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# The size-dependent electromechanical instability of double-sided and paddle-type actuators in centrifugal and Casimir force fields

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## KEYWORDS

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Cylindrical nanowire;  
Size phenomenon;  
Casimir regime.

**Abstract.** The present research is devoted to theoretical study of the pull-in performance of double-sided and paddle-type NEMS actuators fabricated from cylindrical nanowire operating in the Casimir regime and in the presence of the centrifugal force. D'Alembert's principle was used to transform the angular velocity into an equivalent static, centrifugal force. Using the couple stress theory, the constitutive equations of the actuators were derived. The equivalent boundary condition technique was applied to obtain the governing equation of the paddle-type actuator. Three distinct approaches, the Duan-Adomian Method (DAM), Finite Difference Method (FDM), and Lumped Parameter Model (LPM), were applied to solve the equation of motion of these two actuators. This study demonstrates the influence of various parameters, i.e., the Casimir force, geometric characteristics, and the angular speed, on the pull-in performance.

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## 1. Introduction

A double-sided NEMS is constructed from a movable cantilever suspended between two actuating fixed electrodes. The double-sided NEMS has recently attracted much attention due to its promising electromechanical performance such as low power consumption, quick response, etc. [1,2]. The double-sided driven scheme has been proposed for the actuation of gyroscopes and memory elements [3,4]. Electrostatic resonators

possess high stability and resolution based on the double-sided NEMS [3]. These double-sided systems are also attractive for detecting ultra-small mass and sensing mechanical forces [3]. Fu et al. [2] used the energy balance approach for modeling the oscillation of double-sided microbridges. Ke studied the instability of a double-sided resonator [3]. Khan and Akbarzade employed analytical methods to study the oscillation of a driven double-sided miniature resonator [5]. More useful information about double-sided NEMS can be found in [6,7].

The cantilever beam with a rigid plate attached to its free end is called a paddle-type configuration. Recently, several studies have investigated the paddle-type MEMS/NEMS as sensors [8], actuators [9], resonators [10], capacitors, and filters [8-11]. Qian et

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al. [12] developed a capacitive paddle-type NEMS switch fabricated from silicon nanowires. Zhang et al. [10] examined the use of paddle-type resonators for thermal sensing. Tong et al. [11] showed that the paddle-type configuration can eliminate internal stress distribution. Ahmad et al. [13] designed a paddle-type sensor using the COVENTOR software. Other researchers [14,15] investigated a micro-generator consisting of a silicon paddle-type element. Ouakad [8] studied the effect of shock loading on performance of a paddle-type gas sensor. More information about paddle-type NEMS is available in the literature [16,17].

With recent demands for advanced devices, MEMS/NEMS capacitive sensors are increasingly used in developing measurement systems. Measurement of the angular speed is important in rotary systems [18]. There are many promising applications, such as the fault detection of roller bearings [19], balancing rotating equipment [20], measurement of high-speed spindle errors [21], fluidic centrifuges [22], turbomachinery, etc. In these applications, the presence of the centrifugal force plays a major role in determining the electromechanical response of these NEMS sensors. Recently, the paddle-type and double-sided sensors have been considered as angular speed sensors and accelerometers [23–25]. Herein, the effect of the centrifugal force is incorporated into the pull-in instability models of paddle-type and double-sided sensors.

For precise modeling of the pull-in performance of the aforementioned nanostructures, the Casimir force should be included in the model [26–28]. Almost all previous investigations in this area have been devoted to those devices with planar configurations, while only few investigations have considered it in actuators which are fabricated from nanowires with cylindrical geometries [29,30]. A simplistic approach for determining the vacuum fluctuations for compound shapes is the Proximity Force Approximation (PFA) [31,32]. Herein, the PFA is employed to demonstrate the effect of the Casimir force on the physical response of the paddle-type and double-sided NEMS. In addition to the Casimir attraction, the size effect is another crucial phenomenon that should be incorporated into the theoretical model. Experiments have shown a hardening trend in the elastic response of some nanowires as the structure dimensions approach the material length scale [33,34]. This size effect cannot be modeled by classic continuum mechanics. In order to overcome this shortcoming, the non-classical theories, such as non-local elasticity [35], Couple Stress Theory (CST) [36–38], strain gradient theory [39], modified couple stress theory [40], etc., have been developed to consider the size effect on theoretical continuum models. Despite other size-dependent theories, limited research has been conducted on modeling the ultra-small structures using the CST [41–43].

This work presents the influence of the centrifugal force, Casimir effect, and size phenomenon on the electromechanical instability of paddle-type and double-sided nanoactuators. The size effect is modeled using the CST in conjunction with the Euler-Bernoulli beam model. Three distinct solution methods, i.e. the Duan-Adomian Method (DAM), Finite Difference Method (FDM), and Lumped Parameter Model (LPM), are employed to solve the nonlinear governing equations.

## 2. Theory

Figure 1(a) shows the schematic diagram of the typical double-sided NEMS actuator. The voltage difference and initial gap between the movable electrode and upper plane are  $V$  and  $D$ , respectively. The potential difference and initial separation between the nanowire and the lower plane are  $\bar{V}$  and  $\bar{D}$ , respectively. The nanowire of length  $L$  and radius  $r$  is considered. Figure 1(b) depicts the schematic diagram of a typical paddle-type NEMS which is suspended over the fixed plane and deflected by applying electrostatic attraction. The voltage between the paddle and the ground planes is  $V$ . The nanowire of length  $L$  and radius  $r$  is considered. The rigid plate has length  $a$ , width  $b$ , and thickness  $t$ .

The internal resultants in an arbitrary cross-section of the nanowire are shown in Figure 1(c), where  $F$  and  $M$  denote the force and moment at the non-supported end of the nanowire (at  $x = L$ ), respectively. To obtain the equation of motion of the nanowire in the presence of the centrifugal force (Figure 1(d)), the Hamilton's principle is utilized:

$$\delta(U - V) = 0, \quad (1)$$

where  $\delta$  denotes the variation symbol,  $U$  is the strain energy, and  $V$  is the work done by all forces.

### 2.1. Strain energy

In the CST, the strain energy depends on the rotation gradient as well as the strain [44]. Displacement gradient tensor,  $u_{i,j}$ , is decomposed into symmetric strain and skew-symmetric rotation tensors, respectively, as:

$$u_{i,j} = \varepsilon_{ij} + \omega_{ij}, \quad (2)$$

where:

