Must\ be\}.$$

Operations with fuzzy sets within a given universe of discourse are operations with membership functions, which allow relatively easy calculations over fuzzy sets with sufficiently simple membership functions. This can be reached in only a few steps, accepting some limitations in the definitions of membership functions and taking the set of real numbers as the universe of discourse.

Definition 3: Fuzzy number (Delgado, 1998)

A fuzzy number $\tilde{A}$ is a convex normalised fuzzy set $\tilde{A}$ of the real line $\mathbb{R}$.

Two kinds of fuzzy numbers are included in this definition: unimodal (there exists exactly one $x_0 \in \mathbb{R}$ with $\mu_{\tilde{A}}(x_0) = 1$) and flat fuzzy numbers (there exists exactly one interval $[x_1, x_2] \in \mathbb{R}, x_1 \neq x_2$ and $\mu_{\tilde{A}}(x) = 1$).

By means of Zadeh's extension principle, the arithmetic operations of $\mathbb{R}$ extend to $\mathbb{R}_{\tilde{z}}$.

Definition 4: Extension principle (written in reduced form for a single universe of discourse) (Zimmerman, 2001)

Let X be a universe of discourse and $\tilde{A}$ be a fuzzy set in X. $f$ is the mapping from X to a universe $Y, y = f(x)$. Then, the extension principle allows us to define a fuzzy set $\tilde{B}$ in $Y$ by

$$\tilde{B} = f(\tilde{A}) = \{(y, \mu_{\tilde{B}}(y)); y = f(x), x \in X\}$$

where

$$\mu_{\tilde{B}}(y) = \begin{cases} \sup \mu_{\tilde{A}}(x), x \in f^{-1}(y) \;\; if \; f^{-1}(y) \neq 0 \\ 0 \; otherwise \end{cases}$$

Using the definition of a fuzzy number and the extension principle, the membership functions for unary and binary operations can be introduced:

Definition 5: Extended operations (Zimmerman, 2001)

If $\tilde{A}, \tilde{B} \in F(\mathbb{R})$ with $\mu_{\tilde{A}}(x)$ and $\mu_{\tilde{B}}(x)$ are continuous membership functions, then by application of the extension principle for the binary operation $*: \mathbb{R} \otimes \mathbb{R} \rightarrow \mathbb{R}$, the membership function of the fuzzy number $\tilde{A} \circledast \tilde{B}$ is given by $\mu_{\tilde{A} \odot \tilde{B}}(z) = \sup min\{\mu_{\tilde{A}}(x), \mu_{\tilde{B}}(y); z = x * y\}$.

For unary operations $f: X \rightarrow Y, X = X_1$, the extension principle reduces for all $\tilde{A} \in F(\mathbb{R})$ to $\mu_{f(\tilde{A})}(z) = \sup \mu_{\tilde{A}}(x); x \epsilon f^{-1}(z)$.

The computational effort of implementing the extended operations is quite great if general membership functions are considered. Fuzzy sets are just approximate assessments of concepts, properties or data, and thus often it makes no sense to use sophisticated shapes for membership functions. Using specially shaped fuzzy numbers, the problem of computational effort can be significantly diminished. In the following paragraphs, we introduce simplification with trapezoidal fuzzy numbers, which leads to the efficient and useful implementation of fuzzy arithmetic. Without significant loss of generality, we can accept the LR representation of fuzzy numbers. According to Definition 3, we consider unimodal and flat fuzzy numbers. Clearly, the unimodal fuzzy numbers are a special case of flat fuzzy numbers (fuzzy interval), and thus we can consider only fuzzy intervals in the rest of the discussion on fuzzy numbers.

Definition 6: LR-type fuzzy number (Bonissone, 1986)

A fuzzy number $\tilde{A} \in F(\mathbb{R})$ is of the LR-type if there exist continuous and non-increasing shape functions $L$ and $R$ $L: [0,1] \rightarrow [0,1]$, $R: [0,1] \rightarrow [0,1]$ and four parameters a, b, $\alpha$, $\beta$, and the membership function of $\hat{A}$ is

$$\mu_{\tilde{A}}(x) = \begin{cases} L\left(\frac{a-x}{\alpha}\right) \; if \; x\epsilon[a-\alpha, a] \\ 1 \; if \; x \epsilon [a,b] \\ R\left(\frac{x-b}{\beta}\right) \; if \; x\epsilon[b, b+\beta] \\ 0 \; otherwise \end{cases}$$

where the interval [a,b] is the vertex or core of the fuzzy interval $\hat{A}$.

The fuzzy interval of the LR-type can be expressed as $\tilde{A}$ = (a, b, $\alpha$, $\beta$)$_{LR}$ and termed as a generalised trapezoidal fuzzy number. The definition of the generalised trapezoidal number of the LR-type with the linear function L(x) = R(x) = 1 – x leads to the definition of the trapezoidal fuzzy number.

Definition 7: Trapezoidal fuzzy number

A trapezoidal fuzzy number is expressed as $\hat{A}$ = (a, b, $\alpha$, $\beta$) and defined by the linear membership function:

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \frac{a-x}{\alpha} \; if \; a-\alpha \leq x \leq a \\ 1 \; if \; a \leq x \leq b \\ 1 - \frac{x-b}{\beta} \; if \; b \leq x \leq b+\beta \\ 0 \; otherwise \end{cases}$$

The general properties of the basic operations follow from the general definition (Definition 5) of binary operations. The introduction of trapezoidal fuzzy numbers simplifies the operations between them to arithmetic operations over real values.

