# Input Displacement Factor Correction for Three-Phase Three-Level AC to DC PWM-based Boost Rectifier 

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## Original scientific paper


#### Abstract

Pulse width modulation (PWM) strategy for a matrix structured three-phase three-level AC to DC boost rectifier is developed. Such approach has been used on purpose to control the input displacement power factor close to unity. The connection between matrix switching function and PWM requirement very well describes all restrictions that occur in the modulation algorithm. This modulation algorithm enables the input displacement factor correction without an input current sensor. The only necessary control variable is the measured displacement angle between input voltage and input current.


Key words: matrix converter, ac-dc converter, switching function, pulse width modulation, input displacement factor

## 1. INTRODUCTION

A switching matrix analysis in power converter circuits was first investigated in [1] and then more recently, using the generalized high frequency switching strategy, in [2]. Such analyzes offer a pleasant method for understanding the power conversion between sources and sinks through the switching converter circuit. The AC-AC conversion function has been discussed in [2] and [3], where the emphases were done in the modulation strategies and improving the range of three-phase output voltage. In [4] the authors introduced a space vector modulation strategy in the AC to AC converter. In [5] and [6] it is shown that for the hysteresis control of the in-
put phase current and resistive fundamental mains behavior the central point potential can be controlled by an offset of the phase current reference values. This paper deals with the development of the PWM algorithm for the three-phase three-level AC to DC converter based on the switching matrix approach.

## 2. DESCRIPTION OF CONVERTER BY SWITCHING FUNCTIONS

In approaching a design of the switching power converter shown in Figure 1(a) for any application, two related sets of parameters are of paramount


Fig. 1 (a) General switching matrix with current inputs. (b) The rectifier circuit with $M=N=3$
interest. The internal currents and voltages of the converter switching loops will determine the selection of active switching device to be used and also auxiliary passive components needed to enable the devices to operate properly. The switching algorithm used to connect the $M$ input lines with the $N$ output lines is based on Figure 1(a). In order to obtain a precise, quantitative means for describing the converter internal structure and external terminal properties, the switching pattern is needed. This can be conveniently done by defining a function, so called a Switching function which has been defined in [1](1):

$$
H_{i j}=\left\{\begin{array}{lll}
1 & \text { for } \quad S_{i j}=\mathrm{ON}  \tag{1}\\
0 & \text { for } & S_{i j}=\mathrm{OFF}
\end{array} .\right.
$$

The circuit shown in Figure 1(a) will not operate when the voltage sources are short-connected and when the currents have no path to flow. Because of this the switching function must satisfy next condition:

$$
\begin{equation*}
\sum_{j=1}^{N} H_{i j}=1, \quad i=1 \ldots M \tag{2}
\end{equation*}
$$

The switch in the switching matrix connects the $I_{i n i}$ current sources from the set of $M$ input variables with $V_{i n j}$ voltage sources from the set of the $N$ output variables. Whenever the switch is OFF, the blocking voltage appears across it. The voltage impressed on the input line i can be written as:

$$
\begin{equation*}
V_{i n_{i}}=\sum_{j=1}^{N} H_{i j} V_{o u t_{j}}=1, \quad i=1 \ldots M \tag{3}
\end{equation*}
$$

The voltage across the switches can be obtained as a difference between the voltage on $i$ and line $j$ like:

$$
\begin{equation*}
V_{s w_{i j}}=V_{i n_{i}}-V_{o u t_{j}} . \tag{4}
\end{equation*}
$$

Substitution of (3) into (4) gives:

The next step to be defined is the current through the switches. In general the Kirchoff's current low valid for output matrix converter current could be expressed as:

$$
\begin{equation*}
I_{\text {out }_{N}}=-\sum_{j=1}^{N-1} I_{\text {out }_{j}} . \tag{6}
\end{equation*}
$$

The input current could be represented as a linear combination of output currents as follows:

$$
\begin{equation*}
I_{i n_{i j}}=\sum_{j=1}^{N-1} H_{i j} I_{o u t_{j}}-H_{i N} I_{\text {out }_{N}}, \quad i=1 \ldots M \tag{7}
\end{equation*}
$$