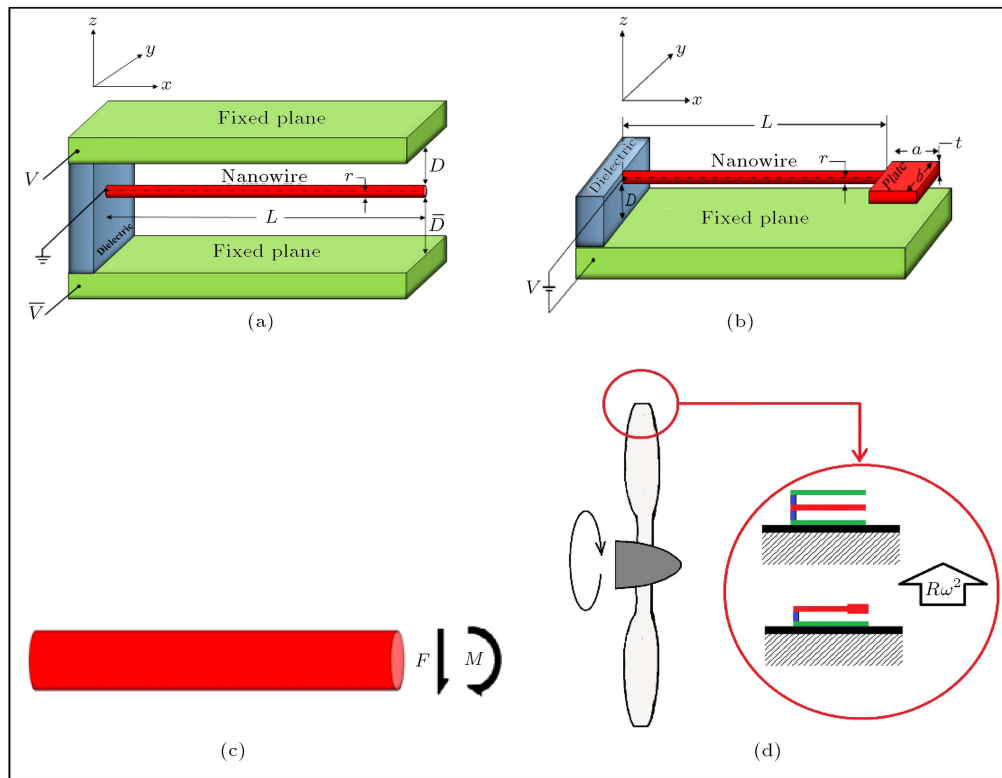
$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}) = \varepsilon_{ji},$$

$$\omega_{ij} = \frac{1}{2}(u_{i,j} - u_{j,i}) = -\omega_{ji}. \quad (3)$$

The rotation vector is:

$$\theta_i = \frac{1}{2}e_{ijk}u_{k,j}, \quad (4)$$

and the gradient of rotation is expressed as:



**Figure 1.** (a) The double-sided NEMS. (b) The paddle-type NEMS. (c) Internal resultants in arbitrary wire cross-section. (d) The nanostructure in the presence of the centrifugal force.

$$\kappa_{ij} = \theta_{j,i} = \frac{1}{2} e_{jkl} u_{l,ki} \quad (5)$$

The strain energy density based on the CST can be expressed as [42,45]:

$$\tilde{u} = \frac{1}{2} \lambda \varepsilon_{ij} \varepsilon_{ij} + \mu \varepsilon_{ij} \varepsilon_{ij} + 2\eta \kappa_{ij} \kappa_{ij} + 2\eta' \kappa_{ij} \kappa_{ij}, \quad (6)$$

where  $\lambda$  and  $\mu$  are the classic Lamé constant and shear modulus, respectively. Moduli  $\eta$  and  $\eta'$  are constants which correspond to the couple stress effects. The classic symmetric Cauchy stress tensor ( $\sigma_{ij}$ ) and the couple stress tensor ( $m_{ij}$ ), respectively, can be derived from Eq. (6) as follows:

$$\sigma_{ij} = \frac{\partial \tilde{u}}{\partial \varepsilon_{ij}} = \lambda \varepsilon_{mm} \delta_{ij} + 2\mu \varepsilon_{ij}, \quad (7a)$$

$$m_{ij} = \frac{\partial \tilde{u}}{\partial \kappa_{ij}} = 4\eta \kappa_{ij} + 4\eta' \kappa_{ji}. \quad (7b)$$

Based on the Euler-Bernoulli beam model, the components of the displacement are [46]:

$$\begin{aligned} u_X(X, Y, Z) &= -Z \frac{\partial w(X)}{\partial X}, & u_Y(X, Y, Z) &= 0, \\ u_Z(X, Y, Z) &= w(X). \end{aligned} \quad (8)$$

By substitution of Eqs. (8) into Eqs. (3), (4), and (7), the following relations for the components of the strain,

stress, gradient of rotation, and couple stress tensors are obtained as follows:

$$\begin{aligned} \varepsilon_{XX} &= -Z \frac{\partial^2 w}{\partial X^2}, \\ \varepsilon_{YY} = \varepsilon_{ZZ} = \varepsilon_{XY} = \varepsilon_{YZ} = \varepsilon_{ZX} &= 0, \end{aligned} \quad (9a)$$

$$\begin{aligned} \sigma_{XX} &= -EZ \frac{\partial^2 w}{\partial X^2}, \\ \sigma_{YY} = \sigma_{ZZ} = \sigma_{YZ} = \sigma_{ZX} = \sigma_{XY} &= 0, \end{aligned} \quad (9b)$$

$$\begin{aligned} \kappa_{XY} &= -\frac{\partial^2 w}{\partial X^2}, \\ \kappa_{XX} = \kappa_{YY} = \kappa_{ZZ} = \kappa_{YZ} = \kappa_{ZY} = \kappa_{ZX} \\ &= \kappa_{XZ} = \kappa_{YX} = 0, \end{aligned} \quad (9c)$$

$$\begin{aligned} m_{XY} &= -4\eta \frac{\partial^2 w}{\partial X^2}, & m_{YX} &= -4\eta' \frac{\partial^2 w}{\partial X^2}, \\ m_{XX} = m_{YY} = m_{ZZ} = m_{YZ} = m_{ZY} = m_{ZX} \\ &= m_{XZ} = 0. \end{aligned} \quad (9d)$$

By substituting Eq. (9) into Eq. (6) and integrating over the length of the nanowire, the strain energy is

calculated as follows:

$$U = \frac{1}{2} \int_0^L (E_{\text{eff}}I + 4A\eta) \left( \frac{\partial^2 w}{\partial X^2} \right)^2 dX, \quad (10)$$

where  $I$  and  $A$  are the second moment and area of the cross-section of the nanowire, respectively. As shown, the strain energy does not depend on  $\eta'$ . According to the previous investigation [45], material constant  $\eta$  can be redefined as:

$$\eta = \mu l^2, \quad (11)$$

where  $l$  is the material length scale parameter.

Note that the effect of large deformation is not considered in the model (see Appendix A).

**2.2. Work by all forces**

The work by all forces is the summation of the work done by the external forces ( $f_{\text{ext}}$ ) and the work done by the internal tractions, i.e. moment ( $M$ ) and force ( $F$ ).

The external forces include the electrostatic and Casimir attractions. The work done by external forces,  $V_{f_{\text{ext}}}$ , is expressed as:

$$V_{f_{\text{ext}}} = \int_0^L \int_0^w f_{\text{ext}} dw dX. \quad (12)$$

The work done by internal moment,  $V_M$ , is obtained as:

$$V_M = \int_0^{\frac{\partial w(L)}{\partial X}} M \left( w(L), \frac{\partial w(L)}{\partial X} \right) \times d \frac{\partial w(L)}{\partial X}. \quad (13)$$

The work done by internal force,  $V_F$ , is determined as:

$$V_F = \int_0^{w(L)} F \left( w(L), \frac{\partial w(L)}{\partial X} \right) \times dw(L). \quad (14)$$

Finally, the overall work done by the aforementioned forces is obtained as:

$$\begin{aligned} V = & V_{f_{\text{ext}}} + V_M + V_F = \int_0^L \int_0^w f_{\text{ext}} dZ dX \\ & + \int_0^{\frac{\partial w(L)}{\partial X}} M \left( w(L), \frac{\partial w(L)}{\partial X} \right) \times d \frac{\partial w(L)}{\partial X} \\ & + \int_0^{w(L)} F \left( w(L), \frac{\partial w(L)}{\partial X} \right) \times dw(L). \end{aligned} \quad (15)$$