The definitions of the basic arithmetic operations for the trapezoidal fuzzy numbers comprise the fuzzy numbers $\tilde{A} = (a, b, \alpha, \beta)$ and $\tilde{B} = (c, d, \gamma, \delta)$.

Table 1: Arithmetic operations for trapezoidal fuzzy numbers

| Operation | Result | Conditions |
|---|---|---|
| $-\tilde{A}$ | $(-b, -a, \beta, \alpha)$ | |
| $\frac{1}{\tilde{A}}$ | $\left(\frac{1}{b}, \frac{1}{a}, \frac{\beta}{b(b+\beta)}, \frac{\alpha}{a(a-\alpha)}\right)$ | $\tilde{A} \neq 0$ |
| $\tilde{A} + \tilde{B}$ | $(a+c, b+d, \alpha+\gamma, \beta+\delta)$ | |
| $\tilde{A} - \tilde{B}$ | $(a-d, b-c, \alpha+\delta, \beta+\gamma)$ | |
| $\tilde{A} \cdot \tilde{B}$ | $(ac, bd, a\gamma + c\alpha - \alpha\gamma, b\delta + d\beta - \beta\delta)$ | $\tilde{A} > 0, \tilde{B} > 0$ |
| | $(ad, bc, d\alpha - a\delta + \alpha\delta, -b\gamma + c\beta - \beta\gamma)$ | $\tilde{A} < 0, \tilde{B} > 0$ |
| | $(bc, ad, b\gamma - c\beta + \beta\gamma, -d\alpha + a\delta - \alpha\delta)$ | $\tilde{A} > 0, \tilde{B} < 0$ |
| | $(bd, ac, -b\delta - d\beta - \beta\delta, -a\gamma - c\alpha + \alpha\gamma)$ | $\tilde{A} < 0, \tilde{B} < 0$ |
| $\frac{\tilde{A}}{\tilde{B}}$ | $\left(\frac{a}{d}, \frac{b}{c}, \frac{a\delta + d\alpha}{d(d+\delta)}, \frac{b\gamma + c\beta}{c(c-\gamma)}\right)$ | $\tilde{A} > 0, \tilde{B} > 0$ |
| | $\left(\frac{a}{c}, \frac{b}{d}, \frac{c\alpha - a\gamma}{c(c-\gamma)}, \frac{d\beta - b\delta}{d(d+\delta)}\right)$ | $\tilde{A} < 0, \tilde{B} > 0$ |
| | $\left(\frac{b}{d}, \frac{a}{c}, \frac{b\delta - d\beta}{d(d+\delta)}, \frac{a\gamma - c\alpha}{c(c-\gamma)}\right)$ | $\tilde{A} > 0, \tilde{B} < 0$ |
| | $\left(\frac{b}{c}, \frac{a}{d}, \frac{-b\gamma - c\beta}{c(c-\gamma)}, \frac{-a\delta - d\alpha}{d(d+\delta)}\right)$ | $\tilde{A} < 0, \tilde{B} < 0$ |

Source: Bonissone and Decker, 1986

At this point, almost everything we need for appraising the attributes has been prepared. Only the term set and the mapping of names into fuzzy numbers is missing. For this we rely on Bonissone and Decker, who propose $L_2$ mapping for this purpose:

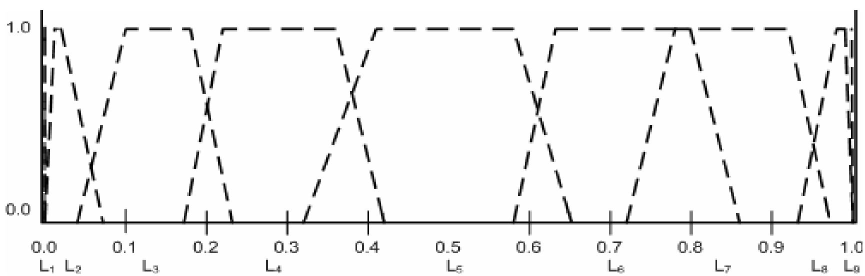
Table 2: Definition of $L_2$ mapping function

| Impossible | Very Unlikely | Very Small Possibility | Minor Possibility | Possible | Significant Possibility | Very Likely | Almost Certain | Certain |
|---|---|---|---|---|---|---|---|---|
| 0 | .01 | .10 | .22 | .41 | .63 | .78 | .98 | 1 |
| 0 | .02 | .18 | .36 | .58 | .80 | .92 | .99 | 1 |
| 0 | .01 | .06 | .05 | .09 | .05 | .06 | .05 | 0 |
| 0 | .05 | .05 | .06 | .07 | .06 | .05 | .01 | 0 |

Source: Bonissone and Decker, 1986

The definition can be illustrated even more comprehensively with a graph of the $L_2$ function:

Figure 4: Graph of the $L_2$ mapping function




Source: Bonissone and Decker, 1986

For the model to work, it will also need a joining operator. Because of the simplicity principle, we have opted, among the many operators, for generalised operators of the weighed mean expressed by the formula:

$$h_\alpha^w(a_1,\dots,a_n) = \left(\sum_{i=1}^n w_i a_i^\alpha\right)^{\frac{1}{\alpha}}, a_i \in [0,1], i \in \mathbb{N}_n, \alpha \in \mathbb{R}(\alpha \neq 0) \quad (1)$$

where for the vector $\vec{w} = (w_1,\dots,w_n)$ holds $\sum_{i=1}^n w_i = 1,\ w_i \geq 0\ \forall i \in \mathbb{N}_n$. The vector $\vec{w}$ is termed the weighed vector, and its components $w_i$ the weights. In the simplest version (equal weights $w_i = \frac{1}{n}$ *and* $\alpha = 1$ it is simply the arithmetic mean.