Equation (6) should be substituted into (7) and finally:

$$
\begin{equation*}
I_{i n_{i j}}=\sum_{j=1}^{N-1} H_{i j} I_{\text {out }_{j}}-H_{i N} \sum_{j=1}^{N-1} I_{\text {out }_{j}}, \quad i=1 \ldots M \tag{8}
\end{equation*}
$$

The main aim followed in this section was to find the expression for estimating the switches currents. The current through switch can be expression by:

$$
\begin{equation*}
I_{s w_{i j}}=H_{i j} I_{i n_{i}}, \quad i=1 \ldots M, \quad j=1 \ldots N \tag{9}
\end{equation*}
$$

It is evident that the current through the switches can be estimated only when the output current is known. From (5) and (9) the internal structure of switching matrix converter was defined.

### 2.1. Power converter circuits analyzed by switching

## Function

In previous section the general approach has been introduced and presented. The circuit shown in Figure 1(b) is appropriate for further analysis. Anyway, the next presumptions will make the analysis easier. The number of input and output lines are equal:

$$
\begin{equation*}
M=N=3 \tag{10}
\end{equation*}
$$

The quantitative assessment of switching converter performance and characteristics for AC to DC converter have been analyzed here are:

- The current sources (voltage sources with inductors) are presented on the input lines.
- The voltage sources/sinks (capacitors) are presented on the output lines.
- The power flow has to be bi-directional.

For output voltage definition it is necessary to presume the next requirements:

- To establish the »output voltage« ( $V_{i n i}$, see Figure $1(\mathrm{~b})$ ) the three output voltage sources $\left(V_{\text {out }}\right)$ are available.
- The blocking voltage on semiconductor switches has to be half of the entire output voltage.


### 2.1.1. Voltage properties

For three-level converter output the voltages will appear on the output capacitors. Because of DC requirements the output voltages could be defined as follows:

$$
\begin{align*}
& V_{\text {out }_{1}}=+V_{o 1} \\
& V_{o u t_{2}}=-V_{o 2}  \tag{11}\\
& V_{o u t_{3}}=0 .
\end{align*}
$$

After defining the voltage external terminal properties, the internal properties will be further considered. The base of this analysis is given in (5) and (9). Let us define the voltages impressed on switches on the first vertical line. First, consider the switch $S_{11}$. After expanding (9) the blocking voltage on the switch is:

$$
\begin{equation*}
V_{\text {sw }_{11}}=\left(H_{11} V_{\text {out }_{1}}+H_{12} V_{\text {out }_{2}}+H_{13} V_{\text {out }_{3}}\right)-V_{\text {out }_{1}} . \tag{12}
\end{equation*}
$$

The blocking voltage on the switch $S_{11}$ has finally two values because of (2):

- When $H_{12}=1$ and $H_{13}=0$, the blocking voltage will be:

$$
\begin{equation*}
V_{s w_{11}}=V_{o u t_{2}}-V_{o u t_{1}}=-V_{o 1}-V_{o 2} . \tag{13}
\end{equation*}
$$

- When $H_{12}=0$ and $H_{13}=1$, the blocking voltage will be:

$$
\begin{equation*}
V_{s w_{11}}=V_{\text {out }_{3}}-V_{\text {out }_{1}}=-V_{\text {o1 }} . \tag{14}
\end{equation*}
$$

From (13) and (14) the next conclusion could be done: When the switch $S_{11}$ is OFF it must block the negative voltage in both case.

For the switch $S_{12}$ the voltage property can be evaluated by:

$$
\begin{equation*}
V_{\text {sw }_{12}}=\left(H_{11} V_{\text {out }_{1}}+H_{13} V_{\text {out }_{3}}\right)-V_{\text {out }_{2}} . \tag{15}
\end{equation*}
$$

The blocking voltage on the switch $S_{12}$ has also two values because of (2):

- When $H_{11}=1$ and $H_{13}=0$, the blocking voltage will be:

$$
\begin{equation*}
V_{s w_{12}}=V_{\text {out }_{1}}-V_{\text {out }_{2}}=V_{o 1}-\left(-V_{o 2}\right) . \tag{16}
\end{equation*}
$$

- When $H_{11}=0$ and $H_{13}=1$, the blocking voltage will be:

$$
\begin{equation*}
V_{s w_{12}}=V_{\text {out }_{3}}-V_{\text {out }_{2}}=-\left(-V_{o 2}\right) . \tag{17}
\end{equation*}
$$

From (16) and (17) the next conclusion could be done: When the switch $S_{12}$ is OFF it must block the positive voltage in both case.