**2.3. The governing equation**

By substituting Eqs. (10) and (15) into Eq. (1), we can obtain the following equation for the variation as follows:

$$\begin{aligned} \delta(V - U) = & \int_0^L \left[ (E_{\text{eff}}I + 4\mu Al^2) \frac{\partial^4 w}{\partial X^4} - f_{\text{ext}} \right] \delta w dX \\ & - (E_{\text{eff}}I + 4\mu Al^2) \frac{\partial^3 w}{\partial X^3} \delta w \Big|_0^L \\ & + (E_{\text{eff}}I + 4\mu Al^2) \frac{\partial^2 w}{\partial X^2} \delta \left( \frac{\partial w}{\partial X} \right) \Big|_0^L \\ & - M \delta \left( \frac{\partial w}{\partial X} \right) \Big|_L - F \delta w \Big|_L = 0. \end{aligned} \quad (16)$$

Hence, the governing equation for the equilibrium of the nanowire is derived from Eq. (16) as follows:

$$(E_{\text{eff}}I + 4\mu Al^2) \frac{d^4 w}{dX^4} = f_{\text{ext}}, \quad (17a)$$

subject to the geometric boundary conditions of:

$$w(0) = 0, \quad \frac{dw}{dX}(0) = 0, \quad (17b)$$

and the natural boundary conditions of:

$$\begin{aligned} (E_{\text{eff}}I + 4\mu Al^2) \frac{d^2 w}{dX^2}(L) &= M, \\ (E_{\text{eff}}I + 4\mu Al^2) \frac{d^3 w}{dX^3}(L) &= -F. \end{aligned} \quad (17c)$$

Eqs. (17) in general can be specialized for each NEMS by substituting their respective formulas for  $f_{\text{ext}}$ ,  $M$ , and  $F$ . This is considered in the following subsections.

*2.3.1. The double-sided structure*

For the double-sided actuator (Figure 1(a)),  $f_{\text{ext}}$  in Eq. (17a) can be defined as:

$$f_{\text{ext}} = f_{\text{elec1}} - f_{\text{elec2}} + f_{\text{Cas1}} - f_{\text{Cas2}} + f_{\text{Cent}}, \quad (18)$$

where  $f_{\text{elec1}}$  and  $f_{\text{Cas1}}$  are the electrical and Casimir attractions between the wire and the upper plane, respectively.  $f_{\text{elec2}}$  and  $f_{\text{Cas2}}$  denote the electrical and Casimir attractions between the wire and the lower plane, respectively.  $f_{\text{Cent}}$  is the centrifugal force.

The electrical force terms in Eqs. (17) can be determined from the capacitive model [47]. The electrostatic energy for a cylindrical conductor parallel to conductive plane ( $E_{\text{elec}}$ ) is given as follows [48]:

$$E_{\text{elec}} = \frac{\pi \epsilon_0 \epsilon_r LV^2}{\text{arccosh} \left( 1 + \frac{D}{r} \right)}, \quad (19)$$

where  $\epsilon_0$  and  $V$  are the permittivity of vacuum and the imposed DC voltage, respectively. Hence, by differentiating the electrostatic energy, the electrical

attraction per unit length of cylinder,  $f_{elec}$ , is derived from Eq. (19) as follows:

$$f_{elec} = \frac{dE_{elec}}{dD} = \frac{\pi \epsilon_0 \epsilon_r V^2}{\sqrt{(D+2r)(D)} \arccos h^2 \left(1 + \frac{D}{r}\right)}. \quad (20)$$

By considering the deflection of nanowire,  $f_{elec1}$  and  $f_{elec2}$  can be derived by replacing  $D$  with  $D_1 - w$  and  $D_2 + w$ , respectively, in Eq. (20). Note that the effect of Charge-tip has not been incorporated while it might be important (see Appendix B).

The Casimir force terms in Eq. (17) can be determined using the PFA. Based on the PFA, the Casimir energy for a conductive cylindrical wire parallel to a conductive plane ( $E^{PFA}$ ) is determined as [31,32]:

$$E^{PFA} = -\frac{\pi^3 \hbar c L}{960} \sqrt{\frac{r}{2D^5}}, \quad (21)$$

where  $\hbar = 1.05457 \times 10^{-34}$  J.s is the reduced Planck's constant,  $c = 2.998 \times 10^8$  m/s is the speed of light, and  $D$  is the gap distance between the nanowire and the plane, respectively. Therefore, the Casimir attraction per unit length of cylinder,  $f_{Cas}$ , is derived by differentiating Eq. (21) as follows:

$$f_{Cas} = \frac{dE^{PFA}}{dD} = \frac{\pi^3 \hbar c}{768} \sqrt{\frac{2r}{D^7}}. \quad (22)$$

Regarding the displacement of the nanowire and replacing  $D$  with  $D - w$  in Eq. (22),  $f_{Cas1}$  can be derived. Similarly,  $f_{Cas2}$  can be obtained by substituting  $D$  with  $\xi D + w$  in Eq. (22).

According to D'Alembert's principal, we can transform an angular speed into an equivalent centrifugal force. Hence, the contribution of the centrifugal force can be modeled by considering the angular velocity of the system. The centrifugal force per unit length of the nanowire, caused by rotation of a rotary machine, is determined as [24,25]:

$$f_{Cent} = \pi \rho r^2 (R \pm D) \omega^2, \quad (23)$$

where  $\rho$ ,  $R$ , and  $\omega$  are the density of the nanowire, the rotary surface radius, and the angular speed of the rotary surface, respectively. For the case of  $R \gg D$ , Eq. (22) is reduced to:

$$f_{Cent} = \pi \rho r^2 R \omega^2. \quad (24)$$

The boundary conditions for the double-sided actuator at the free end are defined as traction free ( $F = M = 0$ ). Hence, Eq. (17c) is reduced to the following relations:

$$\frac{d^2 w}{dX^2}(L) = \frac{d^3 w}{dX^3}(L) = 0. \quad (25)$$

For the double-sided actuator, by using Eqs. (18), (20), (22), and (24), the governing equation for Eq. (17) can finally be written as:

$$\begin{aligned} (1 + \delta) \frac{d^4 \hat{w}}{dx^4} &= \frac{\beta}{\sqrt{(1 - \hat{w}) [1 + k(1 - \hat{w})]} \arccos h^2 (1 + 2k(1 - \hat{w}))} \\ &- \frac{\Xi \beta}{\sqrt{(\xi + \hat{w}) [1 + k(\xi + \hat{w})]} \arccos h^2 (1 + 2k(\xi + \hat{w}))} \\ &+ \frac{\gamma}{(1 - \hat{w})^{\frac{7}{2}}} - \frac{\gamma}{(\xi + \hat{w})^{\frac{7}{2}}} + \bar{\omega}, \end{aligned} \quad (26a)$$

$$\hat{w}(0) = \frac{d\hat{w}}{dx}(0), \quad (26b)$$

$$\frac{d^2 \hat{w}}{dx^2}(1) = \frac{d^3 \hat{w}}{dx^3}(1) = 0, \quad (26c)$$

where the dimensionless parameters in Eq. (24) are defined as follows:

$$\begin{aligned} x &= \frac{X}{L}, \quad \hat{w} = \frac{w}{D}, \quad \xi = \frac{\bar{D}}{D}, \quad \Xi = \left(\frac{\bar{V}}{V}\right)^2, \\ k &= \frac{D}{2r}, \quad \delta = \frac{4\mu Al^2}{E_{eff} I} = \frac{16\mu l^2}{E_{eff} r^2}, \\ \bar{\omega} &= \frac{\pi \rho r^2 L^4 R \omega^2}{E_{eff} I D}, \quad \gamma = \frac{\hbar c \pi^3 r^{\frac{1}{2}} L^4}{384 \sqrt{2} E_{eff} I D^{\frac{9}{2}}}, \\ \beta &= \frac{\epsilon_0 \epsilon_r \pi V^2 L^4}{\sqrt{2} r^{\frac{1}{2}} E_{eff} I D^{\frac{3}{2}}}. \end{aligned} \quad (27)$$

