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# Unified equation for straightforward inversion scheme on vertical electrical sounding data

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A unified apparent resistivity equation is derived encompassing eight electrode configurations for Straightforward Inversion Scheme (SIS) on Vertical Electrical Sounding (VES) data. Suitable parameters are worked out to convert unified equation to a particular configuration viz. Wenner, Pole-Pole, Schlumberger, Pole-Dipole, Radial Dipole, Perpendicular Dipole, Parallel Dipole and Azimuthal Dipole. The derived equation is validated by forward computations of VES data for a 5-layer earth model that compares well with that of VES data obtained using existing individual equations for a particular configuration. SIS scheme is briefly described and a Schlumberger synthetic sounding data is generated using the unified equation over model riddled with equivalence /suppression and subsequently inverted using SIS to show the resolving power of the scheme via-a-vis low resistivity contrast between the layers. Two examples of field Schlumberger soundings from known lithology on a paleochannel in Mario Island in Northern Brazil are also included and the inverted continuous resistivity variation is presented to show the efficacy of the scheme.

*Keywords*: unified equation, straightforward inversion, vertical electrical sounding.

## 1. Introduction

Direct current resistivity methods of geophysical exploration are in extensive use globaly for aquifer mapping (Bhattacharya and Patra, 1966; Zohdy, 1969; Koefoed, 1989) and estimation of aquifer parameters (Kosinky and Kelly, 1981; Sri Niwas and Singhal, 1981, 1985; Mazac et al., 1985; Yadav and Abolfazli, 1998). The physical basis of the resistivity method is based on the relative distribution of impressed current in the earth controlled by subsurface resistivity distribution. Logically the resistivity distribution in a vertically inhomogeneous earth can be derived from distribution of electrical potential at the surface. The tenability of the derived information rests on the basis of a physical theory connecting cause and effect and on an adequate description of the system by a minimal set of parameter to be determined from the data. These constitute the basic statement of an inverse problem. However, the inverse path from data, along the logical path furnished by the physical theory, to causative target is not always as neat as forward path. Coupled with this is the non-linear nature of the inverse resistivity problem. Global optimization methods have been developed to address such problem directly (Sen et al., 1993) but still the quasi-linearization of the inverse problem and use of linear inverse schemes iteratively to obtain inverse solution are popular (Inman et al., 1973; Koefoed, 1979; Constable et al., 1987; Sims and Morgan, 1992; Porsani et al., 2001). The serious concerns in solving inverse problem are existence, uniqueness and stability. Numerical solutions of the inverse problem are invariably non-unique due to possibility of non-retrieval of some parts of the solution from the data (effect of vanished eigen-values). Instability creep in due to infinitesimal variations in the data, caused by the presence of error, resulting in wide fluctuations of the solution due to ill-conditioning of the system matrix (effect of near vanishing eigen-values) determined by condition number (ratio of highest and smallest eigen-values). Ill-conditioned linear systems, wherein some model parameters are linearly dependent (problem of equivalence) results in the existence of near vanishing eigen-values. These endemic problems of inverse solutions violates the basic conditions of well-posed problem enunciated by Hadmard in the $19^{th}$ century and sets a limit to extractable information in terms of the product of resolution and accuracy, so that one can only be enhanced at the cost of other.

Stefanescu (1930) obtained the requisite forward solution, connecting cause and effect over a stratified earth model energized by a point current source placed on the surface, as function of resistivities and thicknesses of horizontal layers stacked one over the other. Langer (1933), for the first time in electrical method, studied the problem of existence and uniqueness of the inverse solution with the conclusion that if the earth resistivity varies continuously with depth and the potential distribution about a current electrode at the surface is completely known, then the inverse potential problem has a unique solution. Slichter (1933) developed a procedure to construct the inverse solution for 1D resistivity sounding data. That remained purely academic for quite a long time due to excessive computational requirement and the interpretation of VES data remained confined to curve matching procedure through theoretical curves prepared using different computational methods (Flathe, 1955; VanDam, 1964; Mooney et al., 1966; Ghosh, 1971). However, due to improved computational facilities currently some canonical form of inverse methods have been distilled and refined to solve resistivity inverse problem reasonably well. In such solutions the non-uniqueness is taken care by designing the best approximate solution excluding the vanished eigen-values. However, a problem often arises from the endemic character of real data kernels in that the small eigen-values decreases very smoothly making it difficult to distinguish between those that are actually vanished from the near vanishing ones. These near vanishing eigen-values are the prime source of instability in the inverse solutions that are tackled by (i) specifying a cutoff eigen-values amounting to reducing the dimension of the eigen-space (Lanczos, 1961; Jupp and Vozoff, 1975) or (ii) by enhancing the near vanishing eigen-values thereby enforcing smoothness (in effect the eigen-values smaller than the smoothing parameter are automatically dropped) on the inverted solution (Marquardt, 1963; Inman, 1975) or a combination of above two (Johansen, 1977). However, these processes extract its price as the model and data resolution deteriorate in (i) and one is forced to seek trade-off between the misfit error and the degree of smoothness in (ii).