For the switch $S_{13}$ the voltage property can be evaluated by:

$$
\begin{equation*}
V_{\text {sw }}^{13} \text { }=\left(H_{11} V_{\text {out }_{1}}+H_{12} V_{\text {out }_{2}}\right)-V_{\text {out }_{3}} . \tag{18}
\end{equation*}
$$

The blocking voltage on the switch $S_{13}$ has also two values because of (2):

- When $H_{11}=1$ and $H_{12}=0$, the blocking voltage will be:

$$
\begin{equation*}
V_{s w_{13}}=V_{o u t_{1}}-V_{\text {out }}^{3} \text { }=V_{o 1} . \tag{19}
\end{equation*}
$$

- When $H_{11}=0$ and $H_{12}=1$, the blocking voltage will be:

$$
\begin{equation*}
V_{s w_{13}}=V_{o u t_{2}}-V_{o u t_{3}}=-V_{o 2} . \tag{20}
\end{equation*}
$$

From (19) and (20) the next conclusion could be done: When the switch $S_{13}$ is OFF it must block the positive and negative voltage in both case.

### 2.1.2. Current properties

The current can be evaluated from (8) and (9). For the switch in the first vertical line (8) could be expanded as follows:

Because of (2) it follows:

$$
\begin{equation*}
I_{i n_{1}}=H_{11} I_{o u t_{1}}=I_{s w_{11}} . \tag{22}
\end{equation*}
$$

or

$$
\begin{equation*}
I_{i n_{1}}=H_{12} I_{o u t_{2}}=I_{s w_{12}} . \tag{23}
\end{equation*}
$$

or

$$
\begin{equation*}
I_{i n_{1}}=H_{13}\left(I_{o u t_{1}}+I_{o u t_{2}}\right)=I_{s w_{13}} . \tag{24}
\end{equation*}
$$

From (22) appears the current property of switch $S_{11}$. According to the DC converter output the output current into voltage source $V_{\text {out }}$ must have positive direction. For the voltage and current properties of the switch $S_{11}$ can be supposed: When the switch $S_{11}$ is OFF it should block the negative voltage, and when it is ON it should conduct the positive current. Such properties are normal for the diode.

From (23) appears the current property of switch $S_{12}$. According to the DC converter output the output current into voltage source $V_{\text {out } 2}$ must have negative direction. For the voltage and current properties of the switch $S_{12}$ can be supposed: When the switch $S_{12}$ is OFF it should block the positive voltage, and when it is ON it should conduct the negative current. Such properties are normal for the reverse connected diode.

To define the properties of the switch $S_{13}$ the next consideration from (24) should be done: Because of condition (2) the switching function $H_{11}=$ $=H_{12}=0$, but the current must have the way to flow. Because of this the switching function in the second vertical line must satisfy next conditions: $H_{22}=1$ and $H_{21}=0$ or $H_{22}=0$ and $H_{21}=1$ and similar for the third vertical line. From this description the next conclusion could be done: When $H_{22}=1$ the switch $S_{13}$ conducts the positive current which has the same direction as current $I_{o u t}$. When the switching function $H_{21}=1$ the switch $S_{13}$ conducts current which has the same direction as current $I_{o u t}$. The current trough switch $S_{13}$ could has both directions. According to this for the voltage and
current properties of the switch $S_{13}$ could be done: When the switch $S_{13}$ is OFF it should block the positive and negative voltage and when it is ON it should conduct the positive and negative current. Such properties are normal for a bi-directional switch. The results of above analysis are summarized in Table 1, where voltage and current characteristics of all switches in first vertical line are defined.

Table 1. The switching device current and voltage properties

|  | $S_{11}$ | $S_{12}$ | $S_{13}$ |
| :--- | :--- | :--- | :--- |
| Current is: | Positive | Negative | Both |
| Blocking voltage is: | Negative | Positive | Both |
| Switching device is: | Diode | Diode | Bi-dir. switch |

Bi-directional switches $S_{13}, S_{23}$ and $S_{33}$ can be constructed in five different combinations of diodes and transistors. After it's substitution in the scheme in Figure 1(a) the five different structures, of the three-phase three-level boost rectifier can be established. The varieties of AC to DC converter types have been depicted in Figure 2(a), (b), (c), (d) and (e) respectively.

