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Fuzzy Model Predictive Control of Electrical Drives with Transmission Elasticity and Backlash

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Original scientific paper

Control strategy based on generalized predictive controller (GPC) is proposed for control of electrical drives with transmission elasticity and backlash. Takagi-Sugeno fuzzy model is used for identification of the two-mass mechanical system with elastic transmission and backlash with negligible friction. It is assumed that only measurement at the load side is available. Since GPC controller requires linear process model, Takagi-Sugeno fuzzy model is linearized by means of instantaneous linearization in each sample instant. This control strategy is then compared to the classical GPC based on linear ARX model by computer simulations and experimentally on a laboratory model of the electrical drive with transmission elasticity and backlash

Key words: electrical drive, transmission elasticity, backlash, model predictive control, Takagi-Sugeno fuzzy model

1 INTRODUCTION

The behavior of the speed control system for electrical drives can be significantly deteriorated when transmission elasticity and backlash are present if proper control algorithms are not applied. It is generally recognized that electrical drives with considerable transmission elasticity tend to produce poorly damped torsional vibrations in the transmitted torque [1] which are, in turn, manifested in poorly damped oscillations of the load speed. Backlash, on the other hand, manifests itself in the discontinuous nature of the transmitted torque [1, 2] thus changing the structure and parameters of the mechanical system.

The compensation of elasticity is usually carried out by introducing the additional feedback paths to the classical PI speed controller [3], or by using more complex controller structure such as state or polynomial (pole placement) controller [4]. If the backlash is also present, the controller output signal is augmented by the additional signal of the backlash compensation term [2]. Both control approaches assume detailed knowledge of the mechanical system in order to obtain desired control system behavior, and therefore are not always applicable.

All methods mentioned so far require either measurement at both sides of the electrical drive or estimation of non-measurable system states. In this paper a different approach to control of electrical drives with combined elasticity and backlash effects is presented. The electrical drive is modeled utilizing the »black box« input-output model based on Takagi-Sugeno fuzzy model [5] with measurement at the load side only. Generalized predictive controller (GPC) [6, 7] is used as load speed controller, and since it requires linear process model, Takagi-Sugeno fuzzy model is linearized by means of instantaneous linearization [8] in each sample instant.

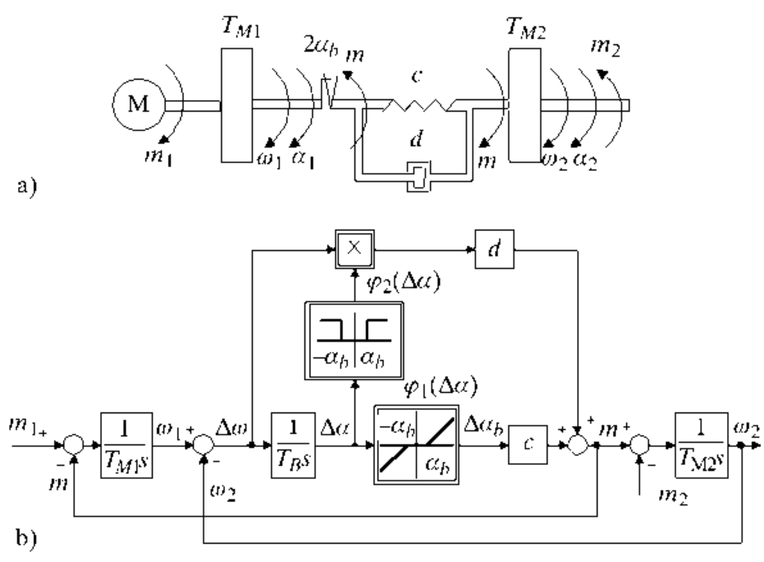

Fig. 1 Laboratory model of the electrical drive: a) structural scheme, b) block diagram

In addition, the behavior of the speed control system with fuzzy model based GPC controller is compared to the behavior of the speed control system with GPC controller based on linear ARX model [9].

