GLASNIK MATEMATIČKI
Vol. 41(61)(2006), 299 – 307

COEFFICIENT ESTIMATES AND PARTIAL SUMS OF A NEW CLASS OF FUNCTIONS

R. K. Raina and Deepak Bansal
Maharana Pratap University of Agriculture & Technology, India

Abstract. This paper investigates boundedness properties of certain classes of functions (which involve partial sums). The usefulness of the main results not only provide unification of results of Choi (where each of the results was proved rather independently), but also generates certain new results. Applications of our main results are pointed out briefly in the concluding section.

1. Introduction and preliminaries

Let $\mathcal{A}$ denote the class of functions $f(z)$ normalized by $f(0) = f'(0) - 1 = 0$, and analytic in the open unit disk $\mathcal{U} = \{z : z \in \mathbb{C},\ |z| < 1\}$. Then $f(z)$ can be expressed as

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \tag{1.1}$$

Let us denote by $\mathcal{M}(\alpha)$ and $\mathcal{N}(\alpha)$ two subclasses of the class $\mathcal{A}$, which are defined (for $\alpha > 1$) as follows:

$$\mathcal{M}(\alpha) = \left\{ f : f \in \mathcal{A}\,; \Re\left(\frac{zf'(z)}{f(z)}\right) < \alpha \quad (z \in \mathcal{U};\ \alpha > 1) \right\}, \tag{1.2}$$

and

$$\mathcal{N}(\alpha) = \left\{ f : f \in \mathcal{A}\,; \Re\left(1 + \frac{zf''(z)}{f'(z)}\right) < \alpha \quad (z \in \mathcal{U};\ \alpha > 1) \right\}. \tag{1.3}$$

2000 *Mathematics Subject Classification.* 30C45.
Key words and phrases. Hadamard product, Ruschweyh derivative, univalent functions, starlike functions, convex functions.

The classes $\mathcal{M}(\alpha)$ and $\mathcal{N}(\alpha)$ were studied recently by Owa and Nishiwaki [3], and also by Owa and Srivastava [4]. In fact, for $1 < \alpha \leqq 4/3$, these classes were investigated earlier by Uralegaddi et al. [7].

It follows from (1.2) and (1.3) that

$$f(z) \in \mathcal{N}(\alpha) \iff zf'(z) \in \mathcal{M}(\alpha)\,. \tag{1.4}$$

If $f, g \in \mathcal{A}$, where $f(z)$ is given by (1.1), and $g(z)$ is defined by

$$g(z) = z + \sum_{n=2}^{\infty} b_n z^n, \tag{1.5}$$

then their Hadamard product (or convolution) $f * g$ is defined (as usual) by

$$(f * g)(z) = z + \sum_{n=2}^{\infty} a_n b_n z^n = (g * f)(z). \tag{1.6}$$

Suppose the functions $\phi(z)$ and $\psi(z)$ are given by

$$\phi(z) = z + \sum_{n=2}^{\infty} \lambda_n z^n, \tag{1.7}$$

and

$$\psi(z) = z + \sum_{n=2}^{\infty} \mu_n z^n, \tag{1.8}$$

where $\lambda_n \geqq \mu_n \geqq 0$ for all $n \in \mathbb{N}\backslash\{1\}$). We say that $f \in \mathcal{A}$ is in $\mathcal{S}_\alpha(\phi, \psi)$ provided that $(f * \psi)(z) \neq 0$, and

$$\Re\left\{\frac{(f * \phi)(z)}{(f * \psi)(z)}\right\} < \alpha \quad (\alpha > 1;\ z \in \mathcal{U}). \tag{1.9}$$

Several new and known subclasses can be obtained from the class $\mathcal{S}_\alpha(\phi, \psi)$ by suitably choosing the functions $\phi(z)$ and $\psi(z)$. We mention below some of these subclasses of $\mathcal{S}_\alpha(\phi, \psi)$ consisting of functions $f(z) \in \mathcal{A}$. Indeed, we have

$$\begin{aligned} &\mathcal{S}_\alpha\left(\frac{z}{(1-z)^{\lambda+2}}, \frac{z}{(1-z)^{\lambda+1}}\right) \\ &= \left\{f : f \in \mathcal{A}\,; \Re\left(\frac{D^{\lambda+1}f(z)}{D^\lambda f(z)}\right) < \alpha \quad (z \in \mathcal{U}; \alpha > 1; \lambda > -1)\right\} \\ &\equiv \Lambda_\alpha(\lambda), \end{aligned} \tag{1.10}$$