As mentioned earlier the problem of non-uniqueness and instability due to vanished and near vanishing eigen-values is the consequence of principle of suppression and principle of equivalence (Koefoed, 1979). Equivalence means that differing layer distributions may have the equal or nearly equal layer thickness – layer resistivity product (transverse resistance) and layer thickness – layer conductivity product (longitudinal conductance) yielding equal or nearly equal VES data. The inverted model would be only one of many acceptable solutions that may be consistent with observed VES data within the accuracy of measurements. The instability occurs due to this linear dependence of layer parameters and error in data.

Logically the problem of non-uniqueness and instability can be overcome if some a-priori knowledge about the nature of one of the model parameters (specially the layer thicknesses) is independently available or can be reasonably assumed. This have already been attempted by few investigators (Parker, 1984; Constable et al., 1987; Sims and Morgan, 1992), however, desired success was not achieved because the thickness could not be taken out of the inversion process through necessary formulation of the forward and inverse problem. Gupta et al. (1997) presented a straightforward inversion scheme (SIS) by assuming that the earth is composed of large numbers of layers of pre-assigned uniform thickness. They have developed recurrence relations for forward and inverse computations. Interestingly in this formulation the non-linear inverse problem reduces to linear one paving way for using minimum norm inversion scheme to obtain reflection coefficients and resistivity of the first layer thereby solving completely the resistivity inverse problem.

There exist a large number of electrode configurations for recording VES, each suitable for particular geological situation. This necessitates corresponding apparent resistivity equations for obtaining inverse solutions. Unified approaches of computing apparent resistivity response using digital filters for any generalized electrode array have been suggested in the past (O'Neill and Merrick, 1984; Das, 1984) that are not useful in SIS. In the following we have developed a unified equation for SIS by combining symmetrical and dipolar apparent resistivity responses hoping that the combined use of SIS and unified equation is promising for inversion of resistivity sounding data.

## 2. Unified Equation

Steafanescu's (1930) equation of electrical potential, $U(r)$ on the surface of the layered earth at distance $r$ from a point current source of strength $I$ is written as:

$$U(r) = \frac{I}{2\pi}\int_0^\infty T(\lambda) J_0(\lambda r)\, d\lambda \tag{1}$$

where $J_0(\lambda r)$ is the Bessel function of first kind and zeroeth order, $T(\lambda)$ is the resistivity transform function at the air-earth interface, and $\lambda$ is the integration variable. Sri Niwas and Israil (1986) used an exponential approximation of $T(\lambda)$, as

$$T(\lambda) = \sum_{j=0}^{\infty} c_j e^{-\xi_j \lambda}, \xi_0 = 0 \tag{2}$$

where $c_j$ is the coefficient of *j-th* approximating function and $\xi_j$ establishes the position of the approximating function along the abscissa, to reduce the integral equation (1) through Lipschitz integral to a simple algebraic equation making numerical computations easy once the $c_j$ and $\xi_j$ are estimated. Gupta et al. (1997) modified the formulation by introducing a layered earth model of uniform thickness $d$, and taking $\xi_j = 2jd$ and developed the following simple equation for the apparent resistivity, $\rho_a(r)$ of a symmetrical four electrode array (Figure 1; $AM = BN = r$; $NA = BM = mr$; $AB/2 = r(m+1)/2$; $m > 1$) as

$$\rho_a(r) = \sum_{j=0}^{\infty} c_j \left[ \frac{mr}{m-1} \left\{ \frac{1}{(r^2 + 4j^2 d^2)^{1/2}} - \frac{1}{(r^2 + j^2 d^2)^{1/2}} \right\} \right] \tag{3}$$

