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Making Sense of Questions in Logic and Mathematics: Mill vs. Carnap

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ABSTRACT: Whether mathematical truths are syntactical (as Rudolf Carnap claimed) or empirical (as Mill actually never claimed, though Carnap claimed that he did) might seem merely an academic topic. However, it becomes a practical concern as soon as we consider the role of questions. For if we inquire as to the truth of a mathematical statement, this question must be (in a certain respect) meaningless for Carnap, as its truth or falsity is certain in advance due to its purely syntactical (or formal-semantical) nature. In contrast, for Mill such a question is as valid as any other. These differing views have their consequences for contemporary erotetic logic.

KEYWORDS: Empirical propositions, erotetic logic, mathematical truth, question, scientific inquiry.

“I thought: are we now back with John Stuart Mill?” (R. Carnap)

1. Plan of Inquiry

The aim of this paper is, first of all, to reveal and contrast the roles played by questions in the work of John Stuart Mill and Rudolf Carnap (sec. 2 and 3), especially in mathematics and logic (sec. 4); and, secondly, to argue (sec. 5) that deciding between the theoretical conceptions of Mill and Carnap implies a decision between the two main approaches to erotetic logic. Questions in mathematics will play the key part in these considerations, for mathematical theorems are expected to be true *a priori*. Thus a question in mathematics makes sense only with respect to an individual searching for knowledge; and how to deal with the knowledge-searching individual is exactly where the two types of accounts in erotetic logic part ways: those which use an intensional operator to model the role of the individual, and those which shift the matter to pragmatics and leave it out of syntax.

2. Questions in General

Needless to say, Mill's theory of logic differs greatly in its nature from that of Carnap. Carnap's theory is formal in the modern sense, something that Mill could not have achieved. Nevertheless, their theories do share a certain feature: they are concerned not only with ready-made propositions, but also with the *questions* that lead to those propositions. The two philosophers are also connected by the general setting in which they examine questions, which are not reduced to linguistic or logical entities, but rather placed within the framework of scientific inquiries.

> Of the various questions which present themselves to our inquiring faculties, some receive an answer from direct consciousness, others, if resolved at all, can only be resolved by evidence. Logic is concerned with these last. But before inquiring into the mode of questions, it is necessary to inquire what are those which offer themselves; what questions are conceivable; what inquiries are there, to which mankind have either obtained, or been able to imagine it possible that they should obtain, an answer. This point is best ascertained by a survey and analysis of Propositions. (Mill, 2002: 12)

Thus the topic of consideration is not questions *per se*, isolated and studied for their own sake, but questions that have *de facto* been of interest. Furthermore, questions are not only located *within* the framework, they also determine its limits:

> With the original data, or ultimate premises of our knowledge; with their number or nature, the mode in which they are obtained, or the tests by which they may be distinguished; logic, in a direct way at least, has, in the sense in which I conceive the science, nothing to do. These questions are partly not a subject of science at all, partly that of a very different science. Whatever is known to us by consciousness, is known beyond possibility of question. (Mill, 2002: 4)

Questions thus separate what belongs to logic from what does not. There are two possibilities for something's not belonging to logic: either the question pertains to something else (to a different science), or there is no question at all.

At first glance, the two passages cited seem to contradict one another. The first speaks of questions which "receive an answer from direct consciousness"; the second states that "[w]hatever is known to us by consciousness, is known beyond possibility of question". However, there is a difference between being the aim of a question and being questionable. For example, if I wonder whether my injured toe still hurts, and press my finger on it to find out, then the result will be an answer to the question that I receive by direct consciousness; but it will not itself be questionable or in any way dubious. The question we *cannot* ask in this context is: Do I really feel pain in my toe? (Yet we clearly do *have* these impressions of direct consciousness, they do exist; and so it makes sense to say that they do not belong to logic.)

But besides these "unposable questions" there are, as I have already observed, still other questions whose answers do not belong to logic, either. And I see no reason to make a distinction between logic and other sciences in this respect. Every science has, according to Mill, its own questions that express the characteristic interests of that field.

