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10-2011

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# **Original Publication Information**

Holwerda, B. W., N. Pirzkal, W. J. G. de Blok, and W. van Driel. "Quantified H i Morphology - V. H i Discs in the Virgo Cluster." 2011. *Monthly Notices of the Royal Astronomical Society* 416(4): 2447-2454.

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# Quantified H I morphology – V. H I discs in the Virgo cluster

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Accepted 2011 March 4. Received 2011 February 7; in original form 2010 October 29

## ABSTRACT

We explore the quantified morphology of atomic hydrogen (H1) discs in the Virgo cluster. These galaxies display a wealth of phenomena in their H1 morphology, e.g., tails, truncation and warps. These morphological disturbances are related to the ram pressure stripping and tidal interaction that galaxies undergo in this dense cluster environment. To quantify the morphological transformation of the H1 discs, we compute the morphological parameters of Concentration, Asymmetry, Smoothness, Gini and  $M_{20}$  and our own  $G_M$  for 51 galaxies in 48 H1 column density maps from the VLA Imaging of Virgo spirals in Atomic gas (VIVA) project.

Some morphological phenomena can be identified in this space of relatively low-resolution H<sub>I</sub> data. Truncation of the H<sub>I</sub> disc can be cleanly identified via the Concentration parameter (C < 1), and Concentration can also be used to identify H<sub>I</sub> deficient discs (1 < C < 5). Tidal interaction is typically identified using combinations of these morphological parameters, applied to (optical) images of galaxies. We find that some selection criteria (Gini– $M_{20}$ , Asymmetry and a modified Concentration– $M_{20}$ ) are still applicable for the coarse (~15 arcsec full width at half-maximum) VIVA H<sub>I</sub> data. We note that Asymmetry is strongly affected by the choice for the centre of these galaxies. The phenomena of tidal tails can be reasonably well identified using the Gini– $M_{20}$  criterion (60 per cent of galaxies with tails identified but with as many contaminants).

Ram pressure does move H<sub>I</sub> discs into and out of most of our interaction criteria: the ram pressure sequence identified by previous authors tracks into and out of some of these criteria (Asymmetry-based and the Gini $-M_{20}$  selections, but not the Concentration $-M_{20}$  or the  $G_{\rm M}$ -based ones). Therefore, future searches for interaction using H<sub>I</sub> morphologies should take ram pressure into account as a mechanism to disturb H<sub>I</sub> discs enough to make them appear as gravitationally interacting. One mechanism would be to remove all the H<sub>I</sub> deficient (C < 5) discs from the sample, as these have undergone more than one H<sub>I</sub> removal mechanism.

**Key words:** galaxies: fundamental parameters – galaxies: interactions – galaxies: kinematics and dynamics – galaxies: spiral – galaxies: structure.

## **1 INTRODUCTION**

The Virgo cluster represents an ideal laboratory for the study of galaxy evolution in a cluster environment. It is one of the nearest galaxy clusters ( $D \sim 17$  Mpc), a relatively populous system (Abell richness class I), consisting of some two thousand catalogued members in the Virgo cluster Catalogue (VCC; Binggeli, Sandage & Tammann 1985). The Virgo Cluster is spiral-rich and dynamically

young, i.e. made of galaxy subgroups which are falling into the main cluster. The most prominent groups are those associated with M86 to the west and M49 to the south. A particularity of the Virgo cluster is that the distribution of its hot intracluster medium is strongly peaked on the central cD galaxy M87 (Böhringer et al. 1994) and overall highly substructured. The Virgo cluster is therefore ideal to study the effects of both gravitational interaction and interaction with the intracluster medium on the atomic gas (H I) discs of spiral members.

Morphological signs of tidal interaction are usually detected in the rest-frame ultraviolet or blue-filter optical images of the

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**Figure 1.** The relation between Gini and Concentration for H I truncated galaxies ( $D_{H_1} \le D_B$ , left panel) and H I deficient galaxies (right panel). Most of the VIVA sample discs are H I deficient ( $def_{H_1} < 1$ ) and we mark those that are extremely so ( $def_{H_1} < 0.5$ ): galaxies with half a dex or more below the typical H I surface density for their type of spiral (Haynes & Giovanelli 1984). Typical values for Concentration are 5 and above for H I discs in WHISP (Holwerda et al. 2011a,c). Therefore, the majority of the H I discs in VIVA are more than typically centrally concentrated; a low Concentration value (C < 5) is a good sign of H I deficiency and, in addition, an extremely low Concentration value (C < 1) identifies half of the truncated H I discs.