### 2.3.2. The paddle-type structure

For the paddle-type actuator, by replacing  $D$  with  $D - w$  in Eqs. (20) and (22),  $f_{ext}$  can be defined as follows:

$$\begin{aligned} f_{ext} &= f_{elec} + f_{Cas} + f_{Cent} \\ &= \frac{\pi \epsilon_0 \epsilon_r V^2}{\sqrt{(D-w+2r)(D-w)} \arccos h^2 \left(1 + \frac{D-w}{r}\right)} \\ &+ \frac{\pi^3 \hbar c}{768} \sqrt{\frac{2r}{(D-w)^7}} + \pi \rho r^2 R \omega^2. \end{aligned} \quad (28)$$

To obtain the appropriate boundary conditions, the distributed forces acting on the plate are replaced with an equivalent concentrated force acting at distance  $\bar{x}$  from the nanowire tip (the force center). The value of  $\bar{x}$  is determined from  $\bar{x} = M/F$  relation. Based on this approach, the boundary conditions at the non-supported end of the nanowire can be obtained as follows:

$$\begin{aligned}
 & (E_{\text{eff}}I + 4\mu Al^2) \frac{d^2 w}{dX^2}(L) \\
 &= \frac{\varepsilon_0 \varepsilon_r}{2} \frac{abV^2 \bar{x}}{(D - w(L) - \bar{x}w'(L))^2} \\
 &+ \frac{\pi^2 \hbar cab \bar{x}}{240(D - w(L) - \bar{x}w'(L))^4} \\
 &+ \frac{\rho a^2 bt R \omega^2}{2}, \tag{29a}
 \end{aligned}$$

$$\begin{aligned}
 & (E_{\text{eff}}I + 4\mu Al^2) \frac{d^3 w}{dX^3}(L) \\
 &= -\frac{\varepsilon_0 \varepsilon_r}{2} \frac{abV^2}{(D - w(L) - \bar{x}w'(L))^2} \\
 &- \frac{\pi^2 \hbar cab}{240(D - w(L) - \bar{x}w'(L))^4} \\
 &- \rho abt R \omega^2. \tag{29b}
 \end{aligned}$$

By substituting Eq. (28) into Eq. (17) and using the dimensionless parameters in Eq. (27), the non-dimensional governing equation of the paddle-type actuator is:

$$\begin{aligned}
 & (1 + \delta) \frac{d^4 \hat{w}}{dx^4} \\
 &= \frac{\beta}{\sqrt{(1 - \hat{w}) [1 + k(1 - \hat{w})] \arccos h^2(1 + 2k(1 - \hat{w}))}} \\
 &+ \frac{\gamma}{(1 - \hat{w})^{\frac{7}{2}}} + \bar{\omega}, \tag{30a}
 \end{aligned}$$

$$\hat{w}(0) = \hat{w}'(0) = 0, \tag{30b}$$

$$\begin{aligned}
 & (1 + \delta) \frac{d^2 \hat{w}}{dx^2}(1) = \vartheta \Omega \xi \tau \left[ \frac{\beta}{2k^{\frac{3}{2}}(1 - \hat{w}(1) - \Omega \xi \tau \hat{w}'(1))^2} \right. \\
 &+ \left. \frac{16\gamma}{5k^{\frac{1}{2}}(1 - \hat{w}(1) - \Omega \xi \tau \hat{w}'(1))^4} + \frac{2\bar{\omega}}{\Omega} \right], \tag{30c}
 \end{aligned}$$

$$\begin{aligned}
 & (1 + \delta) \frac{d^3 \hat{w}}{dx^3}(1) = -\vartheta \left[ \frac{\beta}{2k^{\frac{3}{2}}(1 - \hat{w}(1) - \Omega \xi \tau \hat{w}'(1))^2} \right. \\
 &+ \left. \frac{16\gamma}{5k^{\frac{1}{2}}(1 - \hat{w}(1) - \Omega \xi \tau \hat{w}'(1))^4} + 4\bar{\omega} \right], \tag{30d}
 \end{aligned}$$

where the dimensionless parameters are defined as follows:

$$\begin{aligned}
 \vartheta &= \frac{ab}{2\pi r L}, & \Omega &= \frac{\bar{x}}{a}, \\
 \tau &= \frac{D}{L}, & \xi &= \frac{a}{D}. \tag{31}
 \end{aligned}$$

### 3. Solution methods

#### 3.1. Duan-Adomian Method (DAM)

Recently, Duan has developed a fast decomposition algorithm [49] that is employed in conjunction with the Adomian decomposition method [50] for solving boundary value problems. To analytically solve the governing equations (Eqs. (26) and (30)), we consider the following general fourth-order differential equation with the nonlinearity  $f(x, \hat{w})$  as:

$$\frac{d^4 \hat{w}(x)}{dx^4} = f(x, \hat{w}(x)), \tag{32a}$$

$$\hat{w}(0) = C_1, \quad \hat{w}'(0) = C_2. \tag{32b}$$

The solution of Eq. (32a) can be determined as:

$$\begin{aligned}
 \hat{w}(x) &= \sum_{n=0}^{\infty} \hat{w}_n(x) = C_1 + C_2 x \\
 &+ \frac{1}{2!} C_3 x^2 + \frac{1}{3!} C_4 x^3 \\
 &+ \int_0^x \int_0^x \int_0^x \int_0^x \left( \left[ \sum_{n=0}^{\infty} f_n(x) \right] \right) dx dx dx dx, \tag{33}
 \end{aligned}$$

where  $C_1, C_2, C_3,$  and  $C_4$  are the constants of integration. Note that in the case of Eq. (17), the values of  $C_1$  and  $C_2$  are both zero. Based on the DAM, polynomials  $f_n(x)$  can be determined from the following relation [49,51]:

$$f_n = \sum_{k=1}^n C_n^k h^{(k)}(\hat{w}_0), \tag{34}$$

where terms  $h^{(k)}$  and  $C_n^k$  are determined from the following algorithm [51]:

$$C_n^k = \begin{cases} \hat{w}_n, & k=1, \quad n \geq 1, \\ \frac{1}{n} \sum_{j=0}^{n-k} (j+1) \hat{w}_{j+1} C_{n-1-j}^{k-1}, & 2 \leq k \leq n, \end{cases} \tag{35a}$$

$$h^{(k)} = \frac{\partial^k f}{\partial \hat{w}_0^k}, \quad k \geq 1. \tag{35b}$$