2 SYSTEM DESCRIPTION

The mechanical part of the laboratory model of an electrical drive with emphasized transmission elasticity and backlash is shown in Figure 1 [1, 2].

The variables and constants shown in Figure 1 are:

- ω_1, ω_2 – motor and load speed, respectively,
- T_{M1}, T_{M2} – motor and load mechanical time constant, respectively,
- m, m_1, m_2 – coupling (transmitted) torque, motor torque and load torque, respectively,
- T_B – base time ($T_B = 1$ s),
- $\Delta\omega$ – speed difference,
- $\Delta\alpha$, $\Delta\alpha_b$ – displacement difference and torsional angle, respectively,
- $2\alpha_B$ – backlash angle,
- c – stiffness coefficient of the transmission,
- d – damping coefficient of the transmission (≈ 0).

All system quantities are rated. Therefore, the following state-space model of the electrical drive can be derived:

$$\begin{aligned} \dot{\omega}_1 &= \frac{1}{T_{M1}}(m_1 - m) \\ \dot{\omega}_2 &= \frac{1}{T_{M2}}(m - m_2 - m_{f2}) \\ \Delta\dot{\alpha} &= \frac{1}{T_B}\Delta\omega = \frac{1}{T_B}(\omega_1 - \omega_2) \\ m &= c\varphi_1(\Delta\alpha) + d\Delta\omega\varphi_2(\Delta\alpha) \end{aligned} \tag{1}$$

where $\varphi_1(\Delta\alpha)$ and $\varphi_2(\Delta\alpha)$ are nonlinear functions describing backlash.

The dynamics of the electrical part of the motor and frequency converter can be described by the first-order lag term:

$$G_{ei}(s) = \frac{m_1(s)}{m_{1R}(s)} = \frac{1}{1 + T_{ei}s}, \tag{2}$$

where:

- m_{1R} – speed controller output (motor torque reference),
- T_{ei} – equivalent time constant of the lag term.

3 FUZZY MODEL OF THE SYSTEM

The fuzzy implication R of the Takagi-Sugeno fuzzy model [5] is of the following format:

$$\begin{aligned} &\text{R: IF } (|\Delta\omega_2(k-1)| \subset A_1 \wedge |\Delta\omega_2(k-2)| \subset A_2) \\ &\text{THEN } (\hat{\omega}_2(k) = f(m_{1R}(k-1), \ldots, \omega_2(k-1), \ldots)), \end{aligned} \tag{3}$$

where:

- $\hat{\omega}_2(k)$ – estimated load speed at $t = kT$,
- A_1, A_2 – fuzzy sets in the premise part,
- $f(\ldots)$ – function in the consequence part of the fuzzy rule,
- $\wedge$ – fuzzy AND,

$$|\Delta\omega_2(k-i)| = |\omega_2(k-i) - \omega_2(k-i-1)|.$$

Premise part of the individual rule is defined by fuzzy sets with appropriate membership functions $\mu(\Delta\omega_2)$. The consequence part of the model is described by the following equation:

$$\begin{aligned} \hat{\omega}_2(k) &= f(m_{1R}, \omega_2) = b_1^* m_{1R}(k-1) + \cdots + \\ &+ b_m^* m_{1R}(k-m) + a_1^* \omega_2(k-1) + \cdots a_n^* \omega_2(k-n) \end{aligned} \tag{4}$$

which represents linear (ARX) model of the system valid if certain premise is fulfilled.