stellar disc. A series of morphological parameters, originally developed for galaxy classification, have been used extensively in recent years to identify disturbed galaxies and to infer the merger fractions and rates over cosmological time-scales (Conselice 2003; Conselice et al. 2003, 2008; Lotz, Primack & Madau 2004; Bundy, Ellis & Conselice 2005; Yan et al. 2005; Ravindranath et al. 2006; Trujillo et al. 2007; Lotz et al. 2008; Conselice, Yang & Bluck 2009; Jogee et al. 2009). The observational benefits are that the Hubble Space Telescope observations of high-redshift galaxies and ground-based observations of nearby galaxies (e.g. the Sloan Digital Sky Survey) probe the same wavelength, physical structure and volumes. However, in the nearby Universe, the 21 cm perspective on galaxies is set to catch up to the optical surveys in terms of numbers of galaxies studied. Currently, under construction are two new radio observatories: the South African Karoo Array Telescope (MeerKAT, Jonas 2007; Booth et al. 2009; de Blok et al. 2009) and the Australian SKA Pathfinder (ASKAP, Johnston 2007; Johnston et al. 2007, 2008; Johnston, Feain & Gupta 2009). Together with refurbished observatories such as the Extended Very Large Array (EVLA, Napier 2006) and the APERture Tile In Focus instrument (APERTIF, Verheijen et al. 2008; Oosterloo et al. 2009) on the Westerbork Synthesis Radio Telescope (WSRT), detailed HI maps of thousands of galaxies will become available. The unprecedented volume of data implies that the data reduction and subsequent morphological classification can only be done through automated procedures.

In the previous papers in this series (Holwerda et al. 2009, 2011a,b,c,d), we have shown that the morphological classifiers that are in use to identify mergers in the optical can be extremely useful for the H<sub>I</sub> perspective. The H<sub>I</sub> observations are not of as high resolution as that of the optical disc, but this is compensated for by the larger extent of the atomic gas disc and the relative sensitivity of gas to an interaction – for which much anecdotal evidence already exists in 'The H<sub>I</sub> Rogues Gallery' catalogue (Hibbard et al. 2001).<sup>1</sup>

<sup>1</sup> http://www.nrao.edu/astrores/HIrogues/RoguesLiving.shtml

The sensitivity of H<sub>1</sub> surveys to gas-rich and minor mergers will provide the calibration for higher redshifts where these types of mergers are expected to be the dominant type of galaxy interaction (Lotz et al. 2010a,b). The Square Kilometer Array (Carilli & Rawlings 2004) will subsequently be able to resolve H<sub>1</sub> discs at these higher redshifts.

The H<sub>I</sub> discs of spiral galaxies are also extremely sensitive to ram pressure by the intergalactic or intracluster medium. The Virgo cluster is an ideal laboratory to test the effects of both gravitational interaction and intracluster medium stripping on the quantified morphology of H<sub>I</sub> discs. There are detailed models of the gas stripping sequence (Vollmer 2009), and a uniform H<sub>I</sub> data set from the Very Large Array (VLA) Imaging of Virgo spirals in Atomic gas (VIVA) project (Chung et al. 2009), with detailed notes on individual galaxies identifying morphological phenomena.

In this paper, we explore the distribution of the H<sub>1</sub> morphological parameters from the VIVA column density maps in relation to estimates by Chung et al. (2007, 2009) or Vollmer (2009) of the star formation rate, H<sub>1</sub> deficiency, stripping and gravitational harassment. First, we discuss data and morphological parameters (Sections 2 and 3), but readers already familiar with these can skip to the results and conclusions (Sections 4 and 5).

## 2 VIVA DATA

The VIVA project (Chung et al. 2009) has compiled the largest high-resolution imaging H<sub>1</sub> data base of the Virgo cluster to date. They have published 48 VLA H<sub>1</sub> data cubes and maps with 51 Virgo cluster spiral galaxies. These are some 40 new observations with the remainder from reprocessed archival data. We obtained the zero-order moment maps from http://www.astro.yale.edu/viva/ and converted to H<sub>1</sub> column density maps using the expressions in Walter et al. (2008a). The spatial resolution of the VIVA data is approximately 15 arcsec (or ~1.2 kpc at the distance of Virgo), and the typical sensitivity ranges from ~3 × 10<sup>19</sup> to 5 × 10<sup>19</sup> cm<sup>-2</sup> in the 3 $\sigma$  10 km<sup>-1</sup> s<sup>-1</sup> channel. VIVA data show that (i) the galaxies located in the core of the cluster predominantly have truncated H<sub>1</sub> discs with some gas displaced from the galactic disc (Fig. 1), and (ii)

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**Table 1.** The evaluations from Chung et al. (2009) – whether the H<sub>I</sub> shows evidence of stripping, ram pressure stripping, tidal interaction, a warp or an H<sub>I</sub> tail.

some seven out of the 50 sample galaxies ( $\sim$ 10 per cent) show long, one-sided H1 tails (Chung et al. 2007, Table 1). These tails occur in galaxies located at intermediate projected distances between 0.5 and 1 Mpc from the cluster centre. From the sample listed in table 1 of Chung et al. (2009), we did not use IC 3418 and Holmberg VII, as the former is a non-detection in the VIVA survey and the latter

does not have enough continuous H1 flux to meaningfully compute morphological parameters.

