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Patricius' Phenomenological Theory of Tides and its Modern Relativistic Interpretation

Abstract

This paper brings, for the first time, an interesting modern description of the Patricius' phenomenological theory of tides and its modern relativistic understanding.

Famous historians of science (for ex. M. Jammer, Concepts of Force, *Dover, 1999; First edition published in 1957 by Harvard Press) are emphasizing Patricius' treatise on tides, which had been of primary importance for Kepler in his attempts at formulating the universal character of attraction. Patricius had tried to explain the variety of phenomena of tides in various seas as part of his model of the universe (28th and 29th Books of Patricius'* Pancosmia*). He correctly recognized the Moon and the Sun as two general causes of tides (formulated by Kepler as the lunar theory of tides), but failed to see the role of gravity. Patricius rather ascribed tides, within the framework of his general philosophy, to be caused by light and heat (*lux et calor*). Science (physics) after Patricius explained tides as an effect of gravity (Newton), and later in the 20th century as an effect of spacetime curvature (Einstein). The mathematical description of tides within Newton's theory of gravitation was shown in the paper, along with a more refined calculation of the same phenomenon in curved spacetime within the general relativity theory for the case of weak gravitational fields (Newtonian limit). The general relativistic correction found to be very small compared to the classical Newtonian expression, as one should expect when dealing with weak gravitational fields. Both theories – Newton's and Einstein's – despite their precision and beauty in describing tides, do not, however, describe tides in such detail as Patricius' theory which includes the local features of the phenomenon. At various symposia all over the world dedicated to his miraculous year of 1905 (for ex. 22nd Congress of History of Science, July 24 – 30, 2005, Beijing), Einstein has been recognized as the greatest physicists of the 20th century, and with Newton the two greatest physicists of all times. In the paper Patricius is understood as a direct predecessor of Kepler in the theory of tides, a hundred years before Newton.*

Key Words

tides, gravity, spacetime curvature, phenomenogical theory, Frane Petrić, Johannes Kepler, Isaac Newton, Albert Einstein, Patricius' contributions, Newtonian calculations, Einsteinian calculations

In this paper we try to present Patricius' phenomenological theory of tides as well, as the description of same that followed in the frame of Newton's theory of gravity and Newton's mechanics, in the second half of the 17th century, and in the frame of much more elegant theory of relativity of the 20th century.

Seeing these different descriptions of the same phenomenon one can easily form a picture of the changes in the world of science, or more accurately in the world of thought, which took place from the time of late renaissance up to the present day.

Also Patricius is viewed as a direct predecessor of the great scientists such as Galileo and Kepler and a remote anticipator of some key ideas of modern physics.

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Patricius and his contemporaries

Patricius, also know as Patrizi, in Italy, and Petrić, in his homeland Croatia, born in 1529 on the island of Cres (Cherso), was a late renaissance philosopher who could be characterized through his work as one of the key thinkers between Platonism, which became revived in the time of the renaissance, and the philosophy of nature which appeared at the end of the renaissance.

Our views concerning work and life of Frane Petrić have undergone radical changes in the last decade due to the international and interdisciplinary symposium "The Days of Frane Petrić". Today we have a deeper understanding of Patricius' general philosophy, particularly his cosmological model and basic scientific concepts and pictures elaborated rather well throughout the books of his philosophical system. We have become aware of many dimensions of his complex personality as well of his distinguished stature at the end of the renaissance and at the early beginning of a new theoretical and experimental science.

The philosophy of nature eventually gave birth to physics when the philosophical method was augmented by mathematics and experiment thus becoming scientific method rather than philosophical.

The philosophical system developed by Patricius is therefore not alike those of Platonists Marcello Ficino (1433–1499) and Pico della Mirandola, nor those of Bernardino Telesio (1509–1588) or Giordano Bruno (1548–1600) who were concentrated solely on the philosophy of nature.

So it seems to be the best attitude to refer to Patricius as the renaissance philosopher, describing his work by the time in which he had lived rather than by some contemporary school of thought.

In his time, the time of late renaissance explosion of knowledge, the world of thought was in a desperate need of a new method that could encompass and explain all the known facts, since Aristotelianism and Platonism obviously couldn't.

There were several innovative approaches trying to reconcile one with another, as well as the new approach of the philosophy of nature.