## 3. SWITCHING PATTERN

For establishing the switching pattern the switching matrix approach is very powerful tool. The power supply voltage and input currents are:

$$
\begin{align*}
& V_{A}=\hat{V} \cos (\omega t) \\
& V_{B}=\hat{V} \cos \left(\omega t-\frac{2 \pi}{3}\right)  \tag{25}\\
& V_{C}=\hat{V} \cos \left(\omega t-\frac{4 \pi}{3}\right) \\
& I_{i n 1}=\hat{I} \cos (\omega t+\varphi) \\
& I_{i n 2}=\hat{I} \cos \left(\omega t-\frac{2 \pi}{3}+\varphi\right)  \tag{26}\\
& I_{i n 3}=\hat{I} \cos \left(\omega t-\frac{4 \pi}{3}+\varphi\right) .
\end{align*}
$$

From Figure 1(b) it can be seen that the switching matrix input voltages $V_{i n i}$ can be formed as a combination of output voltages $V_{\text {out } j}$, as follows:

$$
\begin{align*}
& V_{\text {in }_{1}}=H_{11} V_{\text {out }_{1}}+H_{12} V_{\text {out }_{2}}+H_{13} V_{\text {out }_{3}} \\
& V_{\text {in }_{2}}=H_{21} V_{\text {out }_{1}}+H_{22} V_{\text {out }_{2}}+H_{33} V_{\text {out }_{3}}  \tag{27}\\
& V_{\text {in }_{3}}=H_{31} V_{\text {out }_{1}}+H_{32} V_{\text {out }_{2}}+H_{33} V_{\text {out }_{3}} .
\end{align*}
$$



Fig. 2 Six circuit topologies

On the other hand the voltages $V_{i n_{i}}$ can be constructed when the output voltages are connected by the switching matrix with the terminals $u, v$ and $w$ (Figure 2(f)) during the time intervals $t_{1}, t_{-2}$ and $t_{3}$. The input voltages can be defined as:

$$
\begin{align*}
& V_{\text {in }_{1}}=\frac{1}{T_{S}}\left(t_{1} V_{o u t_{1}}+t_{2} V_{o u t_{2}}+t_{3} V_{o u t_{3}}\right) \\
& V_{\text {in }_{2}}=\frac{1}{T_{S}}\left(t_{1} V_{\text {out }}^{2}+t_{2} V_{\text {out }_{3}}+t_{3} V_{\text {out }}^{1} \text { }\right)  \tag{28}\\
& V_{\text {in }_{3}}=\frac{1}{T_{S}}\left(t_{1} V_{\text {out }_{3}}+t_{2} V_{\text {out }_{1}}+t_{3} V_{\text {out }_{2}}\right)
\end{align*}
$$

where $V_{\text {out } 1}, V_{\text {out } 2}$ and $V_{\text {out } 3}$ are the output voltages defined in (11) and $T_{S}=t_{1}+t_{2}+t_{3}$. In (28) it is assumed that the voltage on terminals $u, v$ and $w$ is, in effect, the average value of the three switching events weighted by the time of »dwell« on each of the voltages connected to the output of the rectifier. Because of the input current wave-shape requirement the inductance voltage drop must have the sinusoidal wave-shape as well. Based on this the input voltages first harmonics $V_{i n i}$ must have sinusoidal wave-shape and can be expressed by:

$$
\begin{align*}
& V_{i n_{1}}=\hat{V}_{i n} \cos (\omega t+\beta) \\
& V_{i n_{2}}=\hat{V}_{i n} \cos \left(\omega t-\frac{2 \pi}{3}+\beta\right)  \tag{29}\\
& V_{i n_{3}}=\hat{V}_{i n} \cos \left(\omega t-\frac{2 \pi}{3}+\beta\right)
\end{align*}
$$

where $\beta$ is the phase shift between $V_{i n 1}$ and $V_{A}$. From (26) and (28) it follows:

$$
\begin{align*}
& \frac{t_{1}}{T_{S}}=H_{11}=H_{22}=H_{33}  \tag{30}\\
& \frac{t_{2}}{T_{S}}=H_{12}=H_{23}=H_{31}  \tag{31}\\
& \frac{t_{3}}{T_{S}}=H_{13}=H_{21}=H_{32} \tag{32}
\end{align*}
$$