and the relationships

$$\mathcal{S}_\alpha\left(\frac{z}{(1-z)^2}, \frac{z}{(1-z)}\right) \equiv \mathcal{M}(\alpha) \quad (\alpha > 1), \tag{1.11}$$

and

$$\mathcal{S}_\alpha\left(\frac{z+z^2}{(1-z)^3}, \frac{z}{(1-z)^2}\right) \equiv \mathcal{N}(\alpha) \qquad (\alpha > 1). \tag{1.12}$$

A similar type of a subclass $\Lambda_\alpha(\lambda)$ involving the Ruscheweyh operator D^λ (see [6]), defined by (1.10) above, was also considered by Ali et al. [1], whereas, the subclasses $\mathcal{M}(\alpha)$ and $\mathcal{N}(\alpha)$ are the known classes ([3]) defined by (1.2) and (1.3), respectively.

Let

$$f_N(z) = z + \sum_{k=2}^{N} a_k z^k \qquad (N = 2, 3, 4, ...), \tag{1.13}$$

be a sequence of partial sums of an analytic function of the form (1.1).

In [2], Choi investigated lower bounds on

$$\Re\{f(z)/f_N(z)\}, \Re\{f_N(z)/f(z)\}, \Re\{f'(z)/f'_N(z)\} \text{ and } \Re\{f'_N(z)/f'(z)\}$$

for $\mathcal{M}(\alpha)$ and $\mathcal{N}(\alpha)$, and gave independent proofs of his main results. In this paper we find similar bounds for functions belonging to a general class $\mathcal{S}_\alpha(\phi, \psi)$. Application of our results not only yields the results of Choi [2], but would also lead to some new results. One of such new results is deduced in the concluding section.

Before stating and proving our main results, we derive a sufficient condition giving the coefficient estimates for the function $f(z)$ to belong to the aforementioned class $\mathcal{S}_\alpha(\phi, \psi)$. The result is contained in the following:

Lemma 1.1. *If $f(z) \in \mathcal{A}$ satisfies*

$$\sum_{n=2}^{\infty} (\lambda_n - \alpha\mu_n)|a_n| \leqq (\alpha - 1), \tag{1.14}$$

then $f(z) \in \mathcal{S}_\alpha(\phi, \psi)$, provided that $\lambda_n > \mu_n > 0$, $\langle \mu_n \rangle$ and $\left\langle \frac{\lambda_n}{\mu_n} \right\rangle$ are nondecreasing sequences, and α is such that $1 < \alpha \leqq \frac{1}{2}\left(1 + \frac{\lambda_2}{\mu_2}\right)$.

Proof. Let the condition (1.14) be satisfied for the function $f(z) \in \mathcal{A}$. It is sufficient to show that

$$\left| \frac{\frac{(f*\phi)(z)}{(f*\psi)(z)} - 1}{\frac{(f*\phi)(z)}{(f*\psi)(z)} - (2\alpha - 1)} \right| < 1 \quad (z \in \mathcal{U}).$$

We note that

$$\left|\frac{\frac{(f*\phi)(z)}{(f*\psi)(z)}-1}{\frac{(f*\phi)(z)}{(f*\psi)(z)}-(2\alpha-1)}\right| = \left|\frac{\sum\limits_{n=2}^{\infty}(\lambda_n-\mu_n)a_n z^{n-1}}{(2-2\alpha)+\sum\limits_{n=2}^{\infty}\{\lambda_n-(2\alpha-1)\mu_n\}a_n z^{n-1}}\right|$$

$$\leqq \frac{\sum\limits_{n=2}^{\infty}(\lambda_n-\mu_n)|a_n||z|^{n-1}}{(2\alpha-2)-\sum\limits_{n=2}^{\infty}\{\lambda_n-(2\alpha-1)\mu_n\}|a_n||z|^{n-1}}$$

$$< \frac{\sum\limits_{n=2}^{\infty}(\lambda_n-\mu_n)|a_n|}{(2\alpha-2)-\sum\limits_{n=2}^{\infty}\{\lambda_n-(2\alpha-1)\mu_n\}|a_n|}.$$

The extreme right side expression of the above inequality would remain bounded by 1 if

$$\sum_{n=2}^{\infty}(\lambda_n-\mu_n)|a_n| \leqq (2\alpha-2)-\sum_{n=2}^{\infty}\{\lambda_n-(2\alpha-1)\mu_n\}|a_n|,$$

which leads to the desired inequality (1.14). This completes the proof. □

The aforementioned Lemma 1.1 was also stated in a recent paper of Raina and Bansal [5, p. 3689].