This yields the final expressions that describe the Takagi-Sugeno model for the load speed of the mechanical system with elasticity and backlash:

$$\hat{\omega}_2(k) = \frac{\sum_{i=1}^{n1}\sum_{j=1}^{n2} \mu_i(|\Delta\omega_2(k-1)|)\mu_j(|\Delta\omega_2(k-2)|) f_{ij}}{\sum_{i=1}^{n1}\sum_{j=1}^{n2} \mu_i(|\Delta\omega_2(k-1)|)\mu_j(|\Delta\omega_2(k-2)|)}, \tag{5}$$

where:

$$\begin{aligned} f_{ij} &= b_1^*(i,j) m_{1R}(k-1) + \cdots + b_m^*(i,j) m_{1R}(k-m) + \\ &+ a_1^*(i,j)\omega_2(k-1) + \cdots a_n^*(i,j)\omega_2(k-n), \end{aligned}$$

n_1, n_2 – numbers of membership functions in corresponding sets in the premise part.

It is obvious that ARX model is the special case of Takagi-Sugeno model when membership functions $\mu(\Delta\omega_2) = 1$ and corresponding parameters $b_k^*(i,j)$, $a_k^*(i,j)$ in the consequence part of the rules are equal for every rule.

In order to obtain coefficients in the consequence part of the model (with the assumption that the membership functions in the premise part are invariant) the following matrices should be created [1]:

$$\boldsymbol{\Phi}^T = \begin{bmatrix} \boldsymbol{\beta}(k)m_{1R}(k-1) & \boldsymbol{\beta}(k+1)m_R(k) & \cdots \\ \boldsymbol{\beta}(k)m_{1R}(k-2) & \boldsymbol{\beta}(k+1)m_R(k-1) & \cdots \\ \vdots & \vdots & \ddots \\ \boldsymbol{\beta}(k)\omega_2(k-1) & \boldsymbol{\beta}(k+1)\omega_2(k) & \cdots \\ \boldsymbol{\beta}(k)\omega_2(k-2) & \boldsymbol{\beta}(k+1)\omega_2(k-1) & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix}, \quad (6)$$

$$\boldsymbol{Y}^T = [\omega_2(k) \quad \omega_2(k+1) \quad \cdots],$$

where:

$$\boldsymbol{\beta}(k) = [\beta_{1,1}(k) \cdots \beta_{1,n2}(k) \cdots \beta_{n1,1}(k) \cdots \beta_{n1,n1}(k)]^T$$

and the individual elements are:

$$\beta_{i,j}(k) = \frac{\mu_i(|\Delta\omega_2(k-1)|)\mu_j(|\Delta\omega_2(k-2)|)}{\sum_{i=1}^{n1}\sum_{j=1}^{n2}\mu_i(|\Delta\omega_2(k-1)|)\mu_j(|\Delta\omega_2(k-2)|)}.$$

Parameters in the consequence part of the model are obtained by minimizing the performance criterion [9]:

$$J(\boldsymbol{\theta}) = \frac{1}{2}(\boldsymbol{Y} - \boldsymbol{\Phi\theta})^T(\boldsymbol{Y} - \boldsymbol{\Phi\theta}), \quad (7)$$

which yields

$$\boldsymbol{\theta} = (\boldsymbol{\Phi}^T\boldsymbol{\Phi})^{-1}\boldsymbol{\Phi}^T\boldsymbol{Y}, \quad (8)$$

where

$$\boldsymbol{\theta} = \left[b_1^*(1,1) \cdots b_1^*(n1,n2) \cdots b_m^*(1,1) \cdots a_1^*(1,1) \cdots a_n^*(n1,n2)\right]^T.$$

In order to improve the learning of the Takagi-Sugeno fuzzy model, regularization is introduced in the performance criterion (7):

$$J(\boldsymbol{\theta}) = \frac{1}{2}\left[(\boldsymbol{Y} - \boldsymbol{\Phi\theta})^T(\boldsymbol{Y} - \boldsymbol{\Phi\theta}) + \gamma\boldsymbol{\theta}^T\boldsymbol{\theta}\right], \quad (9)$$

where γ is the regularization coefficient.

