ISSN 0005–1144
ATKAAF 45(1–2), 29–32 (2004)

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Current Density Dominant Mode on Spiral Patch Antennas

UDK 621.396.676
IFAC IA 5.7

Original scientific paper

An elegant and efficient way to calculate and analytically show the main features of spiral patch antennas is presented in this paper. The derivation of an approximate analytical expression for the dominant current density mode on the patch is initially presented, from which it can be easily calculated the patch radiation pattern. Its invariance with frequency scaling, which is the main feature of such log-periodic antennas, is in particular shown, together with a good congruency with experimental results. An extension to higher-order modes for a MoM analysis of the current density is then suggested, which may be useful to study with higher accuracy the other antenna characteristics, such as gain and input impedance at the feed.

Key words: spiral antenna, broadband planar antennas, sinuous antenna

1 INTRODUCTION

The interest in antenna applications with an ultra-wide broadband behavior is currently growing very much, especially for aircraft and vehicle-surveillance systems. Such radiators are usually designed as frequency independent antennas, whose shape is invariant to a scaling in the dimensions. This feature, in fact, implies a self-similar behavior when the frequency is varied that usually grants a bandwidth up to 900 %. For this reason, the Archimedes spiral antennas or other log-periodic antennas have been widely employed for such applications, since their geometry is self-coincident after a scaling in the dimensions.

The spiral shape has been employed for wire antennas [1] and for patch radiator applications [2], showing in both cases a very large bandwidth.

Their analysis and synthesis over a wide or ultra-wide frequency range, however, are not simple tasks, since simulations are still time-consuming. At the moment, in fact, the full-wave numerical analysis of spiral radiators is developed following standard approaches (Method of Moments (MoM), Finite Difference Time Domain (FDTD) method, Finite Element (FE) method, etc.), based on the discretization of the antenna elements in a proper number of sub-domains. The simulations, thus, involve a large number of unknowns and their computational time is consequently very large, due to the wide frequency range of operation and to the small details characterizing the spiral loops that resonate at the higher frequencies. Time saving can be achieved by using a non-uniform discretization or by properly combining the aforementioned numerical techniques with approximate high-frequency approaches, but the results are usually still not sufficient.

Attempts to approach the problem analytically have been performed along the years, but even the simplest case of a spiral wire presents serious difficulties that have discouraged a full-wave analytical solution.

Here in the following, we present an analytical solution for the lowest order dominant current mode on a spiral patch antenna, which may describe in a sufficiently accurate way the radiating properties of a spiral patch and can be used as a first synthesis tool. Moreover, an extension to derive a set of higher-order orthogonal current modes is proposed, allowing in principle an entire-domain MoM analysis of the structure, which would involve a very small number of unknowns, would take into account at the same moment the smallest details and the bigger loops of the patch and would cause a strong time saving in the computation of the antenna properties.

2 THEORETICAL APPROACH

The geometry under analysis is made up of a two-arm spiral patch antenna laying on the $z = 0$ plane, whose geometry is depicted in Figure 1 in a suitable polar reference system of coordinates r and ϕ. The arms are fed at the origin ($r = 0$) and they are supposed to radiate in free space (with permittivity ε_0, permeability μ_0, wave number $k_0 = \omega\sqrt{\varepsilon_0\mu_0}$ under a monochromatic $\varepsilon^{j\omega t}$ excitation at frequency $f = \omega/2\pi$).

Every arm is supposed to be perfectly conducting and its edges are expressed by the following formulas:

$$\varphi(r) = \tau(r) \pm \frac{\delta}{2} = \frac{1}{\beta}\ln\frac{r}{\alpha} \pm \frac{\delta}{2}. \tag{1}$$

The free parameters $\alpha > 0$, β and $\delta > 0$ univocally determine the patch shape and their meaning is readily understood noticing that: for $\beta \to \infty$, the spiral antenna tends to a bow-tie radiator [3] and decreasing β the patch curvature increases; α is simply a scaling factor and δ represents the angular distance of the two edges.