The final results of these calculations, trapezoidal fuzzy numbers, are not suitable for presentation of the results to appraisers. We must therefore map them back to linguistic values. We must find the linguistic value of which the fuzzy equivalent is the closest to the given trapezoidal fuzzy number. For this purpose we need a metric of the fuzzy sets. The Tran-Duckstein's distance takes into account the fuzziness of the fuzzy numbers and is confirmed in practice in environmental vulnerability assessment; we have therefore decided to use it in our approximation of fuzzy numbers with linguistic values. Tran-Duckstein's distance measure comprises the notation of fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ instead of the classical $\tilde{A} = (a, b, \alpha, \beta)$ where the parameters of the fuzzy number are points on the real line where $a_1 = a - \alpha, a_2 = a, a_3 = b, a_4 = b + \beta$.

The Tran-Duckstein distance for LR-type fuzzy numbers
$\tilde{A} = (a_1, a_2, a_3, a_4)_{LR}\ and\ \tilde{B} = (b_1, b_2, b_3, b_4)_{LR}$ (Tran, 2002b):

$$D^2(\tilde{A}, \tilde{B}, f) = \frac{\int_0^1 \left\{ \left[ \left( \frac{A_L(\alpha) + A_U(\alpha)}{2} \right) - \left( \frac{B_L(\alpha) + B_U(\alpha)}{2} \right) \right]^2 + \frac{1}{3} \left[ \left( \frac{A_U(\alpha) - A_L(\alpha)}{2} \right)^2 + \left( \frac{B_U(\alpha) - B_L(\alpha)}{2} \right)^2 \right] \right\} f(\alpha)\, d\alpha}{\int_0^1 f(\alpha)\, d\alpha} \quad (2)$$

where $\alpha$-level interval of fuzzy number $\hat{A}$ is

$$A(\alpha) = (A_L(\alpha), A_U(\alpha)) = (a_2 - (a_2 - a_1) L_A^{-1}(\alpha), a_3 + (a_4 - a_3) R_A^{-1}(\alpha)) \quad (3)$$

and $f$, which serves as a weighting function, is a continuous positive function defined on $[0,1]$. The distance is a weighted sum (integral) of the distances between two intervals at all $\alpha$ levels from 0 to 1. It is reasonable to choose $f$ as an increasing function, indicating greater weight assigned to the distance between two intervals at a higher $\alpha$ level.

With a weighted distance $f(\alpha) = \alpha$, for each $\alpha$ the above definition is for trapezoidal fuzzy numbers $\tilde{A} = (a, b, \alpha, \beta)\ and\ \tilde{B} = (c, d, \gamma, \delta)$ simplified to:

Figure 5: Distance for trapezoidal fuzzy numbers

$$\begin{aligned} D_T^2(\tilde{A}, \tilde{B}, \alpha) = & \left( \frac{a+b}{2} - \frac{c+d}{2} \right)^2 + \frac{1}{3} \left( \frac{a+b}{2} - \frac{c+d}{2} \right) [\beta - \alpha - \delta + \gamma] + \frac{2}{3} \left( \frac{b-a}{2} \right)^2 \\ & + \frac{1}{9} \left( \frac{b-a}{2} \right) [\beta + \alpha] + \frac{2}{3} \left( \frac{d-c}{2} \right)^2 + \frac{1}{9} \left( \frac{d-c}{2} \right) [\delta + \gamma] \\ & + \frac{1}{18} [\beta^2 + \alpha^2 + \delta^2 + \gamma^2] - \frac{1}{18} [\alpha\beta + \gamma\delta] + \frac{1}{12} [\beta\gamma + \alpha\delta + \beta\delta + \alpha\gamma] \end{aligned}$$

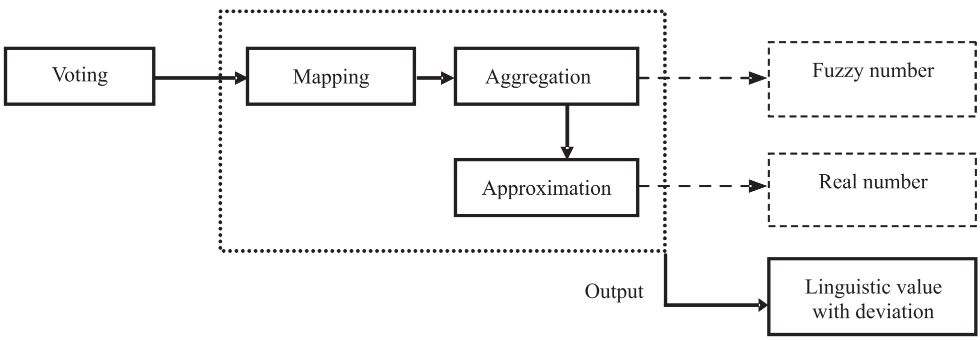
Source: Tran and Duckstein, 2002b

Despite the indistinctness between the notation of $\alpha$ level and the third parameter of a fuzzy number $\tilde{A} = (a, b, \alpha, \beta)$, we keep the classical notation. This way we lose a bit in terms of clarity, but stick with the prevailing manner of fuzzy number representation.