Now, by using Eqs. (33) and (34), the solution to Eq. (26) can be calculated as follows:

$$\begin{aligned} \hat{w} = & \frac{1}{2!}C_3x^2 + \frac{1}{3!}C_4x^3 \\ & + \frac{1}{4!(1+\delta)} \left[ \frac{\beta}{\sqrt{1+k} \arccos h^2(1+2k)} \right. \\ & \left. - \frac{\beta \Xi}{\sqrt{\xi(1+k\xi)} \arccos h^2(1+2k\xi)} + \gamma - \frac{\gamma}{\sqrt{\xi^7}} + \bar{w} \right] x^4 \\ & + \frac{1}{7!(1+\delta)} \left[ \frac{2\beta\sqrt{k}}{(1+k) \arccos h^3(1+2k)} \right. \\ & + \frac{2\beta\sqrt{k}\Xi}{\xi(1+k\xi) \arccos h^3(1+2k\xi)} \\ & + \frac{\beta(1+2k)}{2(1+k)^{3/2} \arccos h^2(1+2k)} \\ & + \frac{\beta\Xi(1+2k\xi)}{2\sqrt{\xi^3}(1+k\xi)^{3/2} \arccos h^2(1+2k\xi)} + \frac{7\gamma}{2} \\ & \left. + \frac{7\gamma}{2\sqrt{\xi^9}} \right] [7C_3x^6 + C_4x^7] + \dots \end{aligned} \quad (36)$$

Similarly, the solution to Eq. (30) can be determined as follows:

$$\begin{aligned} \hat{w} = & \frac{1}{2!}C_3x^2 + \frac{1}{3!}C_3x^3 \\ & + \frac{1}{4!(1+\delta)} \left[ \frac{\beta}{\sqrt{1+k} \arccos h^2(1+2k)} + \gamma + \bar{w} \right] x^4 \\ & + \frac{1}{7!(1+\delta)} \left[ \frac{2\beta\sqrt{k}}{(1+k) \arccos h^3(1+2k)} \right. \\ & + \frac{\beta(1+2k)}{2(1+k)^{3/2} \arccos h^2(1+2k)} \\ & \left. + \frac{7\gamma}{2} \right] [7C_3x^6 + C_4x^7] + \dots \end{aligned} \quad (37)$$

Finally, constants  $C_3$  and  $C_4$  are determined by boundary conditions (Eqs. (26b) and (26c)) and (Eqs. (30b), (30c) and (30d)) for the double-sided and paddle-type nanostructures, respectively.

### 3.2. FDM

According to the finite difference method, the domain is discretized into  $n$  equal segments located between  $(n + 1)$  nodes. For each segment, differential equations (Eq. 17(a)) are discretized as follows:

$$\frac{\hat{w}_{i-2} - 4\hat{w}_{i-1} + 6\hat{w}_i - 4\hat{w}_{i+1} + \hat{w}_{i+2}}{\Delta x^4} = F_i. \quad (38)$$

In the above equation,  $\Delta x$  represents the grid spacing,  $\hat{w}_i$  represents the deflection of the  $i$ th grid, and  $F_i$  is:

$$\begin{aligned} F_i = & \frac{\frac{1}{1+\delta}\beta}{\sqrt{(1-\hat{w}_i)[1+k(1-\hat{w}_i)]} \arccos h^2(1+2k(1-\hat{w}_i))} \\ & - \frac{\frac{1}{1+\delta}\Xi\beta}{\sqrt{(\xi+\hat{w}_i)[1+k(\xi+\hat{w}_i)]} \arccos h^2(1+2k(\xi+\hat{w}_i))} \\ & + \frac{\frac{1}{1+\delta}\gamma}{(1-\hat{w}_i)^{7/2}} - \frac{\frac{1}{1+\delta}\gamma}{(\xi+\hat{w}_i)^{7/2}} + \bar{w}, \quad i = 1, 2, \dots, N. \end{aligned} \quad (39)$$

For the paddle-type actuator, we have Eq. (40) as shown in Box I.

By imposing Eq. (38) on all segments and using the boundary conditions, the following system of algebraic equations is obtained:

$$[\mathbf{A}] \{\hat{w}\} = \{F\}. \quad (41)$$

In the above relation,  $\{\hat{w}\} = [\hat{w}_1, \hat{w}_2, \dots, \hat{w}_n]^T$  and  $\{F\} = [F_1, F_2, \dots, F_n]^T$  are the displacement and force vectors, respectively. In Eq. (41), stiffness matrix  $[\mathbf{A}]$  is calculated as:

$$[\mathbf{A}] = \frac{1}{\Delta x^4} \begin{bmatrix} 7 & -4 & 1 & 0 & 0 & \dots \\ -4 & 6 & -4 & 1 & 0 & \dots \\ 1 & -4 & 6 & -4 & 1 & \dots \\ 0 & 1 & -4 & 6 & -4 & \dots \\ 0 & 0 & 1 & -4 & 6 & \dots \\ 0 & 0 & 0 & 1 & -4 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\ 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \\ -4 & 6 & -4 & 1 & 0 & \dots \\ 1 & -4 & 5 & -2 & 1 & \dots \\ 0 & 1 & -2 & 1 & 0 & \dots \end{bmatrix}_{N \times N}. \quad (42)$$

The above systems of algebraic equation (Eq. (42)) is then numerically solved to determine the nodal deflections.

### 3.3. LPM

The LPM assumes a uniform force distribution along the wire [27]. The mechanical resistance of nanowire is

$$F_i = \frac{\frac{1}{1+\delta}\beta}{\sqrt{(1-\hat{w}_i)[1+k(1-\hat{w}_i)] \arccos h^2(1+2k(1-\hat{w}_i))}} + \frac{\frac{1}{1+\delta}\gamma}{(1-\hat{w}_i)^{\frac{7}{2}}}$$

$$+ \begin{cases} 0, & i = 1, \dots, N-2, \\ \frac{-\frac{\vartheta\Omega\xi\tau}{(1+\delta)\Delta x^2}\beta}{2k^{\frac{3}{2}}(1-\hat{w}_{i+1}-\Omega\xi\tau\frac{\hat{w}_{i+1}-\hat{w}_i}{\Delta x})^2} + \frac{-\frac{16\vartheta\Omega\xi\tau}{(1+\delta)\Delta x^2}\gamma}{5k^{\frac{1}{2}}(1-\hat{w}_{i+1}-\Omega\xi\tau\frac{\hat{w}_{i+1}-\hat{w}_i}{\Delta x})^4} - \frac{2\vartheta\xi\tau\bar{\omega}}{\Delta x^2}, & i = N-1, \\ \frac{[\frac{\vartheta\Omega\xi\tau}{(1+\delta)\Delta x^2} + \frac{\vartheta}{(1+\delta)\Delta x}]\beta}{2k^{\frac{3}{2}}(1-\hat{w}_i-\Omega\xi\tau\frac{\hat{w}_i-\hat{w}_{i-1}}{\Delta x})^2} + \frac{[\frac{16\vartheta\Omega\xi\tau}{(1+\delta)\Delta x^2} + \frac{16\vartheta}{(1+\delta)\Delta x}]\gamma}{5k^{\frac{1}{2}}(1-\hat{w}_i-\Omega\xi\tau\frac{\hat{w}_i-\hat{w}_{i-1}}{\Delta x})^4} + \left[\frac{2\vartheta\xi\tau}{\Delta x^2} + \frac{4\vartheta}{\Delta x}\right]\bar{\omega}, & i = N. \end{cases} \tag{40}$$

Box I

expressed analogously as a linear elastic spring. Hence, the governing equation is simplified to:

$$K w_{tip} = f. \tag{43}$$

In the above equation,  $f$  is the total external forces acting on the nanowire.