At this point one should remember the work of Francis Bacon, a sixteenth century English philosopher, and his *Instauratio magna*, the first part of *New organon,* where a new method of *true induction*, based upon experiment and observation was proposed as an alternative to the existing methods.

The work of Patricius is in general not very different from all those other contemporary attempts. In his system, described completely in his capital work '*Nova de universis philosophia*', especially its fourth and final part *Pancosmia,* one finds several peculiar notions and ideas which can also be found in the works of other thinkers that followed such as Descartes and Jacopo Mazzoni, who was Galileo's mentor. These are ideas that could be considered to be of the utmost importance for the development of the mathematical physics.

Kepler also occupied himself with the phenomenon of tides however he tried to explain the phenomenon on grounds more related to the observations similar like Galileo; note that they are both a seventeenth century scientists.

In his letter to Herwart von Hohenburg (1607), regarding the lunar theory of tides, a model of the phenomenon devised by Kepler, he quoted the work of Patricius.

Kepler stressed out Patricius' lunar theory of tides that was developed upon the prior treatises by Fredericus Chrysogonus, Fredericus Delfinus, Augustinus Caesareus and subsequently Telesio.

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A brilliant journal *Dubrovnik*, no. 1–3,1997, edited by I. Martinović, brings the same paragraph from Kepler's letter to Herwart von Hohenburg in Munich (J.Kepler, *Gessamelte Werke*, ed. Max Casper, vol. 15 (Beck, München 1951), *Briefe* 1604–1607, p. 387), which is also analysed by Jammer, p. 83 of his book.

Alas, the supplement: "Johannes Kepler, Petrić and the nautical literature at the beggining of the XVII century", *Dubrovnik* 1–3,1997, Matrix Croatica, Dubrovnik, p. 290–291, does not include M. Jammer's interpretation of the Patricius' mediate role ("Francisco Patricio inveniuntur") in Kepler's formulation of the notion of force.

Namely, Jammer finds that Patricius played an important mediate role in Kepler's conclusion that the lunar attraction of the sea as well as other heavy bodies is the same (analogous) to the terrestrial attraction.

Kepler actually understood the universal character of attraction (the reciprocal characteristic of gravity), even though he had assumed gravity to be passive (passivity of the stone) instead of active.

Galileo (1564–1642) is considered to be the far greatest predecessor of Isaac Newton (1642–1727), who founded the modern physics, started the mathematization of physics and Kepler (1571–1630) worked out the basis of classical cosmology.

Some of their ideas can be found in the system originally developed by Patricius almost a century before Newton.

Contributions of Patricius

Patricius made numerous contributions to the contemporary world of thought. For example, in *Pancosmia* he introduced both mathematical and physical space separately.

Mathematical space is described as a reality ontologically prior to all bodies. Its basic elements are geometrical points which correspond to units in arithmetic, giving so a geometrical description of arithmetic and putting geometry ahead of arithmetic. In such definition one can certainly spot the platonic element of his system.

Physical space, on the other hand, contains three-dimensional forms (bodies) with resistance. Here we can draw a parallel to a later definition of bodies being geometrically definable, made by Descartes and to the notion of force asserted by Leibniz.

Patricius preferred geometry as a tool for describing physical space rather than arithmetic. Geometry, later analytic geometry introduced by Descartes and modern differential geometry with topology, investigated in the works of Gauss and Riemann, is a fruitful branch of mathematics indeed which can be used to formulate with elegance and ease all great theories of modern physics.

Patricius' phenomenological theory of causes and variety of flowing of the sea was developed in the three books of *Pancosmia*.

In the 28th book "On the variety of flowing into and flowing away of the sea (*De maris affluxus, et refluxus varietate*)" Patricius described the variety of the tide intensity in different seas and oceans of the contemporary known world with an effort of a true explorer. In the same book he also developed the lunar theory of tides based on the prior works of Fredericus Chrysogonus (1472–1538) from Zadar, mathematician Fredericus Delfinus, Augustinus

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Caesareus, Gioralmo Borro and the seaman Nikola Sagroević Sagri from the town of Dubrovnik.

The 29th book "On causes of the flowing of the sea (*De causis affluxus et refluxus maris*)" is about the hierarchy of causes of the tides, which he had derived with his own original methaphisical-scientific method.

Amongst around twenty different causes of the tides, Patricius defined the Moon and the Sun as the two main ones, but not at all the only ones.