The limiting case of this statement is: no question, no science. The questionable constitutes the limits of science. But what about: no science, no question? Are there questions which we may raise, and which do not have any answers in any science? Mill is explicit about this when he writes (in the passage quoted above) of "questions [that] are partly not a subject of science at all." Let us compare all this with Carnap:

> By boundlessness of scientific knowledge we mean: there is no question which in principle [*grundsätzlich*] cannot be answered by science. (Carnap, 1998: 254)

Carnap denies the existence of unanswerable questions. (In what follows, he explains what he means by *grundsätzlich*: every question could be answered, provided there was no temporal or spatial gap between us and what we wanted to know; or else, we would be able to answer the question once the technical means were developed and placed at our disposal.) But perhaps one should not take him too literally here, and grant him a more sympathetic reading. For immediately before the sentence cited above he states that life has many dimensions beyond science, and compares scientific knowledge to a plane: it has no boundaries within itself, but something exists which lies *outside* it:

> Sometimes it is said that the answers to some questions cannot be grasped by concepts, cannot be expressed. But in such cases the question cannot be expressed either. (Carnap, 1998: 254)

In any case, there is no *extralinguistic* approach to questions:

> To recognize this, we will further investigate what the answering of a question consists in. In a strictly logical sense, posing a question means giving a statement and setting the task of establishing that either the statement or its negation is true. (Carnap, 1998: 254)

In his later book *Logische Syntax der Sprache* (1968), Carnap elaborates a formal concept of the question:

> A yes-no question consists in the request to either affirm or negate a certain proposition S_1, that is, to express either S_1 or $\sim S_1$. (Carnap, 1968: 222)

Yes-no questions do not cause any difficulty, fitting perfectly into what Carnap had said in the *Aufbau* (1998). But how does Carnap bring his treatment of questions in the *Aufbau* into accord with that part of the theory in *Logische Syntax* where he formalizes questions like "When is John Stuart here?" – that is, questions which do not simply consist of a given statement that is to be affirmed or denied? He proposes that such a question be understood as a proposition like "John Stuart is here at *t*", where *t* is a variable which is free or bound by a question operator. To answer such a question is, therefore, to divide all propositions of this form into true and false ones. Hence the theory of wh-questions could be seen as a generalization of the theory of yes-no questions. Yet despite appearances it is, all the same, a restriction, a commitment to the basic idea of yes-no questions. Carnap is forced – or thinks he is forced – by his formal theory of logic to limit his understanding of questions in this way. Mill's approach is, of course, free of all such requirements.

3. Questions and Answers

There is also a more structural connection between questions and propositions for both Mill and Carnap:

> The answer to every question which it is possible to frame, must be contained in a Proposition, or Assertation. […] [T]o know the import of all possible propositions, would be to know all questions that can be raised, all matters which are susceptible of being either believed or disbelieved. (Mill, 2002: 12)

Questions and the propositions answering them are theoretically related to one another. Mill's suggestion that knowing all possible answers means knowing all questions is one of the standpoints in erotetic logic – the contemporary logic of questions. Its various proposals are as follows:

(a) The sets of possible answers *determine* the questions.
(b) The sets of possible answers *are* the questions.
(c) The possible answers are more loosely connected to the questions.[1]

[1] A fourth possibility would be: "Questions *determine* the possible answers". However, this would require a (formal) system that starts with questions and derives propositions from them. As far as I know, no such account has yet been developed.

Mill is obviously a proponent of either (a) or (b), though which one cannot be determined. Carnap holds that (b) is the right view. In the example of the when-question above, it is clear that "John Stuart is here at *t*" represents the essence of the question as well as of its answers. This is the first point where interpreting Mill's and Carnap's work has implications for the presuppositions of erotetic logic.

Although a strong theoretical connection between questions and answers is common to the writings of Mill and Carnap alike, for Mill questions are prior to answers in "reality" (in the practice of research). This is not clear in *every* case with Carnap, as we shall see.

Having explained what counts as an answer to a question, we must consider what makes an answer a *right* answer. Roughly speaking, the basic propositions in Carnap's *Konstitutionstheorie* are obtained in a manner quite similar to (empirical) propositions in Mill's theory – namely, by induction. Moreover, both authors share a rather specific understanding of induction: "In short, Carnap appears to be in substantial agreement with J. S. Mill's view that the fundamental type of inductive reasoning is 'from particulars to particulars'" (Nagel, 1963: 802).[2]

Induction need not always lead from a particular case to the general case, or *via the general case* to another particular case. It may be that no universal proposition is concluded. For example, if one has seen a number of white swans and no black ones, then it would be reasonable to conclude that the next swan will also be white, even though one does not have the slightest reason to suppose that *only* white swans exist.