For the cantilever double-sided nanostructure, the elastic stiffness is obtained as [52]:

$$K = \frac{8(E_{eff}I + 4\mu Al^2)}{L^3}. \tag{44}$$

In the case of the paddle-type NEMS, the total deflection is the summation of three values, i.e. the deflection of the wire subjected to:

- a) The uniform load of  $f_{ext}$  along the wire;
- b) A force of  $F$  at the non-supported end;
- c) A moment of  $M$  at the non-supported end.

These three deflections can be modeled by the superposition of three serial springs with the spring constants of  $K_1$ ,  $K_2$ , and  $K_3$ , respectively. Hence, the elastic

stiffness of the paddle-type actuator can be determined as follows:

$$K = \left[ \frac{1}{K_1} + \frac{1}{K_2} + \frac{1}{K_3} \right]^{-1} = \left[ \left( \frac{8(E_{eff}I + 4\mu Al^2)}{L^3} \right)^{-1} + \left( \frac{3f(E_{eff}I + 4\mu Al^2)}{FL^2} \right)^{-1} + \left( \frac{2f(E_{eff}I + 4\mu Al^2)}{ML} \right)^{-1} \right]^{-1}$$

$$= \frac{24f(E_{eff}I + 4\mu Al^2)}{3fL^3 + 8FL^2 + 12ML}. \tag{45}$$

By substituting stiffness (Eqs. (44) and (45)) and force terms into Eq. (43), the governing equations of the nanostructures are readily obtained. Finally, by using dimensionless relations (Eqs. (27) and (31)), the relation between dimensionless wire tip deflection,  $w_{tip}$ , and applied voltage,  $\beta$ , is determined by Eqs. (46) and (47) as shown in Box II.

The pull-in characteristics of the nanoactuators

For the double-sided NEMS actuator:

$$\beta = \frac{8\hat{w}_{tip}(1+\delta) - \frac{\gamma}{(1-\hat{w}_{tip})^{\frac{7}{2}}} + \frac{\gamma}{(\xi+\hat{w}_{tip})^{\frac{7}{2}}} - \bar{\omega}}{\frac{1}{\sqrt{(1-\hat{w}_{tip})[1+k(1-\hat{w}_{tip})] \arccos h^2(1+2k(1-\hat{w}_{tip}))}} - \frac{\Xi}{\sqrt{(\xi+\hat{w}_{tip})[1+k(\xi+\hat{w}_{tip})] \arccos h^2(1+2k(\xi+\hat{w}_{tip}))}}}. \tag{46}$$

For the paddle-type NEMS actuator:

$$\beta = \frac{24\hat{w}_{tip}(1+\delta) - \frac{128\gamma\vartheta+192\vartheta\gamma\Omega\xi\tau}{5k^{\frac{1}{2}}(1-\hat{w}_{tip})^4} - \frac{3\gamma}{(1-\hat{w}_{tip})^{\frac{7}{2}}} - (32\vartheta + 24\vartheta\xi\tau + 3)\bar{\omega}}{\frac{3}{\sqrt{(1-\hat{w}_{tip})[1+k(1-\hat{w}_{tip})] \arccos h^2(1+2k(1-\hat{w}_{tip}))}} + \frac{4\vartheta+6\vartheta\Omega\xi\tau}{k^{\frac{3}{2}}(1-\hat{w}_{tip})^2}}. \tag{47}$$

Box II



are then obtained from Eqs. (46) and (47), as shown in Box II, by setting  $d\beta/d\hat{w}_{tip} = 0$ .

#### 4. Results and discussion

##### 4.1. Double-sided actuator

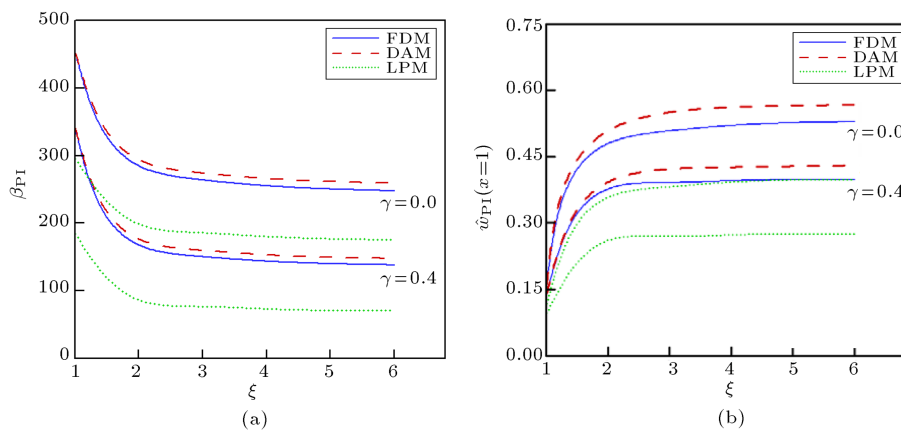
To examine the impact of geometry on the behavior of the double-sided actuator, the variations of  $\beta_{PI}$  and pull-in deflection ( $\hat{w}_{PI}(x = 1)$ ) as a function of  $\xi$  are presented in Figure 2 for various values of Casimir parameter  $\gamma$ . Dimensionless parameter  $\xi$  represents the ratio between  $D$  and  $\bar{D}$ . As shown, decreasing  $\xi$  increases the instability voltage, but at the same time, decreases the instability deflection. This reveals that reduction of the difference between  $D$  and  $\bar{D}$  can stabilize the structure. This is because the upper surface neutralizes the attractive effect of the lower surface.

To show the impact of centrifugal force on the behavior of the double-sided actuator, the variations of  $\beta_{PI}$  and pull-in deflection ( $\hat{w}_{PI}(x = 1)$ ) as a function

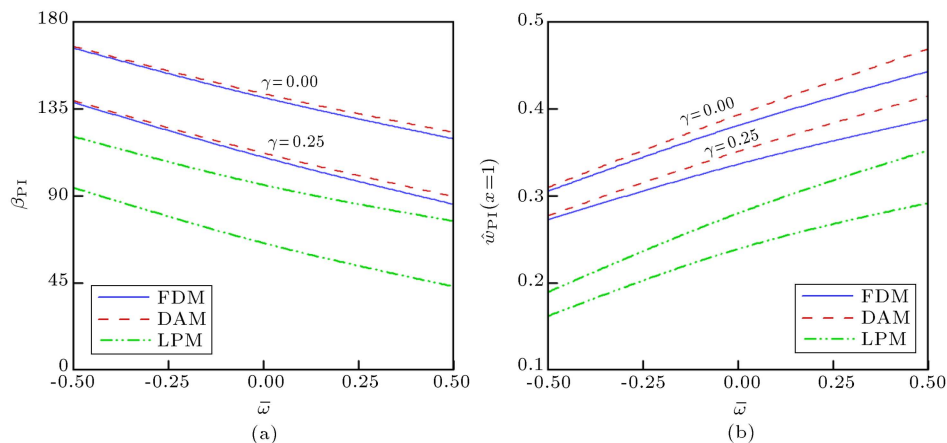
of  $\bar{\omega}$  are illustrated in Figure 3 for various values of Casimir parameter  $\gamma$ . In this figure, both positive and negative values of the centrifugal force are considered. As seen, a negative centrifugal force increases pull-in voltage ( $\beta_{PI}$ ), while a positive centrifugal force decreases the instability voltage of the sensor. In other words, for positive values of the centrifugal force, an increase in the angular speed leads to a decrease of the pull-in voltage. On the other hand, increasing the angular velocity increases the stability threshold of the systems for negative values of the centrifugal force.