This, in turn, modifies the expression (8) to:

$$\boldsymbol{\theta} = (\boldsymbol{\Phi}^T\boldsymbol{\Phi} + \gamma\boldsymbol{I})^{-1}\boldsymbol{\Phi}^T\boldsymbol{Y}. \quad (10)$$

Both the number and the shape of the membership functions define the overlap between individual linear models, thus enabling the generalization and smooth transition between neighboring linear sub-models. The membership functions in the premise part of expression (3) are defined as shown in Figure 2. Such a choice for membership functions is justified because when $\Delta\omega_2(k-1), \Delta\omega_2(k-2) \approx 0$, the load side acceleration in previous time instants was approximately zero. Since acceleration corresponds to the transmitted torque m (when no load torque is present) it is assumed that the system has to some extent entered the regime of backlash (see chapter 2). The extent of the backlash effect is taken into account by the two variables in the premise part of expression (2), resulting in four premises, and the degree of membership defined by the corresponding membership functions.

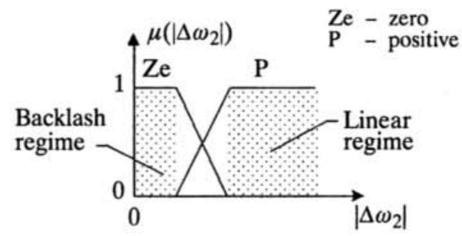

Fig. 2 Choice of the shape and number of membership functions

4 CONTROL SYSTEM STRUCTURE

The structure of the considered speed control loop is depicted by block diagram shown in Figure 3. The GPC can utilize either the proposed fuzzy process model (*sw* in position 1) or the ARX process model (*sw* in position 2).

Electrical drive with elastic transmission and backlash is described by (1) and (2). It is assumed that only load angle α_2 is measured. Digital measuring signal of the load speed ω_{2m} is reconstructed by time-differentiation of the measured angle (transfer function $G_{m\omega}(z)$).

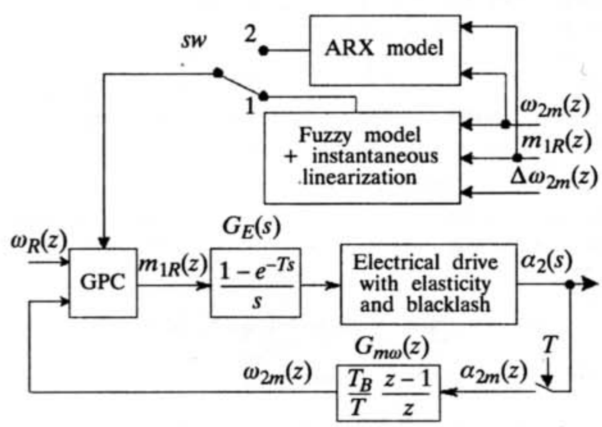

Fig. 3 Control system structure

A. GPC controller

Detailed description of the GPC algorithm can be found in [6, 7], and will be only briefly described here. The basic principle of the GPC algorithm is

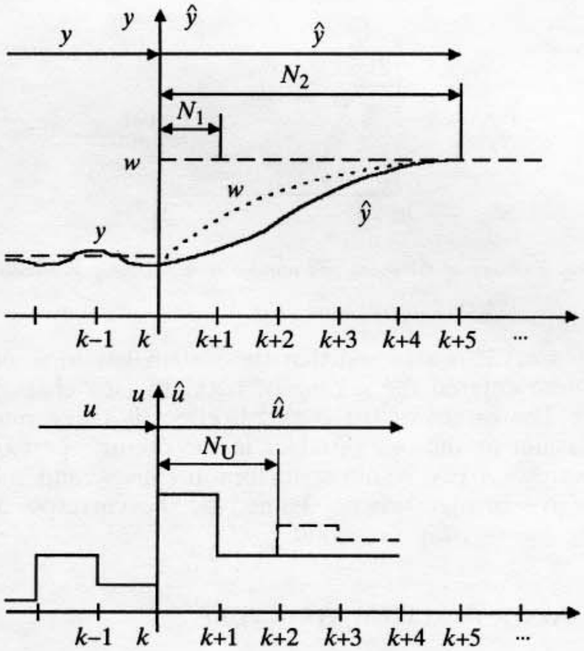

Fig. 4 Basic principle of the GPC

illustrated in Figure 4. The time scale is expressed in terms of sampling period with sample k denoting the present discrete time step.