#### **3 MORPHOLOGICAL PARAMETERS**

Two morphological parameter schemes are now in wide use: Concentration–Asymmetry–Smoothness (CAS; Conselice 2003) and Gini– $M_{20}$  (Lotz et al. 2004). In an image with *n* pixels with intensities I(i, j) at pixel position (i, j), the CAS morphological parameters are defined as

$$C = 5 \log\left(\frac{r_{80}}{r_{20}}\right),\tag{1}$$

$$A = \frac{\sum_{i,j} |I(i,j) - I_{180}(i,j)|}{\sum_{i,j} |I(i,j)|},$$
(2)

$$S = \frac{\sum_{i,j} |I(i,j) - I_S(i,j)|}{\sum_{i,j} |I(i,j)|},$$
(3)

where  $r_f$  is the radius which includes the percentage f of the intensity of the object,  $I_{180}(i, j)$  is the value of the pixel in the rotated image and  $I_S(i, j)$  is the same pixel in the image after smoothing.

Gini and  $M_{20}$  are defined as

$$G = \frac{1}{\bar{I}n(n-1)} \Sigma_i (2i - n - 1) |I_i|,$$
(4)

$$M_{20} = \log\left(\frac{\Sigma_i^k M_i}{M_{\text{tot}}}\right), \text{ for which } \Sigma_i^k I_i < 0.2I_{\text{tot}} \text{ is true},$$
(5)

with  $M_{\text{tot}} = \sum M_i = \sum I_i [(x_i - x_c)^2 + (y_i - y_c)^2]$ , where  $(x_c, y_c)$  is the galaxy's central position and  $I_i$  is the intensity of pixel *i* in a flux-ordered list. Pixel *k* is the pixel marking the 20 per cent point in this list.  $\overline{I}$  is the mean pixel value. The Gini parameter is an indicator of inequality in the distribution of pixel values: Gini = 1 is perfect inequality with one pixel containing all the flux from an object, and Gini = 0 is perfect equality with each pixel containing an equal fraction of the flux. Values in between indicate how concentrated the flux from this object is in a few bright areas. The  $M_{20}$  parameter is an indication whether or not the brightest parts of the object are close to the centre or spread throughout the object.

In addition to these well-established parameters, we compute the Gini coefficient of the second-order moment  $I_i \times [(x_i - x_c)^2 + (y_i - y_c)^2]$ , which we called  $G_M$  (Holwerda et al. 2011b). For all these parameters, we compute the uncertainty by varying the input centre the resolution and randomizing the pixel values. The exception is the Gini parameter as this does not rely on the centre as input. For the Gini error, we jackknife the pixel selection. For a more detailed description of these parameters, we refer the reader to the previous papers in this series (Holwerda et al. 2011b,c) and the original papers introducing these (Conselice 2003; Lotz et al. 2004). We compute these parameters for the area defined by the contour level of  $1 \times 10^{19}$  cm<sup>-2</sup> (the  $1\sigma$  level).

In Holwerda et al. (2011c), we identified several parts of parameter space where tidally interacting galaxies reside for the WHISP<sup>2</sup> sample. To identify the interacting galaxies, we used a subsample of 154 spirals and dwarf galaxies in WHISP, which had independent visual estimates of interaction from either Swaters et al. (2002) or Noordermeer et al. (2005). The criteria from the literature and our

 $<sup>^2</sup>$  Westerbork observations of neutral Hydrogen in Irregular and SPiral galaxies.

#### Morphology HI Parameter Space



**Figure 2.** The spread of morphological parameters. Galaxies with an H<sub>1</sub> tail, as identified in Chung et al. (2007, 2009), are marked with solid black points. The (optical) merger selection criteria from the literature are marked with dashed lines in panel II (equation 7), panel IV (equations 6 and 8), and V and VI (equation 6). Our selection criteria from Holwerda et al. (2011c) are marked with dotted lines: the  $G_M$  criterion in panels I, III, VI and X (equation 9), the  $A-M_{20}$  criterion in panel IV (equation 10) and the  $C-M_{20}$  criterion in panel IX (equation 11).

previous paper are identified in Figs 2 and 3 with dashed and dotted lines, respectively. Criteria from the literature are

$$A > 0.38$$
 (6)

from Conselice (2003), marked by the dashed lines in Figs 2 and 3, panels IV, V and VI.

$$G > -0.115 \times M_{20} + 0.384 \tag{7}$$

from Lotz et al. (2004), marked by the dashed lines in panel II of Figs 2 and 3.

$$G > -0.4 \times A + 0.66 \text{ or } A > 0.4$$
 (8)

from Lotz et al. (2010b), marked by the dashed lines in panel IV of Figs 2 and 3.