## 3. Solution

The overall model encompasses an appraisal module and an optimisation module. With the first, we capture the assessments of the appraisers for individual attributes and aggregate them into the individual appraiser's assessment for an individual project, into assessments for the selections of alternatives (variants) and into group assessments for groups of appraisers. With the help of the optimisation module, we generate a selection of the most suitable variants of projects, among which appraisers choose, according to their own judgement, the one that they find the most suitable. This article intends to present an appraisal module for investment projects, which involves 5 steps, from assembling the linguistic appraisals to the final conversion of fuzzy numbers into linguistic values:

Figure 6: Structure of the appraisal module

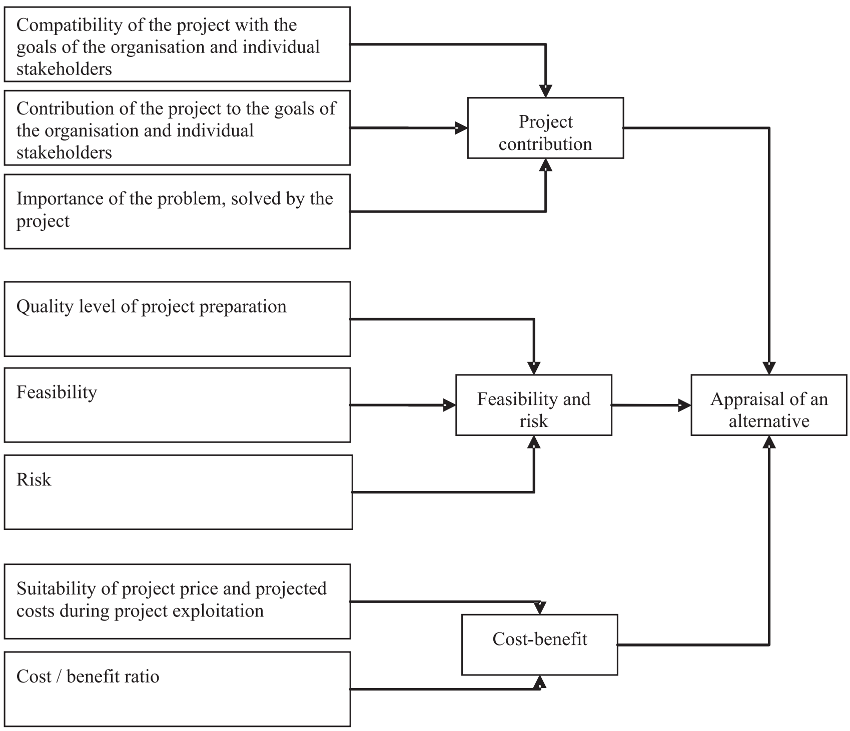



Source: Benčina, 2004

The core of the module is the decision tree that contains knowledge on the structure of the values that determine to what extent an individual alternative is suitable for inclusion in the budget. Starting from the framework of deciding on capital investments in the public sector (GAO/AIMD-98-110, 1998), legally prescribed definitions and the analysis of the method of decision making in local communities in Slovenia, we have determined the structure of the decision tree (Fig. 7).

Merging entry variables into the project appraisal follows the decision tree through the nodes to the root. The leaves and all the nodes including the root each carry three values: linguistic value, approximated deviation and equivalent fuzzy trapezoidal number.

Figure 7: Decision tree of the module of multiple-attribute appraisal of investment projects




Source: Benčina, 2004

Table 3: Linguistic values to fuzzy trapezoidal numbers mapping function

| Reject | Lowest | Very Low | Low | Medium | High | Very High | Highest | Must Be |
|---|---|---|---|---|---|---|---|---|
| 0 | .01 | .10 | .22 | .41 | .63 | .78 | .98 | 1 |
| 0 | .02 | .18 | .36 | .58 | .80 | .92 | .99 | 1 |
| 0 | .01 | .06 | .05 | .09 | .05 | .06 | .05 | 0 |
| 0 | .05 | .05 | .06 | .07 | .06 | .05 | .01 | 0 |
| $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ | $L_6$ | $L_7$ | $L_8$ | $L_9$ |

Source: Benčina, 2004

The linguistic values of the leaves are the direct result of the appraisal process, and the equivalent fuzzy numbers are the images of a simple mapping between the first and the second (Table 3), where the approximated deviation of course gets the value 0. The values of the parent nodes are calculated from the leaves towards the root of the tree as the fuzzy arithmetic mean of the fuzzy values of the children

$$A_{i,j} = \frac{1}{K_{i,j}} \sum_{k} A_{i+1,j,k} \; ; i = I-1,\cdots,1; \; j = 1,\cdots,J_i \, ; k = 1,\cdots K_{i,j} \, ; \qquad (4)$$

where I is the number of levels of the tree and i is the current level of the tree, $J_i$ is the branching of the tree and j is the position of the node at the i-th level and $K_{i,j}$ is the number of children of the parent in question at the level i+1 and k is the position of the child, of the parent in question. The calculated trapezoidal fuzzy numbers are then approximated back to linguistic values so that the closest representative of the linguistic value is found $L_{approx} = \min(D_T(A_{i,j},L_n,\alpha))$; n = 1,…,9. The approximation deviation is the difference of distances to the fuzzy number (0, 0, 0, 0) of the calculated trapezoidal fuzzy number and the fuzzy number image of the linguistic approximation: $AO = D_T(A_{i,j},0,\alpha) - D_T(L_{approx},0,\alpha)$. The linguistic approximations enable an informative representation of the results with the basic values of the linguistic variable (Table 4).