The predictive controller output vector $\hat{\boldsymbol{u}}$ (deg $\hat{\boldsymbol{u}} = N_U$, where N_U is control horizon) calculated by the GPC is such that the predicted process output vector $\hat{\boldsymbol{y}}$ is as close as possible to the desired reference trajectory vector $\boldsymbol{w}$ within the costing horizon, which is defined by the minimum horizon N_1 and maximum horizon N_2 of the performance criterion. The process output is predicted from the process model of previous controller output vector $\boldsymbol{u}$, previous process output vector $\boldsymbol{y}$ and vector $\hat{\boldsymbol{u}}$.

For the electrical drive in question the following process variables correspond to the variables utilized in this general description of the GPC algorithm:

$\hat{\boldsymbol{y}} \equiv \hat{\boldsymbol{\omega}}_2$ – vector of future load speed,

$\hat{\boldsymbol{u}} \equiv \hat{\boldsymbol{m}}_{1R}$ – vector of future motor torque reference,

$\boldsymbol{w} \equiv \omega_R$ – speed reference value.

The GPC algorithm utilizes the ARX process model:

$$\hat{A}(q)y(k) = \hat{B}(q)u(k) \tag{11}$$

where $\hat{A}(q)$ and $\hat{B}(q)$ are polynomials of the order n and m, respectively. $\hat{A}(q)$ is presumed to be monic.

The optimal predictive controller output vector is determined by minimization of the performance criterion:

$$J(\boldsymbol{w},\hat{\boldsymbol{y}},\hat{\boldsymbol{u}}) = \sum_{j=N1}^{N2} \left[\hat{y}(k+j) - w(k+j)\right]^2 + \lambda \sum_{j=1}^{Nu} \left[\hat{u}(k+j-1)\right]^2, \tag{12}$$

where λ is controller output weighting. The first term in the performance criterion refers to the square variation of the predicted process output from the desired reference trajectory, while the second term is added in order to limit the controller output; greater λ yields less active controller output. By minimizing the performance criterion (12), optimal controller output vector $\hat{\boldsymbol{u}}_{\text{opt}}$ is obtained.

The first element of the calculated optimal controller output vector is directed to the controller output. The remaining vector elements are not utilized and the entire procedure is repeated at time $t = (k+1)T$ (principle of receding horizon). The first prediction horizon N_1 is usually chosen to be 1. The choice of second prediction horizon N_2 and the control horizon N_U is arbitrary to some extent: N_2 is usually chosen as to cover the most of the control system's transient, while N_U, which denotes the significance of the future controller outputs shouldn't be greater than $N_2/2$ [6, 7].

B. Instantaneous linearization

The Takagi-Sugeno fuzzy model (5) is of the NARX type (Nonlinear AutoRegressive model with eXogenous inputs) and therefore isn't suitable for the GPC algorithm, since it requires linear ARX model, described by (11). Thus, it is necessary to perform the linearization of the provided Takagi-Sugeno model. Linearization is performed in each sample instant by means of instantaneous linearization [8], i.e. partial differentiation of the nonlinear model. This results in the following expressions for ARX model parameters:

$$\begin{aligned} a_i &= -\left.\frac{\partial \hat{\omega}_2(k)}{\partial \omega_2(k-i)}\right|_k, \quad i = 1,\ldots,n \\ b_j &= \left.\frac{\partial \hat{\omega}_2(k)}{\partial m_{1R}(k-1)}\right|_k, \quad j = 1,\ldots,m \end{aligned} \tag{13}$$

where: a_i – parameters of the $\hat{A}(q)$ polynomial,
b_j – parameters of the $\hat{B}(q)$ polynomial.