The value of $m$ determines the inter-electrode spacing hence the particular symmetrical array. Sri Niwas and Israil (1987) developed apparent resistivity equations for various dipole configurations using following equation (Al'pine, 1958) developed to convert the Schlumberger apparent resistivity $(\rho_{as}(r))$ to various dipole apparent resistivity $(\rho_{aD}(r))$

$$\rho_{aD}(r) = \rho_{as}(r) - fr\,\frac{\partial \rho_{as}(r)}{\partial r} \tag{4}$$

The values of $f$ determine the particular dipole array. By modifying equation (4) for a layered model of uniform thickness the two sets of apparent resistivity equation can be used to derive an unified apparent resistivity equation encompassing *eight* electrode configurations (Figure 1) belonging to symmetric and dipole classes as

$$\rho_a(r)_{unified} = \sum_{j=0}^{\infty} c_j \frac{mr}{m-1}\left(\frac{(1-b)4j^2d^2+r^2}{(4j^2d^2+r^2)^{3/2}} - \frac{(1-b)4j^2d^2+m^2r^2}{(4j^2d^2+m^2r^2)^{3/2}}\right) \tag{5}$$

Equation (5) is converted to the desired configuration by substituting the proper set of values of $m$ and $f$ given in Table 1. The unified equation (5) can be used for efficient computation of apparent resistivity and estimation of $c_j$ for any of the tabulated *eight* electrode configuration.

*Table 1. Tabular values of m and b to be used in equation (5) for desired configuration*

| Configuration | $f$ | $m$ | Symbol |
|---|---|---|---|
| Wenner | 0.0 | 2.0 | $\rho_{aw}$ |
| Schlumberger | 0.0 | 1.1 | $\rho_{as}$ |
| Pole-Pole | 0.0 | $\infty$ | $\rho_{aPP}$ |
| Half-Schlumberger (Pole-Dipole) | 0.0 | 1.1 | $\rho_{aPD}$ |
| Radial Dipole | 0.5 | 1.1 | $\rho_{ar}$ |
| Perpendicular Dipole | 1/3 | 1.1 | $\rho_{ay}$ |
| Parallel Dipole | $\cos^2\theta$ / $(3\cos^2\theta-1)$ | 1.1 | $\rho_{ax}$ |
| Azimuthal Dipole | 0.0 | 1.1 | $\rho_{a\theta}$ |

## 3. Straightforward Inversion Scheme

The uniform thickness layer model considered by Gupta et al. (1997) while developing the SIS found that the coefficient $c_j$ contains information about layer resistivities alone. Thus in equation (2) and in equation (5) the only unknown quantity to be determined from observed data in resistivity transform domain and apparent resistivity domain respectively is $c_j$. For estimation of $c_j$ equation (2) and equation (5) is linear equation. Therefore the choice of uniform thickness layered model reduces the non-linear resistivity inverse problem to a linear one. It is evident that equation (2) or equation (5) represents series up to infinite term that need to be terminated at some stage (say up to $p$ terms) satisfying some desired accuracy »$e_0$« (difference between exact and estimated values of either $T(\lambda)$ or $\rho_a(r)$) criterion. Gupta et al. (1997) derived equation given by $p = 0.5r_{max}/(d\sqrt{e_0})$ to restrict $p$ where $r_{max}$ is the maximum half-current electrode spacing. If apparent resistivity data is recorded at $n$ number of electrode separations ($r_1$, $r_2$, …, $r_n$) then equation (5) can be cast as matrix equation