The appraiser approves each attribute with a linguistic appraisal which represents the degree of trust in the suitability of the project in terms of the given attribute:

The representation in which the approximation deviations are used for marking the deviations up and down (25% < deviation < 50% up or down) is a useful tool for analysing the results and differences between appraisers (Table 5). This is our original solution, distinguished by its clarity, which allows precise and efficient presentation of the results to the decision makers (Benčina, 2004).

The values of all the variables in the tree are merged into an appraisal of the variants of the plan for the development programmes and into joint group appraisals of several appraisers. This aggregation is also done by the simplest approach, with the calculation of the fuzzy arithmetic mean of the fuzzy numbers:

$$A_{i,j} = \frac{1}{|G|\cdot|V|} \sum_{G} \sum_{V} A_{i,j} \, ; G \in P(D); V \in P(A); i = 1,\cdots,I; \; j = 1,\cdots,J_i \, ; \qquad (5)$$

for all subsets of the set of alternatives A and the set of appraisers D for which it is reasonable in the given case. We approximate all the calculated values with linguistic values and equip them with the approximated deviation.

The interpretation of the results is based on the comparison and ranking of the projects. The task is not difficult due to the clarity of the representation. The value of the result depends on the degree of consensus between the groups of political and professional appraisers, and of course on the degree of consensus within the groups. The analysis of deviations is performed in the phase of reporting and coordination within the circle of all appraisers who must unequivocally decide about the required level of consensus and the acceptable maximum deviations. In this manner, all the participants accept joint responsibility for suitable coordination of opinions concerning the final decisions.

The tool for decision-making support that we are presenting is not limited in the number of decision makers, the number of groups of decision makers or the number of investments it covers. Limitations of a practical nature must be observed, since we must plan and realise each application within controllable time limits. The solution is general enough to serve as a suitable basis for constructing a tool for the support of direct e-democracy by which we could, with the help of other suitable approaches and tools, also contribute to the participation of citizens in decision making (Chmura et al., 2004).

We have built the described algorithms and procedures into a web application which allows simple data collection, swift data processing and simple preparation of reports and information.

## 4. Results

The appraisal model was tested in two Slovenian municipalities. The responsible persons in these environments showed great interest in the proposed solution, since this problem is of concern to everyone. It has been proven that the chosen method of appraisal is suitable for the chosen environments. Because of the space constraints we present only the results of the one case study.

Nine representatives of municipal government and four municipal councillors participated in the case we are presenting herein. The appraisals were anonymous insofar as any affiliation with a group. Seven projects that were the subject of current discussion in the municipality were appraised. The results, represented only by the basic linguistic values (Table 4), exhibit a considerable lack of selectivity of the answers at first sight.

Table 4: Joint appraisal of the projects – basic linguistic values only

| | Municipal Council | Municipal Government | Together |
|---|---|---|---|
| All Projects | High | Medium | Medium |
| Project 1 | High | Medium | Medium |
| Project 2 | High | Medium | High |
| Project 3 | Medium | Medium | Medium |
| Project 4 | High | Medium | Medium |
| Project 5 | High | High | High |
| Project 6 | High | Medium | Medium |
| Project 7 | Medium | High | High |

Source: Author

Nevertheless, we soon found out that the appraisers agreed on rejecting Project 3 and agreed to approve Projects 5 and 7. Project 2 also got a High joint appraisal. As can be seen, in this manner we can quickly detect the alternatives that stand out. If we also take into consideration the individual extreme appraisals (Reject and Must Be), we see that Project 3 and Project 4 were given individual rejections, which means that it would also make sense to reject Project 4. What to do with the remaining two projects with a medium value can be deduced from the presentation, with the indicated deviations (Table 5).

Table 5: Projects ranked in terms of the joint appraisal – presentation with deviations

| | Municipal Council | Municipal Governmnt | Together |
|---|---|---|---|
| Project 5 | High | High | High |
| Project 2 | High | Medium→ | ←High |
| Project 7 | Medium | High | ←High |
| Project 1 | High | Medium | Medium→ |
| Project 6 | ←High | Medium | Medium→ |
| Project 4 | ←High | Medium | Medium |
| Project 3 | Medium→ | Medium | Medium |

Source: Author

We find that Project 1 was appraised somewhat higher than Project 2. We get a very clear picture of the appraisers' opinions about the projects with an overview of the values of the nodes of the decision tree (Fig. 7) – area appraisals (Table 6):