We defined three criteria based on the WHISP H I morphologies:  $G_{\rm M} > 0.6,$  (9)

marked by the vertical dotted line in Figs 2 and 3, panels I, III, VI and X,

$$A < -0.2 \times M_{20} + 0.25, \tag{10}$$

marked by the dotted line in Figs 2 and 3, panel V and

$$C > -5 \times M_{20} + 3, \tag{11}$$

which is the dotted line in Figs 2 and 3, panel IX.

These criteria select a merger from its H I morphology for different time-scales. In Holwerda et al. (2011d), we quantified the typical time-scale for a merger to be selected by each of these criteria using H I maps of merger simulations (originally presented in Cox et al. 2006a,b). These can be compared with merger simulations of an isolated disc, evolving passively. The comparison shows how long a merger is selected as well as the level of contamination by passive discs. At the resolution of the WHISP sample, interactions were identifiable as such by their H I morphology for about a Gyr. This is a similar time-scale to the one for optical morphological identifications and better than the close pair method. Our goal is to explore if the above criteria could be applied to H I data in a denser environment, where many other phenomena play a role in the H I morphology, as well as to explore how many of those other phenomena leave a signature in the H I morphology.



### Morphology HI Parameter Space

**Figure 3.** The spread of morphological parameters for galaxies undergoing tidal interaction (black symbols), H<sub>I</sub> stripping (open circles) and ongoing ram pressure (marked with an x), according to Chung et al. (2009), see Table 1. The (optical) merger selection criteria from the literature are marked with dashed lines in panel II (equation 7), panel IV (equations 6 and 8), and V and VI (equation 6). Our selection criteria from Holwerda et al. (2011c) are marked with dotted lines: the  $G_M$  criterion in panels I, III, VI and X (equation 9), the  $A-M_{20}$  criterion in panel V (equation 10) and the  $C-M_{20}$  criterion in panel IX (equation 11). The solid lines connect the ram pressure sequence determined by Vollmer (2009) for a few selected Virgo cluster galaxies.

#### **4 RESULTS: HI MORPHOLOGY**

Figs 2 and 3 show the spread of morphological parameters for the entire VIVA sample with different morphological classifications from Chung et al. (2007, 2009). Table 2 lists all the values. Due to the combination of distance and resolution, the VIVA sample is the lowest physical resolution data to which these morphological parameters have been applied. In Holwerda et al. (2011b,d), we showed that these parameters are relatively invariant with distance out to  $D \sim 20$  Mpc and a resolution of 6 arcsec, so the VIVA sample is effective at the limit of this technique with current observatories.

We used the optical positions (J2000) of the galaxies reported in Chung et al. (2009) as the centre of the galaxy  $(x_c, y_c)$  to compute the parameters that depend on a central position (all except the Gini parameter). Our motivations are that the stellar disc marks the centre of the gravitational potential of a galaxy and kinematic and stellar centres generally align well (see Walter et al. 2008b; Trachternach et al. 2008). Optical positions of the galaxies are a likely starting point for many of the future H<sub>I</sub> surveys, and Chung et al. (2009) reported many tidal H<sub>I</sub> tails and displacements, which could be quantified using our morphological parameters. Thus, it is entirely possible that some of the morphological values (e.g., Asymmetry) are extreme since the centre is not the barycentre of the flux, as is common for optical morphological measurements.

Chung et al. (2009) supply extensive notes on the H<sub>I</sub> morphology and the nature of the H<sub>I</sub> stripping of individual galaxies. We have summarized these evaluations in Table 1. Chung et al. (2009) note which galaxies are currently undergoing or have recently undergone stripping, as well as the likely origin of this stripping ram pressure by the intracluster medium or tidal interaction with a close companion. In addition, they report a parameter for H<sub>I</sub> deficiency (def<sub>H1</sub>; the galaxy's H<sub>I</sub> surface density compared to the value typical for spirals), a measure of H<sub>I</sub> truncation ( $D_{H1}/D_B$ ; the ratio of the H<sub>I</sub> diameter over the optical, *B*-band one) and additional notes