$$v = Gc \tag{6}$$

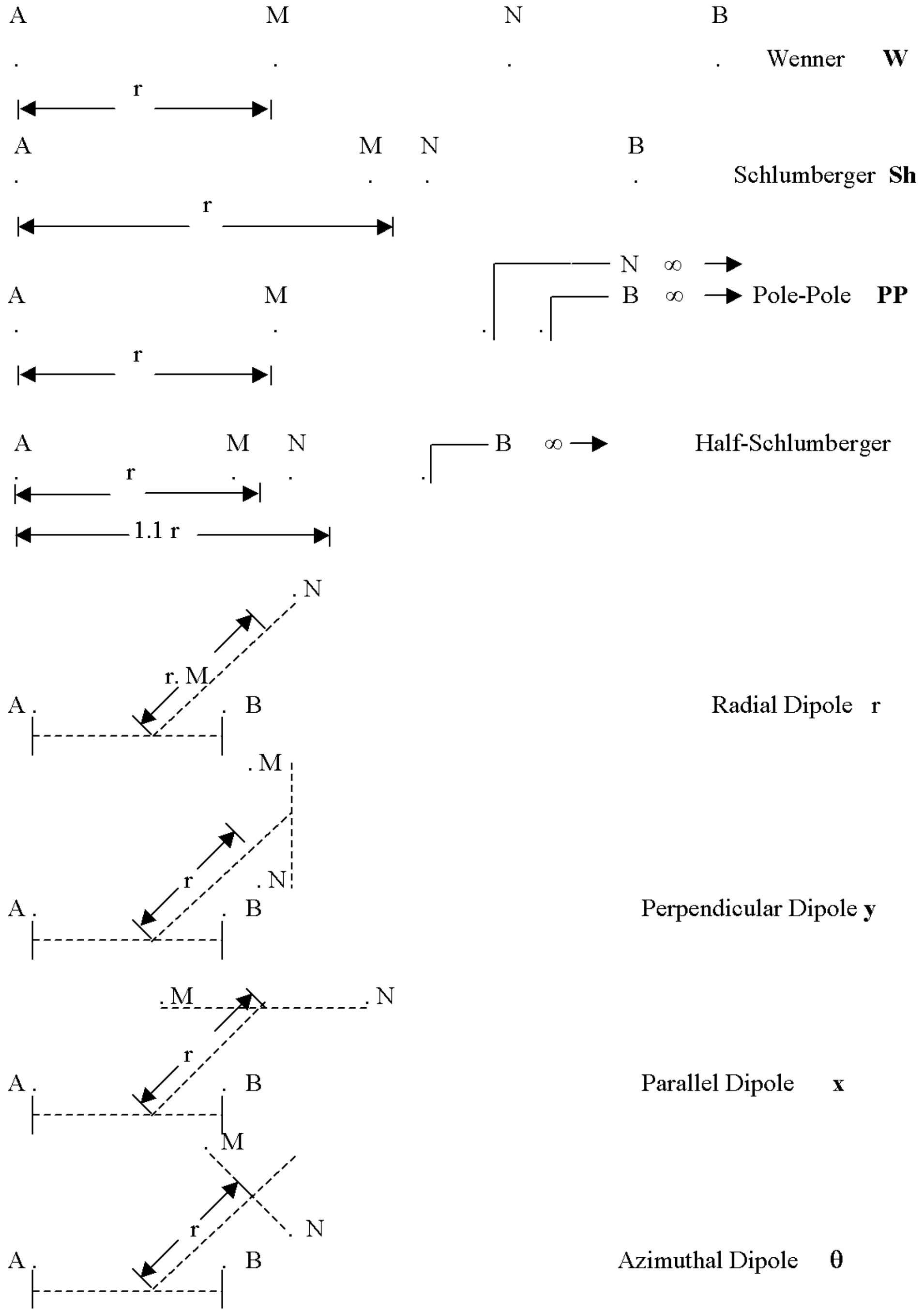


**Figure 1.** Schematic presentation of eight electrode configurations considered.

where column vectors $v$ and $c$ are given by $v = [\rho_a(r_1), \rho_a(r_2),..., \rho_a(r_n)]^t$, $c = [c_1, c_2,..., c_p]^t$, $t$ is transpose operation and the matrix $\boldsymbol{G}$ of order $nxp$ can be composed as its element as

$$G_{j1} = \frac{mr_l}{m-1}\left[\frac{(1-b)4j^2d^2 + r_l^2}{(4j^2d^2 + r_l^2)^{3/2}} - \frac{(1-b)4j^2d^2 + m^2r_l^2}{(4j^2d^2 + m^2r_l^2)^{3/2}}\right]$$

where $j = 1,2,\ldots p$; $l = 1, 2,..n$.