However he did recognize that the influence of the Moon's and Sun's position on the sky as well as their light on the tides is, after all the most important.

Similar by its contents to these two books is also the 30th book of *Pancosmia* "On the other motions of seas and oceans (*Oceani, et Mediterranei motus alii*)".

Most important of all, he arrived to the correct conclusion because his phenomenological theory was based upon exact observations. Patricius had a luxury of possessing detailed descriptions of the seas all over the known world from the sailors and navigators.

Here we can see the beginning of the scientific method, later introduced, with great success, by Galileo and Kepler.

Newton's theory of gravity and tides

Galileo, Mazzoni's disciple, introduced mathematics aided by experiment as a method of describing the physical world. And after him Newton invented calculus and equations of motion in classical Euclidean–Descartes 3-space. That formalism remained generally the same for the entire classical mechanics.

Newtons theory of gravitation enabled exact calculations which gave results in full accordance with Kepler's empirical laws of planet orbits and became the mathematical fundament of classical cosmology.

Among other explanations, Newton's theory of gravitation also provides means for the classical description of tides.

If one uses the Newton's law of gravity to calculate the difference between the force acting on the Earth's centre with respect to its surface (by using certain approximation) one easily obtains the expressions:

$$a = 2G\frac{M_m R_\oplus}{r^3} \quad , \quad a = -2G\frac{M_m R_\oplus}{r^3},$$

for two opposite sides of Earth.

Note that the expressions differ only by the sign describing so a symmetric tide.

Use of the Newtonian mechanics and the Newtonian concept of space and force gives a satisfactory mathematical description of tides. The results are in accordance with the observed facts, for example the different fortitude of influence from the Sun and the Moon.

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Picture depicting Earth, the Moon and the Newtonian understanding of the tides.

Moon

$$F = G \frac{m_M m_Z}{r^2}$$

$$F_{pl} = 2G m_M m \frac{R_Z}{r^3}$$

m=sea mass

Earth
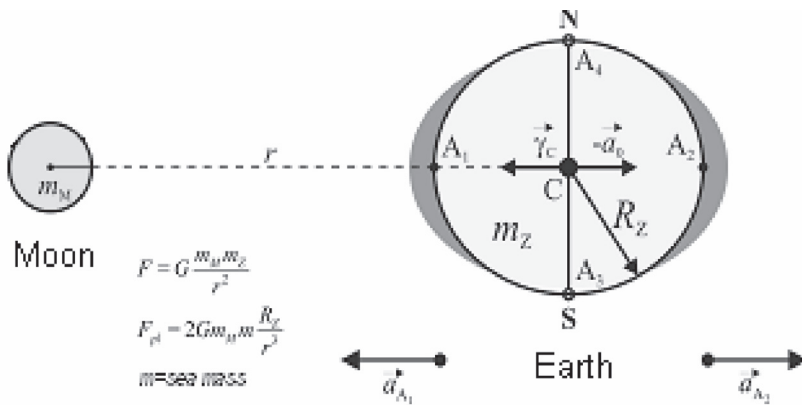
Einstein's understanding of space and tides

At the beginning of the twentieth century, following the works of Lorentz and Poincaré, Albert Einstein made a scientific revolution in the idea of space.

The space earlier, in the classical mechanics, considered to be flat (Euclidean) three dimensional space was more of a purely mathematical entity, a frame for all physical events. However through the works of Gauss, Lobachevski, Riemann and consequently Einstein the space became a real physical entity with its own dynamics.

The space in modern physics is generally Riemanian, i.e. curved, 4-space, described completely by curvature, or to be more specific curvature tensor and related metrics – metric tensor.

The term curvature was forged by Gauss in his *theorema egregium* stating that every 2-space can be described in a way of assigning a real number to its every point-gaussian curvature. Riemann extended that idea to higher dimensional spaces where no ordinary number is sufficient to describe a curvature. When dealing with such spaces one must use a four rank tensor instead.

Geometrization of physics and Patricius

In the light of the history of science, Jammer wrote a special preface to the Dover edition (A new preface prepared for the Dover edition by Max Jammer, Bar Ilan University, Ramat-Gan, Israel, 1999). In the Preface he had brilliantly described the cut between metaphysical concept of force and the scientific understanding of the same, stressing out that the four forces of nature in the Standard model describe interaction of matter particles and/or field quanta, which is ontologically less demanding than the classical Newton's understanding of force. Jammer's Preface and book should give a new incentive to the exploration of Patricius' philosophy of nature, especially his works on concepts of motion and force.