2. Main results

Theorem 2.1. *Let* $1<\alpha\leqq\min\left[\frac{1}{2}\left(1+\frac{\lambda_2}{\mu_2}\right),\frac{\lambda_2+1}{\mu_2+1}\right]$ *and* $z\in\mathcal{U}$. *If* $f(z)\in\mathcal{A}$ *satisfies the coefficient inequality (1.14), then (under the hypotheses of Lemma 1.1)*

$$\Re\left(\frac{f(z)}{f_N(z)}\right)\geqq\frac{\lambda_{N+1}-\alpha\mu_{N+1}-\alpha+1}{\lambda_{N+1}-\alpha\mu_{N+1}}, \tag{2.1}$$

and

$$\Re\left(\frac{f_N(z)}{f(z)}\right)\geqq\frac{\lambda_{N+1}-\alpha\mu_{N+1}}{\lambda_{N+1}-\alpha\mu_{N+1}+\alpha-1}. \tag{2.2}$$

The result is sharp for every N, with the extremal functions given by

$$f(z)=z+\frac{\alpha-1}{\lambda_{N+1}-\alpha\mu_{N+1}}z^{N+1}\quad(N\in\mathbb{N}). \tag{2.3}$$

Proof. We prove (2.1). The proof of (2.2) is similar and will be omitted. We may set

$$\frac{\lambda_{N+1}-\alpha\mu_{N+1}}{\alpha-1}\left[\frac{f(z)}{f_N(z)}-\frac{\lambda_{N+1}-\alpha\mu_{N+1}-\alpha+1}{\lambda_N-\alpha\mu_N}\right]=\frac{1+w(z)}{1-w(z)}.$$

By putting values of $f(z)$, $f_N(z)$ and simplifying, we obtain

$$w(z)=\frac{\frac{\lambda_{N+1}-\alpha\mu_{N+1}}{\alpha-1}\sum\limits_{k=N+1}^{\infty}a_kz^{k-1}}{2+2\sum\limits_{k=2}^{N}a_kz^{k-1}+\frac{\lambda_{N+1}-\alpha\mu_{N+1}}{\alpha-1}\sum\limits_{k=N+1}^{\infty}a_kz^{k-1}},$$

so that $w(0)=0$, and we find that

$$|w(z)|\leqq\frac{\frac{\lambda_{N+1}-\alpha\mu_{N+1}}{\alpha-1}\sum\limits_{k=N+1}^{\infty}|a_k|}{2-2\sum\limits_{k=2}^{N}|a_k|-\frac{\lambda_{N+1}-\alpha\mu_{N+1}}{\alpha-1}\sum\limits_{k=N+1}^{\infty}|a_k|}.$$

Now $|w(z)|\leqq 1$ if and only if

$$\sum_{k=2}^{N}|a_k|+\frac{\lambda_{N+1}-\alpha\mu_{N+1}}{\alpha-1}\sum_{k=N+1}^{\infty}|a_k|\leqq 1. \tag{2.4}$$

This will hold if we show that left-hand side of (2.4) is bounded above by $\sum\limits_{k=2}^{\infty}\frac{\lambda_k-\alpha\mu_k}{1-\alpha}|a_k|$ (in view of (1.14)). This is equivalent to showing that

$$\begin{aligned}&\sum_{k=2}^{N}\left(\frac{\lambda_k-\alpha\mu_k-\alpha+1}{\alpha-1}\right)|a_k|\\&\quad+\sum_{k=N+1}^{\infty}\left(\frac{\lambda_k-\alpha\mu_k-\lambda_{N+1}+\alpha\mu_{N+1}}{\alpha-1}\right)|a_k|\geqq 0.\end{aligned} \tag{2.5}$$