The implementation of SIS is simple having basic steps as:

(i) Keeping in view that in field conditions the actual variation of resistivity in vertical direction is continuous, one can take unit layer thickness ($d$) smaller. As we would not be inverting VES data for thicknesses this choice would not affect the inversion process except in increasing the matrix size. Number of layers ($N$) and number of terms ($p$) have to be fixed a priori.

(ii) Expansion of the reflection function ($R_i$) given by

$$R_{i-1} = \frac{R_i(u) + k_{i-1}}{1 + R_i(u)k_{i-1}}\, u\,, \quad \text{with } k_{i-1} = \frac{\rho_i - \rho_{i-1}}{\rho_i - \rho_{i-1}}\,, \quad i = 1, 2, \ldots N,$$

at each interface as power series in parameter $u = e^{-2\lambda d}$ and coefficient $b$ as

$$R_i(u) = \sum_{j=1}^{\infty} b_{ij} u^j\,.$$

(iii) Development of a recurrence relation equating the coefficients of the same powers of u in the power series of reflection functions of any two successive layers as

$$b_{i-1,j} = (1-k_{i-1}^{\,2})\, b_{i,j-1} - k_{i-1} \sum_{q=2}^{j-1} b_{i,j-q}\, b_{i-1,j} \quad \text{with} \quad b_{i-1,1} = k_{i-1}.$$

(iv) Forward response can be computed using power series at air-earth interface using the recurrence relation

$$c_j = c_0 b_{1,j} + c_1 b_{1,j-1} + c_2 b_{1,j-2} + \ldots + c_{j-2} b_{1,2} + c_{j-1} b_{1,1}$$

and equation (5).

(v) In case of error free data the minimum norm inverse solution of equation (6) is $c = G^t w$ where w $= (G^t G)^{-1} v$. However, in case of error prone data (having average error, $e$) we have to obtain regressed minimum norm inverse solution with $w = (G^t G + e^2 I)^{-1}$ where regression parameter can be taken as $e^2$ and $I$ is the Identity matrix of order $nxn$. The j$^{th}$ component of vecor $c$ can now be expressed as $c_j = \sum_l G_{jl} w_l$, for $j \geq 0; l \geq 1$, and therefore

$$G_{0l} = 1 \text{ for all } l \text{ and } \rho_1 = c_0 = \sum_{l=1}^{n} w_l.$$

(vi) Once the resistivity of the top layer is obtained, other values of resistivities can be estimated successively using the inverse recurrence relation

$$b_{ij} = (1 - k_{i-1}^2)^{-1} [b_{i-1,j+1} - b_{i-1,j}\, b_{i,1}^* - b_{i-1,j-1}\, b_{i,2}^* - \dots - b_{i-1,3} b_{i,j-2}^* b_{i-1,2} b_{i,j-3}^*] \,,$$

with $b_{ij}^* = -k_{i-1} b_{ij}$, as

$$b_{i1} = k_i \quad \text{and hence} \quad \rho_{i+1} = \frac{1 + b_{i1}}{1 - b_{i1}} \rho_i, \quad k_{i-1} \neq \pm 1.$$

(vii) The quality of the inverted resistivity model is established through the misfit error parameter $e_t$, between computed ($\overline{v}$) response and observed ($v$) response as

$$e_t = \sum_{i=1}^{n} \frac{[(\overline{v}_i - v_i)/\overline{v}_i]^2}{n} \,.$$

## 4. Numerical Experiment

### *4.1. Synthetic VES Data*

The applicability of the unified equation in SIS is examined by using a 5-layer geological model given of Gai-shan (1985) with model parameters:

$$\rho_1 = 10\,\Omega m,\ d_1 = 20\ m,\ \rho_2 = 2\,\Omega m,\ d_2 = 20\ m,\ \rho_3 = 5\,\Omega m,$$
$$d_3 = 50\ m,\ \rho_4 = 2\,\Omega m,\ d_4 = 20\ m,\ \rho_5 = 100\,\Omega m.$$

For forward computation we have taken as $d = 2\ m$, $p = 1000$ terms and $N = 100$. The computed VES curves for various electrode configurations up to maximum half-current electrode separation of 800 $m$ are given in Figure 2. These curves are in good agreement with those computed with individual equations. The comparative plots demonstrated that the resolving power of these electrode configurations be placed in increasing order as: Pole-Pole, Wenner, Schlumberger, Perpendicular Dipole, Radial Dipole, and Parallel Dipole ($\theta = 30°$).