Table 6: Appraisal of projects by areas

| | Feasibility and Risk | | | Prices and Costs | | | Contribution of the Project | | |
|---|---|---|---|---|---|---|---|---|---|
| | MC | MG | Joint | MC | MG | Joint | MC | MG | Joint |
| Project 5 | ← High | ← High | High | ← High | → Medium | High | Very High | ← Very High | Very High |
| Project 2 | ← High | ← High | High | High | Medium | Medium | ← High | → Medium | High |
| Project 7 | ← Medium | ← High | Medium | Medium | ← High | Medium | Medium | Very High | High |
| Project 1 | ← High | Medium | Medium | → Medium | → Low | Medium | ← Very High | → Medium | High |
| Project 6 | ← High | Medium | Medium | → Medium | Medium | Medium | High | ← High | High |
| Project 4 | ← High | ← Medium | Medium | ← High | Medium | Medium | ← High | Medium | Medium |
| Project 3 | ← High | → Medium | Medium | → Medium | Medium | Medium | → Medium | Medium | Medium |

Source: Author

The results indicate that the appraisal of the contribution of the project actually determined the ranking of the projects. This probably means that the appraisers are best acquainted with the importance of the projects for the development of the local community and can best distinguish among them on the basis of this property. In the area of feasibility and risk, and price and cost, they would probably need additional information. A detailed analysis of the answers showed that the appraisers used different approaches to make their appraisals, for which reason the results would need to be normalised.

We have also processed the results of the appraisals in more detail and have given a qualitative description of the results (Table 7). The description showed the participants where they disagree and pointed out the further steps to be taken toward coordinating opinions and finding an optimum decision. It was assessed that the situation is in reality just as the results indicated. Since there is no mutual coordination, the level of consensus among the appraisers is low. This makes a systematic approach to deciding on the investment projects all the more necessary. The same is indicated by the opinions of the participants in the study who agree almost unanimously that the current level of cooperation does not allow optimum results and that the experimental model is a suitable tool which they are prepared to use in the future.

Table 7: Qualitative representation of the results

| | Appraisals | | | Level of Consensus | | | Commentary | |
|---|---|---|---|---|---|---|---|---|
| | Σ | MC | MG | MC/MG | MC | MG | Analysis of Appraisals | Classification and Measures |
| Project 5 | H | H | H | C | P | P | Unanimous appraisal of the project; high appraisal in all areas. | Include in the budget. |
| Project 2 | ←H | H | M→ | C | **N** | P | Consensus between groups. Two low appraisals from political appraisers. Low appraisal for the project's contribution. | A positive proposal for inclusion in the budget. The opinion of the municipal council should be checked and the project's contribution thought over. |
| Project 7 | ←H | M | H | **N** | P | P | Disagreement between groups. Political appraisers refused the project, but their opinions vary (two very low appraisals). | Professional services support the project. The municipal council is against it, but not unanimously. Promotion of the project among the municipal councillors must be undertaken. |
| Project 1 | M→ | H | M | **N** | C | **N** | The political appraisers support the project with a considerable level of consensus. Disagreement among professional appraisers and between groups. | A high investment in the project would demand a detailed analysis of the problem. The relations between professional services should be analysed and an adequate level of consensus should be reached. The municipal council must be informed about the issue of feasibility and risk of the project. |
| Project 6 | M→ | ←H | M | C | **N** | **N** | The groups' assessments are close. Some internal disagreements exist, mainly among professionals. The professionals' appraisal of feasibility gives the project a lower mark. The low appraisal of one of the political appraisers stands out. | The possibility of changing the opinion of one of the members of the municipal council should be examined. Disagreements among the professionals must first be analysed. |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| Project 4 | M | ←H | M | P | P | N | The lowest appraisal of feasibility by the professional appraisers. Discrepancies concerning feasibility between the two groups. Completely different opinions among professional appraisers. Low appraisal of the project's contribution. | In view of the results of the project's appraisal the project can justifiably be ranked at the bottom of the priorities. The lowest feasibility appraisal by the professionals indicates that the project has not been well prepared. If we add the low appraisal for the project's contribution we can claim with a good degree of certainty that it cannot be included in the budget. |
| Project 3 | M | M→ | M | C | P | C | The project almost unanimously ranks as last. It got somewhat better appraisals from two political and three professional appraisers. | The starting points of the project must be made clear. The appraisal results indicate that the project cannot be realised within the given framework. |

Legend:

*Stakeholders*: MC – Municipality Council; MG – Municipality Government;
*Consensus appraisals*: C – Consensus, P – Partial consensus, N – No consensus;
*Linguistic appraisals*: R – Reject, LL – Lowest, VL – Very low, L – Low, M – Medium, H – High, VH – Very high, HH – Highest, MB – Must Be.

Source: Author

## 5. Conclusions

With the presented solution, we have constructed a model for support in the process of making decisions on investment projects in local communities and have underpinned it with a software solution. The tool is a simple and effective instrument that enables establishment of a process of decision making on investment projects in municipalities. This approach allows balanced participation in decision making for all involved (professional assistants of the municipal government and municipal council members), permits monitoring of differences in views and establishes an opinion coordination procedure and the search for an optimum solution from the aspect of level of agreement as well as from the aspect of the expected results of realisation of the project. In the phase of appraisal and coordination, the decision makers learn about the differing views regarding individual projects and establish some level of mutual understanding. The final decision making by the municipal council concerning the various budget alternatives is thus supported by the recognition and proposals made by the professional staff in the municipal government. Of course, the execution of the decision-making process does not fully guarantee better and more efficient decisions; it is a good basis, however, for improving the efficiency of decision making in municipalities and more efficient management of funds from the investment part of the budget.