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Table 2.	The morphological	values and errors	of the VI	IVA H I column	density maps.
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Galaxy	Gini	<i>M</i> <sub>20</sub>	C <sub>20/80</sub>	Asymmetry	Smoothness	G <sub>M</sub>
NGC 4064	$0.342 \pm 0.041$	$-0.652 \pm 0.195$	$0.810 \pm 0.430$	$2.000 \pm 0.000$	$0.501 \pm \cdots$	$0.206 \pm \cdots$
NGC 4189	$0.346 \pm 0.011$	$-0.691 \pm 0.019$	$1.156\pm0.106$	$2.000\pm0.000$	$0.154 \pm 0.030$	$0.266\pm0.077$
NGC 4192	$0.444 \pm 0.008$	$-0.965 \pm 0.199$	$7.162\pm2.317$	$1.171\pm0.434$	$0.200\pm0.022$	$0.687\pm0.033$
NGC 4216	$0.398 \pm 0.012$	$-0.633 \pm 0.026$	$0.993\pm0.284$	$2.000\pm0.000$	$0.241\pm0.024$	$0.820\pm0.132$
NGC 4222	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$
NGC 4254	$0.451 \pm 0.004$	$-0.666 \pm 0.008$	$1.580\pm0.075$	$2.000\pm0.000$	$0.035\pm0.012$	$0.232\pm0.022$
NGC 4293	$0.114 \pm \cdots$	$-0.758 \pm 0.089$	$1.933\pm2.181$	$1.605 \pm \cdots$	$1.043 \pm \cdots$	$0.780\pm\cdots$
NGC 4294	$0.493 \pm 0.025$	$-0.730 \pm 0.050$	$2.310\pm0.462$	$2.000\pm0.000$	$0.280\pm0.038$	$0.777\pm0.064$
NGC 4298	$0.525\pm0.009$	$-0.631 \pm 0.017$	$1.749\pm0.246$	$2.000\pm0.000$	$0.229 \pm 0.034$	$0.178 \pm 0.080$
NGC 4299	$0.175\pm\cdots$	$-0.683 \pm 0.136$	$2.299 \pm 1.288$	$2.000\pm0.000$	$0.373\pm\cdots$	$0.279\pm\cdots$
NGC 4302	$0.520 \pm 0.008$	$-0.976 \pm 0.129$	$7.231 \pm 1.485$	$1.932\pm0.487$	$0.228 \pm 0.040$	$0.259 \pm 0.061$
NGC 4321	$0.301 \pm 0.008$	$-0.769 \pm 0.018$	$1.994\pm0.579$	$2.000\pm0.000$	$0.127\pm0.016$	$0.138 \pm 0.106$
NGC 4330	$0.474 \pm 0.018$	$-0.688 \pm 0.110$	$0.362\pm0.105$	$2.000\pm0.000$	$0.499 \pm 0.044$	$0.661 \pm 0.151$
NGC 4351	$0.397\pm0.013$	$-0.693 \pm 0.027$	$1.916 \pm 0.393$	$2.000\pm0.000$	$0.148 \pm 0.030$	$0.249 \pm 0.107$
NGC 4380	$0.210\pm0.009$	$-0.799 \pm 0.061$	$1.725 \pm 0.676$	$2.000\pm0.000$	$0.293 \pm 0.024$	$0.551 \pm 0.151$
NGC 4383	$0.306\pm0.013$	$-0.709 \pm 0.032$	$2.442\pm0.566$	$2.000\pm0.000$	$0.127\pm0.023$	$0.149 \pm 0.116$
NGC 4388	$0.525\pm0.020$	$-0.799 \pm 0.201$	$2.957\pm0.878$	$1.654\pm0.262$	$0.386 \pm 0.089$	$0.546 \pm 0.082$
NGC 4394	$0.188 \pm 0.005$	$-0.764 \pm 0.081$	$2.591 \pm 0.503$	$2.000\pm0.000$	$0.195 \pm 0.008$	$0.094 \pm 0.070$
NGC 4396	$0.482 \pm 0.022$	$-0.686 \pm 0.049$	$1.753 \pm 0.446$	$2.000\pm0.000$	$0.400\pm0.024$	$0.700\pm0.150$
NGC 4405	$0.314 \pm 0.031$	$-0.666 \pm 0.238$	$0.872 \pm 0.609$	$2.000\pm0.000$	$0.445\pm\cdots$	$0.203 \pm \cdots$
NGC 4402	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$
IC 3355	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$
NGC 4419	$0.328\pm\cdots$	$-0.661 \pm 0.184$	$0.907 \pm 0.433$	$2.000 \pm \cdots$	$0.657\pm0.044$	$0.773 \pm 0.182$
NGC 4424	$0.484 \pm 0.021$	$-0.674 \pm 0.042$	$1.074\pm0.302$	$2.000\pm0.000$	$0.328 \pm 0.071$	$0.496 \pm 0.141$
NGC 4450	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$	$0.000\pm0.000$