With appropriate $\beta$ the current through the inductance $L_{A}, L_{B}$ and $L_{C}$ will have no phase shift versus $V_{A, B, C}$. Therefore the switching pattern algorithm must be sensitive on $\beta$ in order to get the rectifier unity power factor operation. In (30) the switching function $H_{33}$ represents the state of the bi-directional switch $S_{33}$ in the third vertical line of the rectifier (Figure $1(\mathrm{~b})$ ). This switch is controllable. Other two switches $S_{11}$ and $S_{22}$ are diodes and they are turned on or off depending on the direction of the current through the switch $S_{33}$. The same conclusion is follows from (8) and (9). There are two other controllable switches $S_{23}$ and $S_{13}$ as well. From (2) follow the conditions when the diodes switching function $H_{I 1}$ and $H_{I 2}$ will be unity. They can be expressed:

$$
\begin{align*}
& H_{11}+H_{12}=1-H_{13} \\
& H_{21}+H_{22}=1-H_{23}  \tag{33}\\
& H_{31}+H_{32}=1-H_{33} .
\end{align*}
$$

From (33) follows that the diodes switches $S_{11}$ and $S_{12}$ in the first vertical line could be active when the bi-directional switch $S_{13}$ is non-active $\left(H_{13}=0\right)$. In this case the new active switch in this vertical line will be defined by the input current direction as:

$$
\begin{align*}
& H_{13}=0 \\
& H_{11}=1 \Rightarrow I_{i n 1}>0  \tag{34}\\
& H_{12}=1 \Rightarrow I_{i n 1}<0
\end{align*}
$$

Let us define the function whose will describe the sign of the input current:

$$
H_{i n_{k}}=\left\{\begin{array}{l}
1 \text { for } I_{i n_{k}} \geq 0  \tag{35}\\
0 \text { for } I_{i n_{k}}<0
\end{array}\right.
$$

where $k=1 \ldots 3$. From (33), (34) and (35) it follows:

$$
\begin{align*}
& H_{i 1}=\left(1-H_{i 3}\right) \cdot H_{i n_{i}} \\
& H_{i 1}=\left(1-H_{i 3}\right) \cdot\left(1-H_{i n_{i}}\right) \tag{36}
\end{align*}
$$

where $i=1 \ldots 3$. Substituting (36) into (27) yields:

$$
\begin{align*}
V_{i n_{1}} & =H_{i n 1}\left(1-H_{13}\right) \cdot V_{o 1}+ \\
& +\left(1-H_{i n 1}\right)\left(1-H_{13}\right) \cdot V_{o 2}+H_{13} V_{o 3} \\
V_{i n_{2}} & =H_{i n 2}\left(1-H_{23}\right) \cdot V_{o 1}+ \\
& +\left(1-H_{i n 2}\right)\left(1-H_{23}\right) \cdot V_{o 2}+H_{23} V_{o 3}  \tag{37}\\
V_{i n_{3}} & =H_{i n 3}\left(1-H_{33}\right) \cdot V_{o 1}+ \\
& +\left(1-H_{i n 3}\right)\left(1-H_{33}\right) \cdot V_{o 2}+H_{33} V_{o 3}
\end{align*}
$$

and comparing with (37) with (27) for $t_{1} / T_{s}, t_{2} / T_{s}$ and $t_{3} / T_{s}$ yields:

$$
\begin{align*}
& \frac{t_{1}}{T_{s}}=H_{33}=1-\frac{q \cos \left(\omega_{i} t+\frac{2 \pi}{3}+\beta\right)}{2 H_{i n 3}-1}  \tag{38}\\
& \frac{t_{2}}{T_{s}}=H_{23}=1-\frac{q \cos \left(\omega_{i} t-\frac{2 \pi}{3}+\beta\right)}{2 H_{i n 3}-1}  \tag{39}\\
& \frac{t_{3}}{T_{s}}=H_{13}=1-\frac{q \cos \left(\omega_{i} t+\beta\right)}{2 H_{i n 3}-1} \tag{40}
\end{align*}
$$

where $q=\hat{V}_{\text {in }} / \hat{V}_{o 1}$. From (29), it is evident that the switching function sum in vertical lines is always a unity (Figure 1(a)).

