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PREDICTIVE RELIABILITY ANALYSIS OF REDUNDANT SHIP NAVIGATIONAL RADAR SYSTEM

In this paper, a general approach to establishing the Markov reliability model of the redundant system is presented. On the basis of this approach, the Markov reliability model of non-repairable and repairable two-component standby redundant systems is established. Finally, a quantitative comparative reliability analysis of the two-component standby redundant ship navigational radar system is performed. The analysis indicates that repairing has a very significant influence on the reliability increase of this system.

Key words: *reliability, standby redundant system, Markov model, navigational radar system*

1. INTRODUCTION

Generally speaking, reliability is defined as the probability that a system will work properly for a certain period of time under a set of operating conditions [4]. Implied in this definition is a clear-cut criterion of failure, on the basis of which we may judge at which point the system is no longer functioning properly. Similarly, the treatment of operating conditions requires an understanding both of loading to which the system is subjected and of the environment within which it operates. Perhaps the most important variable to which we must relate reliability, however, is time.

It is a fundamental tenet of reliability engineering that as the complexity of a system increases, its reliability decreases, unless compensatory measures are taken. Since a frequently used measure of complexity is a number of components in the system, the component reliability must be increased if the number of components in the system has increased.

An alternative to increasing the component reliability is to provide redundancy in a part of the system, or the system as a whole. There are a number of different redundant configurations of the system. One of the redundant types of systems is the standby redundant system [2], which may be non-repaired or repaired.

Two considerable benefits are to be gained by using the standby redundant system. The first is that more than one failure must occur in order for the system to fail. A second is that components can be repaired while the system is on-line. Much higher reliabilities are possible if the failed component has a high probability of being repaired before another one fails. Because the standby redundant system involves dependency between components, the system reliability is nicely analyzed by Markov methods [3, 5].

In this paper, the reliability analysis of the two-component standby redundant ship navigational radar system is evaluated and analyzed on the basis of Markov methods.

2. GENERAL MARKOV RELIABILITY MODEL OF REDUNDANT SYSTEMS

In reliability analysis of the multi-component system, the most pervasive technique is that of estimating the reliability of the system in terms of the reliability of its components. In such an analysis, it is frequently assumed that the component failure and repair properties are mutually independent. In reality, this is often not the case. Therefore, it is necessary to apply the reliability models that take into account the interactions of component failures and repairs.

Many component failure interactions, as well as systems with independent failures, may be modeled effectively as Markov processes [6], provided that the failure and repair rates [4] can be approximated as time-independent.

The aim of Markov analysis is to calculate $p_i(t)$, the probability that a system is in the state i at the time t, where states of the system are defined by a particular combination of operating and failed components of the system. Once this is known, the redundant system reliability can be calculated as a function of time from

$$R(t) = \sum_i p_i(t) \qquad (1)$$

where the sum is taken over all operating states (i.e. over those states for which the system has not failed). Alternately, the reliability may be calculated from

$$R(t) = 1 - \sum_i p_i(t) \qquad (2)$$

where the sum is over all states for which the system has failed. Since at any time the system can only be in one state, we have

$$\sum_i p_i(t) = 1 \tag{3}$$

where the sum is over all possible states of the system.

To determine the $p_i(t)$, we derive a set of Kolmogorov differential equations [6], one for each state of the system.

Let us assume that there are n possible states of the system, and that the vector $\mathbf{p}(t)$ in the form of

$$\boldsymbol{p}(t) = (p_i(t)); \quad i = 1, 2, ..., n$$

represents the vector of all system state probabilities, and that the vector $\boldsymbol{p}'(t)$ in the form of

$$\boldsymbol{p}(t) = (p'_i(t)); \quad i = 1, 2, ..., n$$

represents the vector of all system state probabilities derivation. Then, if the matrix $\boldsymbol{Q}$ in the form of

$$\boldsymbol{Q}[q_{ij}]; \quad i, j = 1, 2, ..., n$$

represents the matrix of the transition rates between all system state pairs, the probability of all system states is determined by the solution of the Kolmogorov system of differential equations in the vector-matrix form of

$$\boldsymbol{p}'(t) = \boldsymbol{p}(t)\boldsymbol{Q} \tag{4}$$

3. MARKOV RELIABILITY MODELS OF TWO-COMPONENT STANDBY REDUNDANT SYSTEMS

The standby redundancy is a widely applied type of redundancy in fault-tolerant systems [1]. By their nature, standby redundant systems involve dependency between components; they are nicely analyzed by Markov methods.