IC 3392	$0.320 \pm 0.029$	$-0.685 \pm 0.263$	$0.367 \pm 0.219$	$2.000\pm0.000$	$0.434 \pm \cdots$	$0.458\pm\cdots$
NGC 4457	$0.287 \pm 0.016$	$-0.716 \pm 0.033$	$0.908 \pm 0.216$	$2.000\pm0.000$	$0.234 \pm 0.034$	$0.108 \pm 0.149$
NGC 4501	$0.331 \pm 0.007$	$-0.603 \pm 0.024$	$2.201 \pm 1.120$	$0.882 \pm 0.547$	$0.185\pm0.016$	$0.501 \pm 0.035$
NGC 4522	$0.398 \pm 0.018$	$-0.660 \pm 0.027$	$0.680 \pm 0.336$	$2.000\pm0.000$	$0.251 \pm 0.045$	$0.478 \pm 0.078$
NGC 4532	$0.591 \pm 0.008$	$-0.693 \pm 0.010$	$1.086 \pm 0.079$	$2.000\pm0.000$	$0.112 \pm 0.056$	$0.256 \pm 0.088$
NGC 4535	$0.322\pm0.007$	$-0.736 \pm 0.048$	$2.303 \pm 0.341$	$2.000\pm0.000$	$0.156 \pm 0.014$	$0.210\pm0.068$
NGC 4533	$0.000\pm0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000\pm0.000$
NGC 4536	$0.371 \pm 0.005$	$-0.734 \pm 0.034$	$2.771 \pm 0.855$	$0.407 \pm 0.451$	$0.097 \pm 0.025$	$0.562 \pm 0.034$
NGC 4548	$0.211 \pm 0.004$	$-0.808 \pm 0.116$	$4.793 \pm 1.165$	$0.518 \pm 0.454$	$0.214 \pm 0.013$	$0.223 \pm 0.066$
NGC 4561	$0.417 \pm 0.008$	$-0.694 \pm 0.009$	$2.407 \pm 0.366$	$2.000 \pm 0.000$	$0.203 \pm 0.032$	$0.395 \pm 0.049$
NGC 4567	$0.424 \pm 0.010$	$-0.710 \pm 0.018$	$1.152 \pm 0.182$	$2.000 \pm 0.000$	$0.115 \pm 0.024$	$0.439 \pm 0.032$
NGC 4568	$0.424 \pm 0.008$	$-1.096 \pm 0.199$	$5.611 \pm 1.421$	$2.000 \pm 0.321$	$0.115 \pm 0.028$	$0.439 \pm 0.044$
NGC 4569	$0.434 \pm 0.010$	$-0.703 \pm 0.019$	$0.429 \pm 0.079$	$1.282 \pm 0.369$	$0.213 \pm 0.021$	$0.519 \pm 0.200$
NGC 4579	$0.218 \pm 0.013$	$-0.705 \pm 0.050$	$1.199 \pm 0.330$	$2.000 \pm 0.000$	$0.203 \pm 0.026$	$0.240 \pm 0.174$
NGC 4580	$0.201 \pm \cdots$	$-0.674 \pm 0.200$	$0.694 \pm 0.386$	$2.000 \pm \cdots$	$0.469 \pm 0.032$	$0.364 \pm \cdots$
NGC 4606	$0.467 \pm 0.018$	$-0.675 \pm 0.039$	$0.108 \pm 0.032$	$2.000 \pm 0.000$	$0.338 \pm 0.067$	$0.669 \pm 0.115$
NGC 4607	$0.463 \pm 0.019$	$-0.667 \pm 0.066$	$0.778 \pm 3.209$	$2.000 \pm 0.000$	$0.337 \pm 0.057$	$0.645 \pm 0.073$
NGC 4651	$0.360 \pm 0.006$	$-0.712 \pm 0.008$	$1.597 \pm 0.105$	$2.000 \pm 0.000$	$0.113 \pm 0.024$	$0.590 \pm 0.022$
NGC 4654	$0.442 \pm 0.007$	$-1.279 \pm 0.258$	$8.395 \pm 1.295$	$1.097 \pm 0.502$	$0.120 \pm 0.021$	$0.522 \pm 0.043$
NGC 4689	$0.249 \pm 0.010$	$-0.739 \pm 0.074$	$2.942 \pm 1.516$	$2.000 \pm 0.000$	$0.126 \pm 0.010$	$0.173 \pm 0.065$
VCC 2062	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$	$0.000 \pm 0.000$
NGC 4694	$0.297 \pm 0.011$	$-0.693 \pm 0.037$	$0.966 \pm 0.501$	$2.000 \pm 0.000$	$0.377 \pm 0.054$	$0.581 \pm 0.099$
NGC 4698	$0.227 \pm 0.003$	$-0.739 \pm 0.014$	$3.115 \pm 0.312$	$2.000 \pm 0.000$	$0.145 \pm 0.008$	$0.583 \pm 0.021$
NGC 4713	$0.398 \pm 0.005$	$-0.695 \pm 0.008$	$2.082 \pm 0.149$	$2.000 \pm 0.000$	$0.110 \pm 0.021$	$0.202 \pm 0.045$
NGC 4772	$0.460 \pm 0.007$	$-0.734 \pm 0.061$	$2.733 \pm 0.424$	$2.000 \pm 0.032$	$0.329 \pm 0.049$	$0.511 \pm 0.075$
NGC 4808	$0.498 \pm 0.010$	$-0.696 \pm 0.016$	$2.883 \pm 0.481$	$2.000 \pm 0.000$	$0.151 \pm 0.049$	$0.417 \pm 0.111$