## 4. VOLTAGE VECTORS

To define the available voltage vectors the scheme in Figure 2(f) is appropriate to define the states of the switches on the next way. For one of the vertical line (vertical legs of the converter) states of the switches are:

$$
\begin{align*}
& 1=\left\{\begin{array}{l}
Q_{A 1}=0 \\
Q_{A 2}=1 \text { for } I_{i n 1}>0
\end{array}\right.  \tag{41}\\
& -1=\left\{\begin{array}{l}
Q_{B 1}=1 \\
Q_{B 2}=0 \\
\text { for } I_{i n 1}<0
\end{array}\right.  \tag{42}\\
& 0=\left\{\begin{array}{l}
Q_{C 1}=0 \\
Q_{C 2}=0 .
\end{array}\right. \tag{43}
\end{align*}
$$



Fig. 3 The voltage vectors hexagon

Table 2. The voltage vectors

| $V_{i}$ | $Q_{A 1}, Q_{A 2}$ | $Q_{B 1}, Q_{B 2}$ | $Q_{C 1}, Q_{C 2}$ | $U_{u v}$ | $U_{v w}$ | $U_{w u}$ | $U_{u N}$ | $U_{v N}$ | $U_{w N}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $V_{0}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $V_{1}$ | 0 | 0 | 1 | 0 | $-V_{D} / 2$ | $V_{D} / 2$ | 0 | 0 | $V_{D} / 2$ |
| $V_{2}$ | 0 | 0 | -1 | 0 | $V_{D} / 2$ | $-V_{D} / 2$ | 0 | 0 | $-V_{D} / 2$ |
| $V_{3}$ | 0 | 1 | 0 | $-V_{D} / 2$ | $V_{D} / 2$ | 0 | 0 | $V_{D} / 2$ | 0 |
| $V_{4}$ | 0 | 1 | 1 | $-V_{D} / 2$ | 0 | $V_{D} / 2$ | 0 | $V_{D} / 2$ | $V_{D} / 2$ |
| $V_{5}$ | 0 | 1 | -1 | $-V_{D} / 2$ | $V_{D}$ | $-V_{D} / 2$ | 0 | $V_{D} / 2$ | $-V_{D} / 2$ |
| $V_{6}$ | 0 | -1 | 0 | $V_{D} / 2$ | $-V_{D} / 2$ | 0 | 0 | $-V_{D} / 2$ | 0 |
| $V_{7}$ | 0 | -1 | 1 | $V_{D} / 2$ | $V_{D}$ | $V_{D} / 2$ | 0 | $-V_{D} / 2$ | $V_{D} / 2$ |
| $V_{8}$ | 0 | -1 | -1 | $V_{D} / 2$ | 0 | $-V_{D} / 2$ | 0 | $-V_{D} / 2$ | $-V_{D} / 2$ |
| $V_{9}$ | 1 | 0 | 0 | $V_{D} / 2$ | 0 | $-V_{D} / 2$ | $V_{D} / 2$ | 0 | 0 |
| $V_{10}$ | 1 | 0 | 1 | $V_{D} / 2$ | $-V_{D} / 2$ | 0 | $V_{D} / 2$ | 0 | $V_{D} / 2$ |
| $V_{11}$ | 1 | 0 | -1 | $V_{D} / 2$ | $V_{D} / 2$ | $-V_{D}$ | $V_{D} / 2$ | 0 | $-V_{D} / 2$ |
| $V_{12}$ | 1 | 1 | 0 | 0 | $V_{D} / 2$ | $-V_{D} / 2$ | $V_{D} / 2$ | $V_{D} / 2$ | 0 |
| $V_{13}$ | 1 | 1 | 1 | 0 | 0 | 0 | $V_{D} / 2$ | $V_{D} / 2$ | $V_{D} / 2$ |
| $V_{14}$ | 1 | 1 | -1 | 0 | $V_{D}$ | $-V_{D}$ | $V_{D} / 2$ | $V_{D} / 2$ | $-V_{D} / 2$ |
| $V_{15}$ | 1 | -1 | 0 | $V_{D}$ | $-V_{D} / 2$ | $-V_{D} / 2$ | $V_{D} / 2$ | $-V_{D} / 2$ | 0 |
| $V_{16}$ | 1 | -1 | 1 | $V_{D}$ | $-V_{D}$ | 0 | $V_{D} / 2$ | $-V_{D} / 2$ | $V_{D} / 2$ |
| $V_{17}$ | 1 | -1 | -1 | $V_{D}$ | 0 | $-V_{D}$ | $V_{D} / 2$ | $-V_{D} / 2$ | $-V_{D} / 2$ |
| $V_{18}$ | -1 | 0 | 0 | $-V_{D} / 2$ | 0 | $V_{D} / 2$ | $-V_{D} / 2$ | 0 | 0 |
| $V_{19}$ | -1 | 0 | 1 | $-V_{D} / 2$ | $-V_{D} / 2$ | $V_{D}$ | $-V_{D} / 2$ | 0 | $V_{D} / 2$ |
| $V_{20}$ | -1 | 0 | -1 | $-V_{D} / 2$ | $V_{D} / 2$ | 0 | $-V_{D} / 2$ | 0 | $-V_{D} / 2$ |
| $V_{21}$ | -1 | 1 | 0 | $V_{D}$ | $V_{D} / 2$ | $V_{D} / 2$ | $-V_{D} / 2$ | $V_{D} / 2$ | 0 |
| $V_{22}$ | -1 | 1 | 1 | $-V_{D}$ | 0 | $V_{D}$ | $-V_{D} / 2$ | $V_{D} / 2$ | $V_{D} / 2$ |
| $V_{23}$ | -1 | 1 | -1 | $-V_{D}$ | $V_{D}$ | 0 | $-V_{D} / 2$ | $V_{D} / 2$ | $-V_{D} / 2$ |
| $V_{24}$ | -1 | -1 | 0 | 0 | $-V_{D} / 2$ | $V_{D} / 2$ | $-V_{D} / 2$ | $-V_{D} / 2$ | 0 |
| $V_{25}$ | -1 | -1 | 1 | 0 | $-V_{D}$ | $V_{D}$ | $-V_{D} / 2$ | $-V_{D} / 2$ | $V_{D} / 2$ |
| $V_{26}$ | -1 | -1 | -1 | 0 | 0 | 0 | $-V_{D} / 2$ | $-V_{D} / 2$ | $-V_{D} / 2$ |