Following the success and mathematical beauty of general relativity, Einstein's great plan was to introduce the geometrical method (Riemann's geometry) to other parts of physics hoping that it would yield the same beautiful and suc-

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cessful theory like GR was. It was a plan to form a geometrical unified field theory. Einstein dedicated the last three decades of his life to that problem but alas he failed to produce a satisfactory solution.

At this point one might reflect on the words of Patricius who favored geometry as a method, as well as Keplers saying *Ubi materia, ibi geometria*, which is utterly true in the frame of GR.

Also, a specific methaphisical concept συνοικείωσις[1] (togetherness and kinship of all parts of the world) was introduced at the beginning of the 28th book of *Pancosmia*. Συνοικείωσις or *sympathia* are because of one mind, one soul, one spirit, one nature, and could be interpreted as intuitive Patricius' formulations of the latter mathematical concepts of motion and force, especially of interactions in modern physics.

Of course both Patricius and Kepler thought of Euclidean geometry. Kepler even tried to impose a geometrical structure of a set of perfect (Plato's) polyhedrons on universe.

In the famous book *Concept of Force* by M. Jammer (Dover, 1999, reprint of the Harvard edition, 1957), on p. 83, stands a reference of "Franciscus Patritius" and his work on tides which served as a basis for Kepler's attempt to formulate the universal character of attraction, a notion usually ascribed to Newton.

Calculation of tides in curved spacetime

According to Einsteinian understanding of spacetime and gravity curvature holds all information of spacetime and the gravitation field. A test particle in curved spacetime moves along a trajectory called the world line.

So using the curvature of spacetime as its most important property one can easily obtain, under the assumption of nonrelativistic velocities, the expression for relative acceleration between two neighbouring world lines.

Such acceleration cannot be transformed away and it is referred to as the tidal force:

$$f^{\,j} = R^{j}_{00k}\, x^{k}$$

This equation is also known as the geodesic deviation equation and conceptually it is similar to classical Newtonian deviation. All we need is to recall the analogy of the curvature tensor to the Laplacian of the gravitational potential.

If one now wishes to calculate the general relativity corrections for the Newtonian expression of tidal phenomenon, in the case of Earth, the Sun and the Moon, one must turn to the Newtonian limit, the limit of weak fields. When dealing with weak fields one can disregard the terms nonlinear in the affine connection (GR analogy of gravitational field) and write the relevant Riemann tensor components as:

$$R^{j}_{00k}\, x^{k} = \Gamma^{j}_{0k,0}\, x^{k} - \Gamma^{j}_{00,k}\, x^{k}$$

Assuming stationary fields the first term, which is in fact a derivative of a field with respect to time, vanishes so we have a final expression for weak stationary fields:

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$$R^{j}_{00k} x^{k} = -\Gamma^{j}_{00,k} x^{k}$$

Since in this case gravitational fields are spherically symmetric, one can use the Schwarzschild solution devised specially for spherically symmetric fields. By doing so and inserting the only affine connection of the aforementioned form one obtains the following expression:

$$a = 2G \frac{M_{m} R_{\oplus}}{r^{3}} \left(1 - 3 \frac{GM_{m}}{c^{2} r} \right),$$

which is in fact the wanted GR correction of the Newtonian expression.

Note that the second term in the bracket in very small compared to the leading unit, as one should expect when dealing with weak fields.

Conclusions and Outlook

Living in the time of the late renaissance Patricius as a philosopher gave considerable contributions to the contemporary world of thought, and it seems he had inspired his illustrious successors, great scientists of the seventeenth century.

Seventeenth century scientific revolution driven by empirical discoveries of Galileo and Kepler was due to the philosophical revolution that occurred in the late renaissance. The quest for the better method gave as a result the scientific method within the philosophy of nature. If we recall Newton's work it was entitled as *Philosophiae Naturalis Principia Mathematica*.

Patricius' concept of mathematical and physical space separately, and favoring of geometry makes him the anticipator of ideas and concepts that arose later in the development of science.

He was occupied with the phenomenon of tides, which he tried to explain within his model of universe (*Pancosmia*, books 28, 29, 30). He had developed the theory based on the observation of the seamen of that time which he had collected with great effort.