5 SIMULATION AND EXPERIMENTAL RESULTS

All simulations and experiments are conducted for the elastic shaft with stiffness coefficient c = 900, mechanical time constants (inertia) on the motor and load side $T_{M1} = 0.147$ s, $T_{M2} = 0.241$ s and the backlash angle $2a_B = 2.43$. Simulations and experiments of proposed control systems were conducted utilizing the Matlab/Simulink software. The control computer was Pentium II based PC with appropriate acquisition cards.

A. Identification results

Identification of both ARX and the Takagi-Sugeno model for the mechanical system with elasticity and backlash was conducted *off-line* utilizing for the excitation signal m_{1R} the Band Limited White Noise with the bandwidth $\Omega_{BLWN} = 314\ \mathrm{s}^{-1}$ and the sampling time $T_S = 2.5$ ms in simulation and on experimental setup. For the ARX model, best results were obtained by utilizing the third order model: models of the greater order showed more significant correlation of the prediction error. According to this result, the Takagi-Sugeno fuzzy model was also chosen to be of the third order (third order ARX model in the consequence part), while the shape of membership functions was varied in order to achieve minimum of the performance criterion.

The important feature of the model that is to be used in the GPC control algorithm is the good long-term prediction, i.e. good prediction of future process outputs based only on the process initial states and previous model outputs. Therefore it is necessary to simulate the behavior of both Takagi--Sugeno model and ARX model as »Output error« (OE) models. The results are shown in Figure 5.

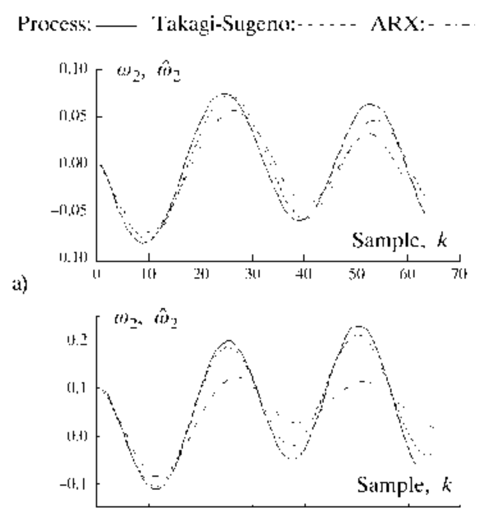

Fig. 5 Long-term prediction of the Takagi-Sugeno model and ARX model: simulation data a) and experimental data b)

As it is shown, long-term prediction is better for the Takagi-Sugeno model because of its ability to learn the non-linear behavior of the system for $\Delta\omega_2 \approx 0$.

B. Speed control

Comparative reference and load step responses for simulation and experiment are shown in Figures 6 to 9. The step reference value change is chosen to be descending to zero: $\omega_R(t) = 0.05\,[1 - S(t)]$. Load step is chosen to be $m_2(t) = 0.2\ S(t - 0.6s)$ in the load speed steady state $\omega_2 = 0$. Since in many cases the speed control loop is the inner loop of the cascade positioning control system with proportional (P) position controller the proposed choice of the speed reference value is justified because in such systems speed reference value tends to become zero.

In both cases (utilization of Takagi-Sugeno model or ARX), the GPC prediction horizons were chosen as follows: $N_1 = 1$, $N_2 = 10$, $N_U = 3$, while the controller output weighting factor was $\lambda = 0.002$.