Through the case studies we demonstrated that with the correct approach it is possible to establish a decision-making procedure that includes the process of opinion coordination in a dialogue between professional arguments and political or stakeholders' demands and desires, in an environment in which the level of cooperation and acceptance of compromise might otherwise be low. The use of a fuzzy approach allows a larger number of decision makers to express their opinions in a very simple way. The simplicity of the model guarantees the willingness to participate in the process. There were no indications of limitations in the use of the model in the public sector in general, and therefore we found that the model could be considered as solid ground for further development of decision making processes in the public sector.

The model we have presented can be used for solving any decision-making problem that lends itself to fuzzy logic. It is especially suitable for use in research in which the same decision-making model is used for a large number of cases (institutions, units, departments), for instance the project that we plan to execute with the help of the described solution among Slovenian local communities. Future activities will involve the further development of the software solution, execution of decision-making projects and appraisals, and empirical research of aggregation functions.

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# Uporaba neizrazite logike pri investicijskom usklađivanju u javnom sektoru

*Jože Benčina*[1]

***Sažetak***

*Prilikom razvojnog procesa ili investicijskog odlučivanja u javnom sektoru, politika i struka često imaju potpuno različita stajališta. Suprotni pogledi, međusobno nerazumijevanje i nemogućnost usvajanja kompromisa pri prijedlogu razvoja ili investicijama česti su uzrok devijacija koje su na kraju izražene u negativnim ekonomskim posljedicama. Učinkovit i uspješan prijedlog, priprema i izdvajanje investicija u javnom sektoru jedan je od ključnih razvojnih čimbenika države obzirom na lokalnu zajednicu. Zbog ovih razloga odlučili smo problem pozornije istražiti i oblikovati sredstvo u obliku sustava za podršku odlučivanja. Da bi ključnim osobama u procesu odlučivanja omogućili iskaz njihovih razmišljanja na prirodan i njima blizak način, omogućili smo im upotrebu metode neizrazite logike - sustava ocjenjivanja s lingvističkim vrijednostima. Namjera ovog članka je predstavljanje značaja i strukture neizrazitog modela usklađivanja mišljenja, a nastao je korištenjem rezultata studije slučaja - dvije slovenske općine. Studije slučaja pokazale su da je u javnom sektoru primjerenim pristupom moguće implementirati postupak odlučivanja kao rezultat procesa usklađivanja mišljenja različitih osoba koje sudjeluju u procesu odlučivanja.*

***Ključne riječi****: investicijski projekt, višekriterijsko odlučivanje, javni sektor, lingvističke vrijednosti, neizrazita logika.*

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## Appendix I – The Data

Table 1: Case study input data - appraisals of criteria for seven projects given by all appraisers

Tablica 1: Ulazni podaci studije primjera: ocjene trinaest ocjenjivača za sedam projekata

| | | MC1 | MC2 | MC3 | MC4 | MG1 | MG2 | MG3 | MG4 | MG5 | MG6 | MG7 | MG8 | MG9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **P1** | C1 | High | Very High | Highest | High | Very High | Very High | Highest | Reject | Highest | Medium | High | High | Low |
| | C2 | High | High | High | Very High | Very High | Very High | Highest | Reject | High | Medium | Medium | High | Medium |
| | C3 | Very High | Very High | Very High | Very High | High | Very High | Highest | Reject | Very High | High | Low | Medium | Low |
| | C4 | Medium | High | Very High | Medium | Very High | Very High | Very High | Reject | Medium | Very High | Medium | Very High | Medium |
| | C5 | Medium | Medium | High | High | Medium | High | Very High | Reject | Highest | High | Very Low | Low | Low |
| | C6 | Low | Medium | High | Medium | Medium | High | Very High | Reject | Low | High | Very Low | Lowest | Lowest |
| | C7 | High | High | High | High | Medium | Medium | Very High | Reject | High | Medium | Low | High | Very Low |
| | C8 | High | High | Very High | Very High | Very High | Very High | Highest | Reject | High | Medium | Medium | Medium | Low |
| **P2** | C1 | High | Highest | High | High | High | High | High | Very High | High | Low | Medium | Highest | Medium |
| | C2 | Medium | Very High | Medium | High | High | High | Medium | High | Medium | Low | Medium | Very High | Medium |
| | C3 | Low | Very High | Medium | High | High | High | Medium | Very High | Low | Low | Low | Very High | Medium |
| | C4 | Medium | Very High | High | High | High | Very High | High | High | High | Low | Medium | Very High | Medium |
| | C5 | Low | High | High | Medium | Medium | High | High | Medium | Very High | Low | High | Very High | Medium |
| | C6 | Low | Highest | High | Medium | Medium | High | Medium | High | High | Low | Medium | High | Medium |
| | C7 | Highest | Highest | High | High | Medium | Low | Medium | High | Highest | Low | Low | Very High | Medium |
| | C8 | Medium | Highest | High | High | High | Medium | High | Very High | High | Low | Medium | Very High | Low |
| **P3** | C1 | Very Low | Medium | Very High | High | Very Low | Very Low | Medium | High | High | Reject | Medium | High | High |
| | C2 | Medium | Medium | High | Very High | Medium | High | Medium | High | High | Reject | High | Very High | Medium |
| | C3 | Low | Medium | High | High | Very Low | Low | Medium | High | Low | Reject | High | High | Medium |
| | C4 | Low | High | High | High | Medium | High | High | High | High | Reject | High | Very High | Medium |
| | C5 | Low | Very High | High | High | Very Low | Low | High | Very High | Highest | Reject | High | High | Medium |
| | C6 | Medium | Very High | Very High | High | Medium | Medium | Medium | High | Low | Reject | Medium | Very High | Medium |
| | C7 | Low | Highest | High | Reject | Low | Low | Medium | High | High | Reject | High | Very High | Low |
| | C8 | Low | High | High | High | Very Low | Very Low | Medium | High | Medium | Reject | Medium | High | Medium |