Because his theory was based on exact observations he made no mistake about the importance of the Sun and the Moon.

Patricius' exploration motives are the causes of the rising and falling of the sea and he approached the challenge with an original philosophical-scientific insight. That insight produced a philosophical interpretation: amongst many causes (more than twenty different causes) of tides the Moon and the Sun were recognized as two general causes, with a stress on the importance of their positions and light.

Patricius failed to see the true nature of gravity (force due to the mass of the celestial bodies) so, in the spirit of his philosophical system, he ascribed that role to the all-pervading light (*Lumen, Lux*) and heat (*Calor*).

His attempt to describe the phenomenon of tides encompasses a wide variety of the sea motions, their variability and local dependence according to the empirical observations, which are very close to the modern observations of tides (stations for the measurements of tides, satellite observations).

1
Patricius' term (metaphor) συνοικείωσιζ = interconnections of all parts of the world (cognateness), most probably forged from two

Greek terms συνοίκησιζ, εωζ = to dwell in a community, κείω, κείων = to split.

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Science, i.e. physics, after Patricius explained the tides as an effect of gravity (Newton) and later as an effect of spacetime curvature (Einstein), however not in the extent Patricius did when he included the local features of the phenomenon.

The phenomenon obvious as the tides was known long before modern physical theories and therefore it is ideal for observing how our concepts on the world around us changed through time. First the proto scientific, phenomenological theory in late renaissance, then later, classical mathematical theory of eighteenth century and finely more general and refined theory like the GR of the twentieth century. Patricius with his system definitely has its place in the history of science.

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SUPPLEMENT

Newtonian calculations:

According to the Newton's law of gravity, the force acting on a point distant by the radius r from the origin of the force is:

$$F = G\frac{Mm}{r^2},$$

therefore the acceleration on that point is:

$$g = G\frac{M}{r^2}.$$

If we have a neighbouring point distant from the first by R, which is small compared to r, (R<<r), we have:

$$a = G\frac{M}{(r \pm R)^2} \approx G\frac{M}{r^2}\left(1 \mp 2\frac{R}{r}\right) = G\frac{M}{r^2} \mp 2G\frac{MR}{r^3}.$$

Now by taking the difference of the accelerations of those two points one obtains the relative acceleration:

$$a_{rel} = \pm 2G\frac{MR}{r^3},$$

which is in fact the tidal force.

Einsteinian calculations:

If one takes the geodesic equation:

$$\frac{d^2 x^{\mu}}{d\tau^2} + \Gamma^{\mu}_{\alpha\beta}\frac{dx^{\alpha}}{d\tau}\frac{dx^{\beta}}{d\tau} = 0$$

and substitute the proper time with the ordinary time like $\frac{d}{d\tau} = \gamma\frac{d}{dt}$ one obtains the equation of the form:

$$\frac{d^2 x^j}{dt^2} + \left(-\frac{dx^j}{dt}\Gamma^0_{\alpha\beta} + \Gamma^j_{\alpha\beta}\right)\frac{dx^{\alpha}}{dt}\frac{dx^{\beta}}{dt} = 0,$$

for $\mu=j$, and of the form:

$$\frac{1}{\gamma}\frac{d\gamma}{dt} + \Gamma^0_{\alpha\beta}\frac{dx^{\alpha}}{dt}\frac{dx^{\beta}}{dt} = 0 \Rightarrow \frac{1}{\gamma}\frac{d\gamma}{dt} = -\Gamma^0_{\alpha\beta}\frac{dx^{\alpha}}{dt}\frac{dx^{\beta}}{dt},$$

for $\mu=0$.

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So one can write the geodesic equation like:

$$\frac{d^2 x^\mu}{dt^2} + \frac{1}{\gamma}\frac{d\gamma}{dt}\frac{dx^\mu}{dt} + \Gamma^\mu_{\alpha\beta}\frac{dx^\alpha}{dt}\frac{dx^\beta}{dt} = 0$$

Denoting the derivative with respect to time as $\dfrac{dx^j}{dt} = v^j$ one has:

$$\frac{dv^k}{dt} - v\Gamma^0_{00} + \Gamma^j_{00} + 2\Gamma^j_{k0}v^k = 0 \Rightarrow \frac{dv}{dt} = -a(1+ax) - 2\omega \times v - \omega \times (\omega \times x) - \frac{d\omega}{dt}\times x + f$$