The simulation results show the inability of the ARX model to describe the nonlinear dynamics to the satisfactory extent, thus causing the limit cycle

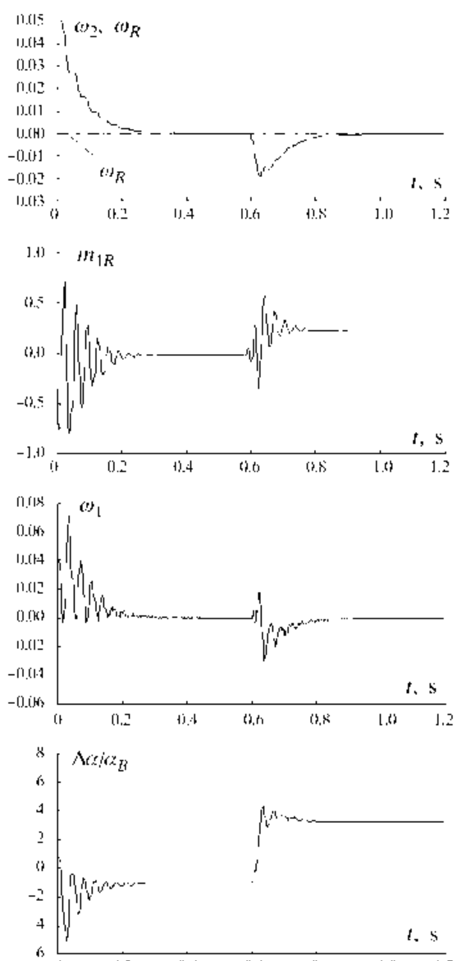

Fig. 6 Reference and load step responses of the speed control system with fuzzy model based GPC controller – simulation

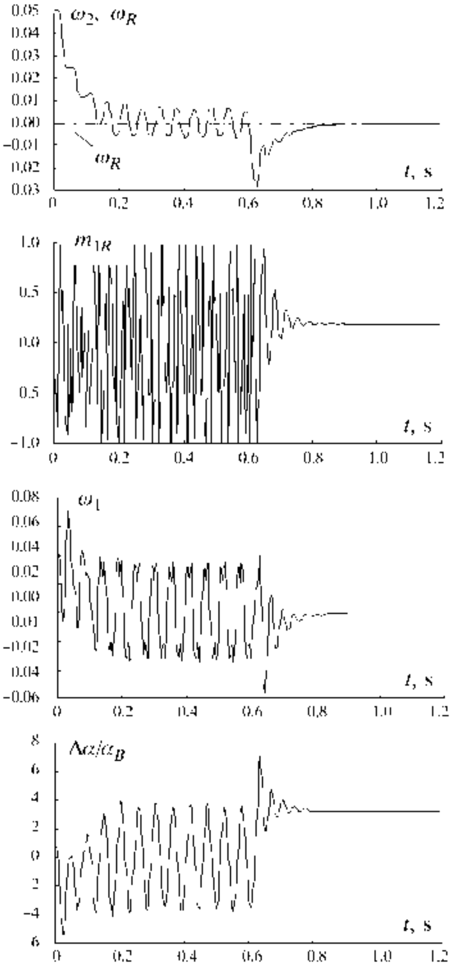

Fig. 7 Reference and load step responses of the speed control system with ARX model based GPC controller – simulation

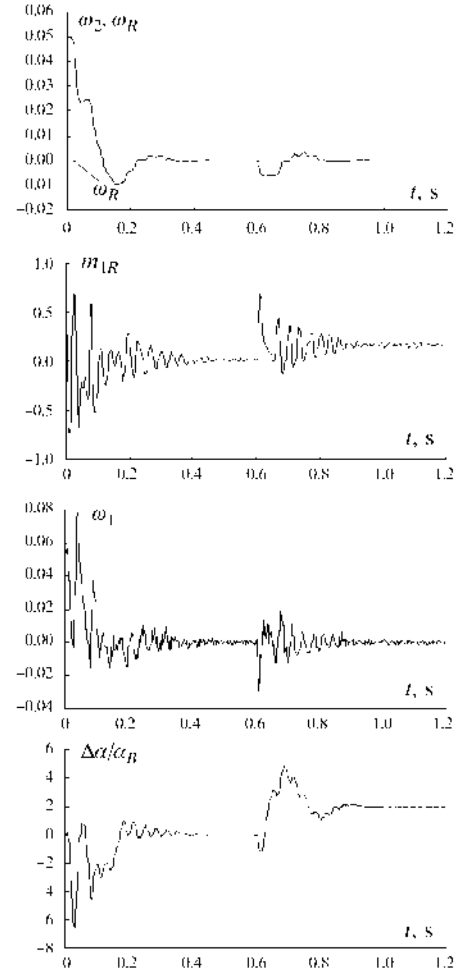

Fig. 8 Reference and load step responses of the speed control system with fuzzy model based GPC controller – experiment