for indicators of warps in the H<sub>I</sub> morphology and velocity maps. Finally, Chung et al. (2009) also list the star formation properties of their sample, as catalogued and classified by Koopmann & Kenney (2004a,b), the level of activity and whether it was recently cutoff (truncated star formation). We have plotted the spread of our morphological parameters with each of these estimates marked to explore if there are specific parts of parameter space that single out the characteristic of an H<sub>I</sub> disc, similar to the separation of interaction and non-interacting galaxies we found in the WHISP data base. Figs 1–3 show the spread in parameters each with certain phenomena marked, so that they can be identified in the morphological parameter space. We discuss several H<sub>I</sub> morphological phenomena noted by Chung et al. below.

#### 4.1 Truncation and deficiency

Fig. 1 shows Gini versus Concentration for  $H_{I}$  deficient and truncated  $H_{I}$  discs. Both parameters are sensitive to how evenly the flux is distributed over the area of the disc and therefore could be sensitive to both these phenomena. To identify  $H_{I}$  truncation, one could use the Concentration of the H<sub>I</sub> emission. We have found that values around or below C = 1 seem to be very indicative of truncation; the radii containing 20 and 80 per cent of the H<sub>I</sub> flux are very close together (Fig. 1, left panel). Given that typical values for Concentration in H<sub>I</sub> are much higher (typically  $C \ge 5$ ; Holwerda et al. 2011c; Holwerda et al., in preparation), this may be a good tracer of truncation in large H<sub>I</sub> surveys. A value of Concentration below unity selects half of the truncated galaxies in the VIVA sample very cleanly (12 out of 24 truncated discs with no contamination from non-truncated discs).

The H<sub>I</sub> deficiency is defined by the surface density compared to the average surface density for spirals (see equation 5 in Chung et al. 2009). The majority of VIVA galaxies are H<sub>I</sub> deficient (44 galaxies with def<sub>H<sub>I</sub></sub> < 1). The low values of Concentration (C < 5) appear to be indicative of this H<sub>I</sub> deficiency. In VIVA, those galaxies with extreme H<sub>I</sub> deficiency (def<sub>H<sub>I</sub></sub> < 0.5) are not much more concentrated than moderately deficient ones (Fig. 1, right panel), at least at this resolution. We note that a  $C \leq 5$  criterion will select some 80 per cent of the H<sub>I</sub> deficient galaxies in the VIVA sample, but their number is contaminated (17 per cent of this selection is not H<sub>I</sub> deficient). In the upcoming large surveys this makes Concentration a quick assessor on whether or not a spiral has a typical H<sub>I</sub> content.

#### 4.2 Interaction, HI tails and ram pressure

Nearly all the spirals in the Virgo cluster are undergoing, or have recently undergone, some type of interaction. Discerning between the dominant mechanism, tidal or ram pressure, was a major focus of the work by Chung et al. (2009) and Vollmer (2009). Several interaction-related morphological phenomena (e.g. warps and H<sub>I</sub> tails) are noted by Chung et al. (2007).

Table 1 lists many galaxies with H1 tails in the VIVA data and several with H1 warps. One would expect that high values for the Asymmetry would identify H1 tails (Fig. 2). However, because we used the optical centres listed by Chung et al. (2009), Asymmetry is already at the maximum value for most of the VIVA galaxies (Fig. 2, panels IV-VII). We note however that the Asymmetry may still be a good indicator of (tidal) H1 tails, but in less extreme or possibly better resolved cases. The  $M_{20}$ -Gini selection criterion (see equation 7) does reasonably separate those galaxies with H1 tails from those without; 60 per cent of the H1 tails identified by Chung et al. (2007, 2009) are selected by this criterion (Fig. 2, panel II). However, about half the objects selected do not have an HI tail. Therefore, the Gini $-M_{20}$  criterion can be used as a likelihood for an H1 tail, but it will suffer from contamination. Warps were identified predominantly by their kinematic signature in Chung et al. (2009) and no morphological parameter or combination seems to be able to identify them in the VIVA data.

Fig. 3 shows the same spread of morphological parameters as Fig. 2 with the estimate we gleaned from Chung et al. (2009) of whether or not the galaxy is stripped, undergoing ram pressure, or undergoing tidal interaction. The sequence of stripping in the Virgo cluster according to Vollmer (2009) is indicated by the solid lines. This sequence of six Virgo galaxies does trace a track into and out of certain interaction criteria defined previously in the literature and in the second paper of this series.

The extreme values for Asymmetry drive almost all of the VIVA galaxies into the Asymmetry-based merger selection space: the Asymmetry criterion from Conselice (2003), the Asymmetry–Gini criterion from Lotz et al. (2010b) and our Asymmetry– $M_{20}$  criterion from Holwerda et al. (2011c) (equations 6, 8 and 10, re-

spectively). The extreme values for Concentration – linked to H<sub>I</sub> deficiency and truncation – similarly make the Concentration– $M_{20}$  selection criterion (equation 11; Holwerda et al. 2011c) inapplicable for merger selection. Only the Gini– $M_{20}$  criterion (equation 7) appears to separate the objects with H<sub>I</sub> tails moderately as discussed above.