Apart from empirical propositions, however, there are also answers to questions which are confirmed as true in a different way: logical theorems. According to Carnap, these are true because we can deduce them in the formal system we are using (or, in late Carnap, they are semantically valid in that system); according to Mill, because they have evidence (see above: "[O]ther [questions], if resolved at all, can only be resolved by evidence. Logic is concerned with these last."). Mill does not really explain what he means by "evidence". He simply opposes it to consciousness or intuition (using these terms indiscriminately), whereby we know what we know directly (for example, our knowledge of being, or having been, hungry at a given moment), and states that what we know by reasoning must be known through evidence. However, I will not deal with this quite complicated topic any further, as it has no relevance to my argumentation below (on Mill's deductive logic, see e.g. Jackson, 1941).

The main points above may be summarized by answering the following question: In which respects do Mill's and Carnap's views coincide,

[2] This is what Aristotle calls *paradeigma*, as opposed to *epagoge*.

and in which do they differ? They both (a) limit meaningful questions by the propositions that might answer them, (b) locate the question in the general context of scientific inquiry, and (c) distinguish between two sorts of reasons which make an answer true and, therefore, between two sorts of questions. However, (a) whereas Carnap places empirical reasons and analytical connections in opposition, Mill's distinction is one of evidence and consciousness, and (b) for Mill there are questions that are outside science, but for Carnap (if taken literally) there are not.

4. Questions in Mathematics

At a symposium held around 1940, Carnap asked: "Are we now back with John Stuart Mill?" He alleged that Mill considered mathematical propositions to be empirical ones. That this is what Carnap really thought is clear from the context in which he voiced this question: he had been attacked by von Mises, Tarski and Quine for having stated that mathematical truths are non-factual and non-empirical.[3]

In some sense, Mill and Carnap may be seen as holding two extreme positions in the discussion on the ontological and epistemic status of mathematical propositions. For Carnap, mathematical propositions lack any empirical content; for Mill, all propositions, and particularly mathematical theorems, derive some of their justification from empirical grounds. But does Mill really claim that mathematical propositions *are* empirical ones?

In Mill's time, the modern understanding of mathematics existed mainly in the field of geometry. Thus if we wish to know what the epistemological status of mathematical theorems is and to understand mathematics as a deductive science on an axiomatic basis, we should study Mill's thoughts on geometry. There is no doubt that Mill regards mathematics as being founded on real, empirical objects:

> We are thinking, all the time, of precisely such objects as we have seen and touched, and with all the properties which naturally belong to them; but, for scientific convenience, we feign them to be invested of all properties, except those which are material to our purpose, and in regard to which we design to consider them. (Mill, 2002: 148)

The first non-empirical element of mathematical theorems is their restriction to a certain aspect, an "insofar". This can be approached from two sides: either by ignoring the inexactness of a real object, or by considering only such objects as are sufficiently exact for the given purpose. There is evidence in Mill's text for both possibilities:

[3] For Carnap's standpoint, see Carnap, 1963: 46f.

> In this relation, of course, the derivative truths of every deductive science must stand to the inductions, or assumptions, on which the science is founded, and which, whether true or untrue, certain or doubtful in themselves, are always supposed certain for the purposes of the particular science. And therefore the conclusions of all deductive sciences were said by the ancients to be necessary propositions. (Mill, 2002: 149)

A second and even more important non-empirical element is included here: supposing, or the will to suppose. This decisive or normative element is a very modern constituent of Mill's conception of mathematical statements[4] (think of the late Wittgenstein, for example):

> In all propositions concerning numbers, a condition is implied, without which none of them would be true […]. The condition is, that 1=1; that all the numbers are numbers of the same or of equal units. Let this be doubtful, and not one of the propositions of arithmetic will hold to be true. […] How can we know that a forty-horse power is always equal to itself, unless we assume that all horses are of equal strength? […] What is commonly mathematical certainty, therefore, which comprises the twofold conception of unconditional truth and perfect accuracy, is not an attribute of all mathematical truths, but of those only which relate to pure Number, as distinguished from Quantity in the more enlarged sense; and only so long as we abstain from supposing that the numbers are a precise index to actual quantities. (Mill, 2002: 170)

Certainty in mathematics depends, accordingly, on the presupposition of mathematical principles. We are free to suppose these principles, or to neglect them; yet only so long as we suppose them, and do not let the truth of a mathematical proposition depend, for example, on physical facts, can a proposition possess the specific certainty of mathematics. However, in order to be useful and meaningful, mathematical principles need to be obtained by induction from empirical facts. This does not necessarily signify a loss of normativity. The idea of induction is also normative, yet not analytic (not "verbal", in Mill's terms). To be more precise, it is not induction but a reliance on causal relations in nature (cf. Scarre, 1998) which constitutes the normative element in Mill's theory of inductive reasoning.