oscillations of the speed control system with GPC controller. The introduction of the constant load torque stabilizes the behavior of the control system because it enforces the alignment on the loose end of the axis (Figure 1a). To the contrary, the Takagi--Sugeno fuzzy model based GPC algorithm stabilizes the mechanical system with elasticity and backlash with satisfactory control effort. The oscillations of the controller output are necessary in order to keep the mechanical system out of the regime of backlash by forcing the alignment on the loose end of the elastic axis.

Since the ARX model is incapable of modeling the dynamics of the mechanical system, speed control experiments were conducted for the utilization of the Takagi-Sugeno model only.

Both reference and load step responses are well damped, although there is an overshoot in the closed control system response, which is probably due to the influence of friction in the bearings of the experimental servo-drive. The proposed GPC based control system is also, to some extent, robust to the change of mechanical system parameters. This is illustrated in Figure 9 where comparative step responses are shown for nominal load inertia and the increase of load inertia by 100 % without changing the controller parameters.

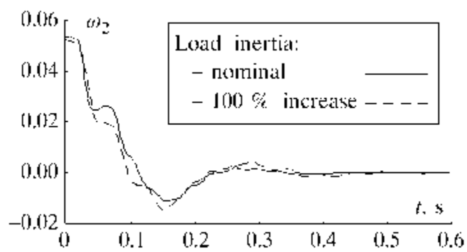

Fig. 9 Load speed reference step responses for different load inertia

6 CONCLUSION

The Takagi-Sugeno fuzzy model was proposed for modeling of electrical drives with transmission elasticity and backlash. It was compared to the linear ARX model through simulation and experimental verification. Both models were used as a basis for the design of the GPC load speed controller. The simulation and experimental results showed that the control system utilizing GPC based on Takagi-Sugeno fuzzy model had favorable behavior, contrary to the case when the ARX model was used.

Since the behavior of the closed speed control loop was affected by the influence of friction, our future research will deal with this problem in the framework of GPC control.

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Prediktivno upravljanje elektromotornim pogonom s elastičnošću i zračnošću u prijenosnom mehanizmu zasnovano na neizrazitom modelu procesa. Predložena je strategija upravljanja elektromotornim pogonom s elastičnošću i zračnosću u prijenosnom mehanizmu zasnovana na primjeni poopćenog prediktivnog regulatora (GPC). Neizraziti model prema Takagiju i Sugenu primijenjen je za identifikaciju dvomasenog mehaničkog sustava s elastičnim prijenosnim mehanizmom, zračnošću i zanemarivim trenjem. Pretpostavlja se da je dostupno samo mjerenje na strani tereta. Kako poopćeni prediktivni regulator (GPC) zahtijeva linearni model za proračun optimalnog upravljačkog signala, neizraziti model prema Takagiju i Sugenu linearizira se primjenom trenutačne linearizacije u svakom koraku uzorkovanja. Ova strategija upravljanja uspoređena je potom s klasičnim poopćenim prediktivnim regulatorom zasnovanim na linearnom ARX modelu, simulacijom i eksperimentalno na laboratorijskom modelu elektromotornog pogona s elastičnošću i zračnosću u prijenosnom mehanizmu.

Ključne riječi: elektromotorni pogon, elestičnost prijenosnog mehanizma, zračnost, modelsko prediktivno upravljanje, Takagi-Sugenov neizraziti model

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