##### 4.2. Paddle-type actuator

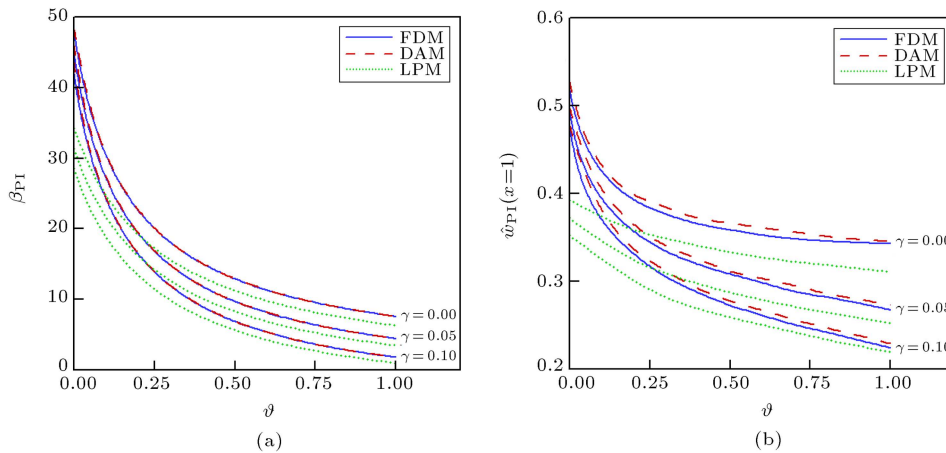
To demonstrate the effect of the paddle geometry on the stability, the variation of the instability characteristics versus geometrical parameter  $\vartheta$  is plotted in Figure 4. Parameter  $\vartheta$  represents the ratio between the rigid plate surface area over the nanowire surface area. As shown, with increasing  $\vartheta$ , instability voltage ( $\beta_{PI}$ ) and deflection ( $\hat{w}_{PI}(x = 1)$ ) decrease. This implies that an increase in the plate surface increases the external forces and reduces the stability threshold. Moreover,



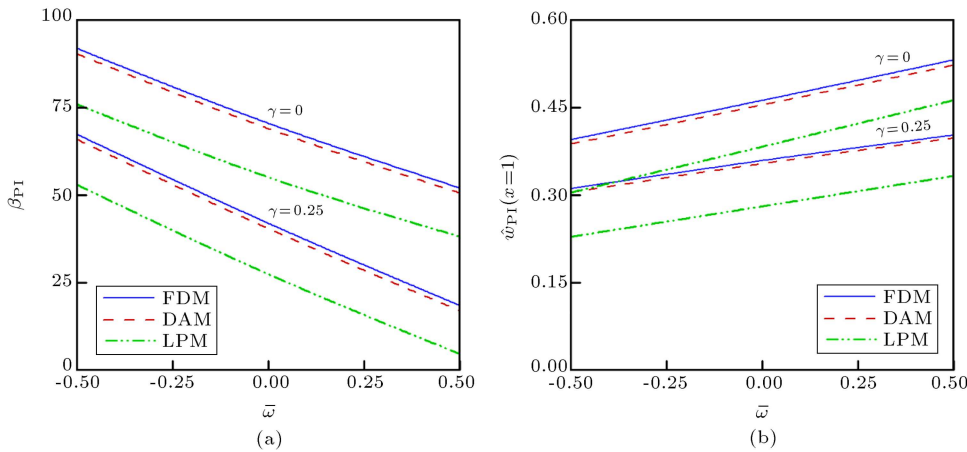
**Figure 2.** The variation of the instability characteristics versus  $\xi$  ( $k = 25$ ,  $\bar{\omega} = 0$ ,  $\Xi = 1$  and  $\delta = 0$  for  $\gamma = 0$  and  $\gamma = 0.4$ ): (a) The pull-in voltage and (b) the pull-in deflection.



**Figure 3.** The variation of the pull-in characteristics versus  $\bar{\omega}$  for different values of  $\gamma$  ( $k = 10$ ,  $\eta = 1.0$ ,  $\delta = 0$  and  $\xi = 1.25$ ): (a) The pull-in voltage and (b) the pull-in deflection.



**Figure 4.** The variation of the pull-in characteristics versus  $\vartheta$  for different values of  $\gamma$  ( $k = 5, \xi = 0.5, \tau = 0.1, \bar{\omega} = 0, \Omega = 0.5$  and  $\delta = 0$ ): (a) The pull-in voltage and (b) the pull-in deflection.



**Figure 5.** The variations of (a)  $\beta_{PI}$  and (b)  $\hat{w}_{PI}(x = 1)$  versus  $\bar{\omega}$  for different values of  $\gamma$  ( $k = 10, \xi = 0.5, \tau = 0.1, \Omega = 0.5, \delta = 0$ , and  $\vartheta = 0.1$ ).

Figure 4(a) and (b) show that the influence of the Casimir force on the stability is more pronounced for higher  $\vartheta$  values.

Figure 5 shows the pull-in characteristics of the sensor as a function of the centrifugal force. This figure depicts that for positive values of the centrifugal force, the pull-in voltage decreases as the angular speed increases. Furthermore, for negative values of the centrifugal force, an increase in the angular velocity increases the instability threshold of the systems.

Finally, note that the DAM is an approximation method. For example, in Figure 2, the difference between the pull-in voltages obtained by 8th order DAM polynomial and the FDM is less than 4%, which is acceptable for most of engineering designs. The results of the DAM can be more close to those of numerical solution by increasing the number of selected series terms [53]. On the other hand, the LPM is a very simple model for modeling the pull-in behavior of NEMS, while LPM results are not very close to the

numerical solution, and they have the advantage of providing a closed-form relation which can interpret the physical phenomena without mathematical complexity. This is very useful for engineers and designers.

### 5. Conclusions

The couple stress theory was employed for modeling the effect of the centrifugal force on the electromechanical instability of paddle-type and double-sided sensors incorporating the size-dependency and the Casimir force. It was found that for positive values of the centrifugal force, pull-in voltage decreases as the angular speed increases. On the other hand, for negative values of the centrifugal force, increase in the angular velocity increases the stability threshold of the systems. The Casimir attraction decreases the pull-in characteristics of the actuators. The good agreement was observed between DAM and FDM, while LPM provided less precise results; however, this model simply explains the instability behavior without mathematical complexity.