Table 1: Continuation

Tablica 1: Nastavak

| | | MC1 | MC2 | MC3 | MC4 | MG1 | MG2 | MG3 | MG4 | MG5 | MG6 | MG7 | MG8 | MG9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **P4** | C1 | Medium | High | Very High | Very High | Reject | Reject | Low | Very High | Very High | Very High | High | Very High | Low |
| | C2 | Medium | High | High | High | Reject | Reject | Low | Very High | Very High | Very High | Very High | Very High | Low |
| | C3 | Low | Medium | High | High | Reject | Reject | Low | Medium | Very High | Very High | Very High | Very High | Low |
| | C4 | Medium | Very High | High | High | Reject | Reject | Low | Low | High | High | High | Highest | Low |
| **P4** | C5 | Very Low | Very High | Very High | Medium | Reject | Reject | Low | Very High | Very High | Medium | Medium | Highest | Low |
| | C6 | Low | Very High | High | Medium | Reject | Reject | Medium | Very High | High | High | Medium | Very High | Low |
| | C7 | High | High | Very High | High | Reject | Reject | Medium | Very High | Very High | Medium | Medium | Very High | Low |
| | C8 | Low | High | High | Very High | Reject | Reject | Medium | Very High | Very High | Very High | High | Highest | Low |
| **P5** | C1 | High | Highest | Highest | Very High | Very High | High | High | Highest | Highest | High | Very High | Highest | High |
| | C2 | High | Highest | Very High | Very High | Very High | High | High | Highest | Highest | High | Highest | Highest | High |
| | C3 | High | Highest | Highest | Very High | Very High | High | High | Highest | Very High | Low | Very High | Highest | High |
| | C4 | Medium | Low | Very High | High | Very High | High | Medium | Very High | High | High | High | Very High | High |
| | C5 | Low | Very High | Highest | High | Very Low | Low | Medium | Medium | Very High | High | High | Highest | High |
| | C6 | Medium | Medium | Highest | High | High | Medium | Medium | Very High | High | Lowest | High | Very High | Medium |
| | C7 | Medium | Medium | Very High | High | Very High | High | Medium | Very High | Very High | Lowest | High | Very High | Medium |
| | C8 | High | Very High | Highest | Very High | Very High | High | High | Highest | Highest | Low | High | Highest | High |
| **P6** | C1 | High | Low | Highest | Highest | High | Medium | High | Very High | Highest | Very Low | Highest | Very High | Very High |
| | C2 | High | Low | Very High | Very High | Very High | High | High | High | Highest | Low | Highest | High | High |
| | C3 | High | Low | Very High | Very High | Medium | Low | High | High | Highest | Very Low | Highest | Medium | High |
| | C4 | Medium | High | Very High | High | High | Medium | High | Very High | High | Lowest | Medium | Low | High |
| | C5 | Low | Medium | Very High | High | Medium | Low | Medium | Very High | High | Very Low | Medium | Very Low | High |
| | C6 | High | Low | High | Medium | High | Medium | Medium | High | High | Reject | High | High | High |
| | C7 | High | Low | High | High | Low | Very Low | High | High | High | Lowest | High | Low | High |
| | C8 | High | Low | Very High | Highest | High | Medium | High | High | Highest | Lowest | Very High | Medium | Very High |

Table 1: Continuation

Tablica 1: Nastavak

| | | MC1 | MC2 | MC3 | MC4 | MG1 | MG2 | MG3 | MG4 | MG5 | MG6 | MG7 | MG8 | MG9 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| **P7** | C1 | Medium | Lowest | High | Highest | Highest | Very High | Very High | Very High | Highest | Very High | Very High | Highest | Medium |
| | C2 | Medium | Lowest | High | Very High | Highest | Very High | Very High | Very High | Highest | Very High | Very High | High | Medium |
| | C3 | High | Lowest | High | Highest | Very High | High | Very High | Very High | Highest | Very High | Very High | Very High | Medium |
| | C4 | Medium | Very Low | Medium | Medium | High | Medium | High | High | High | Medium | Very High | Very High | Low |
| | C5 | Very Low | Very Low | High | High | Very High | Very High | High | High | Very High | Low | Low | Very High | Medium |
| | C6 | Low | Lowest | High | Medium | High | Medium | High | High | Highest | High | High | Medium | Medium |
| | C7 | Very High | Lowest | High | High | Medium | Medium | High | High | High | Medium | Very High | Very High | Reject |
| | C8 | Medium | Lowest | High | Very High | Very High | High | Very High | Very High | Highest | Very High | Very High | Very High | Medium |

Legend: Projects: P1, …, P7; Criteria: C1, …, C8; Municipal counsellor: MC1, …, MC4; Municipal professionals: MG1, .., MG9.
Legenda: Projekti: P1, ..., P7; Kriteriji: C1, …, C8; Općinski savjetnici: MC1, …, MC4; Stručno osoblje: MG1, …, MG9

Source: Author