The log-periodic nature of formula (1) insures that the geometrical shape of the patch is self-invariant after a scaling in its dimensions, apart from a simple rotation. In fact, after the substitution $r' \to kr$, we get from (1):

$$\varphi(r') \to \varphi(r) + \frac{\ln k}{\beta}.$$

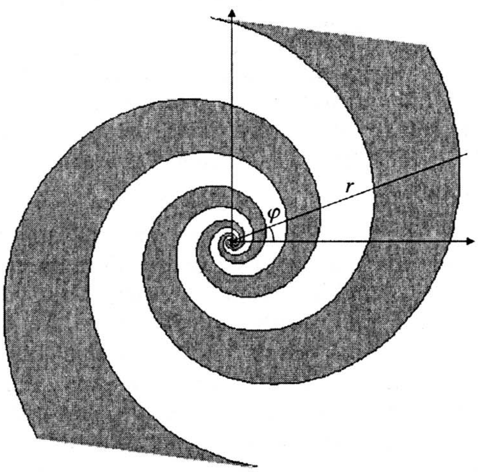

Fig. 1 Geometry of the problem. A two-arm spiral patch in a suitable polar reference system

As also experimentally shown [4–6], at a given frequency the current propagates with no decay before arriving in a region, which in the following we define *active*, where the main contribution to radiation happens. In this region, the main part of the current decays, since power for radiation is lost. As an aside, notice that variations in the patch shape before or after the active regions relative respectively to the higher and lower working frequencies of interest are therefore neglectable.

The dominant current density mode $\mathbf{J}_0(r,\varphi)$ on the patch is the one that mainly contributes to the antenna radiation pattern. It can be derived, in first approximation, by considering the patch as composed of an infinite number of spiral wires, each of them with expression:

$$\varphi(r) = \tau(r) + \phi = \frac{1}{\beta}\ln\frac{r}{\alpha} + \phi. \tag{2}$$

with $-\delta/2 < \phi < \delta/2$.

Each infinitely-thin wire carries an infinitesimal current $\mathrm{d}I = \mathbf{J}_0 \cdot \hat{\mathbf{t}}\mathrm{d}n$, where

$$\hat{\mathbf{t}} = (1+\beta^2)^{-1/2}\hat{\mathbf{r}} + (\beta^4+\beta^2)^{-1/2}\hat{\boldsymbol{\varphi}}$$

is the unit vector directed along the spiral wire and its projections on the polar unit vectors are constant with the coordinates. dn is the infinitesimal transverse section of the wire, orthogonal to it, and described by the equation:

$$\varphi = -\beta\ln\frac{r}{\alpha}\phi_n, \tag{3}$$

which is again a spiral, but revolving in the opposite sense.

The current in each wire can be studied as in [1] and with good approximation propagates along the line with a phase velocity close to the free-space wave number k_0. Moreover, since the current is radiating, its absolute value should decrease along the line when entering the active region, consistently with what found in [1]. The current expression can be given by:

$$\mathrm{d}I = J_{0\tau}\left[e^{-j\gamma l} + Re^{j\gamma l}\right]\mathrm{d}n \tag{4}$$

where R is the reflection coefficient due to the wire termination, which usually can be neglected considering the current attenuation,

$$l = \int_0^r \sqrt{1+\tau'(r)^2}\,\mathrm{d}r = \sqrt{1+\beta^{-2}}\,r$$

is the wire length from the origin to the point of radial coordinate r and $\gamma = k_0 - j\sigma$ is the propagation constant along the wire. The attenuating factor σ takes into account the radiating process and it has been empirically evaluated in the literature for some spiral wires [1] with an average process. Finally, $J_{0\tau}$ is the generic amplitude of the density current on the line, related to the excitation.