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**Appendix A**

**Large deformation theory**

When double-sided actuator exhibits large displacements, the curvature of the deflection should be considered; therefore, governing changes into [54]:

$$(E_{\text{eff}}I + \mu Al^2) \frac{d^2}{dX^2} \left[ \frac{d^2 w}{dX^2} \left( 1 + \left( \frac{dw}{dX} \right)^2 \right)^{-\frac{3}{2}} \right] = f_{\text{ext}}. \tag{A.1}$$

After some manipulations and using Eq. (25), the nondimensional governing equation is obtained as:

$$\begin{aligned} & (1 + \delta) \frac{d^4 \hat{w}}{dx^4} \\ &= \left[ \frac{\beta}{\sqrt{(1-\hat{w}) [1+k(1-\hat{w})]} \arccos h^2(1+2k(1-\hat{w}))} \right. \\ &+ \frac{\gamma}{(1-\hat{w})^{\frac{7}{2}}} \\ &- \frac{\Xi \beta}{\sqrt{(\xi+\hat{w}) [1+k(\xi+\hat{w})]} \arccos h^2(1+2k(\xi+\hat{w}))} \\ &- \left. \frac{\gamma}{(\xi+\xi\hat{w})^{\frac{7}{2}}} + \bar{\omega} \right] \left[ 1 + (\lambda \hat{w}')^2 \right]^2 \\ &+ 3\lambda^2 \frac{3\hat{w}'\hat{w}''\hat{w}''' + \hat{w}'^3}{1 + (\lambda \hat{w}')^2} - \frac{15\lambda^4 \hat{w}'^2 \hat{w}''^3}{\left[ 1 + (\lambda \hat{w}')^2 \right]^2}, \tag{A.2} \end{aligned}$$

where:

$$\lambda = \frac{D}{L}, \tag{A.3}$$

and the prime denotes differentiation with respect to  $x$ . When  $\lambda$  approaches zero, Eq. (A.2) turns into Eq. (24a).

The numerical simulation results shown in Table A.1 reveal that the effect of the large deformation on the pull-in parameters is negligible for  $\lambda < 0.1$ . Table A.1 demonstrates that Eq. (24) has enough accuracy for description of the elastic behavior of double-sided actuator from small to large displacements when

the initial gap to the length ratio is less than 0.1 (i.e.,  $\lambda < 0.1$ ), which is the case in most double-sided actuators designs.

**Appendix B**

**Tip-charge concentration**

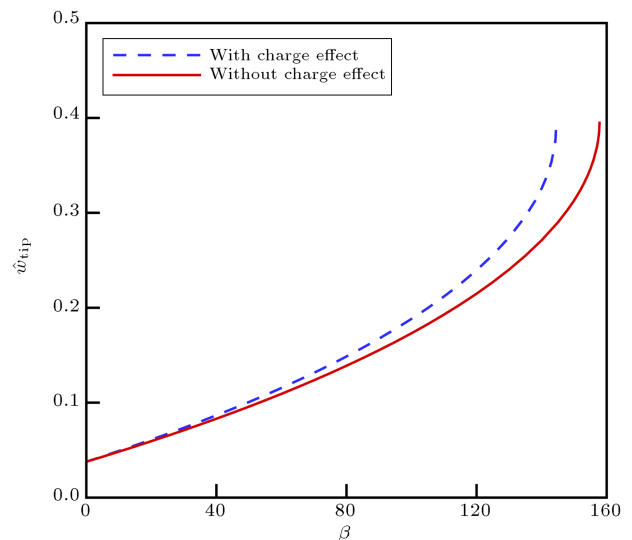
The impact of tip-charge concentration on the deformation of cantilever might be considerable [55]. Thus, the tip-charge concentration force can be written as follows:

$$f_{\text{elec},c} = 0.85[R(D + R)^2]^{\frac{1}{3}} f_{\text{elec}}(L), \tag{B.1}$$

where  $\delta(x)$  is the Dirac delta function.

For the double-sided actuator, by using Eqs. (18), (20), (22), and (B.1), the nondimensional governing equation can be explained by considering the tip-charge concentration as shown in Box B.1.

To examine the impact of tip-charge concentration on the behavior of the double-sided actuator, variation of the tip deflection ( $\hat{w}_{\text{tip}}$ ) versus nondimensional voltage ( $\beta$ ) is presented in Figure B.1. As shown above,



**Figure B.1.** 1 The variation of  $\hat{w}_{\text{tip}}$  versus  $\beta$ : Impact of tip-charge concentration ( $\lambda = 0.1$ ,  $\gamma = 0.3$ ,  $k = 20$ ,  $\xi = 2.0$ ,  $\bar{\omega} = 0$ ,  $\Xi = 1.0$ , and  $\delta = 0$ ).

**Table A.1.** Pull-in parameters of different values of  $k$  using small ( $\lambda = 0$ ) and large deformation ( $\lambda = 0.1$ ) theories when  $\gamma = 0.4$ ,  $\Xi = 2$ ,  $\xi = 1$ , and  $\delta = 0$ .

	$k$	200	150	100	75	50	25	10
<b>Small deformation</b> ( $\gamma = 0$ )	$\beta_{\text{PI}}$	587.57	463.83	330.08	257.94	180.76	96.16	39.79
	$\hat{w}_{\text{PI}}$	0.35263	0.35118	0.34934	0.34820	0.346057	0.34283	0.33743
<b>Large deformation</b> ( $\gamma = 0.1$ )	$\beta_{\text{PI}}$	585.97	462.58	329.20	257.25	180.29	95.91	39.69
	$\hat{w}_{\text{PI}}$	0.35149	0.35056	0.348681	0.346469	0.34598	0.34093	0.33597

$$\begin{aligned}
(1 + \delta) \frac{d^4 \hat{w}}{dx^4} = & \frac{\beta}{\sqrt{(1 - \hat{w}) [1 + k(1 - \hat{w})]} \operatorname{arccos} h^2(1 + 2k(1 - \hat{w}))} \left( 1 + \frac{0.85\lambda}{2k} (1 + 2k)^{\frac{2}{3}} \delta(x - 1) \right) \\
& - \frac{\Xi\beta}{\sqrt{(\xi + \hat{w}) [1 + k(\xi + \hat{w})]} \operatorname{arccos} h^2(1 + 2k(\xi + \hat{w}))} \left( 1 + \frac{0.85\lambda}{2k} (1 + 2k)^{\frac{2}{3}} \delta(x - 1) \right) \\
& + \frac{\gamma}{(1 - \hat{w})^{\frac{7}{2}}} - \frac{\gamma}{(\xi + \hat{w})^{\frac{7}{2}}} + \bar{\omega}, \tag{B.2a}
\end{aligned}$$

$$\hat{w}(0) = \frac{d\hat{w}}{dx}(0), \tag{B.2b}$$

$$\frac{d^2 \hat{w}}{dx^2}(1) = 0,$$

$$\begin{aligned}
\frac{d^3 \hat{w}}{dx^3}(1) = & - \frac{0.85\lambda}{2k} (1 + 2k)^{\frac{2}{3}} \frac{\beta}{\sqrt{(1 - \hat{w}(1)) [1 + k(1 - \hat{w}(1))]} \operatorname{arccos} h^2(1 + 2k(1 - \hat{w}(1)))} \\
& + \frac{0.85\lambda}{2k} (1 + 2k)^{\frac{2}{3}} \frac{\Xi\beta}{\sqrt{(\xi + \hat{w}(1)) [1 + k(\xi + \hat{w}(1))]} \operatorname{arccos} h^2(1 + 2k(\xi + \hat{w}(1)))}. \tag{B.2c}
\end{aligned}$$

Box B.1

considering the tip-charge concentration decreasing the pull-in voltage, the pull-in deflection is not affected significantly.

### Biographies

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**Mariam Keivani.** Her biography was not available at the time of publication.

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