Let us assume that there is a two-component standby redundant system with the following system states:

State 1: The primary component is operational and the standby component is spare.

State 2: The primary component is failed and the standby component has been switched in and is operational.

State 3: The primary and the standby components are failed, i.e. the system is failed.

In addition, let us assume that the failure rate of both the components is λ.

3.1. NON-REPAIRABLE SYSTEM RELIABILITY MODEL

Let us suppose that after a failure the primary component is non-repairable. In this case, the transition rate between State 1 and State 2 and between State 2 and State 3 is λ. However, the transition rate between State 2 and State 1 and between State 3 and State 2 is zero. The state transition diagram for this case is shown in Figure 1.

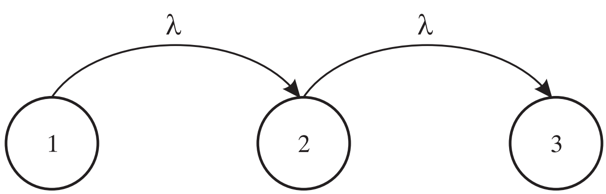

Figure 1. State transition diagram of the non-repairable two-component standby redundant system

For the observed case, the transition rate matrix is in the form of

$$\mathbf{Q} = \begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix} \quad \begin{bmatrix} -\lambda & \lambda & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & 0 \end{bmatrix} \tag{5}$$

and the associated Kolmogorov system of differential equations is in the form of

$$\left(p'_1(t), p'_2(t), p'_3(t)\right) \quad \left(p_1(t), p_2(t), p_3(t)\right) \begin{bmatrix} -\lambda & \lambda & 0 \\ 0 & -\lambda & \lambda \\ 0 & 0 & 0 \end{bmatrix} \tag{6}$$

Since the system States 1 and 2 are operating states and State 3 is a failed state, the general system reliability model is in the form of

$$R(t)=p_1(t)+p_2(t) \tag{7}$$

By solving (6) for $p_1(t)$ and $p_2(t)$ and supposing that $p_1(0)=1$ and $p_2(0)=p_3(0)=0$, the observed system reliability model is finally in the form of

$$R(t)=(1+\lambda t)e^{-\lambda t} \tag{8}$$

3.2. REPAIRABLE SYSTEM RELIABILITY MODEL

Let us assume that after a failure the primary component is repairable with a repair rate μ. In this case, the transition rate between State 2 and State 1 of the observed system is μ, and the state transition diagram is shown in Figure 2.

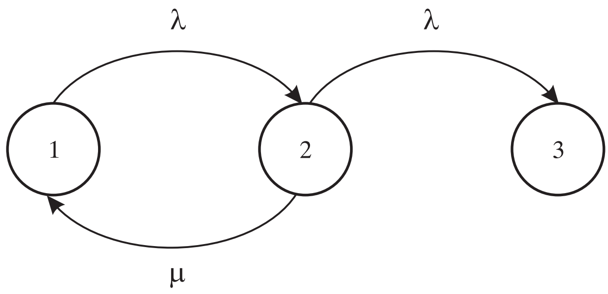

Figure 2. State transition diagram of a repairable two-component standby redundant system

The state transition matrix in this case is in the form of

$$\mathbf{Q}=\begin{bmatrix} q_{11} & q_{12} & q_{13} \\ q_{21} & q_{22} & q_{23} \\ q_{31} & q_{32} & q_{33} \end{bmatrix}=\begin{bmatrix} -\lambda & \lambda & 0 \\ \mu & -(\mu+\lambda) & \lambda \\ 0 & 0 & 0 \end{bmatrix} \tag{9}$$

and the associated Kolmogorov system of differential equations is in the form of

$$\left(p'_1(t), p'_2(t), p'_3(t)\right)=\left(p_1(t), p_2(t), p_3(t)\right)\begin{bmatrix} -\lambda & \lambda & 0 \\ \mu & -(\mu+\lambda) & \lambda \\ 0 & 0 & 0 \end{bmatrix} \tag{10}$$