Fig. 4 The experimental circuit

As can be seen there are 27 states or there are 27 available voltage vectors. This voltage vectors are summarized in Table 2. The voltage vectors which will be established between the converter terminals $u, v$ and $w$ can be expressed with next formula:

$$
\begin{equation*}
V_{k}=\frac{2}{3}\left(V_{u N}+V_{v N} e^{-\frac{2 \pi}{3}}+V_{w N} e^{\frac{2 \pi}{3}}\right) \tag{44}
\end{equation*}
$$

where $k=1,2 \ldots .27$. When we apply the values from Table 2 in (44) the voltage vectors are defined:

$$
\begin{align*}
& V_{1}=\frac{2}{3}\left(0+0 \cdot e^{-\frac{2 \pi}{3}}+0 \cdot e^{\frac{2 \pi}{3}}\right) \\
& V_{15}=\frac{2}{3}\left(\frac{V_{D}}{2}+\frac{V_{D}}{2} \cdot e^{-\frac{2 \pi}{3}}-\frac{V_{D}}{2} \cdot e^{\frac{2 \pi}{3}}\right)  \tag{45}\\
& V_{26}=\frac{2}{3}\left(-\frac{V_{D}}{2}-\frac{V_{D}}{2} \cdot e^{-\frac{2 \pi}{3}}-\frac{V_{D}}{2} \cdot e^{\frac{2 \pi}{3}}\right) .
\end{align*}
$$

From the voltage vector hexagon in Figure 3 follows that a few of different switching combination give the same switching vector. For example, the vectors $V_{8}$ and $V_{9}$ define the same voltage vector. For vectors choosing there are two main constraints:

1. The vectors which define the same inner hexagon point charge the different capacitor. It can be summarized that with the using of the both switching states the balance of the output voltages can be reached.
2. Regarding the position of the current vectors $I_{i n}$ it is not possible to reach all of available vectors. For example, from vectors which describe the hexagon origin only the vector $V_{1}$ can be reached because the vectors $V_{13}$ and $V_{26}$ require that all input currents $I_{i n i}$ should have the same sign. In
the case that the vectors $V_{i n}$ and $I_{i n}$ have the position as is shown in Figure 3, vector $V_{i n}$ can be established from the vectors which are placed in the $-V_{v N}$ and $V_{u N}$ direction. Because $I_{i n 3}$ has negative sign the voltage vector $V_{11}$ can not be realized. Vector $V_{i n}$ can be established only from the voltage vectors which limited the local shaded hexagon (Figure 3). When current vector pass the line $H_{\text {in } 2}$ the next hexagon should be used.

## 5. EXPERIMENTAL RESULTS

In Figure 4 the experimental circuit is shown. The measurement has been done with converter where the input inductances have values: $L_{A}=L_{B}=$ $=L_{C}=5 \mathrm{mH}$, the output capacitances have values $C_{O 1}=C_{O 2}=100 \mu \mathrm{~F}$.

Based on the above description the pulse width modulation algorithm has been implemented on the micro-controller unit. The Hitachi SH7032 micro--controller has been used. By using solutions from (38), (39) and (40) the input phase angle $\varphi$ in (26), can be adjust by computing appropriate $\beta$ in (29). The input values of the PWM algorithm are the desired output voltage $V_{o 1}$ and input angle $\beta$ which can be provided to the PWM algorithm on different ways. In this experiment the control of the input phase angle $\varphi$ has been used as shown in Figure 5.


Fig. 5 The input phase angle control scheme


Fig. 6 The input voltage $V_{A}$ and input current $I_{\text {in } 1}$, $y$-axis 100 V/div, $x$-axis 1 A/div (The laboratory voltage source has been used.) (a) Resistance load $100 \Omega$, (b) Resistance load $66 \Omega$.


Fig. 7 The input voltage $V_{A}$ and input current $I_{\text {in } 1}$, y-axis $100 \mathrm{~V} /$ div, x-axis 1 A/div (The laboratory voltage source has been used.) (a) Resistance load changes from $100 \Omega$ to $66 \Omega$, (b) Resistance load changes from $66 \Omega$ to $100 \Omega$.

In Figures 6(a) and (b) an operation of rectifier in different operating points is shown. Figures 7(a) and (b) show the unity power factor operation of the converter by step load change when RL has been changed from $100 \Omega$ to $60 \Omega$ and vice versa.

## 6. CONCLUSION

Based on the switching matrix converter theory, the topologies of a three-phase three-level AC to DC converter (rectifier) have been developed. This analytical approach enables that from the external terminal requirements it is possible to define the converter internal structure. The switching matrix approach enable the best overview into the modulations algorithm. By switching matrix approach the
appropriate switching sequences have been chosen. It has been also shown that the rectifier unity input displacement factor operation can be achieved with the control of the input displacement angle $\varphi$, without measurement of the input current.

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Korekcija faktora faznog pomaka trofaznog trorazinskog uzlaznog ispravljača zasnovanog na PWM-u. U radu je razvijen PWM algoritam za upravljanje matrično-strukturiranim trofaznim trorazinskim ispravljačem. Takav način upravljanja uzet je radi mogućnosti korekcije faktora pomaka ( $\cos \varphi$ ) na jediničnu vrijednost. Veza između prekidačkih funkcija i zahtjeva na impulsno-širinski modulator jasno opisuje sva ograničenja modulacijskog algoritma. Razvijeni modulacijski algoritam omogućuje korekciju faktora pomaka bez mjerenja ulazne struje pretvarača. Za potrebe te korekcije dovoljno je samo mjerenje faznog kuta između napona i struje na ulazu ispravljača.

Ključne riječi: matrični pretvarač, ispravljač, prekidačka funkcija, impulsno-širinska modulacija, faktor pomaka $\cos \varphi$

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