The second part of the above mentioned equation is written in the classical three-vector form. The last term cannot be transformed away by the suitable choice of the reference frame and it describes the relativistic tidal acceleration:

$$f^{\,j} = R^j_{00k}x^k. \tag{1}$$

Expressing the curvature through affine connections we have:

$$R^\alpha_{\beta\gamma\delta} = \Gamma^\alpha_{\beta\delta\ ,\gamma} - \Gamma^\alpha_{\beta\gamma\ ,\delta} + \Gamma^\alpha_{\gamma\mu}\Gamma^\mu_{\beta\delta} - \Gamma^\alpha_{\delta\mu}\Gamma^\mu_{\beta\gamma}$$

Taking the weak field limit one can disregard the terms non linear in the affine connection and write the relevant curvature tensor components as:

$$R^\alpha_{\beta\gamma\delta} \approx \Gamma^\alpha_{\beta\delta\ ,\gamma} - \Gamma^\alpha_{\beta\gamma\ ,\delta} \Rightarrow R^j_{00k} = \Gamma^j_{0k\,,0} - \Gamma^j_{00\,,k}.$$

Assuming the fields are not time dependent (stationary) we can omit the first term which is in fact a derivative with respect to time so:

$$R^j_{00k} = -\Gamma^j_{00\,,k}.$$

Using the Schwarzschild solution $j=r$ one obtains:

$$\Gamma^j_{00} = \Gamma^r_{00} = \frac{\tilde{G}M(-2\tilde{G}M + r)}{r^3}$$

and according to relation (1):

$$f^{\,j} = \frac{d}{dr}\left(\frac{\tilde{G}M(-2\tilde{G}M + r)}{r^3}\right)R_\oplus \Rightarrow a = 2G\frac{MR_\oplus}{r^3}\left(1 - 3\frac{GM}{c^2 r}\right).$$

The leading term is the Newtonian expression for the tidal acceleration, and the second term in the brackets is the desired GR correction.

We could have also used the tidal tensor, which is actually $\Delta^\mu_\nu = R^\mu_{\alpha\beta\nu}v^\alpha v^\beta$, $f^{\,j} = \Delta^j_k x^k$, and by taking the low velocity limit we would have obtained the same relation for relativistic tidal acceleration (1).

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Die phänomenologische Theorie von Ebbe und Flut des Frane Petrić und ihre moderne relativistische Auffassung

Zusammenfassung

Weltbekannte Wissenschaftshistoriker (z.B. M. Jammer, Concepts of Force, *Dover 1999, Erstauflage 1957 bei Harvard Press) loben Petrić's Abhandlung über Ebbe und Flut, die Keppler bei seinem Formulierungsansatz des universalen Charakters der Gravitation als Grundlage diente. Die Unterschiede zwischen Ebbe und Flut in verschiedenen Meeren suchte Petrić im Rahmen seines Modells des Universums zu erklären (28. und 29. Buch von* Pancosmia). *Er folgerte schlüssig, dass der Mond und die Sonne zwei allgemeine Ursachen von Ebbe und Flut sind (was der lunaren Theorie von Ebbe und Flut bei Keppler entspricht), ohne dabei die Rolle der Gravitation zu erkennen. Vielmehr erklärte Petrić Ebbe und Flut in seinem eigenen philosophischen System als eine Folge von Sonnen- und Wärmeeinwirkungen (*lux et calor). *Die Wissenschaft (Physik) hat nach Petrić Ebbe und Flut als eine Auswirkung der Gravitation (Newton) erklärt, bzw. als Folge der Raumzeitkrümmung (Einstein). In dem Artikel soll die mathematische Beschreibung von Ebbe und Flut im Rahmen von Newtons Gravitationstheorie erklärt werden, aber auch die perfektere Ausrechnung derselben Erscheinung innerhalb der Raumzeitkrümmung gemäß der allgemeinen Relativitätstheorie im schwachen Gravitationsfeld (Newtons Grenze). Die relativistische Korrektion für Ebbe und Flut fällt im Hinblick auf den klassischen Newton'schen Ausdruck geringfügig aus, wie es auch für schwache Gravitationsfelder zu erwarten ist. Weder Newtons noch Einsteins Theorie – so perfekt sie in der mathematischen Beschreibung von Ebbe und Flut auch sein mögen – beschreiben Ebbe und Flut so eingehend, wie es Petrić in seiner phänomenologischen Theorie getan hat, indem er den lokalen Charakter des Phänomens hervorhob. Auf vielen Symposien anlässlich des 100. Jubiläums des wundervollen Jahres 1905 (z.B. auf dem 22. Weltkongress über Wissenschaftsgeschichte, 24.–30. Juli in Peking) wurde Einstein zum größten Physiker des 20. Jahrhunderts erklärt, neben Newton zum größten Physiker aller Zeiten. Umso mehr sollen Petrić's Verdienste gewürdigt werden, nicht nur in Bezug auf seine Theorie des mathematischen und physikalischen Raumes, sondern auch hinsichtlich seiner phänomenologischen Theorie von Ebbe und Flut, die Petrić, als ein unmittelbarer Vorgänger Kepplers und ganze 100 Jahre vor Newton, in sein interessantes philosophisches System der Beschreibung des Universums und der darin stattfindenden Naturerscheinungen eingebaut hat.*