We observe that the first term of the first series in (2.5) is positive if

$$\lambda_2-\alpha\mu_2\geqq\alpha-1.$$

Now expressing

$$\lambda_k-\alpha\mu_k=\mu_k\left(\frac{\lambda_k}{\mu_k}-\alpha\right),$$

which is an increasing sequence for all $k\in\mathbb{N}\setminus\{1\}$, in view of the conditions that $\lambda_n>\mu_n>0$; $\langle\mu_n\rangle$ and $\left\langle\frac{\lambda_n}{\mu_n}\right\rangle$ are nondecreasing sequences, and α is such that $1<\alpha\leqq\frac{1}{2}\left(1+\frac{\lambda_2}{\mu_2}\right)$. This obviously implies that all the other terms in the first series remain positive. Also, the first term of the second series in (2.5) vanishes, and all other terms of this series are positive. Thus if

$$1<\alpha\leqq\min\left[\frac{1}{2}\left(1+\frac{\lambda_2}{\mu_2}\right),\frac{\lambda_2+1}{\mu_2+1}\right],$$

then inequality (2.5) holds true. This completes the proof of (2.1). Finally, it can be verified that the equality in (2.1) is attained for the function given by (2.3), when $z = re^{\pi i/N}$ and $r \to 1^-$. ∎

THEOREM 2.2. *Let* $1 < \alpha \leqq \min\left[\frac{1}{2}\left(1+\frac{\lambda_2}{\mu_2}\right), \frac{\lambda_2+1}{\mu_2+1}\right]$ *and* $z \in \mathcal{U}$. *If* $f(z) \in \mathcal{A}$ *satisfies the condition (1.14) (under the hypotheses of Lemma 1.1), and also if the sequence* $\left\langle \frac{\lambda_k - \alpha\mu_k}{k} \right\rangle_{k=2}^{\infty}$ *is nondecreasing, then*

$$\Re\left(\frac{f'(z)}{f_N'(z)}\right) \geqq \frac{\lambda_{N+1} - \alpha\mu_{N+1} - (\alpha-1)(N+1)}{\lambda_{N+1} - \alpha\mu_{N+1}}, \tag{2.6}$$

and

$$\Re\left(\frac{f_N'(z)}{f'(z)}\right) \geqq \frac{\lambda_{N+1} - \alpha\mu_{N+1}}{\lambda_{N+1} - \alpha\mu_{N+1} + (N+1)(\alpha-1)}. \tag{2.7}$$

The results are sharp for every $n \in \mathbb{N}$, *with the extremal functions given by (2.3).*

PROOF. We prove (2.6). The proof of (2.7) is similar and will be omitted. We may set

$$\frac{\lambda_{N+1} - \alpha\mu_{N+1}}{(\alpha-1)(N+1)}\left[\frac{f'(z)}{f_N'(z)} - \frac{\lambda_{N+1} - \alpha\mu_{N+1} - (\alpha-1)(N+1)}{\lambda_N - \alpha\mu_N}\right] = \frac{1+w(z)}{1-w(z)}.$$

Putting the values of $f'(z), f_N'(z)$, we get

$$|w(z)| \leqq \frac{\frac{\lambda_{N+1}-\alpha\mu_{N+1}}{(\alpha-1)(N+1)} \sum\limits_{k=N+1}^{\infty} k\,|a_k|}{2 - 2\sum\limits_{k=2}^{N} k\,|a_k| - \frac{\lambda_{N+1}-\alpha\mu_{N+1}}{(\alpha-1)(N+1)} \sum\limits_{k=N+1}^{\infty} k\,|a_k|}.$$

Now $|w(z)| \leqq 1$, if and only if

$$\sum_{k=2}^{N} k\,|a_k| + \sum_{k=N+1}^{\infty} \frac{\lambda_{N+1} - \alpha\mu_{N+1}}{(\alpha-1)(N+1)} k\,|a_k| \leqq 1, \tag{2.8}$$

which will hold true if we show that the left-hand side of (2.8) is bounded above by $\sum\limits_{k=2}^{\infty} \frac{\lambda_k - \alpha\mu_k}{\alpha-1}\,|a_k|$ (in view of (1.14)). This is equivalent to showing that

$$\begin{aligned} &\sum_{k=2}^{N} k\left(\frac{\frac{\lambda_k-\alpha\mu_k}{k}}{\alpha-1} - 1\right)|a_k| \\ &\quad + \sum_{k=N+1}^{\infty} k\left(\frac{(N+1)\frac{\lambda_k-\alpha\mu_k}{k} - \lambda_{N+1} + \alpha\mu_{N+1}}{(\alpha-1)(N+1)}\right)|a_k| \geqq 0. \end{aligned} \tag{2.9}$$