The  $G_{\rm M}$  criterion (equation 9) is not contaminated by galaxies undergoing ram pressure stripping but does no longer select tidal interactions well either, something that we attribute to the native VIVA resolution (no visibility times found in Holwerda et al. 2011d, for the VIVA resolution). Because stripping, H1 deficiency and truncation are interlinked, there is the possibility that the selection criteria of Concentration will also select those galaxies that are or have been stripped.

Two effects move H<sub>I</sub> morphologies into the selection criteria: poorer resolution and additional effects such as the ram pressure stripping. Given how the Vollmer sequence tracks into (and out of again) the merger selection criteria, we can conclude that galaxies undergoing ram pressure stripping will sometimes be selected as interacting in a morphological selection in future H<sub>I</sub> surveys, at least those with spatial resolutions similar to VIVA. The addition of some galaxies that are undergoing stripping would, in part, explain the high merger fraction and rate we found in the WHISP survey (Holwerda et al. 2011d). In Holwerda et al. (2011c), we noted that the lower resolution data of the VIVA sample would have some problems in selecting interacting systems from isolated ones, purely on the basis of their H<sub>I</sub> morphology (table B4 in the appendix). However, the success with which tidal H<sub>I</sub> tails could be separated is encouraging.

Therefore, in future medium-resolution H<sub>I</sub> surveys (e.g. WALLABY; Koribalski et al., in preparation), the selection of H<sub>I</sub> discs with disturbed morphology can be used as a first pass in a search for stripping or interacting galaxies, and the merger fraction and rate will have to be corrected for the contamination by stripping galaxies. For example, those galaxies that are H<sub>I</sub> deficient can be removed before a merger rate is determined, but that would bias against denser cluster environments (which do include more other interactions like ram pressure stripping). A higher resolution survey of the Virgo cluster galaxies would resolve the population of tidally interacting galaxies. The separation of ram pressure stripped galaxies from the tidally interacting ones may then be possible without the removal of H<sub>I</sub> deficient ones.

### 4.3 Star formation

We explored the link between the morphological parameter space of H<sub>1</sub> and the star formation and truncation of star formation as listed in table 3 in Chung et al. (2009). Relations are weak as the H<sub>1</sub> gas is undergoing several other effects other than feedback from star formation, but it is interesting to note that those galaxies with little star formation also have lower values of Gini for H<sub>1</sub> (i.e. they are more homogeneous).

## **5 CONCLUDING REMARKS**

In this paper, we present the morphological parameters commonly used in the optical, applied to the H<sub>I</sub> column density maps of Virgo cluster galaxies obtained by the VIVA programme. We find that the resolution of these observations ( $\sim$ 15 arcsec or 1 kpc) is on the coarse side for this type of analysis, but some H<sub>I</sub> morphological phenomena are identifiable, though with limitations. A critical choice is whether or not to use the optical or the H<sub>I</sub> centres for these

galaxies, one that every H<sub>I</sub> survey will have to make. The choice of centre affects measures such as Concentration,  $M_{20}$ ,  $G_M$  and especially Asymmetry. Alternatively, one could use the barycentre of the H<sub>I</sub> emission itself, but the optical centre is most likely the centre of the dark matter halo of the galaxy and this approach would likely miss H<sub>I</sub> displacements like tidal tails. We find that of the H<sub>I</sub> morphological phenomena, H<sub>I</sub> deficiency and truncation can be identified easily by a low value of the Concentration index (Fig. 1). Warps could not be identified using the morphology parameters and H<sub>I</sub> tails can be identified moderately well with the combined Gini and  $M_{20}$  interaction criterion (equation 7 and Fig. 2, panel II) from Lotz et al. (2004, 2008).

Galaxies that are interacting are still identified by some of the morphological criteria from the literature (Conselice 2003; Lotz et al. 2004) and those identified by us specifically for HI in Holwerda et al. (2011c) (equations 6-11). However, galaxies affected by ram pressure stripping are often also selected by these criteria. For example, we note that the ram pressure sequence identified by Vollmer (2009) tracks into the interaction part of the parameter space for many of the morphological selection criteria (solid tracks in Fig. 3). Discerning between the origin of the H1 stripping - ram pressure or tidal - is, however, not possible with these morphological parameters, at least at VIVA's spatial resolution. Thus, any selection of interacting systems in a survey based purely on the H<sub>1</sub> morphology parameters will include those that are undergoing ram pressure stripping in a denser environment. A straightforward correction would be to exclude all H<sub>I</sub> deficient (C < 5) galaxies from a morphological selection of interacting galaxies.

## ACKNOWLEDGMENTS

We wish to thank the anonymous referee for his or her comments, helping us to greatly improve the final manuscript. We thank the VIVA collaboration for making their data public and easily accessible. The National Radio Astronomy Observatory is a facility of the National Science Foundation operated under cooperative agreement by Associated Universities, Inc. We acknowledge support from the National Research Foundation of South Africa. The work of BWH and WJGdB is based upon research supported by the South African Research Chairs Initiative of the Department of Science and Technology and the National Research Foundation.

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