Due to its log-periodic geometry, it is possible to derive analytically where the active region should start in the spiral wire at a given frequency. It can be shown that the radius r at which the radiation is mainly concentrated satisfies the condition:

$$\frac{\ln\frac{r}{\alpha}}{\beta r}=k_0\left(\sqrt{1+\beta^2}-1\right). \qquad (5)$$

A more approximate expression, valid when the spiral can be considered quasi-periodic, i.e. for $\beta<1/\pi$, leads to an explicit expression for the active region:

$$\frac{1}{k_0(1+\beta)}<r<\frac{1}{k_0(1-\beta)}. \qquad (6)$$

The amplitude $J_{0\tau}$ of the current density on each wire can be evaluated imposing that:

- the total flux flowing out from every transverse patch section propagates and decays following (4);
- the current density is singular at the edges, with a limit behavior that should go to infinity as $x^{-1/2}$ along the lines described by equation (1).

Finally, we obtain the expression for the dominant mode current density on the patch:

$$\mathbf{J}_0(r,\varphi)=\hat{\mathbf{t}}\frac{A\sqrt{1+\frac{1}{\beta^2}}}{\sqrt{\sin^2\frac{\delta}{2}-\sin^2\left(\varphi-\frac{1}{\beta}\ln\frac{r}{\alpha}\right)}}\cdot\frac{e^{-jk_0\sqrt{1+\beta^{-2}}\,r}}{rl(r)} \qquad (7)$$

where

$$l(r)=\begin{cases}1 & r<\frac{1}{k_0(1+\beta)}\\ \left[k_0r(1+\beta)\right]^{-c/\sqrt{1+\beta^2}} & r>\frac{1}{k_0(1+\beta)}\end{cases}.$$

In calculating the expression for the attenuation in the active region we have imposed, following [7], that the attenuating factor is proportional to the curvature of the wire, which can be evaluated through its explicit expression:

$$\kappa(r)=\frac{1}{r\sqrt{\beta^2+1}}. \qquad (8)$$

The proportionality constant c can be evaluated experimentally or found imposing that at the end of the active region the current amplitude should not surpass one tenth of its amplitude at the beginning. The amplitude A simply depends on the excitation. It is worth to note that the previous formula collapses to the bow-tie current density when $\beta\to\infty$. In this case, in particular, the slow-wave number $\gamma\to k_0$ and the patch is all in the active region. No attenuation, however, takes place ($l(r)\to 1$) since the patch is not bent.

3 NUMERICAL RESULTS

The analytical expression of the dominant current density mode on a spiral patch given by (7) is a good description of the real current density flowing along the patch if we are interested in the radiation pattern.

In Figure 2 and 3, the radiation patterns of the two-arm spiral patch depicted in Figure 1 for two different working frequencies are shown. It is worth to note how the patterns are omni-directional and substantially invariant with the frequency, as expected from the shape-invariant properties of the spiral patch.

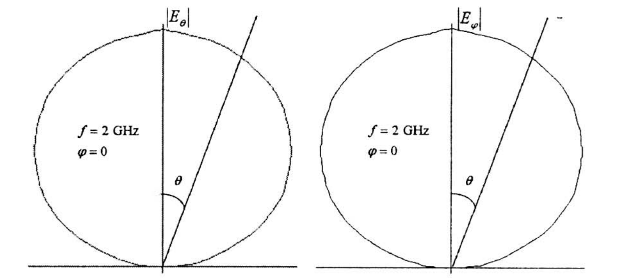

Fig. 2 Radiation patterns for the two-arm spiral patch on the cut $\varphi = 0$ at frequency $f = 2$ GHz

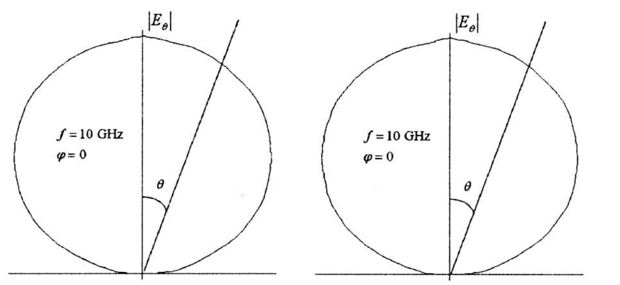

Fig. 3 Radiation patterns for a two-arm spiral patch ($\beta = 0.3$) on the cut $\varphi = 0$ at frequency $f = 10$ GHz

The density plots showing the current densities on the patches for the two different frequencies for which the patterns have been evaluated are shown in Figure 4 and 5. It's visible how the position of the active region moves backward when the frequency is increased, as expected and as widely verified experimentally [4–6].