Schlüsselwörter

Ebbe und Flut, Gravitation, Raumzeitkrümmung, phänomenogische Theorie, Frane Petrić, Johannes Kepler, Isaac Newton, Albert Einstein, die Rolle des Franciscus Patricius, Newtons Kalkulationen, Einsteins Kalkulationen

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La théorie phénoménologique de la marée haute et de la marée basse et son interprétation relativiste moderne

Sommaire

Les célèbres historiens des sciences (dont par exemple M. Jammer: Concepts of force, *Dover, 1999, réimpression de l'édition de Harvard Press de 1957) soulignent l'importance du traité de Petrić sur la marée haute et la marée basse dont s'est servi Kepler dans ses tentatives pour formuler la nature universelle de la gravitation. Les différences entre la marée haute et la marée basse dans des mers différentes ont été expliquées dans le cadre du modèle de l'univers de Petrić (dans les 28ᵉ et 29ᵉ livres de* Pancosmia). *Il a conclu correctement que la Lune et le Soleil sont les deux causes générales de la marée (théorie lunaire des marées formulée par Kepler) sans toutefois reconnaître le rôle de la gravitation. Dans son système philosophique Petrić expliquait la marée haute et la marée basse comme une conséquence de l'influence de la lumière et de la chaleur (*lux et calor). *Après Petrić, la marée haute et la marée basse ont été expliquées par la science (la physique) comme un effet de la gravitation (Newton), c'est à dire comme un effet de la courbure de l'espace-temps (Einstein). Dans cet article une description mathématique de la marée haute et de la marée basse sera présentée dans le cadre de la théorie de la gravitation de Newton, ainsi qu'un calcul soigneusement élaboré du même phénomène dans la courbure de l'espace-temps dans le cadre de la théorie générale de la relativité, en cas d'un champ de*

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gravitation faible (la limite de Newton). La correction relativiste appliquée à la marée haute ou à la marée basse, apparaît peu importante en relation à l'expression classique de Newton, ce à quoi on s'attendait dans le cas des champs de gravitation faibles. Pourtant, les deux théories, celle de Newton et celle d'Einstein, quelque parfaites qu'elles soient dans leur description des marées, n'expliquent pas la marée haute et basse d'une manière aussi détaillé que l'avait fait Petrić dans sa théorie phénoménologie en soulignant le caractère local du phénomène. De nombreux colloques dans le monde entier dédiés à l'année miraculeuse 1905 (par exemple le 22ᵉ congrès international de l'histoire des Sciences qui a eu lieu du 24 au 30 juillet 2005 à Bei-Jing) ont désigné Einstein comme étant le plus grand physicien du 20ᵉ siècle, et avec Newton, un des plus grands physiciens de tous les temps. Par conséquent il est très important d'insister sur le rôle de Petrić, non seulement pour ses théories d'espace mathématique et physique, mais aussi pour sa théorie phénoménologique de la marée haute et de la marée basse que Petrić, qui a été le prédécesseur de Kepler, anticipant Newton d'une centaines d'années, a incorporé dans son système philosophique original de l'univers et des phénomènes naturels.

Mots clés

Marée haute et marée basse, gravitation, courbure de l'espace-temps, théorie phénoménologique, Frane Petrić, Johannes Kepler, Isaac Newton, Albert Einstein, le rôle de Patricius, le calcul de Newton, le calcul d'Einstein