This is true due to the condition implied on α, and the increasingness of the sequence $\left\langle \frac{\lambda_k - \alpha\mu_k}{k} \right\rangle_{k=2}^{\infty}$. ∎

3. Applications

In this section we shall consider some applications of our main results (Theorems 2.1 and 2.2). By appealing to (1.9), and comparing the functions $\phi(z)$ and $\psi(z)$ according to (1.10), we note that the sequences $\langle\lambda_n\rangle$ and $\langle\mu_n\rangle$ are, respectively, given by

$$\lambda_n = \binom{\lambda+n}{n-1} \qquad (\forall\, n \in \mathbb{N}\backslash\{1)\},$$

and

$$\mu_n = \binom{\lambda+n-1}{n-1} \qquad (\forall\, n \in \mathbb{N}\backslash\{1\}).$$

We observe that the sequence $\left\langle \frac{\lambda_k - \alpha\mu_k}{k} \right\rangle_{k=2}^{\infty}$ involved in Theorem 2.2 remains nondecreasing provided that

$$1 < \alpha \leqq \min\left[\frac{2\lambda+3}{2\lambda+2}, \frac{\lambda+3}{\lambda+2}\right]; \qquad \lambda \geqq 0,$$

and thus Theorems 2.1 and 2.2 give the following results:

Corollary 3.1. *Let* $1 < \alpha \leqq \min\left[\frac{2\lambda+3}{2\lambda+2}, \frac{\lambda+3}{\lambda+2}\right]$ *and* $z \in \mathcal{U}$. *If* $f(z) \in \mathcal{A}$ *satisfies the coefficient inequality*

$$\sum_{n=2}^{\infty} \mathcal{B}(\alpha, n)\,|a_n| \leqq \alpha - 1,$$

then

$$\Re\left(\frac{f(z)}{f_N(z)}\right) \geqq \frac{\mathcal{B}(\alpha, N+1) - \alpha + 1}{\mathcal{B}(\alpha, N+1)} \qquad (\lambda > -1), \tag{3.1}$$

$$\Re\left(\frac{f_N(z)}{f(z)}\right) \geqq \frac{\mathcal{B}(\alpha, N+1)}{\mathcal{B}(\alpha, N+1) + \alpha - 1} \qquad (\lambda > -1), \tag{3.2}$$

$$\Re\left(\frac{f'(z)}{f'_N(z)}\right) \geqq \frac{\mathcal{B}(\alpha, N+1) - (\alpha-1)(N+1)}{\mathcal{B}(\alpha, N+1)} \qquad (\lambda \geqq 0), \tag{3.3}$$

and

$$\Re\left(\frac{f'_N(z)}{f'(z)}\right) \geqq \frac{\mathcal{B}(\alpha, N+1)}{\mathcal{B}(\alpha, N+1) + (N+1)(\alpha-1)} \qquad (\lambda \geqq 0), \tag{3.4}$$

where

$$\mathcal{B}(\alpha, n) = \frac{\Gamma(n+\lambda)}{\Gamma(n)\,\Gamma(\lambda+1)}\left[\frac{n+\lambda}{\lambda+1} - \alpha\right].$$

The result is sharp for every N, with extremal functions given by

$$f(z) = z + \frac{\alpha - 1}{\mathcal{B}(\alpha, N+1)} z^{N+1} \qquad (N \in \mathbb{N}). \tag{3.5}$$

Similarly, by selecting the functions $\phi(z)$ and $\psi(z)$ in (1.9) according to (1.11) and (1.12), we are led to the following results:

Corollary 3.2. *Let $1 < \alpha \leqq 3/2$ and $z \in \mathcal{U}$. If $f(z) \in \mathcal{A}$ satisfies the coefficient inequality*

$$\sum_{n=2}^{\infty} (n - \alpha)\, |a_n| \leqq \alpha - 1,$$

then

$$\Re\left(\frac{f(z)}{f_N(z)}\right) \geqq \frac{N + 2 - 2\alpha}{N + 1 - \alpha}, \tag{3.6}$$

$$\Re\left(\frac{f_N(z)}{f(z)}\right) \geqq \frac{N + 1 - \alpha}{N}, \tag{3.7}$$

$$\Re\left(\frac{f'(z)}{f'_N(z)}\right) \geqq \frac{2(N+1) - \alpha(N+2)}{N + 1 - \alpha}, \tag{3.8}$$

and

$$\Re\left(\frac{f'_N(z)}{f'(z)}\right) \geqq \frac{N + 1 - \alpha}{N + 1 - \alpha + (N+1)(\alpha - 1)}. \tag{3.9}$$

The result is sharp for every N, with extremal functions given by

$$f(z) = z + \frac{\alpha - 1}{N + 1 - \alpha} z^{N+1} \qquad (N \in \mathbb{N}). \tag{3.10}$$

Corollary 3.3. *Let $1 < \alpha \leqq 3/2$ and $z \in \mathcal{U}$. If $f(z) \in \mathcal{A}$ satisfies the coefficient inequality*

$$\sum_{n=2}^{\infty} n\,(n - \alpha)\, |a_n| \leqq \alpha - 1,$$

then

$$\Re\left(\frac{f(z)}{f_N(z)}\right) \geqq \frac{N(N + 2 - \alpha) - 2(\alpha - 1)}{(N+1)(N + 1 - \alpha)}, \tag{3.11}$$

$$\Re\left(\frac{f_N(z)}{f(z)}\right) \geqq \frac{(N+1)(N + 1 - \alpha)}{N(N + 2 - \alpha)}, \tag{3.12}$$

$$\Re\left(\frac{f'(z)}{f'_N(z)}\right) \geqq \frac{N + 2 - 2\alpha}{N + 1 - \alpha}, \tag{3.13}$$

and

$$\Re\left(\frac{f'_N(z)}{f'(z)}\right) \geqq \frac{N + 1 - \alpha}{N}. \tag{3.14}$$

The result is sharp for every N, with extremal functions given by

$$f(z) = z + \frac{\alpha - 1}{(N+1)(N+1-\alpha)} z^{N+1} \qquad (N \in \mathbb{N}). \tag{3.15}$$

REMARK 3.4. Results of Corollary 3.1 are believed to be new, whereas, the results of Corollaries 3.2 and 3.3 were earlier obtained (as separate theorems) by Choi ([2, Theorems 3, 4 and 6]).

ACKNOWLEDGEMENT.

The present investigation was supported by AICTE (Govt. of India), New Delhi.

References

[1] R. M. Ali, M. H. Khan, V. Ravichandran and K. G. Subramanian, *A class of multivalent functions with positive coefficients defined by convolution*, J. Inequal. Pure Appl. Math. **6** (2005), 1-9.

[2] J. H. Choi, *Univalent functions with positive coefficients involving a certain fractional integral operator and its partial sums*, Fract. Calc. Appl. Anal. **1** (1998), 311-318.

[3] S. Owa and J. Nishiwaki, *Coefficient estimates for certain classes of analytic functions*, J. Inequal. Pure Appl. Math. **3** (2002), 1-5.

[4] S. Owa and H. M. Srivastava, *Some generalized convolution properties associated with certain subclasses of analytic functions*, J. Inequal. Pure Appl. Math. **3** (2002), 1-13.

[5] R. K. Raina and D. Bansal, *Coefficient estimates and subordination properties associated with certain classes of functions*, Int. J. Math. Math. Sci. **22** (2005), 3685-3695.

[6] S. Ruscheweyh, *New criteria for univalent functions*, Proc. Amer. Math. Soc. **49** (1975), 109-115.

[7] B. A. Uralegaddi, M. D. Ganigi and S. M. Sarangi, *Univalent function with positive coefficients*, Tamkang J. Math. **25** (1994), 225-230.

R. K. Raina
Department of Mathematics
Maharana Pratap University of Agriculture and Technology
College of Technology and Engineering
Udaipur -313001, Rajasthan
India
E-mail: rainark_7@hotmail.com

D. Bansal
Department of Mathematics
Maharana Pratap University of Agriculture and Technology
College of Technology and Engineering
Udaipur -313001, Rajasthan
India
E-mail: deepakbansal_79@yahoo.com

Received: 23.6.2005.
Revised: 10.2.2006.