Note that all the forward responses in Figure 2 seem to depict a 3-layer model. This is due to the fact that the resistivity values of the second (2 Ωm), third (5 Ωm) and fourth (2 Ωm) layers are not significantly different. This

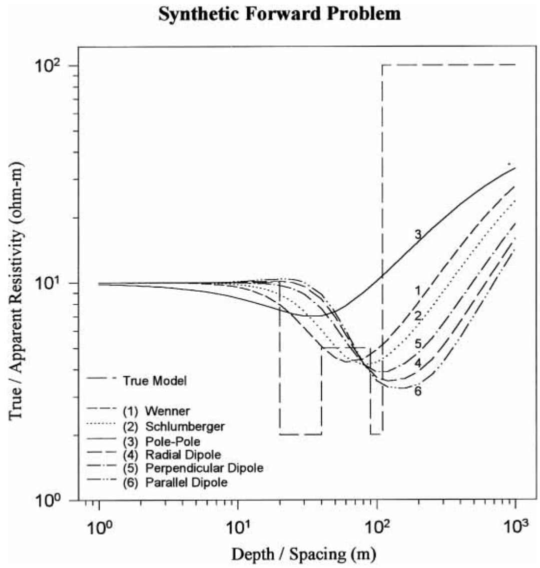


**Figure 2.** 5-layer true model and computed synthetic VES data over it for eight electrode configurations using unified equation.

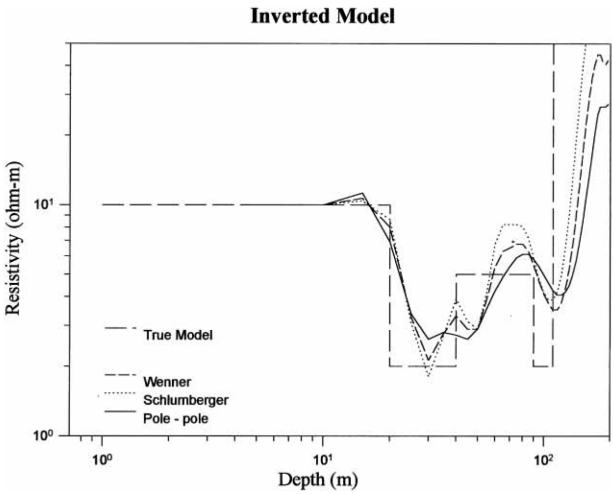


**Figure 3.** True model and SIS inverted models using 5-layer synthetic VES data for Wenner, Schlumberger and Pole-Pole configurations.

however, may cause constraint on choosing an initial guess model and thereby on iterative inversion scheme. In SIS the only free parameter is the unit layer thickness. For fixing the total number of layers considerable help can be taken