Notice that the results here presented are in very good agreement with those available experimentally in the literature (e.g. [1–6]).

The expression for the dominant current density mode expressed by (7) allows several fast and accurate evaluations for a quick design of spiral patches. If, however, there is a need of a more accurate analysis of the antenna, for instance to evaluate the input impedance with more precision,

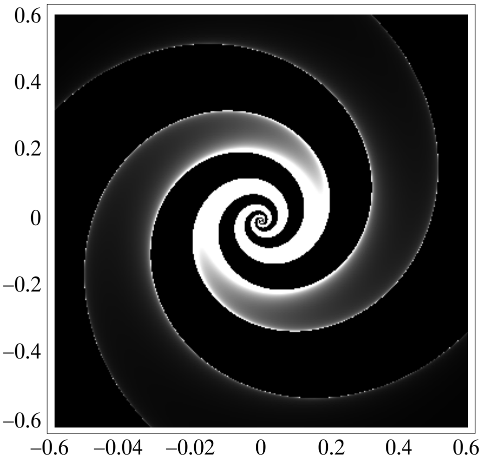

Fig. 4 Current density plots for a two-arm spiral patch ($\beta = 0.3$) on the cut $\varphi = 0$ at frequency $f = 2$ GHz

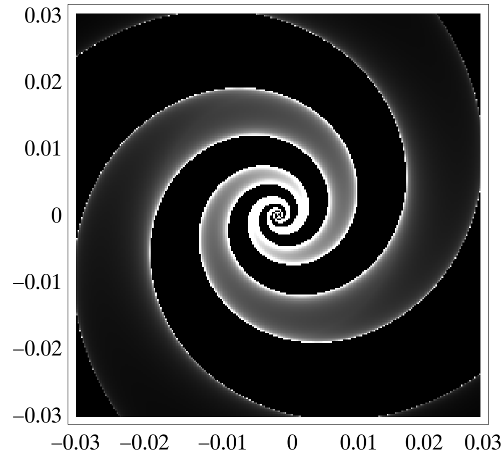

Fig. 5 Current density plots for a two-arm spiral patch ($\beta = 0.3$) on the cut $\varphi = 0$ at frequency $f = 10$ GHz

such expression can be easily extended adopting Chebyshev polynomials to evaluate a set of orthogonal entire-domain current density modes, from which a MoM analysis can be easily performed. This work is under progress and will be presented by the authors in a future paper.

4 CONCLUSIONS

An elegant and efficient analytical expression for the dominant current density mode on spiral patch antennas has been derived. Such derivation has been briefly presented and then some numerical results underlining the broadband features of spiral patches have been shown. The results are in good agreement with experimental results from previous papers in the literature. An extension to higher-order modes for a MoM analysis of the current density is also suggested, which may be useful to study with more accuracy the other antenna characteristics, such as gain and input impedance at the feed.

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Dominantni mod gustoće struje na spiralnoj patch anteni. U radu je prikazan privlačan i djelotvoran pristup proračunu i analitičkom prikazu spiralnih patch antena. Prvo je izveden aproksimativni analitički izraz za dominantni mod gustoće struje na spiralnom patchu. Pomoću izvedenog izraza moguće je jednostavno izračunati dijagrame zračenja patch antene. Proračunom je potvrđeno da se dijagrami zračenja ovakve log-periodičke antene ne mijenjaju s frekvencijom. Utvrđeno je dobro podudaranje proračuna dijagrama zračenja s rezultatima mjerenja. Predloženo je proširenje za više modove gustoće struje pogodne za analizu patch antene pomoću metode momenata. Primjenom metode momenata uz predloženo proširenje višim modovima može se s većom točnošću proučavati i druge osobine antene kao npr. dobitak i ulazna impedancija.

Ključne riječi: spiralna antena, širokopojasna planarna antena, menadrirajuća spiralna antena

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Received: 2004–10–23