Let us now consider the role of *questions* aimed at mathematical or logical truths. How did this problem arise, and how there can be "new" theorems in mathematics when all mathematical theorems are true *a priori*? A conflict only occurs when we consider the enterprise of *finding* such theorems, i.e. research work. Therefore, *questions* are the point at which the origin of mathematical truth becomes relevant.

[4] The normative impact of Mill's considerations is discussed in Skorupski, 1998: 53.

Following Carnap, one can pose such questions, formulating them in a syntactically correct way. Hence these are no *Scheinfragen*, in the sense of metaphysical questions; yet in another respect they are, however, meaningless, as their answers are set in advance. A mathematical proposition is true or false due to syntactical reasons (or formal-semantical ones), and does not require any empirical information. Thus a question inquiring about the truth or falsity of such a proposition is not a real question, at least not in logic (although perhaps it is one in psychology). Following Mill, of course, such questions make perfect sense, if only because their empirical grounds have to be evaluated.

5. Consequences of Choosing a System of Erotetic Logic

Questions in mathematics are the crucial point in discussions about rival theories of questions, with intensional conceptions on the one side, and those which operate with bound or unbound "query variables" on the other.

Aqvist (1965), Hintikka (1976), Kubinski (1980), Lewis and Lewis (1975) and several other authors argue that a question aimed at the truth of a sentence p means a proposition including a certain modal operator, such as: "Let it turn out to be the case that I know that p." (In formal notation: ?p:=!Kp, where "!" means "Let it turn out to be the case" and "K" means "I know that".) Some variants are: "Bring it about that I know…", "Bring it about that I believe…", or "Tell me truly if…".[5]

For Carnap, such intensional approaches are quite indispensable, because asking a question like ?(2+3=5) or ?($p \rightarrow p$), where "?(…)" means an arbitrary formalization of that question within a formal system, would be pointless if there were no difference between being an answer and being known as an answer – and this would be the case for all mathematical questions. Therefore, in order to make the formal expression of a question in mathematics meaningful, one must introduce a formal equivalent for the person who wants to know. This can be done by using a modal operator.

For Mill, on the other hand, these questions might have three aims:

(1) the empirical basis,
(2) the adequateness of restriction to a certain aspect, and
(3) acceptance of the underlying norms.

Consider the example: "Is there an angle in a triangle greater than 180°?" In this instance (1) and (2) seem quite clear, while (3) may be stated as follows: Do we accept the presuppositions and means leading to the con-

[5] For an overview of these accounts, see e.g. Harrah, 2002.

clusion that there is no such angle? Fulfillment of these three requirements gives an exact characterization of the usual manner of justification of and in formal systems. (We judge a mathematical proof to be correct if we accept the premises and each step [i.e. the method] leading to the result.)

Let us now turn to someone we might not expect here:

> To ask a question means to bring into the open. The openness of what is in question consists in the fact that the answer is not settled. (Gadamer, 1975: 326)

The context in which this passage from *Truth and Method* is located offers many interesting connections with formal logic; much of what Gadamer says can even be translated into meta-logical statements. But here I will only borrow his idea of "openness" for my purposes. If a question is to be open, then ?(2+3=5), as Carnap understands it, is not a question; we must introduce an intensional operator (such as "knowing that") into the logical system in order to express the idea of openness. (But one could argue that in doing so we lose this very same notion of openness.) None of these problems arise with Mill, since for him there is simply no question without openness. Whether we are asking about an empirical fact or a mathematical theorem, there are always questionable ingredients: empirical grounds, norms, or at least the acceptance of some evidence.

But now what about formalization? Mill's standpoint saves us from the problem by spreading it to anything and everything. By putting a question one also questions part of the framework, the formal system in which the question is located. The concept of a "question *within* a fixed (formal) framework" is simply impossible from Mill's point of view, even in mathematics. In this respect, questions do not differ from propositions; however, questions need openness in order to be meaningful, while propositions do not. Choosing the means of formalization always constitutes an essential part of the subject of a question. Thus mathematical questions do not generate a need for extensions of erotetic logic like those proposed by Hintikka, Kubinski and others, since all questions, including mathematical ones, have a subject (which is not set in advance), namely, the system or framework in which they are expressed. Therefore, the distinction between "being an answer" and "being known as an answer" is not crucial *in this context*.[6] Thus being "back with John Stuart Mill" would mean surpassing some of the presuppositions of modern erotetic logic.

[6] In line with Mill's thinking, one may introduce the formalization of questions into a logical system, which may be regarded as a formal component, and does *not* itself require openness in any sense.

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