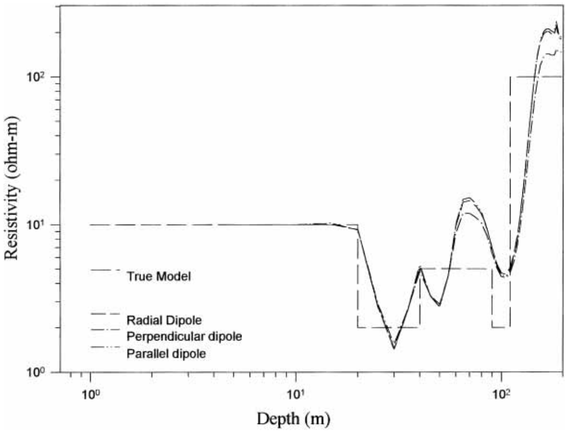


**Figure 4.** True model and SIS inverted using 5-layer synthetic VES data for different dipole configurations.

from the concept of 'depth of investigation' (Roy and Apparao, 1971) using maximum outer effective electrode separation (e.g. Wenner, 0.11 AB; Schlumberger, 0.125 AB; etc). For inverting the synthetic data set given in Figure 2 SIS has been used. Total thickness was fixed as 200 $m$ with three different values of $d$ as 1 m, 2 m, and 5 m thereby fixing the number of layers to 200, 100 and 40 respectively. The regression parameter chosen is 0.005 that implies 0.5 % error in data due to truncation. The inverted models for $d = 5$ m only, are given in Figure 3 (For Schlumberger, Wenner and Pole-Pole Configurations) and Figure 4 (for various dipoles), as we observe that the inverted models for $d = 1$ m and 2 m do not differ significantly. The misfit error $e_t$ was computed and can be placed in increasing order of magnitude as 0.0706 (Wenner), 0.0739 (Schlumberger), 0.166 (Perpendicular Dipole), 0.172 (Azimuthal Dipole), 0.185 (Radial Dipole) and 0.239 (Pole-Pole). These misfit errors are significantly small and all the five layers are noticeably resolved in the inverted model.

### *4.2. Field VES Data*

Encouraged with the successful inversion of synthetic VES data we have used the unified equation and SIS for inverting two field VES data SUV 1.2 and SUV 2.2 recorded on a paleochannel in the Marajó Island, Northern Brazil. The observed values using Schlumberger configuration is given in Table 2. These VES were recorded (Porsani and Rijo, 1993) near a well 42 m deep having following lithology with thickness given in ().

*Clay (5 m) – fresh water bearing fine sand (20 m) – wet clay (4 m) – not so fine sand having saline water (13 m).*

*Table 2. Field VES data from Marajó Island, Northern Brazil.*

| AB/2 (m) | $\rho_{as}$ (ohmm), SUV 1.2 | $\rho_{as}$ (ohmm), SUV 2.2 |
|---|---|---|
| 1.5 | 29.8 | 12.4 |
| 2.0 | 27.8 | 9.6 |
| 3.0 | 23.1 | 8.9 |
| 4.0 | 22.3 | 9.1 |
| 5.0 | 19.8 | 10.5 |
| 7.0 | 16.1 | 12.4 |
| 10.0 | 18.1 | 15.4 |
| 15.0 | 21.1 | 17.1 |
| 20.0 | 20.9 | 24.9 |
| 20.0 | 21.7 | 20.6 |
| 30.0 | 21.9 | 27.9 |
| 30.0 | 24.3 | 24.0 |
| 40.0 | 22.0 | 23.6 |
| 60.0 | 16.9 | – |
| 60.0 | 18.1 | 16.3 |
| 80.0 | 11.5 | 11.9 |
| 80.0 | 12.5 | 10.5 |
| 100.0 | 7.5 | 8.5 |
| 140.0 | 5.3 | 6.9 |
| 140.0 | 4.9 | 5.7 |
| 200.0 | 5.2 | 5.5 |
| 200.0 | 4.7 | 5.5 |
| 300.0 | 5.8 | 6.9 |
| 400.0 | 7.3 | 8.0 |

The maximum half-current electrode separation was 400 $m$, and therefore for inverting these data we have taken solution domain of 110 $m$ consisting of $N$ = 110 with $d$ = 1 $m$. It may be mentioned that the error free scheme could not invert both data sets thereby forcing us to use regression parameter. VES No. SUV 1.2 was successfully inverted with misfit error 0.208. It is interesting to note that this data set was successfully inverted for assumed error more than 1% with increased misfit error (for example 5% error gives misfit error 0.284). This means that data does not contain error greater than 1%. The inverted models with resistivity as continuous function of depth are shown in Figure 5. The data set of VES No. SUV 2.2 could be successfully inverted with misfit error 0.30 by choosing the regression parameter corresponding to the data error of 8% indicating that this data set is erroneous comparatively to SUV 1.2. There was wild fluctuation of inverted solution with assumed error of 5%. The inverted continuous model is given in Figure 6. Thus while choos-