Since the general system reliability model has the form (7), by solving (10) for $p_1(t)$ and $p_2(t)$, the observed system reliability model is in the form of

$$R(t)=\frac{s_1 e^{-s_2 t}-s_2 e^{-s_1 t}}{s_1-s_2} \tag{11}$$

where

$$s_1=-\frac{2\lambda+\mu}{2}+\frac{\sqrt{\mu^2+4\lambda\mu}}{2}$$

$$s_2=-\frac{2\lambda+\mu}{2}-\frac{\sqrt{\mu^2+4\lambda\mu}}{2}$$

4. RELIABILITY OF THE TWO-COMPONENT STANDBY REDUNDANT SHIP NAVIGATIONAL RADAR SYSTEM

Let us assume that there are two radar devices which are components of the two-component standby redundant ship navigational radar system. One of the radar devices is a primary component and the other radar device is a back-up component of the observed radar system. If the failure rate of both the radar devices is the same and is λ, and the observed radar system is non-repairable, the reliability function of this system is in the form of (8) and is graphically presented in Figure 3.

In the second case, let us assume that the primary radar device for the observing radar system can be repaired on-line. For simplicity, we assume that the failure of the backup radar device in the standby mode and the switching failures can be neglected. If the failure rate of both the radar devices is λ and the repair rate of the backup radar device is μ, the system reliability function is in the form of (11) and is graphically presented in Figure 3 for $\mu=10\lambda$, $\mu=50\lambda$, and $\mu=100\lambda$

Plots in Figure 3 show that the reliability of the non-repairable two-radar device standby redundant navigational radar system is relatively low and that it very rapidly decreases with time. However, if the observed radar system is re-

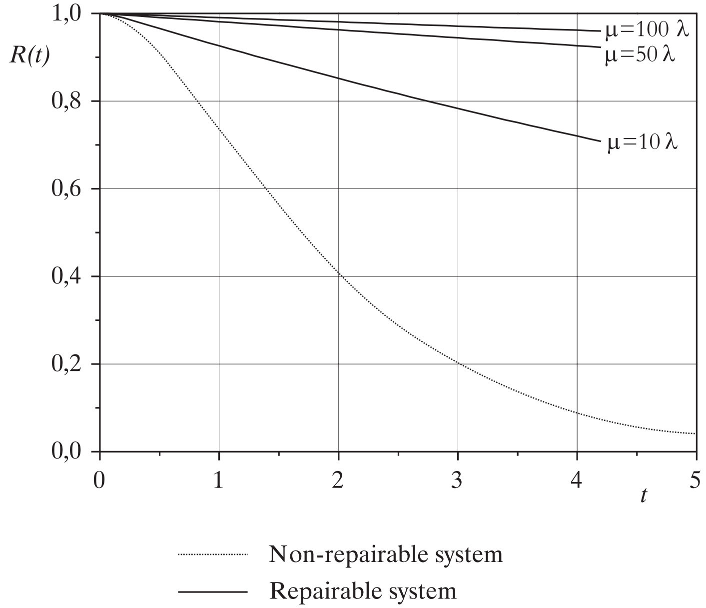

Figure 3. Reliability of the two-radar device standby redundant ship navigational radar system

pairable, the system reliability is incomparably higher than the reliability of the non-repairable system and very slowly decreases with time.

5. CONCLUSION

In general, the reliability of the system increases if the reliability of its components increases as well. An alternative to increasing the component reliability is providing redundancy in the system. Standby or backup redundant systems are a widely applied type of redundancy in fault-tolerant systems. In addition, the repair of the standby redundant system components significantly increases with the system reliability. Since the standby redundant systems are dependent on the component states, Markov methods are suitable for modelling and analyzing the reliability of these systems.

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Sažetak

PREDIKTIVNA ANALIZA POUZDANOSTI REDUNDANTNOG BRODSKOG NAVIGACIJSKOG RADARSKOG SUSTAVA

U radu je predstavljen opći pristup uspostavljanju Markovljeva modela pouzdanosti redundantnih sustava. Na temelju tog pristupa uspostavljen je Markovljev model pouzdanosti neobnovljivog i obnovljivog dvokomponentnog standby redundantnog sustava. Nadalje, obavljena je kvantitativna komparativna analiza pouzdanosti dvokomponentnog standby redundantnog brodskog navigacijskog radarskog sustava. Analiza pokazuje da obnavljanje sustava značajno utječe na porast njegove pouzdanosti.

Ključne riječi: *pouzdanost, standby redundantni sustav, Markovljev model, navigacijski radarski sustav*

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