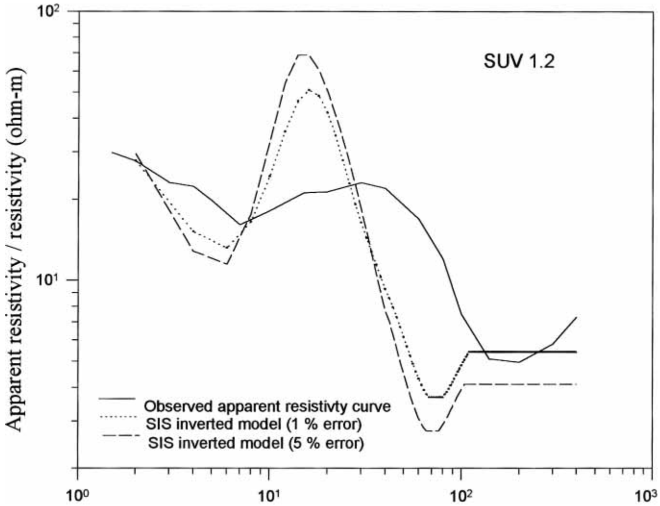


**Figure 5.** SIS inverted model using VES SUV 1.2 data from Marajó Island, Northern Brazil.

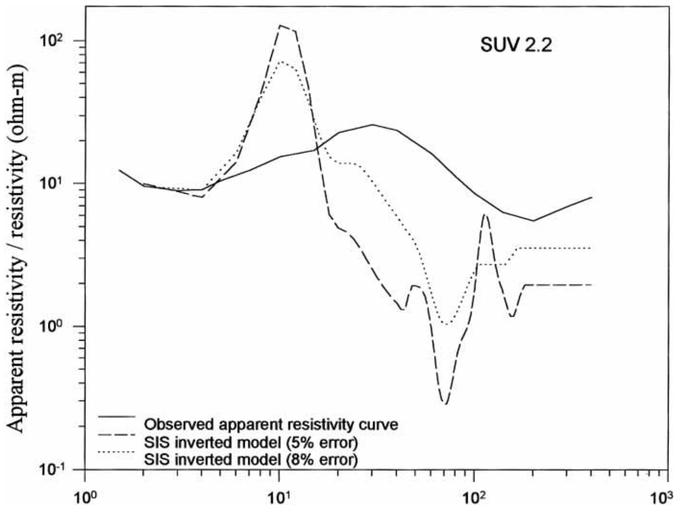


**Figure 6.** SIS inverted model using VES SUV 2.2 data from Marajó Island, Northern Brazil.

ing regression parameter this fact should be kept in mind that higher the parameter more will be the misfit error. It is to be noted that in both the synthetic and field examples the thin conducting layer at relatively large depth are clearly resolved with SIS. The usage of unified equation would broaden its domain of applicability for group of eight electrode configurations without changing the code.

## 5. Conclusions

It has already been established that straightforward inversion scheme (SIS) of Gupta et al. (1997) is a linear scheme to obtain nearly continuous subsurface resistivity variation from Vertical Electrical Sounding (VES) data. In this article a unified apparent resistivity equation capable of handling eight linear, symmetric and dipole electrode configurations is derived and used for forward modeling and inversion using SIS. The efficacy of combination of unified equation – SIS has been demonstrated through the inversion of synthetic and field VES data. On the basis of discussion herein in conclusion, we affirm that unified equation – SIS combination is proved to be a useful interpretational tool having multi-resolution capability for different depth scale investigations. It is robust, is insensitive to the problem of equivalence, and free from the bias that may be introduced by the requirement of an initial model for prompting the inversion process.

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SAŽETAK

## Opća jednadžba za neposrednu inverznu shemu podataka vertikalnog električnog sondiranja

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Izvedena je opća jednadžba za prividnu otpornost koja uključuje osam konfiguracija elektroda primjenjenih za neposrednu inverznu shemu (Straightforward Inversion Scheme (SIS)) podataka vertikalnog električnog sondiranja (VES). Nađeni su odgovarajući parametri koji omogućuju primjenu opće jednadžbe za određenu konfiguraciju (Wenner, pol-pol, Schlumberger, pol-dipol, radijalan dipol, okomit dipol, paralelni dipol i azimutalni dipol).

Izračun za VES podatke za 5-slojni model podzemlja, dobro se slaže s podacima dobivenim upotrebom postojećih individualnih jednadžbi za određenu konfiguraciju, čime potvrđuje dobivenu jednadžbu. Opisana su dva primjera Schlumbergerovog sondiranja polja poznate litologije na paleo-kanalu na Mario otoku u sjevernom Brazilu i prikazana je varijacija invertiranog kontinuiranog otpora kako bi se pokazala efikasnost sheme.

*Ključne riječi*: opća jednadžba, neposredna inverzija, vertikalno električno sondiranje

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